Effect of magnons on the temperature dependence and anisotropy of spin-orbit torque

Frank Freimuth [1,2,*] Stefan Blügel,[1] and Yuriy Mokrousov[1,2]

[1]*Peter Grünberg Institut and Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, 52425 Jülich, Germany*
[2]*Institute of Physics, Johannes Gutenberg University Mainz, 55099 Mainz, Germany*

We investigate the influence of magnons on the temperature dependence and the anisotropy of the spin-orbit torque (SOT). For this purpose, we use third-order perturbation theory in the framework of the Keldysh formalism to derive suitable equations to compute the magnonic SOT. We find several contributions to the magnonic SOT, which depend differently on the spin-wave stiffness $\mathcal{A}$ and the temperature T, with the dominating contribution scaling like $T^2/\mathcal{A}^2$. Based on this formalism, we compute the magnonic SOT in the ferromagnetic Rashba model. For large Rashba parameters, the magnonic SOT is strongly anisotropic, and for small quasiparticle broadening, it may become larger than the nonmagnonic SOT.

I. INTRODUCTION

Spin-orbit torques (SOTs) allow us to excite magnetization dynamics and to switch the magnetization in magnetic bits, which may be used for magnetoresistive random-access memory applications [1]. Therefore, they have become a cornerstone in spintronics research (see Ref. [2] for a recent review). A magnetic bilayer, such as Co/Pt, is composed of a magnetic layer (Co) on a heavy metal layer (Pt). When an electric current is applied in plane along the x direction in the magnetic bilayer, the torque exerted on the magnetization $\hat{\mathbf{M}}$ due to the SOT consists of the fieldlike torque $\propto \hat{\mathbf{M}} \times \hat{\mathbf{e}}_y$ and the antidamping torque $\propto \hat{\mathbf{M}} \times [\hat{\mathbf{M}} \times \hat{\mathbf{e}}_y]$. Additional contributions, which depend neither like $\hat{\mathbf{M}} \times \hat{\mathbf{e}}_y$ nor like $\hat{\mathbf{M}} \times [\hat{\mathbf{M}} \times \hat{\mathbf{e}}_y]$ on the magnetization direction $\hat{\mathbf{M}}$ have been found both experimentally [3] and theoretically [4,5]. They are referred to as the anisotropy of the SOT.

Electronic structure calculations describe many properties of the SOTs measured in experiments correctly [6–8]. However, the SOT measured in Ta/CoFeB/MgO nanowires exhibits a strong increase of the fieldlike component with increasing temperature, suggesting thermally induced excitation processes to be at play, which have not yet been considered in microscopic calculations of the SOT [9–11]. The same observation is made in Pt/Co/C [12], Pt/Hf/FeCoB/MgO, and W/Hf/FeCoB/MgO [11]. A second phenomenon that calls for extensions of the existing theoretical models is the strong anisotropy found in experiments [3], which contrasts the often roughly isotropic or only weakly anisotropic SOT obtained in first-principles electronic structure calculations [13].

In this paper, we extend our formalism for calculations of the SOT [7] by including magnons. Theoretical approaches to compute the effect of magnons on the electrical conductivity in models use rate equations [14], the Boltzmann equation [15], and diagrammatic perturbation theory [16,17].

First-principles methods are based on the disordered-local-moment approach [18] or on the Kubo-Landauer formalism applied to a large supercell with spin disorder [19]. In this paper, we make use of the torque operator $\mathcal{T}$ to include the effect of magnons. In previous works, we showed that the torque operator may be used to compute the response to magnetization dynamics [20] and to calculate the effects of magnetic texture [21,22]. Here, we use perturbations of the form $\mathcal{T}_x \sin(\mathbf{q} \cdot \mathbf{r} - \omega_{\text{mag}} t)$ to consider the wave vectors $\mathbf{q}$ and the frequencies ω_{mag} of magnons. We employ the Keldysh nonequilibrium formalism to assess the SOT in the presence of these perturbations by magnons.

This paper is structured as follows. In Sec. II A, we develop the equations suitable to compute the magnonic SOT. Several contributions to the magnonic SOT are detailed in Appendix A. The necessary integrals of the magnon dispersion are dealt with in Sec. II B. In Sec. II C, we explain how the magnonic torque scales with temperature and spin-wave stiffness. In Sec. II D, we generalize the formalism for general magnetization directions, which is necessary for the calculation of the anisotropy of the SOT. In Sec. III, we present our results on the magnonic torque in the ferromagnetic Rashba model. Additional results for various different parameters are presented in Appendix B. This paper ends with a summary in Sec. IV.

II. FORMALISM

A. SOT in the presence of magnons

The one-magnon state is described by the normalized magnetization:

$$\hat{\mathbf{M}}(\mathbf{r}, t) = \begin{Bmatrix} \eta \cos[\mathbf{q} \cdot \mathbf{r} - \omega_{\text{mag}}(\mathbf{q})t] \\ \eta \sin[\mathbf{q} \cdot \mathbf{r} - \omega_{\text{mag}}(\mathbf{q})t] \\ 1 - \frac{\eta^2}{2} \end{Bmatrix}, \tag{1}$$

where η determines the cone angle of the magnon, $\omega_{\text{mag}}(\mathbf{q})$ is the dispersion, and $\mathbf{q}$ is the magnon wave vector. From the solution of the Heisenberg model, it is known that M_z is reduced

*Corresponding author: f.freimuth@fz-juelich.de

in the one-magnon state by the factor $1 - 1/N_{\mathrm{mag}}$, where N_{mag} is the number of sites. Consequently, $1 - \eta^2/2 = 1 - 1/N_{\mathrm{mag}}$, and therefore,

$$\eta = \sqrt{\frac{2}{N_{\mathrm{mag}}}}. \tag{2}$$

We consider the ferromagnetic ground state with magnetization in the z direction as the unperturbed reference state (in Sec. II D, we will generalize the formalism to general magnetization directions) and add the perturbation term:

$$\delta H_{\mathrm{mag}}(t) = \mu_{\mathrm{B}} \Omega^{\mathrm{xc}}(\mathbf{r}) \eta \sigma_x \cos{[\mathbf{q} \cdot \mathbf{r} - \omega_{\mathrm{mag}}(\mathbf{q})t]} + \mu_{\mathrm{B}} \Omega^{\mathrm{xc}}(\mathbf{r}) \eta \sigma_y \sin{[\mathbf{q} \cdot \mathbf{r} - \omega_{\mathrm{mag}}(\mathbf{q})t]}$$
$$= \eta \mathcal{T}_y \cos{[\mathbf{q} \cdot \mathbf{r} - \omega_{\mathrm{mag}}(\mathbf{q})t]} - \eta \mathcal{T}_x \sin{[\mathbf{q} \cdot \mathbf{r} - \omega_{\mathrm{mag}}(\mathbf{q})t]}, \tag{3}$$

to the Hamiltonian to compute the electronic states in the presence of the magnon, Eq. (1). Here, $\Omega^{\mathrm{xc}}(\mathbf{r}) = \frac{1}{2\mu_{\mathrm{B}}}[V_{\mathrm{minority}}^{\mathrm{eff}}(\mathbf{r}) - V_{\mathrm{majority}}^{\mathrm{eff}}(\mathbf{r})]$ is the exchange field, i.e., the difference between the effective potentials of minority and majority electrons, μ_{B} is the Bohr magneton, and $\mathcal{T}$ is the torque operator [7]. We include the effect of this perturbation, Eq. (3), on the electronic states with the help of the Keldysh nonequilibrium formalism. A single perturbation by Eq. (3) leads to a response that oscillates spatially proportional to the cos and sin so that its spatial average is zero. We therefore consider the quadratic response to the perturbation Eq. (3). A sampling over the magnon distribution is performed in the course of the derivation.

The perturbation by the applied electric field is given by

$$\delta H_{\mathrm{em}}(t) = e\mathbf{v} \cdot \mathbf{A}(t), \tag{4}$$

where

$$\mathbf{A}(t) = \frac{1}{2}\left[\frac{\mathbf{E}_0 e^{-i\omega t}}{i\omega} - \frac{\mathbf{E}_0 e^{i\omega t}}{i\omega}\right] = -\frac{\mathbf{E}_0 \sin(\omega t)}{\omega}, \tag{5}$$

is the vector potential, $\mathbf{v}$ is the velocity operator, and e is the elementary positive charge. We will take the limit $\omega \to 0$ at the end of the calculation to extract the direct current response to the applied electric field.

Since we need the response quadratic in δH_{mag} and linear in δH_{em}, we take the third-order perturbation from the Dyson equation [23]:

$$G_3^< = G_{\mathrm{eq}}^{\mathrm{R}} \frac{\delta H_{\mathrm{tot}}(t_1)}{\hbar} G_{\mathrm{eq}}^{\mathrm{R}} \frac{\delta H_{\mathrm{tot}}(t_2)}{\hbar} G_{\mathrm{eq}}^{\mathrm{R}} \frac{\delta H_{\mathrm{tot}}(t_3)}{\hbar} G_{\mathrm{eq}}^< + G_{\mathrm{eq}}^{\mathrm{R}} \frac{\delta H_{\mathrm{tot}}(t_1)}{\hbar} G_{\mathrm{eq}}^{\mathrm{R}} \frac{\delta H_{\mathrm{tot}}(t_2)}{\hbar} G_{\mathrm{eq}}^< \frac{\delta H_{\mathrm{tot}}(t_3)}{\hbar} G_{\mathrm{eq}}^{\mathrm{A}}$$
$$+ G_{\mathrm{eq}}^{\mathrm{R}} \frac{\delta H_{\mathrm{tot}}(t_1)}{\hbar} G_{\mathrm{eq}}^< \frac{\delta H_{\mathrm{tot}}(t_2)}{\hbar} G_{\mathrm{eq}}^{\mathrm{A}} \frac{\delta H_{\mathrm{tot}}(t_3)}{\hbar} G_{\mathrm{eq}}^{\mathrm{A}} + G_{\mathrm{eq}}^< \frac{\delta H_{\mathrm{tot}}(t_1)}{\hbar} G_{\mathrm{eq}}^{\mathrm{A}} \frac{\delta H_{\mathrm{tot}}(t_2)}{\hbar} G_{\mathrm{eq}}^{\mathrm{A}} \frac{\delta H_{\mathrm{tot}}(t_3)}{\hbar} G_{\mathrm{eq}}^{\mathrm{A}}, \tag{6}$$

where $G_{\mathrm{eq}}^{\mathrm{R}}$, $G_{\mathrm{eq}}^{\mathrm{A}}$, and $G_{\mathrm{eq}}^<$ are the retarded, advanced, and lesser Green's functions of the unperturbed system, respectively, and

$$\delta H_{\mathrm{tot}}(t) = \delta H_{\mathrm{mag}}(t) + \delta H_{\mathrm{em}}(t). \tag{7}$$

In Eq. (6), we suppressed the two time arguments that each Green's function has for notational convenience. Additionally, we suppressed the time integrations over the intermediate times t_1, t_2, and t_3 for notational brevity. How these time integrals are performed is clarified in the following Eq. (8). The time integration of the product of four Green's functions is given by

$$\int dt_1 dt_2 dt_3 \exp{(-i\Omega_1 t_1)} \exp{(-i\Omega_2 t_2)} \exp{(-i\Omega_3 t_3)} G_{\mathrm{eq}}^{\alpha}(t, t_1) G_{\mathrm{eq}}^{\alpha'}(t_1, t_2) G_{\mathrm{eq}}^{\alpha''}(t_2, t_3) G_{\mathrm{eq}}^{\alpha'''}(t_3, t)$$
$$= \frac{\exp{[-i(\Omega_1 + \Omega_2 + \Omega_3)t]}}{2\pi} \int d\Omega\, G_{\mathrm{eq}}^{\alpha}(\Omega) G_{\mathrm{eq}}^{\alpha'}(\Omega - \Omega_1) G_{\mathrm{eq}}^{\alpha''}(\Omega - \Omega_1 - \Omega_2) G_{\mathrm{eq}}^{\alpha'''}(\Omega - \Omega_1 - \Omega_2 - \Omega_3), \tag{8}$$

where $\alpha = \mathrm{R}, \mathrm{A}, <$ and Ω_i may take the values $\pm\omega$ and $\pm\omega_{\mathrm{mag}}$ ($i = 1, 2,$ and 3). The following frequency combinations may contribute to the magnonic SOT: Case 1: $\Omega_1 = \pm\omega$ and $\Omega_2 = -\Omega_3 = \pm\omega_{\mathrm{mag}}$. Case 2: $\Omega_2 = \pm\omega$ and $\Omega_1 = -\Omega_3 = \pm\omega_{\mathrm{mag}}$. Case 3: $\Omega_3 = \pm\omega$ and $\Omega_1 = -\Omega_2 = \pm\omega_{\mathrm{mag}}$.

