# Thermal dissipation in the quantum Hall regime in graphene

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(Received 24 December 2020; accepted 30 August 2021; published 10 September 2021)

It is widely accepted that both backscattering and dissipation cannot occur in topological systems because of the topological protection. Here we show that thermal dissipation can occur in the quantum Hall regime in graphene in the presence of dissipation sources, although the Hall plateaus and the zero longitudinal resistance still survive. Dissipation appears along the downstream chiral flow direction of the constriction in the Hall plateau regime, but it occurs mainly in the bulk in the Hall plateau transition regime. In addition, dissipation processes are accompanied with the evolution of the energy distribution from nonequilibrium to equilibrium. This indicates that topology neither prohibits the appearance of dissipation nor prohibits entropy increasing, which opens a new topic on the dissipation in topological systems.

DOI: 10.1103/PhysRevB.104.115411

# I. INTRODUCTION

Topological systems have been attracting extensive and ongoing interests [1-6] because of potential applications in low-dissipation electronic devices and topologically protected edge or surface states [4–6]. The quantum Hall (OH) effect is a prime phenomenon in topological systems [7-10], which was first discovered in a two-dimensional electron gas (2DEG) system [7]. With the influence of a strong perpendicular magnetic field, the energy spectrum forms a series of impurity broadened discrete Landau levels, where the extended states exist in the center and the localized states exist at the edge of the band [11-13]. This special energy band leads to many peculiar properties. For example, it supports topologically protected chiral edge states, which are characterized by the topological properties of the wave function's distribution in two-dimensional lattice momentum space and the corresponding topological invariant is TKNN number, also known as the Chern number [5,14].

When the Fermi energy is between the Landau levels, the chiral edge states exhibit quantized Hall resistance and the corresponding longitudinal resistance is zero [15]. The QH effect can occur in macroscopic systems. In the experiment, the quantized Hall resistance is very specified and it is insensitive to the details of the sample, which leads to the establishment of a new metrological standard [16,17]. Moreover, the charge carriers in the chiral edge states are very resistant to scattering and are expected to be used in dissipationless electronic devices [8].

Graphene, a kind of two-dimensional topological material with an isolated single-layer hexagonal lattice of carbon atoms, has been found to be an ideal material for the realization of the QH effect recently [18–20]. Graphene has a unique band structure with a linear dispersion relation near the Dirac points, which leads to many fancy properties [21–23]. Its quasiparticles obey the massless Dirac-type equation and its Hall plateaus are assumed at the half-integer values [18,19,24].

The dissipation sources (e.g., the electron-phonon interaction, impurities) inevitably exist in real systems and could lead to thermal dissipation with the energy transfer from electric energy to heat energy in charge transport processes [25,26]. Usually, dissipation is associated with the generation of resistance [27]. The longitudinal resistance is zero in the Hall plateau regime but nonzero in the Hall plateau transition regime. Therefore, although there are dissipation sources, one would expect that dissipation cannot occur in the Hall plateau regime, but only in the Hall plateau transition regime. However, it was shown that dissipation can take place at the QH edges through resonant impurities and electron-phonon interactions [28,29]. In particular, using a high-sensitive noncontact nanothermometer [30-32], Marguerite et al. [33] reported an amazing experiment that dissipative transport happened along the chiral QH edge states in graphene but in the bulk no dissipation occurred in either the QH plateau or plateau transition regimes. In view of this, a thorough and reliable analysis of thermal dissipation accompanied with energy relaxation and electron redistribution is urgent in the QH effect.

In this paper, motivated by the recent experiment by Marguerite *et al.* [33], we theoretically study the thermal dissipation of a six-terminal graphene device under a perpendicular magnetic field B. The thermal dissipation processes require energy transfer from the electronic system to the environment. Here we simulate the dissipation sources by

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introducing the Büttiker's virtual leads [34], where the energy of moving electrons leaks into these virtual leads and induces thermal dissipation. By using the tight-binding model and the Landauer-Büttiker formalism together with the nonequilibrium Green's function, the local heat generation and equivalent temperature are calculated. Our results indicate that thermal dissipation can occur in the QH plateau regime, deviating from the principle that backscattering and dissipation cannot happen in the QH effect due to the topological protection. The thermal dissipation mainly appears along the downstream chiral flow direction of the constriction, with the relaxation length determined by the dissipation strength. These features are in excellent agreement with the recent experiment [33]. On the other hand, in the plateau transition regime, the thermal dissipation primarily appears in the bulk of graphene, which is the same as previous physical intuition [8,33].

The rest of the paper is organized as follows. In Sec. II, we describe the model of the system and give the details of our calculations. In Sec. III, we show the numerical results and some discussions. Finally, a brief conclusion is presented in Sec. IV.

#### **II. MODEL AND METHOD**

In the tight-binding representation, the Hamiltonian of a six-terminal graphene device [see Fig. 1(a)] can be written as [35-38]

$$H = \sum_{\mathbf{i}} \epsilon_{\mathbf{i}} c_{\mathbf{i}}^{\dagger} c_{\mathbf{i}} - \sum_{\langle \mathbf{ij} \rangle} t e^{i\phi_{\mathbf{i}j}} c_{\mathbf{i}}^{\dagger} c_{\mathbf{j}} + H_d.$$
(1)

