Collective magnetic and plasma excitations in Josephson ψ junctions

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We show that Josephson ψ junctions with a half-metallic (HM) weak link coupled to superconducting (S) electrodes through ferromagnetic (F) layers host collective excitations of the magnetic moment and the Josephson phase. This results in a shift of the ferromagnetic resonance frequency, anomalies in the current-voltage characteristics, and the appearance of additional magnetic anisotropy in the F layers. In contrast to previously studied S/F/S junctions, coupling between magnetic and plasma modes emerges even in the long wavelength limit. Such coupling is shown to enable controllable magnetization reversal in the F layer governed by a dc current pulse, which provides an effective mechanism for the magnetic moment manipulation in the devices of superconducting spintronics.

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I. INTRODUCTION

Josephson junctions with ferromagnetic (F) interlayers between the superconducting (S) electrodes are known to support a variety of exotic quantum phenomena [1,2]. The conversion of spin-singlet Cooper pairs into triplet ones at S/F interfaces results in an anomalous local increase in the electronic density of states at the Fermi level [3], the appearance of long-range Josephson currents [4–7], the formation of π junctions [8–10], etc. Even more unusual phenomena arise in S/F/S multilayered systems with broken inversion symmetry resulting in strong spin-orbit coupling (SOC). The most remarkable feature of such sandwiches is the so-called anomalous Josephson effect, i.e., the emergence of a nonzero phase $\varphi_0 \neq \pi$ between the superconducting electrodes in the ground state so that near the superconducting critical temperature the relation between the current I and phase φ takes the form $I = I_c \sin(\varphi - \varphi_0)$ (where I_c is the critical current) [11–15]. Being integrated into the superconducting loops, such φ_0 junctions play the role of phase batteries producing spontaneous currents [16-20], which is expected to bring new functionality to the devices of rapid single-flux quantum logics [21].

During the past decade both conventional S/F/S and φ_0 junctions acquired much attention also because of their unusual dynamic properties. Specifically, in multilayered S/F/S junctions the spin waves in the F layer become coupled to the dynamics of the Josephson phase difference φ (plasmalike waves) provided the propagation vector along the junction plane is nonzero [22]. Experimentally, hybridized waves are expected to reveal themselves mainly through substantial changes in the number, position, and shape of Fiske and Shapiro steps on the *I-V* characteristics of S/F/S junctions [22–32]. Another remarkable manifestation of such coupling between plasmalike and spin waves is the possibility to stimulate a magnetization reversal in the F layer by applying current through the junction [24,33-36]. This may provide an effective mechanism for the magnetic moment control in the devices of superconducting spintronics [37,38]. A similar hybridization of Josephson phase oscillations and precession of the magnetic moment is predicted for φ_0 junctions [39–46]. The ground-state phase φ_0 is determined by the angle θ between the magnetic moment **M** orientation inside the ferromagnet and the unit vector **n** along the direction of the broken inversion symmetry so that $\varphi_0 \propto \sin \theta$. As a result, the Josephson energy $E = (\Phi_0 I_c/2\pi c)[1 - \cos(\varphi - \varphi)]$ φ_0)] becomes dependent on the magnetic moment orientation which results not only in the influence of the magnetic order on the superconducting current but also in the backaction of the Josephson current on the magnetization direction. Consequently, outside the equilibrium, magnetic and superconducting excitations in the φ_0 junction become coupled to each other, giving rise to collective oscillations. Similarly to the S/F/S junctions, the collective plasmalike and spin excitations provide the possibility to realize the ultrafast reversal of the magnetization direction controlled by short current pulses, which is promising for the design of memory cells.