To make the equations more compact, we introduce the Keldysh Green's function:

$$\hat{G}_{\mathrm{eq}}(\Omega) = \begin{bmatrix} G_{\mathrm{eq}}^{\mathrm{R}}(\Omega) & G_{\mathrm{eq}}^<(\Omega) \\ 0 & G_{\mathrm{eq}}^{\mathrm{A}}(\Omega) \end{bmatrix}. \tag{9}$$

In case j, we obtain ($j = 1, 2, 3$):

$$\hat{G}_{3,j} = \lim_{\omega \to 0} \frac{1}{8i\omega} \sum_{u=\pm} \sum_{u'=\pm} u \hat{G}_{3,j}(u, u'), \tag{10}$$

where

$$\hat{G}_{3,1}(u, u') = \frac{\eta^2 e\mathbf{E}_0}{2\pi\hbar^3} \cdot \sum_{\gamma=x,y} \int d\Omega\, \hat{G}_{\mathrm{eq}}(\Omega) \mathbf{v} \hat{G}_{\mathrm{eq}}(\Omega - u\omega) \mathcal{T}_\gamma \hat{G}_{\mathrm{eq}, -u'\mathbf{q}}(\Omega - u\omega - u'\omega_{\mathrm{mag}}) \mathcal{T}_\gamma \hat{G}_{\mathrm{eq}}(\Omega - u\omega), \tag{11}$$

in case 1,

$$\hat{G}_{3,2}(u, u') = \frac{\eta^2 e \mathbf{E}_0}{2\pi\hbar^3} \cdot \sum_{\gamma=x,y} \int d\Omega \hat{G}_{\mathrm{eq}}(\Omega) \mathcal{T}_\gamma \hat{G}_{\mathrm{eq},-u'\mathbf{q}}(\Omega - u'\omega_{\mathrm{mag}}) \mathbf{v}_{-u'\mathbf{q}}$$

$$\times \hat{G}_{\mathrm{eq},-u'\mathbf{q}}(\Omega - u\omega - u'\omega_{\mathrm{mag}}) \mathcal{T}_\gamma \hat{G}_{\mathrm{eq}}(\Omega - u\omega), \tag{12}$$

in case 2, and

$$\hat{G}_{3,3}(u, u') = \frac{\eta^2 e \mathbf{E}_0}{2\pi\hbar^3} \cdot \sum_{\gamma=x,y} \int d\Omega \hat{G}_{\mathrm{eq}}(\Omega) \mathcal{T}_\gamma \hat{G}_{\mathrm{eq},-u'\mathbf{q}}(\Omega - u'\omega_{\mathrm{mag}}) \mathcal{T}_\gamma \hat{G}_{\mathrm{eq}}(\Omega) \mathbf{v} \hat{G}_{\mathrm{eq}}(\Omega - u\omega), \tag{13}$$

in case 3. Green's functions and velocity operators that carry the momentum subscript $-u'\mathbf{q}$ are shifted in momentum space by $-u'\mathbf{q}$.

Summing up cases 1, 2, and 3, we obtain

$$\hat{G}_3 = \sum_{j=1}^{3} \hat{G}_{3,j} = \lim_{\omega\to 0} \frac{1}{8i\omega} \sum_{u=\pm} \sum_{u'=\pm} u \hat{G}_3(u, u'), \tag{14}$$

where

$$\hat{G}_3(u, u') = \frac{\eta^2 e}{2\pi\hbar^3} \int d\Omega \sum_{\gamma=x,y} [\hat{G}_\Omega \mathbf{v} \cdot \mathbf{E}_0 \hat{G}_{\Omega-u\omega} \mathcal{T}_\gamma \hat{G}_{\Omega-u\omega-u'\omega_{\mathrm{mag}},-u'\mathbf{q}} \mathcal{T}_\gamma \hat{G}_{\Omega-u\omega}$$

$$+ \hat{G}_\Omega \mathcal{T}_\gamma \hat{G}_{\Omega-u'\omega_{\mathrm{mag}},-u'\mathbf{q}} \mathbf{v}_{-u'\mathbf{q}} \cdot \mathbf{E}_0 \hat{G}_{\Omega-u\omega-u'\omega_{\mathrm{mag}},-u'\mathbf{q}} \mathcal{T}_\gamma \hat{G}_{\Omega-u\omega}$$

$$+ \hat{G}_\Omega \mathcal{T}_\gamma \hat{G}_{\Omega-u'\omega_{\mathrm{mag}},-u'\mathbf{q}} \mathcal{T}_\gamma \hat{G}_\Omega \mathbf{v} \cdot \mathbf{E}_0 \hat{G}_{\Omega-u\omega}]. \tag{15}$$

Here, to save space, we introduced the notation $\hat{G}_\Omega = \hat{G}_{\mathrm{eq}}(\Omega)$. The SOT due to $\hat{G}_3$ is given by

$$\mathbf{T}_{\mathrm{mag}} = i\mathrm{Tr}[\mathcal{T} G_3^<]. \tag{16}$$

An important consistency check is that Eq. (16) predicts a SOT of zero when there is no spin-orbit interaction. This may be seen as follows: In the absence of spin-orbit interaction, the velocity operator is diagonal in spin-space, and the Green functions $\hat{G}_\Omega$ are diagonal in spin-space as well. In contrast, every torque operator causes a transition from spin-up to spin-down or from spin-down to spin-up. Since the number of torque operators in all summands is three, taking the trace in spin-space will yield zero when there is no spin-orbit coupling included in the calculation.

In the nonmagnonic SOT, the application of an electric field generates a nonequilibrium spin density perpendicular to the magnetization, which exerts a torque on the magnetization. The magnonic SOT described by Eqs. (14)–(16) corresponds to processes where these nonequilibrium spins are additionally flipped two times by a magnon. In the presence of spin-orbit interaction, two consecutive spin flips by a magnon constitutes a nontrivial process because the nonequilibrium spins may precess in the spin-orbit field in between the two spin flips. This leads to a modification of the nonequilibrium spin density by the magnons.

For the numerical evaluation of Eq. (15), it is convenient to perform a Taylor expansion in $\mathbf{q}$ and ω_{mag} as follows:

$$G_3^<(u, u') = G_3^{<,(0,0)}(u, u') + G_3^{<,(1,0)}(u, u') + G_3^{<,(2,0)}(u, u') + G_3^{<,(0,1)}(u, u') + G_3^{<,(0,2)}(u, u') + \dots, \tag{17}$$

where

$$G_3^{<,(i,j)}(u, u') \propto (u'\omega_{\mathrm{mag}})^i q^j, \tag{18}$$

i.e., $G_3^{<,(i,j)}(u, u')$ is ith order in ω_{mag} and jth order in $\mathbf{q}$ in the Taylor expansion of $G_3^<(u, u')$. A priori, it is unclear whether all terms in the expansion Eq. (17) contribute to the magnonic SOT. Therefore, we will evaluate them separately so that we can compare their magnitudes later.

The contributions to $G_3^{<,(i,j)}(u, u')$ may be further distinguished according to the order of the derivative of the Fermi function that they contain. Derivatives of the Fermi function are produced when the derivatives $\partial/\partial\omega$ or $\partial/\partial\omega_{\mathrm{mag}}$ act on the lesser Green's functions $G_{\mathrm{eq}}^<(\Omega - u\omega)$, $G_{\mathrm{eq}}^<(\Omega - u'\omega_{\mathrm{mag}})$, or $G_{\mathrm{eq}}^<(\Omega - u\omega - u'\omega_{\mathrm{mag}})$ because $G_{\mathrm{eq}}^<(\Omega) = f(\Omega)[G_{\mathrm{eq}}^A(\Omega) - G_{\mathrm{eq}}^R(\Omega)]$ contains the Fermi function $f(\Omega)$. While we use the derivative $\partial/\partial\omega$ to take the $\omega \to 0$ limit, the $\partial/\partial\omega_{\mathrm{mag}}$ derivatives are necessary for the Taylor expansion in ω_{mag} according to Eqs. (17) and (18). Following the standard notation used in linear response theory, we label terms that contain f with a superscript II (so-called *lesser-two*) and terms that contain f' with a superscript I (so-called *lesser-one*). However, due to the Taylor expansion in ω_{mag}, we will also encounter higher derivatives of f that do not occur in standard linear response theory. We denote terms that involve the second derivative f'' with a superscript III and terms that involve the third derivative f''' with a superscript IV.

At zeroth order in ω_{mag} and **q**, the lesser-one contribution from Eq. (15) is given by

$$G_3^{<,\text{I},(0,0)}(u,u') = -u\omega e \int d\Omega \sum_{\gamma=x,y} f'(\hbar\Omega) \frac{\eta^2 \mathbf{E}_0}{2\pi\hbar^2} \cdot \left[G_\Omega^R \mathbf{v} G_\Omega^S \mathcal{T}_\gamma G_\Omega^A \mathcal{T}_\gamma G_\Omega^A + G_\Omega^R \mathbf{v} G_\Omega^R \mathcal{T}_\gamma G_\Omega^S \mathcal{T}_\gamma G_\Omega^A \right.$$

$$\left. + G_\Omega^R \mathbf{v} G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \mathcal{T}_\gamma G_\Omega^S + G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^S \mathcal{T}_\gamma G_\Omega^A + G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^R \mathcal{T}_\gamma G_\Omega^S + G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^S \right], \quad (19)$$

where $G_\Omega^S = G_\Omega^A - G_\Omega^R$.

Here, $G_3^{<,\text{I},(0,0)}(u,u')$ still needs to be summed over the populated magnon modes. Also, $G_3^{<,\text{I},(0,0)}(u,u')$ itself depends on the magnons only through η. The effect of summing $G_3^{<,(0,0)}(u,u')$ over the magnon modes is therefore the multiplication by the number of magnons. We Taylor-expand only the electronic lesser Green's function in terms of ω_{mag} and **q** and not the Bose-Einstein distribution function. Therefore, we introduce the integral:

$$I^{(0,0)}(T) = \frac{1}{N_{\text{mag}} A_{\text{mag}}} \sum_{\mathbf{q}} F[\omega_{\text{mag}}(\mathbf{q}), T] = \int \frac{d^2q}{(2\pi)^2} F[\omega_{\text{mag}}(\mathbf{q}), T], \quad (20)$$

where $F[\omega_{\text{mag}}(\mathbf{q}), T]$ is the Bose-Einstein distribution function, and A_{mag} is the area occupied by one magnetic site. For example, in the case of Co/Pt magnetic bilayers, A_{mag} is the area of the unit cell. This integral is evaluated below in Sec. II B.

Plugging Eq. (19) into Eq. (14), summing over magnon modes, and using Eq. (16) to evaluate the torque, we obtain therefore

$$\mathbf{T}_{\text{mag}}^{\text{I},(0,0)} = -\frac{A_{\text{mag}} I^{(0,0)}(T)}{2\pi\hbar^2} \int d\Omega \sum_{\gamma=x,y} f'(\hbar\Omega) \text{Tr}\left\{ \mathcal{T} G_\Omega^R \left[\mathbf{v} G_\Omega^S \mathcal{T}_\gamma G_\Omega^A \mathcal{T}_\gamma G_\Omega^A + \mathbf{v} G_\Omega^R \mathcal{T}_\gamma G_\Omega^S \mathcal{T}_\gamma G_\Omega^A \right.\right.$$

$$\left.\left. + \mathbf{v} G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \mathcal{T}_\gamma G_\Omega^S + \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^S \mathcal{T}_\gamma G_\Omega^A + \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^R \mathcal{T}_\gamma G_\Omega^S + \mathcal{T}_\gamma G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^S \right] \cdot \mathbf{E}_0 e \right\}, \quad (21)$$

where we made use of $\eta^2 = 2/N_{\text{mag}}$ [see Eq. (2)].

Similarly, we may extract the lesser-two contribution from Eq. (15) and evaluate the corresponding torque, which is given in Eq. (A1) in Appendix A.