The first and second terms describe the graphene system including central scattering region (the blue dotted lines surrounding region) and six semi-infinitely long terminals.  $c_{i}^{\dagger}(c_{i})$ is the creation (annihilation) operator at site  $\mathbf{i}, \epsilon_{\mathbf{i}}$  is the on-site energy, and t is the nearest-neighbor coupling. Here  $\mathbf{i} = (x, y)$ is also the spatial position coordinate, as shown in Fig. 1(a). The magnetic field B is expressed as  $\phi_{ij} = \int_{i}^{j} \vec{A} \cdot d\vec{l} / \phi_0$ , with the vector potential  $\vec{A} = (-By, 0, 0)$  in leads 1, 4, and the central region;  $\vec{A} = (0, Bx, 0)$  in leads 2, 3, 5, 6; and  $\phi_0 = \hbar/e$ the flux quantum, see Appendix A. We stress that our results still hold in square-lattice systems (or 2DEG systems). The third term  $H_d = \sum_{\mathbf{i},k} \epsilon_k a_{\mathbf{i}k}^{\dagger} a_{\mathbf{i}k} + (t_k a_{\mathbf{i}k}^{\dagger} c_{\mathbf{i}} + \text{H.c.})$  represents the Hamiltonian of Büttiker's virtual leads and their couplings to central sites, which is used to simulate the dissipation sources.  $a_{ik}^{\dagger}(a_{ik})$  is the creation (annihilation) operator of the electrons in the virtual lead  $\mathbf{i}$ ,  $t_k$  is the coupling strength between the virtual leads and the graphene. In fact, the virtual leads can be used as dephasing probes, voltage probes, or temperature probes, which depend on the boundary conditions of the virtual leads. When the virtual leads are set as dephasing probes, electrons can lose phase memories by going into and coming back from the virtual leads, and the electric currents in these virtual leads are zero [39]. When they are set as voltage probes, the electric currents in the virtual leads are also zero and the voltage in the virtual leads are studied. When they are set as temperature probes, both the heat currents and the electric currents are zero. Here we use the virtual leads to simulate the dissipation sources, in this case electrons can lose



FIG. 1. (a) Schematic diagram for a six-terminal graphene device with a narrow constriction on the left side (i.e., the narrow constriction is between lead 1 and leads 2, 6). The region surrounded by the red (or green) solid lines contains dissipation sources and the region surrounded by the blue dotted lines is the central scattering region. In this diagram, the sizes of device are N = 16, M = 8, W = 4,  $L_1 = 4$ ,  $L_p = 2$ ,  $L_q = 4$ ,  $L_3 = 3$ , and  $L_4 = 4$ . (b) and (c) show the Hall resistance  $R_H$  and longitudinal resistance  $R_{xx}$  vs the magnetic field *B* with different dissipation strengths  $\Gamma_d$  for M = N and M = 8, respectively.  $L_q = 80$ , the dissipation sources are in the red solid lines surrounding region and the electric currents  $J_i$  in the virtual leads are zero.

energy by going into and coming back from the virtual leads, i.e., the electric currents are zero and we focus on the heat currents in the virtual leads. In addition, we consider that the dissipation sources only exist in the region enclosed by the red (or green) solid lines in Fig. 1(a), where each site is attached by a virtual lead.

By using the multiprobe Landauer-Büttiker formula, the electric current and the heat current in real leads r (r = 1, 2,..., 6) and virtual ones **i** can be expressed as [40–42]

$$J_{p} = \frac{2e}{h} \sum_{q} \int T_{pq}(E) [f_{p}(E) - f_{q}(E)] dE,$$
  
$$Q_{p} = -\frac{2}{h} \sum_{q} \int (E - \mu_{p}) T_{pq}(E) [f_{p}(E) - f_{q}(E)] dE, \quad (2)$$

where  $p, q \in r$ , or **i**. For convenience, we define the electric current  $J_p$  flows from lead p to central region as the positive direction, while the heat current  $Q_p$  flows from the central region to lead p as the positive direction. Here, the heat current  $Q_p$  is induced by the flow of the electric current, so we will call it the current-induced local heat generation in the following. In Eq. (2),  $T_{pq}(E) = \text{Tr}[\mathbf{\Gamma}_p \mathbf{G}^r \mathbf{\Gamma}_q \mathbf{G}^a]$  is the transmission coefficient from lead q to lead p, and  $\sum_{q(q \neq p)} T_{pq}(E) = \sum_{q(q \neq p)} T_{qp}(E)$  because of the conservation of current. The Green's function

 $\mathbf{G}^{r}(E) = [\mathbf{G}^{a}(E)]^{\dagger} = [E\mathbf{I} - \mathbf{H}_{cen} - \sum_{p} \boldsymbol{\Sigma}_{p}^{r}]^{-1}$  and the linewidth function  $\boldsymbol{\Gamma}_{p}(E) = i[\boldsymbol{\Sigma}_{p}^{r}(E) - \boldsymbol{\Sigma}_{p}^{a}(E)]$ .  $\mathbf{H}_{cen}$  is the Hamiltonian of the central region.  $\boldsymbol{\Sigma}_{p}^{r}(E) = [\boldsymbol{\Sigma}_{p}^{a}(E)]^{\dagger}$  is the retarded self-energy due to the coupling to lead p. For real leads p ( $p \in r$ ), the self-energy  $\boldsymbol{\Sigma}_{p}^{r}$  can be calculated numerically [42]. For virtual leads p ( $p \in \mathbf{i}$ ),  $\boldsymbol{\Sigma}_{p}^{r} = -\frac{i}{2}\Gamma_{d}$ , with the dissipation strength  $\Gamma_{d} = 2\pi\rho t_{k}^{2}$  and  $\rho$  the density of states in the virtual leads [39]. We assume that  $\Gamma_{d}$  is independent of the energy, i.e., in the wide-band approximation.  $f_{p}(E) = [e^{(E-\mu_{p})/k_{B}T_{p}} + 1]^{-1}$  is the Fermi distribution function in lead p, with the temperature  $\mathcal{T}_{p}$ , the chemical potential  $\mu_{p} = E_{F} + eV_{p}$ , and the voltage  $V_{p}$  in lead p.