A further advancement in the field of spontaneous Josephson effects is associated with the implementation of fully spin-polarized ferromagnets which are often called halfmetals (HMs) [47,48]. Although singlet Cooper pairs consisting of two electrons with opposite spins cannot penetrate a half-metal directly, experiments on the S/HM/S systems demonstrate the existence of the Josephson transport through the HM layer [49,50]. This unusual observation is attributed to the spin-active interfaces of the half-metal which transform the spin structures of Cooper pairs, converting them from the spin-singlet state to a triplet one [51,52]. A controllable singlet-triplet conversion can be achieved in complex multilayered S/F/HM/F/S structures with noncoplanar orientation of the magnetic moments in the three ferromagnetic layers (see, e.g., Refs. [53–58]). The theoretical calculations within different approaches predict the anomalous Josephson effect for the S/F/HM/F/S junctions with a current-phase relation of the form $I(\varphi) = I_c \sin(\varphi - \psi - \pi)$, where the critical current $I_c > 0$, and $I_c \propto |\sin \vartheta_1 \sin \vartheta_2|$ (ϑ_1 and ϑ_2 are the angles between the magnetic moments in the F₁ and F₂ layers and the *z* axis). Remarkably, the spontaneous phase ψ is equal to the angle between the projections of magnetic moments in the two F layers to the plane perpendicular to the spin quantization axis of the half-metal and does not depend on all other system parameters [59–61]. Note that for the parallel orientation of the magnetic moment projections the junction should be in the π state [59,62,63]. This additional π shift can be taken into account with the minus sign in the current-phase relation so that it takes the form

$$I(\varphi) = -I_c \sin(\varphi - \psi). \tag{1}$$

The most striking feature of the S/F/HM/F/S ψ junctions which contrasts to the properties of usual φ_0 junctions is the Josephson phase accumulation accompanying the magnetic moment rotation in one of the F layers. Recently, this Berry phase effect was shown to allow the controllable pumping of magnetic flux into a superconducting loop containing a ψ junction without application of any out-of-plane external magnetic field [64].

In this paper we show that the peculiar coupling between magnetic moments and the Josephson phase in ψ junctions gives rise to collective magnetic and plasma excitations which substantially differ from the ones in conventional S/F/S systems and φ_0 junctions. Specifically, in contrast to the S/F/S structures, ψ junctions enable coupling between magnetic and plasma oscillations even for a zero wave vector along the junction, which results in measurable shifts in the ferromagnetic resonance frequency as well as in anomalies on the current-voltage characteristics. Moreover, we show that in the rf superconducting quantum interference device (SQUID) geometry, when the S/F/HM/F/S ψ junction is integrated into the superconducting loop, the coupling between the Josephson phase and magnetic degrees of freedom effectively renormalized the anisotropy in one of two F layers, producing an additional easy-axis direction. Finally, we demonstrate controllable magnetization reversal in one of the F layers under the effect of an external dc current pulse flowing through the junction, which provides an effective tool for magnetic moment manipulation in the devices of superconducting spintronics.



FIG. 1. Sketch of the Josephson ψ junction based on the multilayered S/F/HM/F/S structure.

The paper is organized as follows. In Sec. II we calculate the resonance frequencies of the collective magnetic and plasma excitations in a S/F/HM/F/S junction and compare the results with the ones previously obtained for S/F/S systems. In Sec. III we analyze the additional induced magnetic anisotropy in the ψ junction integrated into the superconducting loop. In Sec. IV we demonstrate magnetic moment reversal in the S/F/HM/F/S junction under the effect of a dc current. Finally, in Sec. V we summarize our results.

II. COLLECTIVE EXCITATIONS IN AN ISOLATED JOSEPHSON JUNCTION

We start from an analysis of the collective magnetic and superconducting excitations in an isolated $S/F_1/HM/F_2/S$ junction where the spin quantization axis in the half-metal is directed along the *z* axis, the magnetic moment in the F_1 layer is fixed along the *x* axis, while the magnetization **M** in the F_2 layer can change its direction (see Fig. 1). For simplicity we will restrict ourselves to the case when the temperature is well below the Curie temperature so that the absolute value of the magnetization *M* has reached the saturation value M_0 and is almost constant.