The next contribution to the Taylor expansion is $G_3^{<,\text{I},(1,0)}(u,u')$ [see Eq. (17)]. According to Eq. (18), we have $G_3^{<,\text{I},(1,0)}(u,u') \propto u'\omega_{\text{mag}}$. Since we need to sum over $u' = \pm 1$, this does not contribute to the magnonic SOT. The following contribution $G_3^{<,\text{I},(2,0)}(u,u')$ [see Eq. (17)] requires us to extract the terms quadratic in ω_{mag} from Eq. (15). We obtain

$$\hat{G}_3^{(2,0)}(u,u') = -\frac{eu\omega[\omega_{\text{mag}}]^2}{4\pi} \frac{\eta^2}{\hbar^3} \int d\Omega \sum_{\gamma=x,y} \hat{G}_\Omega \mathbf{E}_0$$

$$\cdot \left[\mathbf{v} \frac{\partial \hat{G}_\Omega}{\partial \Omega} \mathcal{T}_\gamma \frac{\partial^2 \hat{G}_\Omega}{\partial \Omega^2} \mathcal{T}_\gamma \hat{G}_\Omega + \mathbf{v} \hat{G}_\Omega \mathcal{T}_\gamma \frac{\partial^3 \hat{G}_\Omega}{\partial \Omega^3} \mathcal{T}_\gamma \hat{G}_\Omega + \mathbf{v} \hat{G}_\Omega \mathcal{T}_\gamma \frac{\partial^2 \hat{G}_\Omega}{\partial \Omega^2} \mathcal{T}_\gamma \frac{\partial \hat{G}_\Omega}{\partial \Omega} + \mathcal{T}_\gamma \frac{\partial^2 \hat{G}_\Omega}{\partial \Omega^2} \mathbf{v} \frac{\partial \hat{G}_\Omega}{\partial \Omega} \mathcal{T}_\gamma \hat{G}_\Omega \right.$$

$$+ \mathcal{T}_\gamma \frac{\partial^2 \hat{G}_\Omega}{\partial \Omega^2} \mathbf{v} \hat{G}_\Omega \mathcal{T}_\gamma \frac{\partial \hat{G}_\Omega}{\partial \Omega} + \mathcal{T}_\gamma \hat{G}_\Omega \mathbf{v} \frac{\partial^3 \hat{G}_\Omega}{\partial \Omega^3} \mathcal{T}_\gamma \hat{G}_\Omega + \mathcal{T}_\gamma \hat{G}_\Omega \mathbf{v} \frac{\partial^2 \hat{G}_\Omega}{\partial \Omega^2} \mathcal{T}_\gamma \frac{\partial \hat{G}_\Omega}{\partial \Omega} + 2\mathcal{T}_\gamma \frac{\partial \hat{G}_\Omega}{\partial \Omega} \mathbf{v} \frac{\partial^2 \hat{G}_\Omega}{\partial \Omega^2} \mathcal{T}_\gamma \hat{G}_\Omega$$

$$\left. + 2\mathcal{T}_\gamma \frac{\partial \hat{G}_\Omega}{\partial \Omega} \mathbf{v} \frac{\partial \hat{G}_\Omega}{\partial \Omega} \mathcal{T}_\gamma \frac{\partial \hat{G}_\Omega}{\partial \Omega} + \mathcal{T}_\gamma \frac{\partial^2 \hat{G}_\Omega}{\partial \Omega^2} \mathcal{T}_\gamma \hat{G}_\Omega \mathbf{v} \frac{\partial \hat{G}_\Omega}{\partial \Omega} \right]. \quad (22)$$

From this, we extract the lesser-one contribution:

$$G_3^{<,\text{I},(2,0)}(u,u') = -\int d\Omega f'(\hbar\Omega) \sum_{\gamma=x,y}$$

$$\times G_\Omega^R \left[\mathbf{v} G_\Omega^S \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^A}{\partial \Omega^2} \mathcal{T}_\gamma G_\Omega^A + 2\mathbf{v} \frac{\partial G_\Omega^R}{\partial \Omega} \mathcal{T}_\gamma \frac{\partial G_\Omega^S}{\partial \Omega} \mathcal{T}_\gamma G_\Omega^A + 3\mathbf{v} G_\Omega^R \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^S}{\partial \Omega^2} \mathcal{T}_\gamma G_\Omega^A + 2\mathbf{v} G_\Omega^R \mathcal{T}_\gamma \frac{\partial G_\Omega^S}{\partial \Omega} \mathcal{T}_\gamma \frac{\partial G_\Omega^A}{\partial \Omega} \right.$$

$$+ \mathbf{v} G_\Omega^R \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^R}{\partial \Omega^2} \mathcal{T}_\gamma G_\Omega^S + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^S}{\partial \Omega} \mathbf{v} \frac{\partial G_\Omega^A}{\partial \Omega} \mathcal{T}_\gamma G_\Omega^A + \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^R}{\partial^2 \Omega} \mathbf{v} G_\Omega^S \mathcal{T}_\gamma G_\Omega^A + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^S}{\partial \Omega} \mathbf{v} G_\Omega^A \mathcal{T}_\gamma \frac{\partial G_\Omega^A}{\partial \Omega}$$

$$+ \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^R}{\partial^2 \Omega} \mathbf{v} G_\Omega^R \mathcal{T}_\gamma G_\Omega^S + 3\mathcal{T}_\gamma G_\Omega^R \mathbf{v} \frac{\partial^2 G_\Omega^S}{\partial \Omega^2} \mathcal{T}_\gamma G_\Omega^A + 2\mathcal{T}_\gamma G_\Omega^R \mathbf{v} \frac{\partial G_\Omega^S}{\partial \Omega} \mathcal{T}_\gamma \frac{\partial G_\Omega^A}{\partial \Omega} + \mathcal{T}_\gamma G_\Omega^R \mathbf{v} \frac{\partial^2 G_\Omega^S}{\partial \Omega^2} \mathcal{T}_\gamma G_\Omega^S$$

$$+ 2\mathcal{T}_\gamma G_\Omega^S \mathbf{v} \frac{\partial^2 G_\Omega^A}{\partial \Omega^2} \mathcal{T}_\gamma G_\Omega^A + 4\mathcal{T}_\gamma \frac{\partial G_\Omega^R}{\partial \Omega} \mathbf{v} \frac{\partial G_\Omega^S}{\partial \Omega} \mathcal{T}_\gamma G_\Omega^A + 2\mathcal{T}_\gamma G_\Omega^S \mathbf{v} \frac{\partial G_\Omega^A}{\partial \Omega} \mathcal{T}_\gamma \frac{\partial G_\Omega^A}{\partial \Omega} + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^R}{\partial \Omega} \mathbf{v} G_\Omega^S \mathcal{T}_\gamma \frac{\partial G_\Omega^A}{\partial \Omega}$$

$$\left. + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^R}{\partial \Omega} \mathbf{v} \frac{\partial G_\Omega^R}{\partial \Omega} \mathcal{T}_\gamma G_\Omega^S + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^S}{\partial \Omega} \mathcal{T}_\gamma G_\Omega^A \mathbf{v} \frac{\partial G_\Omega^A}{\partial \Omega} + \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^R}{\partial \Omega^2} \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^S \right] \cdot \mathbf{E}_0 \frac{e}{4\pi} u\omega[\omega_{\text{mag}}]^2 \frac{\eta^2}{\hbar^2}. \quad (23)$$

Equation (23) depends on the magnons through η^2 and through ω_{mag}^2. Consequently, to perform the sampling over magnon modes, we introduce the integral:

$$I^{(2,0)}(T) = \int \frac{d^2q}{(2\pi)^2} [\hbar\omega_{\text{mag}}(\mathbf{q})]^2 F(\mathbf{q}, T), \tag{24}$$

which we discuss below in Sec. II B. Thus, employing Eqs. (14) and (16) yields the following contribution to the SOT after summing over the magnon modes:

$$
\begin{aligned}
\mathbf{T}_{\text{mag}}^{\text{I},(2,0)} = -\frac{A_{\text{mag}} I^{(2,0)}(T)}{4\pi\hbar^4} & \int d\Omega \sum_{\gamma=x,y} f'(\hbar\Omega) \\
\times \operatorname{Tr}\Bigg\{ \boldsymbol{\mathcal{T}} G_\Omega^{\text{R}} \Bigg[& \mathbf{v} G_\Omega^{\text{S}} \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^{\text{A}}}{\partial\Omega^2} \mathcal{T}_\gamma G_\Omega^{\text{A}} + 2\mathbf{v} \frac{\partial G_\Omega^{\text{R}}}{\partial\Omega} \mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{S}}}{\partial\Omega} \mathcal{T}_\gamma G_\Omega^{\text{A}} + 3\mathbf{v} G_\Omega^{\text{R}} \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^{\text{S}}}{\partial\Omega^2} \mathcal{T}_\gamma G_\Omega^{\text{A}} + 2\mathbf{v} G_\Omega^{\text{R}} \mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{S}}}{\partial\Omega} \mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{A}}}{\partial\Omega} \\
& + \mathbf{v} G_\Omega^{\text{R}} \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^{\text{R}}}{\partial\Omega^2} \mathcal{T}_\gamma G_\Omega^{\text{S}} + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{S}}}{\partial\Omega} \mathbf{v} \frac{\partial G_\Omega^{\text{A}}}{\partial\Omega} \mathcal{T}_\gamma G_\Omega^{\text{A}} + \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^{\text{R}}}{\partial^2\Omega} \mathbf{v} G_\Omega^{\text{S}} \mathcal{T}_\gamma G_\Omega^{\text{A}} + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{S}}}{\partial\Omega} \mathbf{v} G_\Omega^{\text{A}} \mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{A}}}{\partial\Omega} \\
& + \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^{\text{R}}}{\partial^2\Omega} \mathbf{v} G_\Omega^{\text{R}} \mathcal{T}_\gamma G_\Omega^{\text{S}} + 3\mathcal{T}_\gamma G_\Omega^{\text{R}} \mathbf{v} \frac{\partial^2 G_\Omega^{\text{S}}}{\partial\Omega^2} \mathcal{T}_\gamma G_\Omega^{\text{A}} + 2\mathcal{T}_\gamma G_\Omega^{\text{R}} \mathbf{v} \frac{\partial G_\Omega^{\text{S}}}{\partial\Omega} \mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{A}}}{\partial\Omega} + \mathcal{T}_\gamma G_\Omega^{\text{R}} \mathbf{v} \frac{\partial^2 G_\Omega^{\text{R}}}{\partial\Omega^2} \mathcal{T}_\gamma G_\Omega^{\text{S}} \\
& + 2\mathcal{T}_\gamma G_\Omega^{\text{S}} \mathbf{v} \frac{\partial^2 G_\Omega^{\text{A}}}{\partial\Omega^2} \mathcal{T}_\gamma G_\Omega^{\text{A}} + 4\mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{R}}}{\partial\Omega} \mathbf{v} \frac{\partial G_\Omega^{\text{S}}}{\partial\Omega} \mathcal{T}_\gamma G_\Omega^{\text{A}} + 2\mathcal{T}_\gamma G_\Omega^{\text{S}} \mathbf{v} \frac{\partial G_\Omega^{\text{A}}}{\partial\Omega} \mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{A}}}{\partial\Omega} + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{R}}}{\partial\Omega} \mathbf{v} G_\Omega^{\text{S}} \mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{A}}}{\partial\Omega} \\
& + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{R}}}{\partial\Omega} \mathbf{v} \frac{\partial G_\Omega^{\text{R}}}{\partial\Omega} \mathcal{T}_\gamma G_\Omega^{\text{S}} + 2\mathcal{T}_\gamma \frac{\partial G_\Omega^{\text{S}}}{\partial\Omega} \mathcal{T}_\gamma G_\Omega^{\text{A}} \mathbf{v} \frac{\partial G_\Omega^{\text{A}}}{\partial\Omega} + \mathcal{T}_\gamma \frac{\partial^2 G_\Omega^{\text{R}}}{\partial\Omega^2} \mathcal{T}_\gamma G_\Omega^{\text{R}} \mathbf{v} G_\Omega^{\text{S}} \Bigg] \Bigg\} \cdot \mathbf{E}_0 e.
\end{aligned} \tag{25}
$$

Similarly, we obtain $\mathbf{T}_{\text{mag}}^{\text{II},(2,0)}$ from the lesser-two contribution to the Green's function, which is given in Eq. (A2) in Appendix A. Due to the derivatives with respect to ω_{mag}, there are additionally the contributions $\mathbf{T}_{\text{mag}}^{\text{III},(2,0)}$ and $\mathbf{T}_{\text{mag}}^{\text{IV},(2,0)}$ from the lesser-three and lesser-four Green's functions, respectively. The explicit expressions are given in Eqs. (A3) and (A4) in Appendix A.