By using the Green's function, the energy distribution function at the site **i** can be expressed as

$$F_{\mathbf{i}}(E) = \frac{n(\mathbf{i}, E)}{\text{LDOS}_{\mathbf{i}}(E)},$$
(3)

where  $n(\mathbf{i}, E) = -\frac{i}{2\pi} \mathbf{G}_{\mathbf{ii}}^{<}(E)$  is the electron density per unit energy at the lattice site  $\mathbf{i}$  and  $\text{LDOS}_{\mathbf{i}}(E) = -\frac{1}{\pi} \text{Im} \mathbf{G}_{\mathbf{ii}}^{r}(E)$ is the local density of states at the lattice site  $\mathbf{i}$  [41]. According to the Keldysh equation, the lesser Green's function  $\mathbf{G}_{\mathbf{ii}}^{<}(E) = \sum_{p} \mathbf{G}_{\mathbf{ip}}^{r} \boldsymbol{\Sigma}_{p}^{<} \mathbf{G}_{p\mathbf{i}}^{a}$  and the lesser self-energy  $\boldsymbol{\Sigma}_{p}^{<}(E) = -f_{p}(E)[\boldsymbol{\Sigma}_{p}^{r}(E) - \boldsymbol{\Sigma}_{p}^{a}(E)].$ 

When a small bias  $V_b$  is applied between leads 1 and 4 with  $V_1 = V_b$  and  $V_4 = 0$ , the current flows along the longitudinal direction. It is reasonable to assume that the transmission coefficient  $T_{pq}(E)$  is approximately independent of energy E since the bias is small. The electric currents in leads 2, 3, 5, 6 are zero and their voltages will be calculated. In addition, the electric currents in the virtual leads are also zero because electrons go into and come back from the virtual leads, only losing energy. The Fermi distribution function is  $f_p(E) = \theta(\mu_p - E)$  at zero temperature. Thus, Eq. (2) can be expressed as

$$J_{p} = \frac{2e^{2}}{h} \sum_{q \neq p} T_{pq}(E_{F})(V_{p} - V_{q}),$$
  
$$Q_{p} = -\frac{2e^{2}}{h} \sum_{q \neq p} T_{pq}(E_{F}) \left(V_{p}V_{q} - \frac{1}{2}V_{p}^{2} - \frac{1}{2}V_{q}^{2}\right).$$
(4)

Combining  $J_p = 0$  ( $p \in \{i, 2, 3, 5, 6\}$ ) with Eq. (4), the voltages  $V_p$ , the current-induced local heat generation  $Q_p$  in these leads and the longitudinal current ( $J = J_1 = -J_4$ ) can be obtained. The current J is proportional to the bias  $V_b$  and the heat generation  $Q_p$  to  $V_b^2$ . The local heat generation  $Q_L(x, y)$  at position  $\mathbf{i} = (x, y)$  is  $Q_L(x, y) = Q_i$ . Finally, the longitudinal resistance  $R_{xx} = R_{14,23} = (V_2 - V_3)/J$  and the Hall resistance  $R_H = R_{14,26} = (V_2 - V_6)/J$  can be calculated straightforwardly. The local heat generation  $Q_L(x, y)$  will be studied in Sec. III A.

On the other hand, if the thermal conductivity between the sample and environment is poor, the local heat generation  $Q_i$  disappears and the local electron temperature  $\mathcal{T}_i$ (i.e., the temperature in the virtual leads) rises. In this case, the virtual leads act as both the dissipation sources and the temperature detection terminals (temperature probes), and both  $J_i$  and  $Q_i$  are zero. Notice that although both  $J_i$  and  $Q_i$  are zero in the virtual leads, the heat current density  $\sum_{q} (E - \mu_p) T_{pq}(E) [f_p(E) - f_q(E)]$  and the electric current density  $\sum_{q} T_{pq}(E) [f_p(E) - f_q(E)]$  are usually nonzero. So the dissipation can occur and the virtual leads still play the role of dissipation sources. Provided that the six real leads have the same temperature ( $T_1 = T_2 = T_3 = T_4 = T_5 = T_6 = T$ ). At low temperature and small voltage, the current  $J_p$  and the current-induced local heat generation  $Q_p$  in Eq. (2) can be reduced as

$$J_{p} = \frac{2e^{2}}{h} \sum_{q \neq p} T_{pq}(E_{F})(V_{p} - V_{q}),$$
  
$$Q_{p} = -\frac{2}{h} \sum_{q \neq p} T_{pq}(E_{F}) \left[ \frac{\pi^{2}}{6} k_{B}^{2} (\mathcal{T}_{p}^{2} - \mathcal{T}_{q}^{2}) - \frac{1}{2} e^{2} (V_{p} - V_{q})^{2} \right].$$
(5)

Here  $k_B$  is the Boltzmann constant and  $\mathcal{T}_p$  is the temperature in lead p. In the expression of the current-induced local heat generation  $Q_p$  in Eq. (5), the linear term  $\Delta T_p \equiv T_p - T$ exists, but the linear term  $(V_p - V_q)$  disappears because we now focus on the heat generation at the low temperature and small voltage, see Appendix B for details. This is different from some previous literature on thermoelectric effects and Peltier effect in linear response, which includes the linear term  $(V_p - V_q)$  [43,44]. Notice that the temperature  $\mathcal{T}_p$   $(p \in \mathbf{i})$  in the virtual leads may not be equal to the background temperature  $\mathcal{T}$ . Then by using the boundary conditions that the net currents  $J_p$  flowing through real leads 2, 3, 5, 6 and all the virtual leads are zero and the heat generation  $Q_p$  in all the virtual leads are also zero, the local equivalent electron temperature  $\mathcal{T}_{i}$  and voltage  $V_p$  ( $p \in r$  or **i**) can be obtained. Also notice that the electron at the site i of the graphene is in nonequilibrium. Here  $\mathcal{T}_{\mathbf{i}}$  is the temperature of the virtual lead  $\mathbf{i}$ , which is in equilibrium. Because the virtual leads are in thermal contact with graphene and the heat current  $Q_i$  from the graphene to the virtual leads is zero, we can use the temperature  $T_i$  as an equivalent temperature of the local nonequilibrium electron at the site i. In Sec. III B, we will numerically study the local equivalent temperature  $T_i$ .