To describe the dynamics of the Josephson phase we use the resistively shunted junction (RSJ) model introducing the normal state resistance R_N and the capacity *C* of the junction [66],

$$\frac{\partial^2 \varphi}{\partial t^2} + \frac{\omega_p}{O} \frac{\partial \varphi}{\partial t} - \omega_p^2 \sin\left(\varphi - \psi\right) = 0, \qquad (2)$$

where $\omega_p = \sqrt{2\pi I_c c/(\Phi_0 C)}$ is the plasma frequency, and $Q = \omega_p R_N C$ is the quality factor.

The magnetization dynamics can be described by the Landau-Lifshitz-Gilbert equation,

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma [\mathbf{H}_{\text{eff}} \times \mathbf{M}] + \frac{\alpha}{M_0} \left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \right], \quad (3)$$

where γ is the electron gyromagnetic ratio, α is the phenomenological damping constant, and the effective field \mathbf{H}_{eff} is determined by the derivative of the system free energy *F* accounting for both the Josephson and magnetic contributions:

$$\mathbf{H}_{\rm eff} = -\frac{1}{V} \frac{\partial F}{\partial \mathbf{M}}.$$
 (4)

The specific form of the free energy functional F depends on the type of magnetic anisotropy in the F₂ ferromagnet. Let us consider the case when the F₂ later has an easy-axis anisotropy along the x axis. Then the system free energy has the form

$$F = E_J [1 + \cos{(\varphi - \psi)}] - \frac{K_{\parallel} V}{2} \left(\frac{M_x}{M_0}\right)^2.$$
 (5)

Here, $E_J = \Phi_0 I_c / (2\pi c)$ is the Josephson energy, $K_{\parallel} > 0$ is the magnetic anisotropy constant, V is the volume of the F₂ layer, and M_x is the projection of the magnetization **M** to the easy axis.

One sees that the system's ground state is doubly degenerate: In equilibrium the magnetization is directed parallel or antiparallel to the *x* axis so that $M_x = \nu M_0$ where $\nu = \pm 1$. The

corresponding values of the ground-state Josephson phase are $\varphi_0^{\nu} = \pi$ for $\nu = +1$ and $\varphi_0^{\nu} = 0$ for $\nu = -1$.

In the case of strong magnetic anisotropy $(K_{\parallel}V \gg E_J)$ deviations of the vector **M** direction from the *x* axis are small. In this case one may put $M_x \approx \nu M_0$ and the angle ψ entering the current-phase relation of the junction takes the value $\psi \approx M_y/M_0$ for $\nu = 1$ and $\psi \approx \pi - M_y/M_0$ for $\nu = -1$.

Taking the derivative in (4) we get the expressions for the effective field,

$$\mathbf{H}_{\rm eff} = \frac{K_{\parallel} M_x}{M_0^2} \mathbf{\hat{x}}_0 - \frac{E_J}{V} \sin\left(\varphi - \psi\right) \frac{\partial \psi}{\partial M_y} \mathbf{\hat{y}}_0, \tag{6}$$

and in the vicinity of the two ground states,

$$\mathbf{H}_{\rm eff}^{\nu} = \frac{\nu K_{\parallel}}{M_0} \hat{\mathbf{x}}_0 - \frac{E_J}{VM_0} \sin\left(\varphi - \frac{\nu M_y}{M_0}\right) \hat{\mathbf{y}}_0.$$
 (7)

Using the system of Eqs. (2) and (3) let us first determine the resonant frequencies of the Josephson junction neglecting damping, i.e., considering the case $\alpha \to 0$ and $Q \to \infty$. It is convenient to introduce the unit vector $\mathbf{m} = \mathbf{M}/M_0$ directed along the magnetization. Then, considering the oscillatory process where all values m_y , m_z , and $\delta \varphi = \varphi - \varphi_0^v$ are proportional to $e^{i\omega t}$ and linearizing Eqs. (2) and (3) for the corresponding complex amplitudes (which will be further indicated by the tilde), we obtain