The next contribution to the Taylor expansion is $G_3^{<,\text{I},(0,1)}(u, u')$ [see Eq. (17)]. Since it is linear in $\mathbf{q}$, the average over magnon modes evaluates to zero for it. The next nonzero contribution is therefore $G_3^{<,\text{I},(0,2)}(u, u')$. The Taylor expansion of Eqs. (11)–(13) up to second order in $\mathbf{q}$ and up to zeroth order in ω_{mag} yields the lesser-one contributions:

$$
\begin{aligned}
\hat{G}_{3,1}^{<,\text{I},(0,2)}(u, u') = & -u\omega \frac{1}{4\pi} \frac{\eta^2}{\hbar^2} e \sum_{\gamma=x,y} \sum_{ij} q_i q_j \int d\Omega f'(\hbar\Omega) G_\Omega^{\text{R}} \mathbf{E}_0 \\
& \cdot \Bigg[\mathbf{v} G_\Omega^{\text{S}} \mathcal{T}_\gamma \frac{\partial^2 G_{\Omega,\mathbf{q}}^{\text{A}}}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{A}} + \mathbf{v} G_\Omega^{\text{R}} \mathcal{T}_\gamma \frac{\partial^2 G_{\Omega,\mathbf{q}}^{\text{S}}}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{A}} + \mathbf{v} G_\Omega^{\text{R}} \mathcal{T}_\gamma \frac{\partial^2 G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{S}} \Bigg],
\end{aligned} \tag{26}
$$

in case 1,

$$
\begin{aligned}
\hat{G}_{3,2}^{<,\text{I},(0,2)}(u, u') = & -u\omega \sum_{ij} \sum_{\gamma=x,y} \int d\Omega f'(\hbar\Omega) \frac{1}{4\pi} q_i q_j \frac{\eta^2}{\hbar^2} e \mathbf{E}_0 \cdot G_\Omega^{\text{R}} \mathcal{T}_\gamma \\
\times \Bigg[& \frac{\partial^2 G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i \partial q_j} \mathbf{v} G_\Omega^{\text{S}} \mathcal{T}_\gamma G_\Omega^{\text{A}} + \frac{\partial^2 G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i \partial q_j} \mathbf{v} G_\Omega^{\text{R}} \mathcal{T}_\gamma G_\Omega^{\text{S}} + \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i} \mathbf{v} \frac{\partial G_{\Omega,\mathbf{q}}^{\text{S}}}{\partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{A}} \\
& + \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i} \mathbf{v} \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{S}} + \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_j} \mathbf{v} \frac{\partial G_{\Omega,\mathbf{q}}^{\text{S}}}{\partial q_i} \mathcal{T}_\gamma G_\Omega^{\text{A}} + \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_j} \mathbf{v} \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i} \mathcal{T}_\gamma G_\Omega^{\text{S}} + G_\Omega^{\text{R}} \mathbf{v} \frac{\partial^2 G_{\Omega,\mathbf{q}}^{\text{S}}}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{A}} \\
& + G_\Omega^{\text{R}} \mathbf{v} \frac{\partial^2 G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{S}} + \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i} \frac{\partial \mathbf{v}}{\partial q_j} G_\Omega^{\text{S}} \mathcal{T}_\gamma G_\Omega^{\text{A}} + \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_j} \frac{\partial \mathbf{v}}{\partial q_i} G_\Omega^{\text{S}} \mathcal{T}_\gamma G_\Omega^{\text{A}} + G_\Omega^{\text{R}} \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial G_{\Omega,\mathbf{q}}^{\text{S}}}{\partial q_i} \mathcal{T}_\gamma G_\Omega^{\text{A}} \\
& + G_\Omega^{\text{R}} \frac{\partial \mathbf{v}}{\partial q_i} \frac{\partial G_{\Omega,\mathbf{q}}^{\text{S}}}{\partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{A}} + \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i} \frac{\partial \mathbf{v}}{\partial q_j} G_\Omega^{\text{R}} \mathcal{T}_\gamma G_\Omega^{\text{S}} + \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_j} \frac{\partial \mathbf{v}}{\partial q_i} G_\Omega^{\text{R}} \mathcal{T}_\gamma G_\Omega^{\text{S}} + G_\Omega^{\text{R}} \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_i} \mathcal{T}_\gamma G_\Omega^{\text{S}} \\
& + G_\Omega^{\text{R}} \frac{\partial \mathbf{v}}{\partial q_i} \frac{\partial G_{\Omega,\mathbf{q}}^{\text{R}}}{\partial q_j} \mathcal{T}_\gamma G_\Omega^{\text{S}} \Bigg],
\end{aligned} \tag{27}
$$

in case 2, and

$$\hat{G}_{3,3}^{<,\text{I},(0,2)}(u,u') = -u\omega e\mathbf{E}_0 \cdot \sum_{ij} q_i q_j \sum_{\gamma=x,y} \int d\Omega f'(\hbar\Omega) G_\Omega^R \mathcal{T}_\gamma \frac{\partial^2 G_{\Omega,\mathbf{q}}^R}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^S \frac{1}{4\pi} \frac{\eta^2}{\hbar^2}, \tag{28}$$

in case 3.

These lesser-one Green's functions depend on the magnons through η^2 and through $q_i q_j$. Consequently, we use the integral:

$$I_{ij}^{(0,2)}(T) = \int \frac{d^2q}{(2\pi)^2} q_i q_j F(\mathbf{q}, T), \tag{29}$$

to average over the magnon distribution. This integral is discussed below in Sec. II B. Using Eqs. (14) and (16) and summing over the magnon modes, we obtain the torque $\mathbf{T}_{\text{mag}}^{\text{I},(0,2)}$ from the lesser-one Green's functions in Eqs. (26)–(28). The explicit expression is given in Eq. (A5) in Appendix A. Similarly, we obtain the torque Eq. (A6) from the lesser-two Green's function.

We introduce the torkance tensors $t_{\text{mag},ij}^{I,(J,K)}$ [7] so that

$$\mathbf{T}_{\text{mag}}^{I,(J,K)} = \sum_{i=1}^{3} \sum_{j=1}^{2} \hat{\mathbf{e}}_i t_{\text{mag},ij}^{I,(J,K)} E_{0,j}, \tag{30}$$

where $\mathbf{E}_0 = (E_{0,x}, E_{0,y}, 0)$ is the applied in-plane electric field (applied in the plane of the magnetic bilayer, therefore no z component), $\hat{\mathbf{e}}_i$ is the unit vector along the ith Cartesian direction, $I = \text{I}, \text{II}, \text{III}, \text{IV}, J = 0, 2,$ and $K = 0, 2$. When periodic boundary conditions are used, the Green's functions depend on a k-point, which we suppress for notational simplicity. In this case, an additional k integration is necessary, i.e., we use

$$t_{\text{mag},ij}^{\text{tot}} = \int \frac{d^2k}{(2\pi)^2} \left[t_{\text{mag},ij}^{\text{I},(0,0)} + t_{\text{mag},ij}^{\text{II},(0,0)} + t_{\text{mag},ij}^{\text{I},(2,0)} + t_{\text{mag},ij}^{\text{II},(2,0)} \right.$$
$$\left. + t_{\text{mag},ij}^{\text{III},(2,0)} + t_{\text{mag},ij}^{\text{IV},(2,0)} + t_{\text{mag},ij}^{\text{I},(0,2)} + t_{\text{mag},ij}^{\text{II},(0,2)} \right], \tag{31}$$

to obtain the total torkance.

B. Integrals over magnon modes

In the previous subsection, we introduced integrals over magnon modes in Eqs. (20), (24), and (29). To evaluate these integrals, we assume that the magnon dispersion is given by

$$\omega_{\text{mag}}(\mathbf{q}) = \mathcal{A}q^2 + \mathcal{C}, \tag{32}$$

where $\mathcal{A}$ is the spin-wave stiffness, and $\mathcal{C}$ is the spin-wave gap. In principle, the $\mathbf{q}$ integrals should be restricted to the first Brillouin zone in $\mathbf{q}$ space, the volume of which is reciprocal to A_{mag}. However, for the examples considered here, one introduces only a small error by waiving the restriction to the first Brillouin zone and integrating instead over the full $\mathbf{q}$ space. Therefore, we integrate in the following over the full $\mathbf{q}$ space, which has the advantage that the integrals are given then by analytical expressions.

The first integral is

$$I^{(0,0)}(T) = \int \frac{d^2q}{(2\pi)^2} F[\omega_{\text{mag}}(\mathbf{q}), T]$$
$$= \frac{1}{2\pi} \int \frac{q\,dq}{\exp\left[\frac{\hbar\omega_{\text{mag}}(q)}{k_B T}\right] - 1}$$
$$= \frac{1}{4\pi\mathcal{A}} \int_{\mathcal{C}}^{\infty} \frac{d\omega_{\text{mag}}}{\exp\left(\frac{\hbar\omega_{\text{mag}}}{k_B T}\right) - 1}$$
$$= \frac{1}{4\pi\mathcal{A}} \frac{k_B T}{\hbar} \int_{\hbar\mathcal{C}/(k_B T)}^{\infty} \frac{d\xi}{e^\xi - 1}$$
$$= \frac{1}{4\pi\mathcal{A}} \frac{k_B T}{\hbar} \left\{ \frac{\hbar\mathcal{C}}{k_B T} - \log\left[\exp\left(\frac{\hbar\mathcal{C}}{k_B T}\right) - 1\right] \right\}, \tag{33}$$

which diverges when the magnon gap $\mathcal{C}$ goes to zero. Here, k_B is the Boltzmann constant.

The second integral is (assuming $\mathcal{C} = 0$)

$$I^{(2,0)}(T) = \int \frac{d^2q}{(2\pi)^2} [\hbar\omega_{\text{mag}}(\mathbf{q})]^2 F[\omega_{\text{mag}}(\mathbf{q}), T]$$
$$= \frac{\hbar^2}{2\pi\mathcal{A}} \left[\frac{k_B T}{\hbar}\right]^3 \zeta(3), \tag{34}$$

where ζ denotes the ζ function, i.e., $\zeta(3) \approx 1.202$.

For the isotropic dispersion of Eq. (32), the third integrals satisfy $I_{xx}^{(0,2)}(T) = I_{yy}^{(0,2)}(T) = I^{(0,2)}(T)/2$, with (assuming $\mathcal{C} = 0$)

$$I^{(0,2)}(T) = \int \frac{d^2q}{(2\pi)^2} q^2 F(\mathbf{q}, T)$$
$$= \frac{\pi}{24\mathcal{A}^2} \left[\frac{k_B T}{\hbar}\right]^2. \tag{35}$$

In Table I, we list the values of these integrals for various ferromagnets. For the spin-wave stiffnesses, we took bulk values from the literature [24–26]. Here, A_{mag} is the in-plane area of the unit cell per magnetic atom. These areas are $A_{\text{mag}} = 4.109\,\text{Å}^2$ in the case of Fe, $A_{\text{mag}} = 2.723\,\text{Å}^2$ in the case of Co, and $A_{\text{mag}} = 3.107\,\text{Å}^2$ in the case of Ni. The data

TABLE I. Integrals $I^{(0,0)}(T)A_{\text{mag}}$, $I^{(2,0)}(T)A_{\text{mag}}$, and $I^{(0,2)}(T)A_{\text{mag}}$ at temperature $T = 300\,\text{K}$ for various ferromagnets. In the case of Mn-13K, the temperature is $T = 13\,\text{K}$.