## **III. NUMERICAL RESULTS AND DISCUSSIONS**

In the numerical calculations, we set the hopping energy t = 2.75 eV, the on-site energy  $\epsilon_i = 0.2t$  (i.e., the energy of Dirac point), and the Fermi energy  $E_F = 0$ . Taking into account the spin degeneracy, we will use  $h/2e^2$  as the resistance unit. The corresponding filling factors are taken as odd integers ( $\nu = 1, 3, 5,...$ ) instead of even integers ( $\nu = 2, 6,$ 10,...), which are actually in accordance with the experiment result [33]. The zigzag edge graphene ribbon is considered [Fig. 1(a)] and the results still hold for the armchair one. The magnetic field is expressed as the Peierls phase [37,45]:  $2\phi =$  $(3\sqrt{3}/2)a^2B/\phi_0$ , with  $(3\sqrt{3}/2)a^2B$  the magnetic flux threading a single hexagon and the unit of B being  $4\hbar/(3\sqrt{3ea^2})$ , where a = 0.142 nm is the lattice constant of graphene. The device sizes are N = 120, W = 30,  $L_1 = 10$ ,  $L_p = 10$ ,  $L_q =$  $180, L_3 = 50, L_4 = 50, B = 0.02$ , and the temperature  $\mathcal{T} = 0$ for all leads, which will be used in the calculations unless stated otherwise.

### A. Hall resistance and thermal dissipation

Figures 1(b) and 1(c) show the Hall resistance  $R_H$  and the longitudinal one  $R_{xx}$  as functions of magnetic field *B* with different dissipation strengths  $\Gamma_d$ . In the absence of dissipation sources ( $\Gamma_d = 0$ ) and narrow constriction,  $R_H$  increases and  $R_{xx}$  oscillates with increasing *B*. At large *B*, the expected plateaus at  $R_H = (1/\nu)h/2e^2$  with filling factors  $\nu = 1, 3,$ 5,... are found, and  $R_{xx}$  is zero except in the plateau transition regime, owing to the formation of Landau levels and chiral edge states [41,46]. In particular, the QH plateaus and zero  $R_{xx}$  remain well in the case of either dissipation sources or narrow constriction exist. These phenomena are consistent with previous experimental [7,16,18–20] and theoretical works [8,39,41,47]. That is, the QH effect is topologically protected, so no dissipation and no backscattering are naively expected.

Next, we focus on the heat generation in the presence of dissipation sources. Without the narrow constriction, i.e., M = N = 120, the current-induced local heat generation  $Q_L$ is almost zero in the QH plateau regime because the charge carriers flow along the topologically protected chiral edge states. This means that thermal dissipation and backscattering cannot occur as expected. However, the results are different in the presence of narrow constriction (M = 8).

Figures 2(a) and 2(b) plot the local heat generation  $Q_L$ at M = 8 in the QH plateau regime with B = 0.02 (v =1). When B = 0.02, the magnetic length  $l_B = \sqrt{\frac{\hbar}{eB}} \approx 8a \sim$ M, thus backscattering will happen at the constriction and the electrons will be in nonequilibrium states after passing through the constriction, leading to voltage drop and work generation at the constriction. In Fig. 2, the dissipation sources (virtual leads) are added in the region surrounded by the red solid lines, and  $Q_L$  is zero in the region with no dissipation sources. From Figs. 2(a) and 2(b), one can see that  $Q_L$  is quite large at the constriction region and the device edges, indicating the emergence of thermal dissipation, contrary to the intuition that the QH effect is dissipationless. Furthermore, thermal dissipation mainly appears along the downstream chiral flow direction of the constriction and is very weak in the bulk. These results are in good agreement with the recent experiment [33]. When away from the constriction, the heat generation  $Q_L$  is gradually declined with a relaxation length  $\lambda$ . For small dissipation strength  $\Gamma_d$ ,  $\lambda$  is very long and  $Q_L$  is almost the same along the downstream channel [Fig. 2(a)], similar to the experimental results [33]. While for large  $\Gamma_d$ ,  $\lambda$  is short and  $Q_L$  decays significantly [Fig. 2(b)]. With the increase of dissipation strength  $\Gamma_d$ , the total local heat generation  $Q_{LT} = \sum_{x,y} Q_L(x, y)$  in the virtual leads increases. Notice that although the thermal dissipation appears, the QH plateaus and zero longitudinal resistance  $R_{xx}$  remain well [Figs. 1(b) and 1(c)].

Figures 2(c) and 2(d) plot the local heat generation  $Q_L$  for the higher filling factor  $\nu = 3$  (B = 0.01). For the higher  $\nu$ , the thermal dissipation increases significantly since there are more edge states. From Figs. 2(c) and 2(d), we can see that  $Q_L$  mainly appears along the downstream chiral flow direction of the constriction, and is still very weak in the bulk. When away from the constriction, the heat generation  $Q_L$  is gradually declined with a relaxation length  $\lambda$ . With the increase