$$i\omega\tilde{m}_y = -\nu\omega_{\parallel}\tilde{m}_z,\tag{8}$$

$$i\omega\tilde{m}_z = \nu\omega_{\parallel}\tilde{m}_y - \beta_{\parallel}\omega_{\parallel}(\delta\tilde{\varphi} - \nu\tilde{m}_y), \qquad (9)$$

$$-\omega^2 \delta \tilde{\varphi} + \omega_p^2 (\delta \tilde{\varphi} - \nu \tilde{m}_y) = 0, \qquad (10)$$

where $\omega_{\parallel} = \gamma K_{\parallel}/M_0$ is the ferromagnetic resonance frequency (we assume $\omega_{\parallel} > \omega_p$) and $\beta_{\parallel} = E_J/(K_{\parallel}V)$ is the dimensionless parameter characterizing the ratio between the Josephson and the anisotropy energies (the above assumption of a strong magnetic anisotropy field requires $\beta_{\parallel} \ll 1$). The system (8)–(10) may have a nontrivial solution provided

$$\omega^4 - \omega^2 \left[\omega_p^2 + \omega_{\parallel}^2 (1 + \beta_{\parallel}) \right] + \omega_p^2 \omega_{\parallel}^2 = 0.$$
 (11)

This equation has two real roots for an arbitrary choice of system parameters,

$$\omega^{2} = \frac{\omega_{p}^{2} + \omega_{\parallel}^{2}(1+\beta_{\parallel}) \pm \sqrt{\left[\omega_{p}^{2} + \omega_{\parallel}^{2}(1+\beta_{\parallel})\right]^{2} - 4\omega_{p}^{2}\omega_{\parallel}^{2}}}{2}.$$
(12)

In the limit $\beta_{\parallel} \ll (1 - \omega_p^2 / \omega_{\parallel}^2)$ the two resonance frequencies ω_1 and ω_2 read

$$\omega_{1} \approx \omega_{p} \left(1 - \frac{\beta_{\parallel}}{2} \frac{\omega_{\parallel}^{2}}{\omega_{\parallel}^{2} - \omega_{p}^{2}} \right), \quad \omega_{2} \approx \omega_{\parallel} \left(1 + \frac{\beta_{\parallel}}{2} \frac{\omega_{\parallel}^{2}}{\omega_{\parallel}^{2} - \omega_{p}^{2}} \right).$$
(13)

Remarkably, the hybridization of the magnetic and superconducting excitations in the S/F/HM/F/S junction substantially differs from a similar phenomenon in S/F/S junctions previously discussed in Ref. [22]. The key feature specific to the ψ junction is the mixing of magnetic and plasma waves even at zero wave vector while in S/F/S structures such mixing arises only for the waves propagating along

the junction. As a consequence, in contrast to the S/F/S junction, in the long wavelength limit the coupling between magnetic and plasma oscillation in the ψ junction results in a shift of the ferromagnetic resonance frequency (see the above expression for ω_2) which provides an effective tool for the experimental observation of the predicted collective excitations. Note that the lateral size of the experimentally fabricated S/F/S Josephson junctions with the composite F layer is typically small and lies in the range 0.5–20 μ m [6,7,65], which is much smaller than the typical values of the Josephson length $\lambda_J \sim 100 \ \mu$ m. This means that in the experimentally achievable structures only uniform plasma oscillations with a zero wave vector should be generated.

Also, the interaction between the oscillation of the magnetic moment and Josephson phase should give rise to anomalies in the current-voltage characteristics of ψ junctions. To demonstrate this we consider the limit when the current flowing through the junction is much larger than I_c . In this case the voltage U across the junction is almost constant so that the Josephson phase linearly depends on time: $\varphi = \omega_J t$, where $\omega_J = 2eU/\hbar$. We restrict ourselves to the case of small deviations of the magnetic moment vector from its equilibrium direction assuming $m_x, m_y \propto \beta_{\parallel} \ll 1$ and neglecting the contributions $\sim O(\beta_{\parallel}^2)$. Then in the presence of damping (for $\alpha \neq 0$) we may write Eq. (3) describing the dynamics of magnetic moment in the form