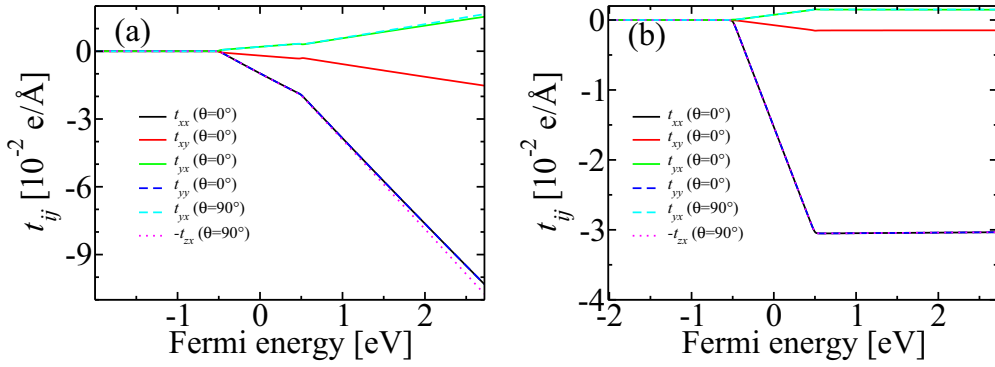|	$\mathcal{A}$ (meVÅ^2)	$I^{(0,0)}A_{\text{mag}}$	$I^{(2,0)}A_{\text{mag}}$ (eV^2)	$I^{(0,2)}A_{\text{mag}}$ (Å^{-2})
Fe	307	0.153	4.424×10^{-5}	3.814×10^{-3}
Co	539	5.78×10^{-2}	1.67×10^{-5}	8.20×10^{-4}
Ni	433	8.203×10^{-2}	2.372×10^{-5}	1.450×10^{-3}
Mn-13K	56	3.923×10^{-2}	4.808×10^{-8}	5.246×10^{-4}

FIG. 1. (a) Magnonic spin-orbit torque (SOT) and (b) nonmagnonic SOT for $\alpha^R = 72$ meV Å and $\Gamma = 25$ meV.

for Mn correspond to a monolayer of Mn on W(001) [27] with $A_{\text{mag}} = 10.018$ Å^2. For the first integral $I^{(0,0)}$, we used a magnon gap of 0.1 meV, which ensures convergence, while the values of the second and third integrals are almost not affected by this small gap of 0.1 meV, and therefore, their values are almost identical to the analytical expressions above with $\mathcal{C} = 0$. Since the scanning-tunneling microscopy (STM) experiments on Mn/W(001) were performed at $T = 13$ K, we set the temperature in the integrals to $T = 13$ K (Mn-13K). As the spin-wave stiffness of the Mn monolayer is much smaller than the spin-wave stiffnesses of Fe, Co, and Ni, the integrals $I^{(0,0)}$ and $I^{(0,2)}$ in Mn at $T = 13$ K are similar in size to the ones of Fe, Co, and Ni at $T = 300$ K.

C. Dependence on temperature

Putting together the results from the previous two subsections, we find that the three magnonic contributions to the SOT exhibit the following scaling behavior with respect to temperature T and spin-wave stiffness $\mathcal{A}$:

$$t^{(0,0)}_{\text{mag},ij} \propto \frac{T}{\mathcal{A}} \left\{ \frac{\hbar \mathcal{C}}{k_B T} - \log \left[\exp \left(\frac{\hbar \mathcal{C}}{k_B T} \right) - 1 \right] \right\}, \quad (36)$$

$$t^{(2,0)}_{\text{mag},ij} \propto \frac{T^3}{\mathcal{A}}, \quad (37)$$

and

$$t^{(0,2)}_{\text{mag},ij} \propto \frac{T^2}{\mathcal{A}^2}. \quad (38)$$

In the ferromagnetic Rashba model, $t^{(0,2)}_{\text{mag},ij}$ is the dominant contribution. It depends quadratically on the temperature. A scaling $\propto T^{d/2+1}$, where d is the dimensionality of the system, has also been found for the spin-wave-induced correction to the conductivity of ferromagnets [17]. This strong temperature dependence resembles the one measured in experiments [9–12].

Even though the relaxation time τ depends on temperature through phonon and magnon scattering, we do not express the relaxation time in terms of the temperature here because interfacial disorder is expected to provide major scattering channels as well in magnetic bilayers. Therefore, we treat temperature and relaxation time τ as independent parameters because the latter can be controlled independently of temperature by tuning the disorder in the system.

Spin disorder usually increases the electrical resistivity [18,19] due to the additional scattering channels, which may be described effectively by a simple reduction of the relaxation time. In contrast, the magnonic SOT discussed here cannot simply be accounted for by this reduction of the relaxation time.

D. Generalizations of the formalism to treat the anisotropy of SOT

In Sec. II A, we assumed that the magnetization is oriented in the z direction. To compute the anisotropy of the SOT, it is necessary to generalize this for general magnetization directions. It is effective to express the magnetization direction in
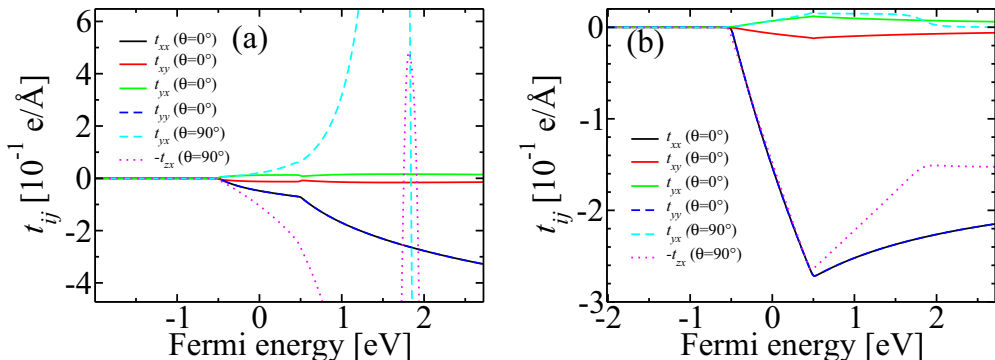

FIG. 2. (a) Magnonic spin-orbit torque (SOT) and (b) nonmagnonic SOT for $\alpha^R = 720$ meV Å and $\Gamma = 25$ meV.
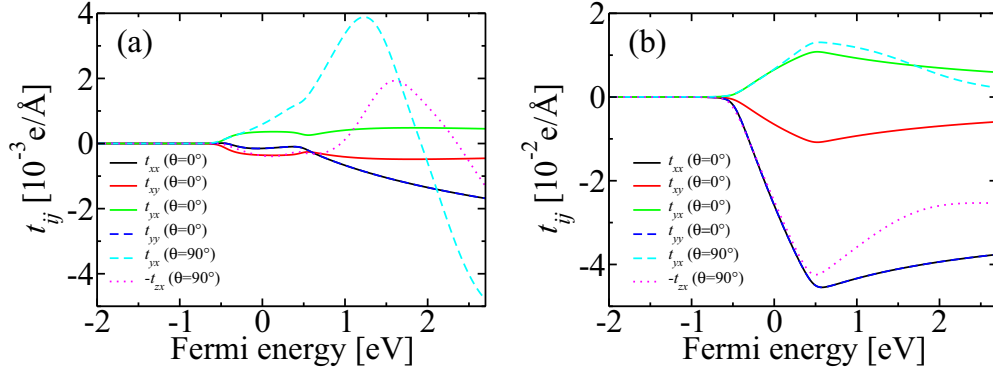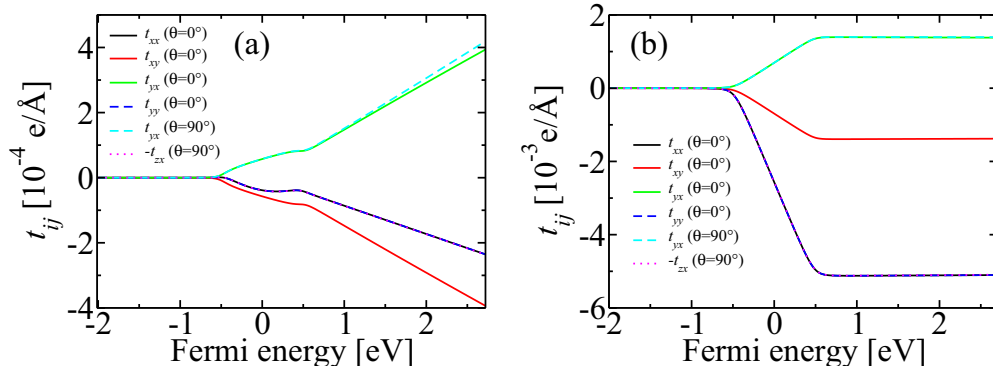
FIG. 3. (a) Magnonic spin-orbit torque (SOT) and (b) nonmagnonic SOT for $\alpha^R = 720$ meV Å and $\Gamma = 136$ meV.

spherical coordinates:

$$\hat{\mathbf{M}} = \begin{bmatrix} \sin(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\theta) \end{bmatrix}. \qquad (39)$$

To discuss the anisotropy of the SOT, it is convenient to project the torques onto the unit vectors $\hat{\mathbf{e}}_\theta = \partial\hat{\mathbf{M}}/\partial\theta$ and $\hat{\mathbf{e}}_\phi = \partial\hat{\mathbf{M}}/\partial\phi/\sin(\theta)$ of the spherical coordinate system because the torques are perpendicular to the magnetization [3].

Equations (21), (25), and (A1)–(A6) become valid for general magnetization direction if the following replacement is made:

$$\sum_{\gamma=x,y} \rightarrow \sum_{\gamma=\theta,\phi}, \qquad (40)$$

where

$$\mathcal{T}_\theta = \sum_{\gamma=x,y,z} \hat{\mathbf{e}}_\theta \cdot \hat{\mathbf{e}}_\gamma \mathcal{T}_\gamma, \qquad (41)$$

and

$$\mathcal{T}_\phi = \sum_{\gamma=x,y,z} \hat{\mathbf{e}}_\phi \cdot \hat{\mathbf{e}}_\gamma \mathcal{T}_\gamma. \qquad (42)$$

III. MAGNONIC SOT IN THE FERROMAGNETIC RASHBA MODEL

In this section, we study the magnonic SOT numerically in the ferromagnetic Rashba model [28]:

$$H_{\mathbf{k}} = \frac{\hbar^2}{2m^*}k^2 + \alpha^R(\mathbf{k} \times \hat{\mathbf{e}}_z) \cdot \sigma + \frac{\Delta V}{2}\sigma \cdot \hat{\mathbf{M}}, \qquad (43)$$

where α^R is the Rashba parameter, $\hat{\mathbf{M}}$ is the magnetization direction, and ΔV is the exchange splitting. We set the mass m^* to the electron mass and the exchange splitting to $\Delta V = 1$ eV. We use the expressions given in Sec. II A and in Appendix A to compute the magnonic SOT. For the integrals $I^{(0,0)}A_{\mathrm{mag}}$, $I^{(2,0)}A_{\mathrm{mag}}$, and $I^{(0,2)}A_{\mathrm{mag}}$, in these expressions, we take the values provided in Table I for the case of Co. We introduce a broadening parameter Γ, which may be used to model the effect of disorder, i.e., we use

$$G_\Omega^R = \hbar[\hbar\Omega - H_{\mathbf{k}} + i\Gamma]^{-1}, \qquad (44)$$

$$G_\Omega^A = \hbar[\hbar\Omega - H_{\mathbf{k}} - i\Gamma]^{-1}, \qquad (45)$$

and $G_\Omega^S = G_\Omega^A - G_\Omega^R$ for the Green's functions in Eqs. (21), (25), and (A1)–(A6). To compare the magnonic SOT to the nonmagnonic one, we also compute the nonmagnonic SOT according to the equations in Ref. [7].