FIG. 2. (a)–(d) show the local heat generation  $Q_L$  vs lattice position (x, y) in the QH plateau regime with a narrow constriction on the left side [i.e., the narrow constriction is between lead 1 and leads 2, 6 in Fig. 1(a)]. B = 0.02 ( $\nu = 1$ ) in (a), (b), 0.01 ( $\nu = 3$ ) in (c), (d), and dissipation strengths  $\Gamma_d = 0.005$  in (a), (c), 0.02 in (b), (d). (e) shows  $Q_L$  at B = 0.02 for the device with a narrow constriction on the right side [i.e., the narrow constriction is between lead 4 and leads 3, 5 in Fig. 1(a)] and  $\Gamma_d = 0.02$ . (f) shows  $Q_L$  in the Hall plateau transition regime (B = 0.0116) with  $L_q = 80$ , W = 60, and  $\Gamma_d = 0.005$ . The dissipation sources are added in the region surrounded by the red solid lines and the electric currents  $J_i$  in the virtual leads are zero.

of dissipation strength  $\Gamma_d$ , the total local heat generation  $Q_{LT}$ increases but the relaxation length  $\lambda$  decreases. These results are similar to the case of filling factor  $\nu = 1$ . In addition, for the higher filling factor, the thermal dissipation is slightly delocalized to the edge of the system because the higher edge states are more extended. Figure 2(e) shows the local heat generation  $Q_L$  for the situation that the constriction locates at the right side. Now the thermal dissipation mainly occurs at the lower edge, which is still aligned with the downstream chiral flow direction of the constriction.

However, in the Hall plateau transition regime, one can see from Fig. 2(f) that thermal dissipation mainly occurs in the bulk as expected because the Fermi energy locates at the spatially extended Landau level. The thermal dissipation is slightly larger in the upper part than the lower one, as the



FIG. 3. Local heat generation  $Q_L$  vs lattice position (x, y) with  $L_p = 30$ ,  $L_q = 80$ . The dissipation strength (a)  $\Gamma_d = 0.005$ , (b) 0.01, (c) 0.02, and (d) 0.05. The dissipation sources are added in the region surrounded by the red solid lines and the electric currents  $J_i$  in the virtual leads are zero.

electrons move toward the upper part under the magnetic field, which is different from the recent experiment where the thermal dissipation occurs along both the downstream and upstream directions with no visible chirality [33]. This experimental phenomenon may originate from the edge reconstruction in graphene [48–50], which induces additional nontopological counterpropagating channels [51]. Our results predict that thermal dissipation mainly occurs in the bulk in the plateau transition regime if there is no edge reconstruction in graphene or 2DEG systems (e.g., GaAs/AlGaAs heterostructures). It is worth mentioning that edge reconstruction has little effect on thermal dissipation in the QH plateau regime because the Fermi energy  $E_F$  locates far from the Landau levels in this case.

Now, we offer a detailed discussion that  $L_p = 10$  in Fig. 2 is reasonable and the local heat generation is almost zero in the bulk in the Hall plateau regime. Figure 3 shows the local heat generation  $Q_L$  for  $L_p = 30$  and the magnetic field B = 0.02. When B = 0.02, the system is in the first QH plateau regime. From Fig. 3, we can see that for the case of  $L_p = 30$ , which is much larger than  $l_B$ , the thermal dissipation along the left edge still occurs, and mainly appears in the region with x < 10, regardless of the dissipation strength  $\Gamma_d$ . Therefore, it is reasonable to assume  $L_p = 10$ . What's more, there is no thermal dissipation in the bulk in the Hall plateau regime. The thermal dissipation mainly occurs along the downstream chiral flow direction of the narrow constriction, with a relaxation length  $\lambda$  related to the dissipation strength. With the increase of dissipation strength  $\Gamma_d$ , the total local heat generation  $Q_{LT}$  in the virtual leads increases, and the relaxation length reduces.



FIG. 4. The local heat generation  $Q_L$  vs lattice position (x, y) for dissipation strengths  $\Gamma_d = 0.01$  in (a) and  $\Gamma_d = 0.05$  in (b).  $Q_L$  along the gray or white dotted lines in (a) and (b) are shown in Figs. 5(a) and 5(b), respectively. The dissipation sources are added in the region surrounded by the green solid lines and the electric currents  $J_i$  in the virtual leads are zero.

These results are completely the same as those in Figs. 2(a) and 2(b).

In the discussion above, the dissipation sources exist at the constriction, so the transmission coefficient of the constriction will be affected by  $\Gamma_d$ . To eliminate the interaction between the constriction and the dissipation sources, here we consider the situation that dissipation sources only appear in the region enclosed by the green solid lines in Fig. 1(a). From Figs. 4(a) and 4(b), we can see a large heat generation  $Q_L$  still appears along the downstream chiral flow direction (the upper edge of the system) when  $\Gamma_d \neq 0$ , and the heat generation is almost zero along the upstream chiral flow direction (the lower edge of the system), just like Figs. 2(a) and 2(b).

Then, we study the thermal dissipation in more details in the QH plateau regime. The curves in Figs. 5(a) and 5(b)are extracted from Fig. 4 by fixing the transverse location y or the longitudinal location x. Figure 5(a) shows the local heat generation  $Q_L$  versus longitudinal location x with a fixed transverse location y for different  $\Gamma_d$ . For small dissipation strength  $\Gamma_d$ , the local heat generation  $Q_L$  is almost the same regardless of x. For large  $\Gamma_d$ ,  $Q_L$  is dramatically declined with increasing x. Figure 5(b) plots the local heat generation  $Q_L$ versus transverse location y. It is obvious that the thermal dissipation is almost zero at the lower boundary (the upstream direction) and mainly occurs at the upper boundary (the downstream direction). Furthermore, the thermal dissipation obviously reduces with the increase of longitudinal location x. For example,  $Q_L$  at x = 50 is much larger than that at x = 100, and  $Q_L$  at x = 150 is nearly zero. In addition, at a fixed x, the thermal dissipation oscillates with y. The wave function  $|\Psi(y)|^2$  of the chiral edge state is also shown in Fig. 5(b).  $|\Psi(y)|^2$  oscillates with y, which is very similar to the curve  $Q_L$ -y. This indicates that thermal dissipation originates from the topologically protected chiral edge states.