$$\frac{\partial m_y}{\partial t} = \frac{\omega_{\parallel}}{1 + \alpha^2} [-\nu m_z - \alpha m_y - \alpha \beta_{\parallel} \sin(\omega_J t)], \quad (14)$$

$$\frac{\partial m_z}{\partial t} = \frac{\omega_{\parallel}}{1 + \alpha^2} [\nu m_y - \alpha m_z + \nu \beta_{\parallel} \sin(\omega_J t)].$$
(15)

The solution of these equations is somewhat similar to the one previously obtained for the φ_0 junctions [39],

$$m_y(t) = (\omega_- - \omega_+)\sin(\omega_J t) + (\alpha_+ + \alpha_-)\cos(\omega_J t), \quad (16)$$

where

$$\omega_{\pm} = \frac{\beta_{\parallel}}{2\omega_{\parallel}} \frac{\omega_J \pm \omega_{\parallel}}{\Omega_{\pm}}, \quad \alpha_{\pm} = \frac{\alpha \beta_{\parallel} \omega_J}{2\omega_{\parallel} \Omega_{\pm}}, \quad (17)$$

$$\Omega_{\pm} = \frac{(\omega_J \pm \omega_{\parallel})^2 + (\alpha \omega_J)^2}{\omega_{\parallel}^2}.$$
 (18)

The obtained solution for $m_y(t)$ allows us to calculate the superconducting current $I_s = -\nu I_c \sin(\omega_J t - \nu m_y)$. Remarkably, it has a dc component I_s^{dc} which arises due to the damping:

$$I_{s}^{\rm dc} = \frac{\alpha \beta_{\parallel} I_{c} \omega_{J}}{4\omega_{\parallel}} \left(\frac{1}{\Omega_{+}} + \frac{1}{\Omega_{-}} \right). \tag{19}$$

This dc contribution to the electric current experiences resonance behavior for the frequencies $\omega_J = \pm \omega_{\parallel}$ which should produce the steps in the current-voltage characteristics at $U = \pm \hbar \omega_{\parallel}/(2e)$ similar to the Shapiro steps arising under the effect of microwave radiation.

III. INDUCTANCE-INDUCED MAGNETIC ANISOTROPY IN THE LOOP GEOMETRY

The coupling between magnetic and plasma excitations in $S/F_1/HM/F_2/S$ junctions gives rise to the peculiar renormalization of the magnetic anisotropy in a ferromagnetic layer provided the ψ junction is integrated into a closed superconducting loop (the geometry analogous to the rf SQUID). To demonstrate this we again consider the S/F₁/HM/F₂/S structure shown in Fig. 1 where the spin quantization axis inside the half-metal coincides with the *z* axis, the magnetization in the F₁ layer is directed along the *x* axis, while the F₂ layer reveals the easy *xy*-plane magnetic anisotropy, making the presence of the magnetization component M_z energetically unfavorable. When such a junction is embedded into the superconducting loop of the inductance *L* the system Gibbs free energy takes the form

$$F = E_J \left[1 + \cos\left(\varphi - \psi\right) + \frac{\varphi^2}{2\lambda} \right] + \frac{K_\perp V}{2} \left(\frac{M_z}{M_0} \right)^2, \quad (20)$$

where $\lambda = 2\pi cLI_c/\Phi_0$ is the dimensionless loop inductance, $K_{\perp} > 0$ is the magnetic anisotropy constant, and **M** is the magnetization vector in the F₂ layer.

Because of the finite loop inductance the system has only one ground state which corresponds to $M_z = M_y = 0$, $M_x = -M_0$ ($\psi = \pi$), and $\varphi = 0$. This is equivalent to the effective renormalization of the magnetic anisotropy: In the loop geometry there appears an additional weak easy-axis anisotropy along the x axis.