In Fig. 1, we show the SOTs when the Rashba parameter and the broadening are $\alpha^R = 72$ meV Å and $\Gamma = 25$ meV, respectively. The magnonic SOT shown in Fig. 1(a) is larger than the nonmagnonic one shown in Fig. 1(b) for this choice

FIG. 4. (a) Magnonic spin-orbit torque (SOT) and (b) nonmagnonic SOT for $\alpha^R = 72$ meV Å and $\Gamma = 136$ meV.
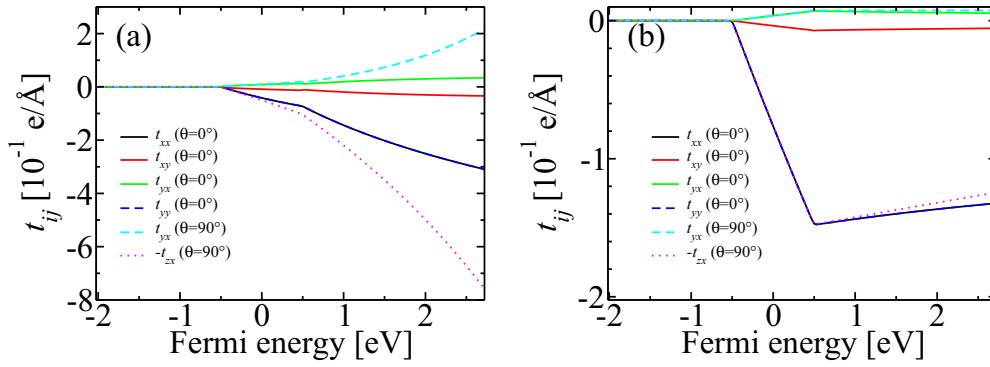
FIG. 5. (a) Magnonic spin-orbit torque (SOT) and (b) nonmagnonic SOT for $\alpha^{R} = 360$ meV Å and $\Gamma = 25$ meV.
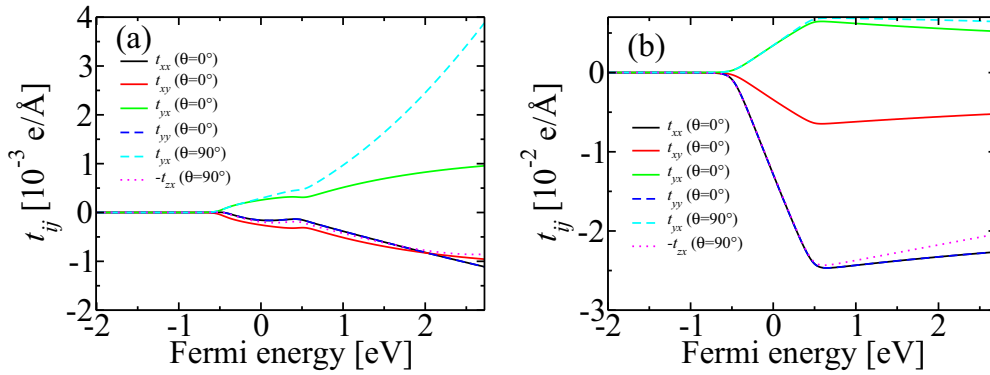
FIG. 6. (a) Magnonic spin-orbit torque (SOT) and (b) nonmagnonic SOT for $\alpha^{R} = 360$ meV Å and $\Gamma = 136$ meV.
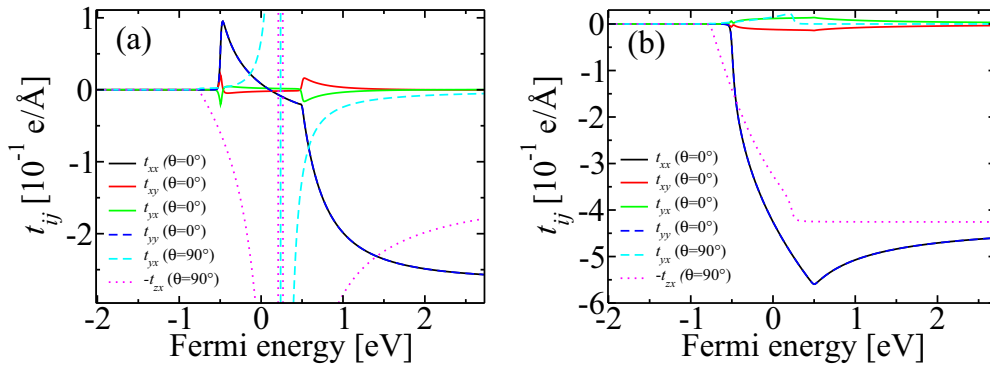
FIG. 7. (a) Magnonic spin-orbit torque (SOT) and (b) nonmagnonic SOT for $\alpha^{R} = 2$ eV Å and $\Gamma = 25$ meV.
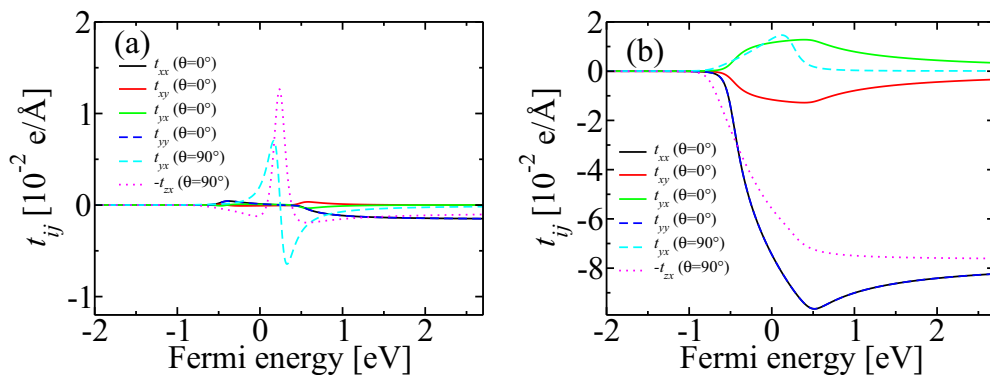
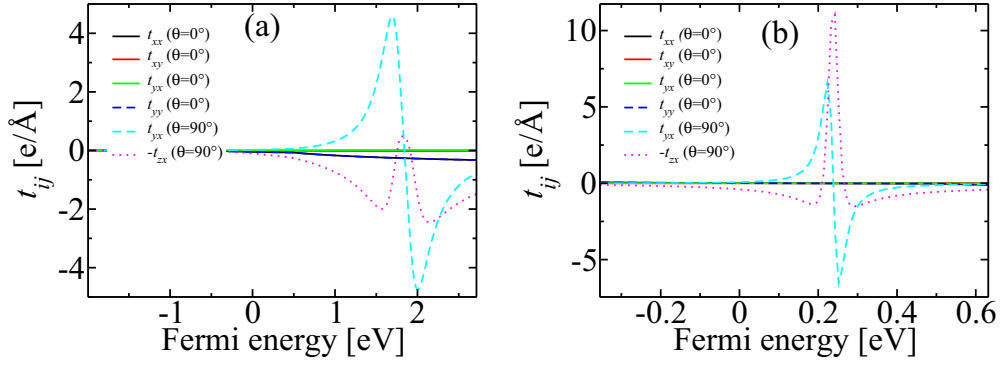FIG. 8. (a) Magnonic spin-orbit torque (SOT) and (b) nonmagnonic SOT for $\alpha^{R} = 2$ eV Å and $\Gamma = 136$ meV.

FIG. 9. Magnonic spin-orbit torque (SOT) for (a) $\alpha^R = 720$ meV Å and $\Gamma = 25$ meV and (b) $\alpha^R = 2$ eV Å and $\Gamma = 25$ meV.

of parameters. To study the anisotropy of the SOT, we show the torkances for $\theta = \phi = 0$, i.e., $\hat{\mathbf{M}}$ along $\hat{\mathbf{e}}_z$, and for $\theta = 90°$, $\phi = 0$, i.e., $\hat{\mathbf{M}}$ along $\hat{\mathbf{e}}_x$. We call the SOT anisotropic if $t_{xx}(\theta = 0°) = -t_{zx}(\theta = 90°)$ or $t_{yx}(\theta = 0°) = t_{yx}(\theta = 90°)$ are not satisfied. While the magnonic SOT shows a small anisotropy, the anisotropy of the nonmagnonic SOT is invisible to the eye.

In Fig. 2, we show the SOTs when the Rashba parameter and the broadening are $\alpha^R = 720$ meV Å and $\Gamma = 25$ meV, respectively. Since the magnonic SOT is much larger for $\theta = 90°$ than it is for $\theta = 0°$, it is out of scale for several ranges of the Fermi energy in Fig. 2(a). We show the full range of the magnonic SOT at $\theta = 90°$ in Fig. 9(a). The nonmagnonic SOT shown in Fig. 2(b) is rather isotropic up to the Fermi energy 0.5 eV, where its anisotropy starts to become significant. In contrast, for the magnonic SOT in Fig. 2(a), the relation $t_{yx}(\theta = 0°) = t_{yx}(\theta = 90°)$ is satisfied approximately only up to the Fermi energy of 0 eV, where its anisotropy starts to increase rapidly. The relation $t_{xx}(\theta = 0°) = -t_{zx}(\theta = 90°)$ is satisfied approximately only for very small Fermi energies up to -0.3 eV.

In Fig. 3, we show the SOTs at the same Rashba parameter $\alpha^R = 720$ meV Å but at a larger broadening of $\Gamma = 136$ meV. In agreement with the expectation [7] for the nonmagnonic torque, we find that $t_{xy} \propto \Gamma^0$, $t_{yx} \propto \Gamma^0$, $t_{xx} \propto \Gamma^{-1}$, $t_{yy} \propto \Gamma^{-1}$, and $t_{zx} \propto \Gamma^{-1}$ are approximately satisfied when we compare Figs. 3(b) and 2(b). In contrast, the magnonic SOT depends much stronger on Γ, and it is roughly one order of magnitude smaller than the nonmagnonic one at this value of the broadening of $\Gamma = 136$ meV.

In Appendix B, we provide the plots of the SOT for several additional choices of parameters, which confirms the trends that we discussed above using three examples. In general, we find that the magnonic torque is sizable in comparison with the nonmagnonic one if the broadening parameter Γ is small, i.e., when the disorder is small. Additionally, we find that the anisotropy of the magnonic

SOT may become gigantic if the Rashba parameter is large.

IV. CONCLUSIONS

Using third-order perturbation theory within the framework of the Keldysh nonequilibrium formalism, we derive suitable equations to assess the magnonic contributions to the SOT. In comparison with the purely electronic SOT, its magnonic counterpart depends more strongly on the temperature. We distinguish several contributions to the magnonic SOT, which depend differently on the spin-wave stiffness $\mathcal{A}$ and the temperature T. The dominating contribution scales like $T^2/\mathcal{A}^2$, which leads to a strong temperature dependence of the magnonic contribution to the SOT, in agreement with experimental observations. We compute the magnonic SOT in the ferromagnetic Rashba model. It exhibits a strong anisotropy when the Rashba parameter is large, and it becomes larger than the nonmagnonic SOT when the quasiparticle broadening becomes small. Since the magnonic SOT is sizable in comparison with its purely electronic counterpart, magnons may therefore explain both the strong temperature dependence and the anisotropy of the SOT found in some experiments.

APPENDIX A: ADDITIONAL CONTRIBUTIONS

The torque from the lesser-two Green's function at the zeroth order in ω_{mag} and $\mathbf{q}$ is given by

$$\mathbf{T}_{\text{mag}}^{\text{II},(0,0)} = \frac{A_{\text{mag}} I^{(0,0)}(T)}{4\pi\hbar^3} \int d\Omega \sum_{\gamma=x,y} f(\hbar\Omega)$$

$$\times \text{Tr}\{\mathcal{T}[G_\Omega^A \mathcal{T}_\gamma G_\Omega^A \mathcal{T}_\gamma G_\Omega^A \mathbf{v} G_\Omega^A G_\Omega^A + G_\Omega^A \mathcal{T}_\gamma G_\Omega^A \mathbf{v} G_\Omega^A \mathcal{T}_\gamma G_\Omega^A G_\Omega^A + G_\Omega^A \mathcal{T}_\gamma G_\Omega^A \mathbf{v} G_\Omega^A G_\Omega^A \mathcal{T}_\gamma G_\Omega^A$$

$$+ G_\Omega^A \mathbf{v} G_\Omega^A \mathcal{T}_\gamma G_\Omega^A \mathcal{T}_\gamma G_\Omega^A G_\Omega^A + G_\Omega^A \mathbf{v} G_\Omega^A \mathcal{T}_\gamma G_\Omega^A G_\Omega^A \mathcal{T}_\gamma G_\Omega^A + G_\Omega^A \mathbf{v} G_\Omega^A G_\Omega^A \mathcal{T}_\gamma G_\Omega^A \mathcal{T}_\gamma G_\Omega^A$$

$$
\begin{aligned}
&- G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} G^R_\Omega G^R_\Omega - G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega G^R_\Omega - G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} G^R_\Omega G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \\
&- G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathcal{T}_\gamma G^R_\Omega G^R_\Omega - G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega G^R_\Omega \mathcal{T}_\gamma G^R_\Omega - G^R_\Omega \mathbf{v} G^R_\Omega G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \big] \cdot \mathbf{E}_0 e \big\}.
\end{aligned} \tag{A1}
$$