When dissipation sources only appear in the region enclosed by the green solid lines in Fig. 1(a), the transmission coefficient through the narrow constriction is  $T = \sum_{q=2}^{6} T_{q1} + \sum_{i} T_{i1}$ . Figure 5(c) shows the transmission coefficient *T* versus the magnetic field *B*. *T* is noninteger and depends on the magnetic field *B*. For B = 0.02 (in the first Hall plateau regime), T = 0.4858, and for B = 0.0116 (in



FIG. 5. (a) The local heat generation  $Q_L$  vs longitudinal location x under fixed y = 115 for different  $\Gamma_d$ . (b) shows  $Q_L$  vs transverse location y with  $\Gamma_d = 0.05$  for different x, and the wave function  $|\Psi(y)|^2$  vs location y. (c) The transmission coefficient T of the narrow constriction vs the magnetic field B for different  $\Gamma_d$ . (d) shows the total local heat generation  $Q_{LT}$  versus dissipation strength  $\Gamma_d$ . In (a)–(c), the dissipation sources exist in the region surrounded by the green solid lines in Fig. 1(a). In (d), the red and green curves correspond to the dissipation sources in the regions enclosed by the red and green solid lines, respectively. The electric currents  $J_i$  in the virtual leads are zero in (a)–(d).

the Hall plateau transition regime), T = 0.5466. In addition, the transmission coefficient T will be hardly affected by the dissipation strength  $\Gamma_d$ . Figure 5(d) plots the total local heat generation  $Q_{LT}$  versus the dissipation strength  $\Gamma_d$ . Without the dissipation sources ( $\Gamma_d = 0$ ), the total local heat generation  $Q_{LT}$  is zero. With the increase of  $\Gamma_d$ ,  $Q_{LT}$  increases monotonically. For dissipation sources only exist at the sample [in the green solid lines surrounding region in Fig. 1(a)],  $Q_{LT}$  at large  $\Gamma_d$  limit tends to a saturation value,  $Q_{LT}^{\text{max}} = \frac{1}{2}T(1 - 1)$  $T V_b^2 2e^2/h$ , see the green curve in Fig. 5(d). For example,  $Q_{LT} \approx 0.1249 V_b^2 2e^2/h$  at  $\Gamma_d = 0.2$ , which is very close to  $Q_{LT}^{\max} \approx 0.125 V_b^2 2e^2/h$ . On the other hand, when the dissipation sources exist at both the narrow constriction and the sample [in the red solid lines surrounding region in Fig. 1(a)], the total local heat generation  $Q_{LT}$  is much larger than that of the dissipation sources in the green solid lines surrounding region, but is less than  $2Q_{LT}^{max}$ . In addition, our numerical results also indicate that the total heat generation ( $Q_T = Q_{LT}$  +  $\sum_{p=1}^{6} Q_p$ ) in the whole device is equal to the Joule heating  $JV_b$ since the electric current  $J_p$  ( $p \in \{i, 2, 3, 5, 6\}$ ) is zero and the energy of the system is conserved, demonstrating the validity of our numerical results.

#### **B.** Equivalent temperature

In the above, the temperature of the environment (the virtual leads) is equal to the electronic system (the real leads). However, if the thermal conductivity between the sample and the environment is poor, the local heat generation  $Q_i$  disap-



FIG. 6. Equivalent temperature  $k_B \Delta T_i$  ( $\Delta T_i \equiv T_i - T$ ) vs lattice position  $\mathbf{i} = (x, y)$  with (a)  $\Gamma_d = 0.005$ , (b) 0.01, (c) 0.02, and (d) 0.05. The background temperature  $k_B T/(eV_b) = 0.1$ . The dissipation sources exist in the region surrounded by the red solid lines in Fig. 1(a) and both electric currents  $J_i$  and heat currents  $Q_i$  in the virtual leads are zero. The light cyan area is added artificially to show the part without coupling of the virtual leads.

pears, thus the local electron temperature rises. Figures 6 and 7 show the equivalent temperature  $\Delta T_i (\Delta T_i \equiv T_i - T)$  versus the position i = (x, y) for the dissipation sources in the red and green solid lines surrounding regions in Fig. 1(a), respectively. The equivalent temperature along the downstream chiral flow direction (the upper edge of the system) increases significantly because of the chiral heat transport in the QH plateau regime [52,53]. However, at the upstream direction (the lower edge of the system), the local temperature  $T_i$  is almost equal to



FIG. 7. Equivalent temperature  $k_B \Delta T_i (\Delta T_i \equiv T_i - T)$  vs lattice position  $\mathbf{i} = (x, y)$  with (a)  $\Gamma_d = 0.01$  and (b) 0.05. The background temperature  $k_B T / (eV_b) = 0.1$ . The dissipation sources exist in the region surrounded by the green solid lines in Fig. 1(a) and both electric currents  $J_i$  and heat currents  $Q_i$  in the virtual leads are zero. The light cyan area is added artificially to show the part without coupling of the virtual leads.



FIG. 8. (a)–(d) show distribution function F vs energy E for different x under fixed y = 115. In (a) and (b), the temperature of the virtual leads are the same as the real leads,  $J_i = 0$ ,  $\Gamma_d = 0.01$ in (a) and 0.05 in (b). In (c) and (d), the virtual leads act as the temperature detecting terminals with  $J_i = 0$  and  $Q_i = 0$ ,  $\Gamma_d = 0.05$ ,  $k_B T/(eV_b) = 0.01$  in (c) and 0.1 in (d). The dissipation sources are in the green solid lines surrounding region.

the background temperature  $\mathcal{T}$ . What's more, the equivalent temperature does not reduce with the increase of the longitudinal location x and is almost independent of the dissipation strength  $\Gamma_d$ . For example, when the dissipation strength  $\Gamma_d =$  $0.005, k_B \Delta \mathcal{T}_i \approx 0.1931 eV_b$  at the upper edge, which is almost the same as  $k_B \Delta \mathcal{T}_i \approx 0.1926 eV_b$  at  $\Gamma_d = 0.05$  [Fig. 6]. Recent experimental work [31] developed a temperature detection technique using an ultrasensitive scanning nanothermometer with a superconducting quantum interference device placed on a tip. This technique allows one to obtain a spatial temperature variation with submicrokelvin sensitivity.