To describe the dynamics of **M** out of equilibrium it is convenient to introduce the spherical coordinates in a way that θ is the angle between **M** and the *xy* plane while ψ is the polar angle in the *xy* plane: $M_z = M_0 \sin \theta$, $M_x = M_0 \cos \theta \cos \psi$, and $M_y = M_0 \cos \theta \sin \psi$. In the case of strong magnetic anisotropy one may assume $\theta \ll 1$. The effective field (4) takes the form

$$\mathbf{H}_{\rm eff} = -\frac{K_{\perp}M_z}{M_0^2}\hat{\mathbf{z}}_0 - \frac{E_J}{V}\sin\left(\varphi - \psi\right)\frac{\partial\psi}{\partial\mathbf{M}},\qquad(21)$$

where

$$\frac{\partial \psi}{\partial \mathbf{M}} = \frac{M_x \mathbf{y}_0 - M_y \mathbf{x}_0}{M_x^2 + M_y^2} \approx \frac{1}{M_0^2} (M_x \mathbf{y}_0 - M_y \mathbf{x}_0).$$
(22)

Then the Landau-Lifshitz-Gilbert equation (3) can be written in the form of two coupled equations for the angles θ and ψ :

$$\frac{\partial\theta}{\partial t} = \frac{\gamma E_J}{VM_0} \sin\left(\varphi - \psi\right), \quad \frac{\partial\psi}{\partial t} = -\frac{\gamma K_\perp}{M_0}\theta.$$
(23)

At the same time, the dynamics of the Josephson phase is described the RSJ model equation accounting for the finite loop inductance,

$$\frac{\partial^2 \varphi}{\partial t^2} + \frac{\omega_p}{Q} \frac{\partial \varphi}{\partial t} + \omega_{LC}^2 \varphi - \omega_p^2 \sin\left(\varphi - \psi\right) = 0, \qquad (24)$$

where $\omega_{LC} = (LC)^{-1/2}$ is the resonance frequency of the *LC* circuit.

Considering the $e^{i\omega t}$ processes in the absence of damping $(Q \to \infty)$ and linearizing Eqs. (23) and (24) near the ground state, assuming the values φ , θ , and $\delta \psi = \psi - \pi$ to be small, we find the following characteristic equation which determines the resonance frequencies of the loop with embedded

 ψ junction,

$$\omega^{4} - \omega^{2} \left(\omega_{LC}^{2} + \omega_{p}^{2} + \Omega_{\perp}^{2} \right) + \Omega_{\perp}^{2} \omega_{LC}^{2} = 0, \qquad (25)$$

where $\Omega_{\perp}^2 = \gamma^2 E_J K_{\perp} / (V M_0^2)$. Equation (25) always has two real solutions:

$$\omega^{2} = \frac{\omega_{LC}^{2} + \omega_{p}^{2} + \Omega_{\perp}^{2} \pm \sqrt{\left(\omega_{LC}^{2} + \omega_{p}^{2} + \Omega_{\perp}^{2}\right)^{2} - 4\Omega_{\perp}^{2}\omega_{LC}^{2}}}{2}.$$
(26)

For loops with a large inductance (when the frequency ω_{LC} is smaller than all other frequencies) the small collective magnetic and plasma excitations are characterized by two resonance frequencies:

$$\omega_1^2 \approx \Omega_\perp^2 + \omega_p^2 + \omega_{LC}^2 \frac{\omega_p^2}{\Omega_\perp^2 + \omega_p^2}, \quad \omega_2^2 \approx \omega_{LC}^2 \frac{\Omega_\perp^2}{\Omega_\perp^2 + \omega_p^2}.$$
(27)

Thus, the *LC*-circuit resonance frequency of the rf SQUID based on ψ junctions becomes dependent not only on the geometry of the Josephson junction (affecting its capacity) but also on its current-phase relation.