The torque from the lesser-two Green's function at the second order in ω_{mag} and at the zeroth order in $\mathbf{q}$ is given by

$$
\mathbf{T}^{\mathrm{II},(2,0)}_{\mathrm{mag}} = \frac{A_{\mathrm{mag}} I^{(2,0)}(T)}{4\pi \hbar^5} \int d\Omega \sum_{\gamma=x,y} f(\hbar\Omega)
$$

$$
\begin{aligned}
\times \operatorname{Tr}\bigg\{ \mathcal{T}\bigg[& G^R_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^R_\Omega + G^R_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^R_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \\
& + G^R_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^R_\Omega + G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} \frac{\partial^2}{\partial\Omega^2} G^R_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^R_\Omega + G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} \frac{\partial^3}{\partial\Omega^3} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \\
& + G^R_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^R_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega + G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^R_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^R_\Omega + G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma \frac{\partial^3}{\partial\Omega^3} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \\
& - G^A_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^A_\Omega \mathcal{T}_\gamma G^A_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^A_\Omega - G^A_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^A_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^A_\Omega \mathcal{T}_\gamma G^A_\Omega - G^A_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^A_\Omega \mathbf{v} G^A_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega \\
& - G^A_\Omega \mathcal{T}_\gamma G^A_\Omega \mathbf{v} \frac{\partial^2}{\partial\Omega^2} G^A_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega - G^A_\Omega \mathcal{T}_\gamma G^A_\Omega \mathbf{v} \frac{\partial^3}{\partial\Omega^3} G^A_\Omega \mathcal{T}_\gamma G^A_\Omega - G^A_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^A_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^A_\Omega \mathcal{T}_\gamma G^A_\Omega \\
& - G^A_\Omega \mathbf{v} G^A_\Omega \mathcal{T}_\gamma \frac{\partial^2}{\partial\Omega^2} G^A_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega - G^A_\Omega \mathbf{v} G^A_\Omega \mathcal{T}_\gamma \frac{\partial^3}{\partial\Omega^3} G^A_\Omega \mathcal{T}_\gamma G^A_\Omega - 2 G^A_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^A_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega \\
& - 2 G^A_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega \mathbf{v} \frac{\partial^2}{\partial\Omega^2} G^A_\Omega \mathcal{T}_\gamma G^A_\Omega + 2 G^R_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^R_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^R_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^R_\Omega \\
& + 2 G^R_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^R_\Omega \mathbf{v} \frac{\partial^2}{\partial\Omega^2} G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \bigg] \bigg\} \cdot \mathbf{E}_0 e.
\end{aligned} \tag{A2}
$$

Additionally, there are the following torques from the lesser-three and lesser-four Green's functions:

$$
\mathbf{T}^{\mathrm{III},(2,0)}_{\mathrm{mag}} = -\frac{A_{\mathrm{mag}} I^{(2,0)}(T)}{4\pi \hbar^3} \int d\Omega \sum_{\gamma=x,y} f''(\hbar\Omega)
$$

$$
\begin{aligned}
\times \operatorname{Tr}\bigg\{ \mathcal{T}\bigg[& G^R_\Omega \mathcal{T}_\gamma G^S_\Omega \mathcal{T}_\gamma G^A_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^A_\Omega + G^R_\Omega \mathcal{T}_\gamma G^S_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^A_\Omega \mathcal{T}_\gamma G^A_\Omega + G^R_\Omega \mathcal{T}_\gamma G^S_\Omega \mathbf{v} G^A_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega + G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} G^S_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega \\
& + G^R_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^R_\Omega \mathcal{T}_\gamma G^S_\Omega \mathcal{T}_\gamma G^A_\Omega + G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma G^S_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^A_\Omega + 3 G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} \frac{\partial}{\partial\Omega} G^S_\Omega \mathcal{T}_\gamma G^A_\Omega \\
& + 3 G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^S_\Omega \mathcal{T}_\gamma G^A_\Omega + 2 G^R_\Omega \mathcal{T}_\gamma \frac{\partial}{\partial\Omega} G^R_\Omega \mathbf{v} G^S_\Omega \mathcal{T}_\gamma G^A_\Omega \bigg] \bigg\} \cdot \mathbf{E}_0 e,
\end{aligned} \tag{A3}
$$

and

$$
\mathbf{T}^{\mathrm{IV},(2,0)}_{\mathrm{mag}} = -\frac{A_{\mathrm{mag}} I^{(2,0)}(T)}{4\pi \hbar^2} \int d\Omega \sum_{\gamma=x,y} f'''(\hbar\Omega) \operatorname{Tr}\big\{ \mathcal{T}\big[G^R_\Omega \mathcal{T}_\gamma G^R_\Omega \mathbf{v} G^S_\Omega \mathcal{T}_\gamma G^A_\Omega + G^R_\Omega \mathbf{v} G^R_\Omega \mathcal{T}_\gamma G^S_\Omega \mathcal{T}_\gamma G^A_\Omega \big] \big\} \cdot \mathbf{E}_0 e. \tag{A4}
$$

The torque from the lesser-one Green's function at the second order in q and zeroth order in ω_{mag} is given by

$$
\mathbf{T}^{\mathrm{I},(0,2)}_{\mathrm{mag}} = -\sum_{ij} \frac{e A_{\mathrm{mag}} I^{(0,2)}_{ij}(T)}{4\pi \hbar^2} \int d\Omega
$$

$$
\begin{aligned}
\times \sum_{\gamma=x,y} \operatorname{Tr}\bigg\{ \mathcal{T} f'(\hbar\Omega) G^R_\Omega \mathbf{E}_0 \cdot \bigg[& \mathbf{v} G^S_\Omega \mathcal{T}_\gamma \frac{\partial^2 G^A_{\Omega,\mathbf{q}}}{\partial q_i \partial q_j} \mathcal{T}_\gamma G^A_\Omega + \mathbf{v} G^R_\Omega \mathcal{T}_\gamma \frac{\partial^2 G^S_{\Omega,\mathbf{q}}}{\partial q_i \partial q_j} \mathcal{T}_\gamma G^A_\Omega + \mathbf{v} G^R_\Omega \mathcal{T}_\gamma \frac{\partial^2 G^R_{\Omega,\mathbf{q}}}{\partial q_i \partial q_j} \mathcal{T}_\gamma G^S_\Omega \\
& + \mathcal{T}_\gamma \frac{\partial^2 G^R_{\Omega,\mathbf{q}}}{\partial q_i \partial q_j} \mathbf{v} G^S_\Omega \mathcal{T}_\gamma G^A_\Omega + \mathcal{T}_\gamma \frac{\partial^2 G^R_{\Omega,\mathbf{q}}}{\partial q_i \partial q_j} \mathbf{v} G^R_\Omega \mathcal{T}_\gamma G^S_\Omega + \mathcal{T}_\gamma \frac{\partial G^R_{\Omega,\mathbf{q}}}{\partial q_i} \mathbf{v} \frac{\partial G^S_{\Omega,\mathbf{q}}}{\partial q_j} \mathcal{T}_\gamma G^A_\Omega \\
& + \mathcal{T}_\gamma \frac{\partial G^R_{\Omega,\mathbf{q}}}{\partial q_i} \mathbf{v} \frac{\partial G^R_{\Omega,\mathbf{q}}}{\partial q_j} \mathcal{T}_\gamma G^S_\Omega + \mathcal{T}_\gamma \frac{\partial G^R_{\Omega,\mathbf{q}}}{\partial q_j} \mathbf{v} \frac{\partial G^S_{\Omega,\mathbf{q}}}{\partial q_i} \mathcal{T}_\gamma G^A_\Omega + \mathcal{T}_\gamma \frac{\partial G^R_{\Omega,\mathbf{q}}}{\partial q_j} \mathbf{v} \frac{\partial G^R_{\Omega,\mathbf{q}}}{\partial q_i} \mathcal{T}_\gamma G^S_\Omega
\end{aligned}
$$

$$
+ \mathcal{T}_\gamma G_\Omega^R \mathbf{v} \frac{\partial^2 G_{\Omega,\mathbf{q}}^S}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^A + \mathcal{T}_\gamma G_\Omega^R \mathbf{v} \frac{\partial^2 G_{\Omega,\mathbf{q}}^R}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^S + \mathcal{T}_\gamma \frac{\partial G_{\Omega,\mathbf{q}}^R}{\partial q_i} \frac{\partial \mathbf{v}}{\partial q_j} G_\Omega^S \mathcal{T}_\gamma G_\Omega^A
$$

$$
+ \mathcal{T}_\gamma \frac{\partial G_{\Omega,\mathbf{q}}^R}{\partial q_j} \frac{\partial \mathbf{v}}{\partial q_i} G_\Omega^S \mathcal{T}_\gamma G_\Omega^A + \mathcal{T}_\gamma G_\Omega^R \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial G_{\Omega,\mathbf{q}}^S}{\partial q_i} \mathcal{T}_\gamma G_\Omega^A + \mathcal{T}_\gamma G_\Omega^R \frac{\partial \mathbf{v}}{\partial q_i} \frac{\partial G_{\Omega,\mathbf{q}}^S}{\partial q_j} \mathcal{T}_\gamma G_\Omega^A
$$

$$
+ \mathcal{T}_\gamma \frac{\partial G_{\Omega,\mathbf{q}}^R}{\partial q_i} \frac{\partial \mathbf{v}}{\partial q_j} G_\Omega^R \mathcal{T}_\gamma G_\Omega^S + \mathcal{T}_\gamma \frac{\partial G_{\Omega,\mathbf{q}}^R}{\partial q_j} \frac{\partial \mathbf{v}}{\partial q_i} G_\Omega^R \mathcal{T}_\gamma G_\Omega^S + \mathcal{T}_\gamma G_\Omega^R \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial G_{\Omega,\mathbf{q}}^R}{\partial q_i} \mathcal{T}_\gamma G_\Omega^S
$$

$$
+ \mathcal{T}_\gamma G_\Omega^R \frac{\partial \mathbf{v}}{\partial q_i} \frac{\partial G_{\Omega,\mathbf{q}}^R}{\partial q_j} \mathcal{T}_\gamma G_\Omega^S + \mathcal{T}_\gamma \frac{\partial^2 G_{\Omega,\mathbf{q}}^R}{\partial q_i \partial q_j} \mathcal{T}_\gamma G_\Omega^R \mathbf{v} G_\Omega^S \Bigg] \Bigg\}.
\tag{A5}
$$

Additionally, we obtain the following Fermi sea contribution:

$$
\mathbf{T}_{\text{mag}}^{\text{II},(0,2)} = \sum_{ij} \frac{A_{\text{mag}} I_{ij}^{(0,2)}(T)}{4\pi \hbar^3} \int d\Omega
$$

$$
\times \sum_{\gamma=x,y} \text{Tr} \Bigg\{ \boldsymbol{\mathcal{T}} f(\hbar\Omega) \Bigg[G_\Omega^R \mathbf{v} \frac{\partial}{\partial \Omega} G_\Omega^R \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^R + G_\Omega^R \mathbf{v} G_\Omega^R \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^R
$$

$$
+ G_\Omega^R \mathbf{v} G_\Omega^R \mathcal{T}_\gamma \frac{\partial^3}{\partial q_i \partial q_j \partial \Omega} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^R - G_\Omega^A \mathbf{v} \frac{\partial}{\partial \Omega} G_\Omega^A \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^A
$$

$$
- G_\Omega^A \mathbf{v} G_\Omega^A \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^A - G_\Omega^A \mathbf{v} G_\Omega^A \mathcal{T}_\gamma \frac{\partial^3}{\partial q_i \partial q_j \partial \Omega} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^A
$$

$$
+ G_\Omega^R \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^R \bigg|_{q=0} \mathbf{v} \frac{\partial}{\partial \Omega} G_\Omega^R(e,0) \mathcal{T}_\gamma G_\Omega^R + G_\Omega^R \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^R \bigg|_{q=0} \mathbf{v} G_\Omega^R(e,0) \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^R
$$

$$
+ G_\Omega^R \mathcal{T}_\gamma G_\Omega^R(e,0) \mathbf{v} \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^R + G_\Omega^R \mathcal{T}_\gamma G_\Omega^R(e,0) \mathbf{v} \frac{\partial^3}{\partial q_i \partial q_j \partial \Omega} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^R
$$