## C. Evolution of the energy distribution

In this subsection, let us discuss why thermal dissipation can occur at the topologically protected chiral edge states in the QH plateau regime. When the electrons injected from lead 1 arrive at the narrow constriction, some of them will be reflected back and the others flow through the constriction, leading to a noninteger transmission coefficient [Fig. 5(c)]. Subsequently, the electron distribution at the downstream edge states of the constriction is nonequilibrium. For example, at zero temperature, the distribution function of the downstream edge states satisfies F(E) = 1 for energy E < 0, F(E) = T(E) for  $0 < E < eV_b$ , and F(E) = 0 for  $eV_b < E$ [see Figs. 8(a) and 8(b)]. In other words, the downstream edge states are completely occupied for E < 0, partially occupied for  $0 < E < eV_b$ , and empty for  $eV_b < E$ . In the presence of dissipation sources, these nonequilibrium electrons will tend to be equilibrium, so the thermal dissipation appears. The maximum thermal dissipation is the energy difference between the nonequilibrium states and the equilibrium ones, i.e.,  $Q_{LT}^{\text{max}} = \frac{1}{2}T(1-T)V_b^2 2e^2/h$ . At  $B = 0.02, T \approx 0.5$ , and  $Q_{LT}^{\text{max}} \approx 0.125 V_b^2 2e^2/h$ . Our numerical results show that  $Q_{LT} \approx 0.1249 V_b^2 2e^2/h$  at  $\Gamma_d = 0.2$  [see Fig. 5(d)], which is very close to  $Q_{LT}^{max}$ . Here are two points worth mentioning. (i) Although the thermal dissipation occurs in the QH plateau regime, backscattering cannot take place because electrons transfer from the high energy edge state to the low one with unchanged propagating direction during the dissipation processes. As a result, the QH plateaus and zero  $R_{xx}$  can survive well [Figs. 1(b) and 1(c)]. (ii) The thermal dissipation and entropy generation always occur as long as the electron distribution is nonequilibrium, no matter whether the system is topologically protected or not. In a word, topology only protects no backscattering but cannot protect no dissipation.

Figures 8(a) and 8(b) display the distribution  $F_i(E)$  at different locations x along the downstream direction. When the electrons pass through the constriction, they are located in severe nonequilibrium states and the corresponding distribution strongly deviates from the Fermi distribution function, as can be seen from the curves of x = 5 in Figs. 8(a) and 8(b). In the dissipation region, the local heat and entropy generate, which is accompanied with the decrease of higher-energy electrons and the increase of lower-energy electrons, then the distribution function will evolve gradually from nonequilibrium to equilibrium with increasing x. After a long distance, the distribution will turn back to the equilibrium Fermi distribution [Fig. 8(b)]. But for small  $\Gamma_d$ , the distribution cannot return to the Fermi distribution even at x = 160 [Fig. 8(a)].

Figures 8(c) and 8(d) show the distribution  $F_i(E)$  versus energy E for poor thermal conductivity between the sample and the environment. In this case, the equilibrium distribution in the virtual leads is different from the nonequilibrium distribution in graphene lattice site. Under a certain energy, the occupancy of electrons in graphene and in the virtual leads is usually different, thus the electrons will flow from the one with more occupancy to the one with less occupancy. As a result, the dissipation will occur and the nonequilibrium distribution F of the two-step shape also evolves gradually into the equilibrium Fermi distribution with higher temperature, although the local heat generation is zero. For small x (near the narrow constriction), the distribution of the electron is severely nonequilibrium with a two-step shape. Along the +xdirection, the electron relaxation process occurs due to the presence of the dissipation sources, and the nonequilibrium distribution evolves gradually into the equilibrium Fermi distribution. Without the energy loss, the temperature of the final equilibrium distribution rises. This indicates that dissipation and entropy can still increase.

### **IV. CONCLUSION**

In summary, the thermal dissipation processes in the QH regime in graphene are studied. We find that thermal dissipation can occur in the QH regime, and the relaxation length is affected by dissipation strength. The thermal dissipation mainly appears along the downstream chiral flow direction of the constriction in the QH plateau regime, although the Hall plateaus and the zero longitudinal resistance remain well. But in the QH plateau transition regime, thermal dissipation mainly occurs in the bulk. In addition, for the poor thermal conductivity case, the local heat generation is zero thus the local electron temperature rises, and it is not affected by the dissipation strength. Furthermore, accompanying the thermal

dissipation, the energy distribution of electrons evolves gradually from nonequilibrium distribution to equilibrium Fermi distribution. This work shows that added dissipation sources does not destroy behavior emerging from topological states such as the QH effect, and topology does not prohibit the emergence of dissipation and the increase of entropy.

### ACKNOWLEDGMENTS

This work was financially supported by National Key R and D Program of China (Grant No. 2017YFA0303301), NSF-China (Grants No. 11921005 and No. 11874428), the Strategic Priority Research Program of Chinese Academy of Sciences (Grant No. XDB28000000), and Beijing Municipal Science & Technology Commission (Grant No. Z191100007219013). We acknowledge the Highperformance Computing Platform of Peking University for providing computational resources.