IV. MANIPULATION OF MAGNETIC MOMENT BY JOSEPHSON CURRENT

The coupling between the magnetization direction and the Josephson phase in ψ junctions allows the manipulation of the magnetic moment by an external dc current flowing through the junction. To illustrate the mechanism of such magnetic moment manipulation let us again consider the $S/F_1/HM/F_2/S$ junction shown in Fig. 1. For simplicity we consider the situation when the magnetization in the F_2 layer has a strong *xy* easy-plane anisotropy and additional weak easy-axis *x* anisotropy. This ensures that the critical current of the Josephson junction remains almost constant since **M** tends to remain in the *xy* plane but, at the same time, there are two equilibrium magnetization directions, namely, parallel and antiparallel to the *x* axis.

In the presence of an external electric current I flowing through the junction the system energy reads [66]

$$E = E_{J} \left[1 + \cos\left(\varphi - \psi\right) - \frac{\varphi I}{I_{c}} \right] + \frac{V(K_{\perp}M_{z}^{2} - KM_{x}^{2})}{2M_{0}^{2}},$$
(28)

where $K_{\perp} > 0$, K > 0, and we assume that $K_{\perp} \gg K$, E_J/V which guarantees that the dynamics of the magnetic moment in the F₂ layer occurs in the vicinity of the *xy* plane so that the critical current $I_c \approx \text{const.}$

The dynamics of the magnetization M is described by Eq. (3) with the following effective field H_{eff} :

$$\mathbf{H}_{\rm eff} = \frac{KM_x}{M_0^2} \mathbf{x}_0 - \frac{K_\perp M_z}{M_0^2} \mathbf{z}_0 - \frac{E_J}{V} \sin(\varphi - \psi) \frac{M_x \mathbf{y}_0 - M_y \mathbf{x}_0}{M_x^2 + M_y^2}.$$
(29)

In the case $K_{\perp} \gg K$, E_J/V one may neglect the contributions $\propto (M_z/M_0)^2$ and put $M_x^2 + M_y^2 \approx M_0^2$. Then, introducing the unit vector $\mathbf{m} = \mathbf{M}/M_0$ and dimensionless time variable



FIG. 2. The reversal of the magnetization direction under the effect of the current pulse. For calculations we took $\alpha = 0.1$, $\kappa = 0.02$, $\beta_{\perp} = 0.02$, and w = 5. The gray region shows the time interval when the external current $I = 2I_c$ was applied. After $\tau = 20$ the current was switched off.

 $\tau = (\gamma K_{\perp} t / M_0)(1 + \alpha^2)^{-1}$, we obtain the following set of equations,

$$\dot{m}_x = m_y m_z + \alpha \kappa m_x m_y^2 - \beta_\perp \sin(\varphi - \psi)(m_x m_z - \alpha m_y),$$

$$\dot{m}_y = -(1 + \kappa)m_x m_z - \alpha \kappa m_x^2 m_y$$

$$-\beta_\perp \sin(\varphi - \psi)(m_y m_z + \alpha m_x),$$

$$\dot{m}_z = \kappa m_x m_y - \alpha (1 + \kappa)m_x^2 m_z - \alpha m_y^2 m_z + \beta_\perp \sin(\varphi - \psi),$$

(30)

where $\beta_{\perp} = E_J/(K_{\perp}V) \ll 1$, $\kappa = K/K_{\perp} \ll 1$, and the dot symbol denotes the derivative with respect to τ .

In the limit when the Josephson junction has a low capacity C the relation between the transport current I flowing through the junction and the Josephson phase φ takes the form [66]

$$\frac{I}{I_c} = \frac{1}{\omega_p Q} \frac{\partial \varphi}{\partial t} - \sin(\varphi - \psi).$$
(31)

Introducing the dimensionless current $i = I/I_c$ and considering the dynamics in dimensionless time τ one may rewrite Eq. (31) in the form

$$i = w\dot{\varphi} - \sin(\varphi - \psi), \tag{32}$$

where

$$w = \frac{1}{1+\alpha^2} \frac{\gamma K_\perp}{M_0} \frac{1}{\omega_p Q}.$$
(33)

The above equations (30) and (32) determine the collective dynamics of the ψ junction under the effect of external current.