$$
- G_\Omega^A \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^A \bigg|_{q=0} \mathbf{v} \frac{\partial}{\partial \Omega} G_\Omega^A(e,0) \mathcal{T}_\gamma G_\Omega^A - G_\Omega^A \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^A \bigg|_{q=0} \mathbf{v} G_\Omega^A(e,0) \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^A
$$

$$
- G_\Omega^A \mathcal{T}_\gamma G_\Omega^A(e,0) \mathbf{v} \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^A - G_\Omega^A \mathcal{T}_\gamma G_\Omega^A(e,0) \mathbf{v} \frac{\partial^3}{\partial q_i \partial q_j \partial \Omega} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^A
$$

$$
- 2 G_\Omega^A \mathcal{T}_\gamma \frac{\partial}{\partial q_i} G_\Omega^A \bigg|_{q=0} \mathbf{v} \frac{\partial}{\partial q_j} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^A - 2 G_\Omega^A \mathcal{T}_\gamma \frac{\partial}{\partial q_i} G_\Omega^A \bigg|_{q=0} \mathbf{v} \frac{\partial^2}{\partial q_j \partial \Omega} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^A
$$

$$
+ 2 G_\Omega^R \mathcal{T}_\gamma \frac{\partial}{\partial q_i} G_\Omega^R \bigg|_{q=0} \mathbf{v} \frac{\partial}{\partial q_j} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^R + 2 G_\Omega^R \mathcal{T}_\gamma \frac{\partial}{\partial q_i} G_\Omega^R \bigg|_{q=0} \mathbf{v} \frac{\partial^2}{\partial q_j \partial \Omega} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^R
$$

$$
+ G_\Omega^R \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^R \mathbf{v} \frac{\partial}{\partial \Omega} G_\Omega^R - G_\Omega^A \mathcal{T}_\gamma \frac{\partial^2}{\partial q_i \partial q_j} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^A \mathbf{v} \frac{\partial}{\partial \Omega} G_\Omega^A
$$

$$
+ 2 G_\Omega^R \mathcal{T}_\gamma \frac{\partial}{\partial q_i} G_\Omega^R \bigg|_{q=0} \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial}{\partial \Omega} G_\Omega^R \mathcal{T}_\gamma G_\Omega^R + 2 G_\Omega^R \mathcal{T}_\gamma \frac{\partial}{\partial q_i} G_\Omega^R \bigg|_{q=0} \frac{\partial \mathbf{v}}{\partial q_j} G_\Omega^R \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^R
$$

$$
+ 2 G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial}{\partial q_i} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^R + 2 G_\Omega^R \mathcal{T}_\gamma G_\Omega^R \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial^2}{\partial q_i \partial \Omega} G_\Omega^R \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^R
$$

$$
- 2 G_\Omega^A \mathcal{T}_\gamma \frac{\partial}{\partial q_i} G_\Omega^A \bigg|_{q=0} \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial}{\partial \Omega} G_\Omega^A \mathcal{T}_\gamma G_\Omega^A - 2 G_\Omega^A \mathcal{T}_\gamma \frac{\partial}{\partial q_i} G_\Omega^A \bigg|_{q=0} \frac{\partial \mathbf{v}}{\partial q_j} G_\Omega^A \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^A
$$

$$
- 2 G_\Omega^A \mathcal{T}_\gamma G_\Omega^A \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial}{\partial q_i} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma \frac{\partial}{\partial \Omega} G_\Omega^A - 2 G_\Omega^A \mathcal{T}_\gamma G_\Omega^A \frac{\partial \mathbf{v}}{\partial q_j} \frac{\partial^2}{\partial q_i \partial \Omega} G_\Omega^A \bigg|_{q=0} \mathcal{T}_\gamma G_\Omega^A \Bigg] \cdot \mathbf{E}_0 e \Bigg\}.
\tag{A6}
$$

APPENDIX B: ADDITIONAL PLOTS OF THE SOT

In this Appendix, we provide additional plots of the SOT for different Rashba and broadening parameters. In Fig. 4, we show the SOT for the Rashba and broadening parameters of $\alpha^R = 72$ meV Å and $\Gamma = 136$ meV, respectively. In contrast to Fig. 1, the magnonic SOT is roughly an order of magnitude smaller than the nonmagnonic one due to the larger broadening. In Fig. 5, we show the SOTs for the Rashba and broadening parameters of $\alpha^R = 360$ meV Å and $\Gamma = 25$ meV, respectively. The anisotropy of the magnonic SOT is much larger than the one of the nonmagnonic SOT due to the Rashba parameter, which is larger than in Figs. 1 and 4. In Fig. 6, we show the SOTs at the same Rashba parameter but with

a larger broadening of $\Gamma = 136$ meV. In contrast to Fig. 5, where the magnonic SOT is larger than the nonmagnonic one, the magnonic SOT is smaller here due to the larger broadening. In Fig. 7, we show the SOT for the Rashba and broadening parameters of $\alpha^R = 2$ eV Å and $\Gamma = 25$ meV, respectively. The anisotropy of the magnonic SOT is gigantic at this large value of the Rashba parameter, and it is much larger than the anisotropy of the nonmagnonic SOT. When the broadening is increased to 136 meV, the magnonic SOT for $\theta = 90°$ is still sizable in comparison with the nonmagnonic SOT for Fermi energies ~ 0, while it is suppressed otherwise, as shown in Fig. 8. In Fig. 9(a), we replot Fig. 2 with a different scale of the vertical axis to show the full range of the magnonic SOT for $\theta = 90°$. Similarly, we replot Fig. 7(a) in Fig. 9(b).

[1] S. Bhatti, R. Sbiaa, A. Hirohata, H. Ohno, S. Fukami, and S. N. Piramanayagam, Spintronics based random access memory: A review, Mater. Today **20**, 530 (2017).

[2] A. Manchon, J. Železný, I. M. Miron, T. Jungwirth, J. Sinova, A. Thiaville, K. Garello, and P. Gambardella, Current-induced spin-orbit torques in ferromagnetic and antiferromagnetic systems, Rev. Mod. Phys. **91**, 035004 (2019).

[3] K. Garello, I. M. Miron, C. O. Avci, F. Freimuth, Y. Mokrousov, S. Blügel, S. Auffret, O. Boulle, G. Gaudin, and P. Gambardella, Symmetry and magnitude of spin-orbit torques in ferromagnetic heterostructures, Nat. Nanotech. **8**, 587 (2013).

[4] J.-P. Hanke, F. Freimuth, B. Dupé, J. Sinova, M. Kläui, and Y. Mokrousov, Engineering the dynamics of topological spin textures by anisotropic spin-orbit torques, Phys. Rev. B **101**, 014428 (2020).

[5] K. D. Belashchenko, A. A. Kovalev, and M. van Schilfgaarde, First-principles calculation of spin-orbit torque in a Co/Pt bilayer, Phys. Rev. Mater. **3**, 011401(R) (2019).

[6] P. M. Haney, H.-W. Lee, K.-J. Lee, A. Manchon, and M. D. Stiles, Current-induced torques and interfacial spin-orbit coupling, Phys. Rev. B **88**, 214417 (2013).

[7] F. Freimuth, S. Blügel, and Y. Mokrousov, Spin-orbit torques in Co/Pt(111) and Mn/W(001) magnetic bilayers from first principles, Phys. Rev. B **90**, 174423 (2014).

[8] C. Ciccarelli, L. Anderson, V. Tshitoyan, A. J. Ferguson, F. Gerhard, C. Gould, L. W. Molenkamp, J. Gayles, J. Zelezny, L. Smejkal, Z. Yuan, J. Sinova, F. Freimuth, and T. Jungwirth, Room-temperature spin-orbit torque in NiMnSb, Nat. Phys. **12**, 855 (2016).

[9] X. Qiu, P. Deorani, K. Narayanapillai, K.-S. Lee, K.-J. Lee, H.-W. Lee, and H. Yang, Angular and temperature dependence of current induced spin-orbit effective fields in Ta/CoFeB/MgO nanowires, Sci. Rep. **4**, (2014).

[10] J. Kim, J. Sinha, S. Mitani, M. Hayashi, S. Takahashi, S. Maekawa, M. Yamanouchi, and H. Ohno, Anomalous temperature dependence of current-induced torques in CoFeB/MgO heterostructures with Ta-based underlayers, Phys. Rev. B **89**, 174424 (2014).

[11] Y. Ou, C.-F. Pai, S. Shi, D. C. Ralph, and R. A. Buhrman, Origin of fieldlike spin-orbit torques in heavy metal/ferromagnet/oxide thin film heterostructures, Phys. Rev. B **94**, 140414(R) (2016).

[12] D. Li, J. Yun, S. Chen, B. Cui, X. Guo, K. Wu, Y. Zuo, D. Yang, J. Wang, and L. Xi, Joule heating and temperature effects on current-induced magnetization switching in perpendicularly magnetized Pt/Co/C structures, J. Phys. D: Appl. Phys. **51**, 265003 (2018).

[13] F. Mahfouzi and N. Kioussis, First-principles study of the angular dependence of the spin-orbit torque in Pt/Co and Pd/Co bilayers, Phys. Rev. B **97**, 224426 (2018).

[14] T. Kasuya, Electrical resistance of ferromagnetic metals, Prog. Theor. Phys. **16**, 58 (1956).

[15] D. Goodings, Electrical resistivity of ferromagnetic metals at low temperatures, Phys. Rev. **132**, 542 (1963).

[16] R. Misra, A. F. Hebard, K. A. Muttalib, and P. Wölfle, Spin-wave-mediated quantum corrections to the conductivity of thin ferromagnetic films of gadolinium, Phys. Rev. B **79**, 140408(R) (2009).

[17] J. Danon, A. Ricottone, and P. W. Brouwer, Spin-wave-induced correction to the conductivity of ferromagnets, Phys. Rev. B **90**, 024405 (2014).

[18] J. Kudrnovský, V. Drchal, I. Turek, S. Khmelevskyi, J. K. Glasbrenner, and K. D. Belashchenko, Spin-disorder resistivity of ferromagnetic metals from first principles: the disordered-local-moment approach, Phys. Rev. B **86**, 144423 (2012).

[19] A. L. Wysocki, K. D. Belashchenko, J. P. Velev, and M. van Schilfgaarde, Calculations of spin-disorder resistivity from first principles, J. Appl. Phys. **101**, 09G506 (2007).

[20] F. Freimuth, S. Blügel, and Y. Mokrousov, Direct and inverse spin-orbit torques, Phys. Rev. B **92**, 064415 (2015).

[21] F. Freimuth, S. Blügel, and Y. Mokrousov, Berry phase theory of Dzyaloshinskii-Moriya interaction and spin-orbit torques, J. Phys.: Condens. Matter **26**, 104202 (2014).

[22] F. Freimuth, R. Bamler, Y. Mokrousov, and A. Rosch, Phase-space berry phases in chiral magnets: Dzyaloshinskii-Moriya interaction and the charge of skyrmions, Phys. Rev. B **88**, 214409 (2013).

[23] J. Rammer and H. Smith, Quantum field-theoretical methods in transport theory of metals, Rev. Mod. Phys. **58**, 323 (1986).

[24] C. Loong, J. M. Carpenter, J. W. Lynn, R. A. Robinson, and H. A. Mook, Neutron scattering study of the magnetic excitations in ferromagnetic iron at high energy transfers, J. Appl. Phys. **55**, 1895 (1984).

[25] P. Buczek, A. Ernst, and L. M. Sandratskii, Different dimensionality trends in the Landau damping of magnons in iron, cobalt, and nickel: Time-dependent density functional study, Phys. Rev. B **84**, 174418 (2011).

[26] H. A. Mook, R. M. Nicklow, E. D. Thompson, and M. K. Wilkinson, Spin-wave spectrum of nickel metal, J. Appl. Phys. **40**, 1450 (1969).

[27] P. Ferriani, K. von Bergmann, E. Y. Vedmedenko, S. Heinze, M. Bode, M. Heide, G. Bihlmayer, S. Blügel, and R. Wiesendanger, Atomic-Scale Spin Spiral with a Unique Rotational Sense: Mn Monolayer on W(001), Phys. Rev. Lett. **101**, 027201 (2008).

[28] A. Manchon, H. C. Koo, J. Nitta, S. M. Frolov, and R. A. Duine, New perspectives for Rashba spin-orbit coupling, Nat. Mater. **14**, 871 (2015).