## APPENDIX A: PHASES $\phi_{ij}$ IN THE LONGITUDINAL LEAD

In the numerical calculation, the Hamiltonian of the leads 2, 3, 5, 6 is required to obey the translational invariance along y direction, and the Hamiltonian of the leads 1, 4 needs to obey the translational invariance along x direction. So we choose the vector potential  $\vec{A} = (-By, 0, 0)$  in leads 1, 4 and the central region,  $\vec{A} = (0, Bx, 0)$  in the longitudinal leads 2, 3, 5, 6. In Fig. 9, we show the phases  $\phi_{ij}$  in the lead 2 and the coupling between lead 2 and the central region in detail, which are used in our calculation. We can see that each hexagonal lattice has a phase of  $2\phi$  along the clockwise direction, which is equal to the number of the flux quantum  $(3\sqrt{3}/2)a^2B/\phi_0$ , with  $(3\sqrt{3}/2)a^2B$  the magnetic flux threading a single hexagon. It is easy to prove that how to choose the phase  $\phi_{ij}$  has no effect on the results, as long as the number of the flux quantum (the sum of the phase  $\phi_{ij}$  along the clockwise direction of the hexagonal lattice) in all hexagonal lattices is  $2\phi$ . Choosing different phases  $\phi_{ij}$  is equivalent to take the different gauge, so the results are the same. In addition, for the leads 3, 5, 6 and their couplings to the central region, we also guarantee that each hexagonal lattice has a phase of  $2\phi$  along the clockwise direction and the translational invariance along y direction of leads 2, 3, 5, 6 is satisfied in our calculation.

## APPENDIX B: DISAPPEARANCE OF THE LINEAR TERM OF THE VOLTAGE DIFFERENCE $\Delta V_{pq}$ IN THE EXPRESSION OF THE HEAT GENERATION

In Eq. (2) in the main text,  $Q_p$  is electronic heat current from the central region to the lead p, which includes the heat currents caused by the temperature difference, by the Peltier effect, and by the flow of electric current. When a small bias is applied between leads 1 and 4, the electric current flows through the device, then the thermal dissipation occurs and the Joule heat generates while in the presence of the dissipation sources [25,26]. Notice that the current-induced local heat generation is proportional to  $(\Delta V_{pq})^2 (\Delta V_{pq} \equiv V_p - V_q)$ . This is essentially different from the heat current caused by the Peltier effect, which is proportional to  $\Delta V_{pq}$ .



FIG. 9. The zoomed-in figure of the lead 2 and the coupling between lead 2 and the central region in Fig. 1(a). The arrows and values represent the phase  $\phi_{ij}$  from the site **i** to **j**. The absence of arrow and value in some lines indicates that their phases are zero.

Now let us discuss under what conditions the currentinduced local heat generation [the quadratic term  $(\Delta V_{pq})^2$ ] dominates the heat current  $Q_p$ . To clearly show this issue, we set that the temperatures of all leads are equal ( $\mathcal{T}_p = \mathcal{T}$ ). Expanding the Fermi function up to the second-order term, we have

$$f_p(E) \approx f(E) - eV_p f'(E) + \frac{e^2 V_p^2}{2} f''(E),$$
 (B1)

where  $f(E) = \frac{1}{e^{\epsilon}+1}$ ,  $f'(E) = -\frac{1}{k_B T} \frac{e^{\epsilon}}{(e^{\epsilon}+1)^2}$ , and  $f''(E) = \frac{1}{k_B^2 T^2} \frac{e^{\epsilon} - e^{-\epsilon}}{(e^{\epsilon}+1)^2(e^{-\epsilon}+1)^2}$  with  $\epsilon = \frac{E-E_F}{k_B T}$ . Considering the Sommerfeld expansion for the transmission coefficient

$$T_{pq}(E) = T_{pq}(E_F) + (E - E_F)T'_{pq},$$
 (B2)

with  $T'_{pq} = \frac{dT_{pq}(E)}{dE}|_{E=E_F}$ . Substituting Eqs. (B1) and (B2) into Eq. (2),  $Q_p$  changes into

$$Q_{p} = \frac{2}{h} \sum_{q} \int dE (E - \mu_{p}) [T_{pq}(E_{F}) + (E - E_{F})T'_{pq}] \\ \times \left[ e(V_{p} - V_{q})f'(E) - (e^{2}/2)(V_{p}^{2} - V_{q}^{2})f''(E) \right] \\ = \frac{2}{h} \sum_{q} \left\{ -\frac{\pi^{2}}{3}k_{B}^{2}\mathcal{T}^{2}e\Delta V_{pq}T'_{pq} + \frac{1}{2}e^{2}(\Delta V_{pq})^{2}T_{pq}(E_{F}) \right. \\ \left. + \frac{1}{2}e^{3}V_{p}(V_{p}^{2} - V_{q}^{2})T'_{pq} \right\}.$$
(B3)

At the small voltage difference  $\Delta V_{pq}$  limit and the finite temperature  $\mathcal{T}$ , the leading term of  $Q_p$  is  $-\frac{\pi^2}{3}k_B^2\mathcal{T}^2e\Delta V_{pq}T'_{pq}$ , which is the linear term  $\Delta V_{pq}$ , depends on  $T'_{pq}(E_F)$  [not  $T_{pq}(E_F)$ ], and describes the Peltier effect [43,44]. However, when both the voltage difference  $\Delta V_{pq}$  and the temperature  $\mathcal{T}$  are small, the leading term of  $Q_p$  in Eq. (B3) is the second term,  $\frac{1}{2}(e\Delta V_{pq})^2T_{pq}(E_F)$ . This term is proportional to  $(\Delta V_{pq})^2$ , depends on  $T_{pq}(E_F)$ , and describes the Joule heat

generation by the electric current. Because the present work studies the heat generation and assumes the small temperature and small voltage, the term  $\frac{1}{2}(e\Delta V_{pq})^2 T_{pq}(E_F)$  emerges in Eqs. (4) and (5) in the main text.

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