Interestingly, the application of a current pulse to the ψ junction may result in a reversal of the magnetization direction. Figure 2 illustrates such a reversal process. In the initial equilibrium state the magnetization is antiparallel to the x axis so that $m_x = -1$. Then at $\tau = 5$ the transport current flowing through the junction becomes switched from zero to $I = 2I_c$, remains constant until $\tau = 20$, and then is switched off. The subsequent relaxation dynamics ends up in

a new magnetic state with $m_x = 1$. For typical S/F/S structures with $\gamma K_{\perp}/M_0 \sim 10^{11} \text{ s}^{-1}$ the timescale of the magnetic moment switching is of the order of 10^{-10} s. Note that application of the second pulse will return the magnetic moment back to the state with $m_x = -1$. The described mechanism of magnetic switching driven by the current pulse can be used, e.g., provides a possibility to perform precise control of the magnetic state in the devices of superconducting spintronics.

V. CONCLUSION

To sum up, we predict the hybridization of the magnetic moment and Josephson phase oscillations in S/F/HM/F/S junctions. Such a coupling between two types of excitations does not require the propagation of the wave along the junction and occurs even in the long wavelength limit in contrast to conventional S/F/S junctions. This provides the possibility to observe collective excitations by a shift in the ferromagnetic resonance (FMR) frequency. In typical S/F/S junctions $\omega_{\parallel} \sim$ 10^{11} s^{-1} and $\omega_p \sim 10^{10} \text{ s}^{-1}$ so that $\omega_p \leq \omega_{\parallel}$. At the same time, the parameter $\beta_{\parallel} = E_J/(K_{\parallel}V)$ may vary in a broad range from small values typical for junctions with a large distance between the superconducting electrodes (which results in the decrease of both E_J and V^{-1}) up to $\beta_{\parallel} \sim 100$ for the permalloy weak links [39]. Thus, the increase of the FMR frequency due to the coupling with Josephson excitations can be comparable with ω_{\parallel} . Note that very recent measurements showed anomalously large FMR frequency shifts in S/F/S junctions [67–70]. We hope that the application of similar experimental techniques to the S/F/HM/F/S junctions with an alternating F/HM/F weak link thickness would provide direct evidence of the predicted collective magnetic end plasma excitations in ψ junctions.

Also, we find that integration of the S/F/HM/F/S ψ junction into the superconducting loop can substantially change the effective magnetic anisotropy in one of the F layers. For example, in addition to the initial easy-plane anisotropy the F layer can acquire a weak easy-axis anisotropy which favors the specific orientation of the magnetic moment in the easy plane. In this case the coupling between magnetic and plasma oscillations also results in a shift of the resonance frequency of the *LC* circuit which becomes dependent not only on the geometrical parameters of the Josephson junction but also on its current-phase relation.

Finally, we have demonstrated the possibility to reverse the magnetization direction in one of the F layers of the S/F/HM/F/S structure by applying the pulse of a dc current. Interestingly, such a reversal is accompanied by the switching between the 0 and π state of the Josephson junction since the ground-state phase is determined by the magnetic moment orientation. This provides promising mechanisms for the controllable manipulation of the magnetic moment and current-phase relation in the Josephson devices of superconducting spintronics.

Note that similar phenomena can arise also in the Josephson S/F/F'/F/S systems where the exchange field in the central F' layer is slightly smaller than the Fermi energy. Such structures are relatively easy to fabricate as compared to the half-metal based sandwiches. However, in this case the dependence of the spontaneous Josephson phase ψ on the angle between the magnetic moments in the two F layers becomes nonlinear and can even reveal hysteresis behavior, which should affect the resonance frequencies of the collective excitations as well as the specific regimes of the magnetic moment switching. Thus, it would be interesting to analyze the collective magnetic and plasma excitations in S/F/F'/F/S structures.

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