Fate of symmetry protected coherence in open quantum system

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We investigate the fate of coherence in the dynamical evolution of symmetry protected quantum systems. Under the formalism of system plus bath for open quantum systems, antiunitary symmetries exhibit significant differences from the unitary ones in protecting initial coherence. Specifically, taking advantage of the Lindblad master equation, we find that a pure state in the symmetry protected degenerate subspace will be decoherent even though both the system Hamiltonian and system-environment interaction respect the same antiunitary symmetry. In contrast, the coherence will persist when the protecting symmetry is unitary. We provide an elaborate classification table to illustrate what kinds of symmetry combinations are able to preserve the coherence of the initial state, which is confirmed in several concrete models of single spin-3/2 systems. It can also be applied to the Haldane phase in interacting topological systems. Our results could be helpful in exploring possible experimental realization of stable time-reversal symmetric topological states.

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I. INTRODUCTION

Symmetry is one of the greatest unifying themes in modern physics and plays a fundamental role in classifying quantum phases of matter.

In condensed matter physics, the existence of the Periodical Table in topological free fermions is a great example in which the anomalous quantum Hall effect, topological insulators and topological superconductors, and many other interesting topological phenomena are unified into a single framework of classification theory [1–3]. Despite the high degree of universality on classification theory, previous research on symmetry analysis and topology in condensed matter mainly focused on the isolated systems [4–9] or those systems coupled to non-Hermitian potentials [10–17]. However, realistic quantum systems inevitably couple to external degrees of freedom, which, in principle, should be described by open quantum systems.

Recently, efforts have been made to study the classification problem on steady states in Lindblad equations [18,19]. The key point for experimentally searching stable symmetry protected topological states is maintaining the coherence of quantum states in the presence of surroundings. Thus a natural question is whether these symmetry protected topological states can survive from the decoherence induced by environment. In particular, one of the central issues for the open quantum system is the decoherence dynamics concerning how the quantum coherence evolves and vanishes, which is particularly important for quantum information and quantum computation [20]. A crucial question is how to prevent decoherence which is an unavoidable roadblock to quantum information processing.

Therefore, diagnosing the stability of coherence of symmetry protected quantum states has a fundamental importance in theory and practice. Symmetry analysis plays a vital role in this issue. According to Wigner's theorem [21], a symmetry transformation is either unitary or antiunitary. Recent theoretical research revealed that the coherence of states underlying many symmetry protected features could be fragile when the symmetry is antiunitary, even though both the environment and system-environment interaction respect the same symmetry as that of the system Hamiltonian [22]. But these features are always robust as long as the protected symmetry is unitary. Strikingly, when applying this to topological systems protected by time-reversal symmetry, one concludes that the topological phases could be unstable to the perturbation of environment [22]. Understanding these nonequilibrium dynamics is of fundamental interest and has potential applications in quantum information.

This work investigates the fate of coherence in degenerate subspace protected by the unitary or antiunitary symmetry. When a system is weakly coupled to the environment, its dissipative dynamics under the Born-Markovian approximation in this open system are governed by the Lindblad master equation. Indeed, the anomalous decoherence or degeneracy breaking in antiunitary symmetry has already arisen from the non-Hermitian linear response theory [23]. At this level, dynamical evolution of density matrix in degenerate subspace is able to distinguish the difference between antiunitary and unitary symmetries in maintaining coherence [24]. Besides, the decoherence is also related to the number of coupling channels between system and environment when all coupling operators are Hermitian ones [22]. We will classify the maintenance of coherence with different symmetry combinations respected by the system and system-environment interaction. Furthermore, we take several representative examples in spin-3/2 systems to illustrate our classification. We also provide a general proof about the origin of decoherence by taking advantage of non-Hermitian linear response theory.

The rest of this paper is organized as follows. Section II introduces the general formalism. In Sec. III, we carry out research into the roles of symmetry in dynamical evolution

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and give the classification of coherence of symmetry protected states. And then, in Sec. IV, taking advantage of the Lindblad master equation, we exemplify the classification table with spin-3/2 models by calculating the density matrix and von Neumann entropy in the degenerate subspace. Section V provides a brief summary and outlook. The analytic proof and application to interacting topological phase are included in the Appendixes.

II. GENERAL FORMALISM

In this section, we derive a general formalism from a microscopical Hamiltonian to study the dynamical evolution in the system protected by unitary or antiunitary symmetry. To investigate the robustness of coherence in degenerate space, we consider a quantum system coupled to a Markov bath whose Hamiltonian reads

$$\hat{H}_T = \hat{H}_S + \hat{H}_B + \hat{H}_{SB}, \tag{1}$$

where $\hat{H}_S$, $\hat{H}_B$ are Hamiltonians belonging to the system and bath, respectively, and $\hat{H}_{SB}$ denotes the interaction between them. The coupling part $\hat{H}_{SB}$ can be decomposed as

$$\hat{H}_{SB} = \sum_{j=1}^{M} \hat{\mathcal{A}}_j^\dagger \otimes \hat{\mathcal{B}}_j + \hat{\mathcal{A}}_j \otimes \hat{\mathcal{B}}_j^\dagger, \tag{2}$$

where $\hat{\mathcal{A}}_j$, $\hat{\mathcal{B}}_j$ are operators belonging to system and bath, respectively, and M denotes the number of coupling channels.

Let us focus on the situation that $\hat{H}_S$ has related symmetry in consideration, and then a natural question is whether the symmetry protected feature such as degeneracy is maintained or not. It is generally expected that the symmetry protected feature would be destroyed if the system-bath coupling $\hat{H}_{SB}$ breaks the related symmetry, and it survives as long as $\hat{H}_{SB}$ respects the same symmetry. This intuitive principle seems to be an unquestionable fact. However, recently, in their seminal work [22], McGinley and Cooper found that it can fail when the respecting symmetry is antiunitary even when each part ($\hat{\mathcal{A}}_j$, $\hat{\mathcal{B}}_j$, and $\hat{H}_B$) obeys the same symmetry. This unexpected discovery is deeply rooted in Schur's Lemma for antiunitary groups [25,26] and immediately indicates the fragility of time-reversal symmetry protected topological edge states [24]. Here we will investigate the fate of coherence in the degenerate subspace of the system. For concreteness, we consider the following total Hamiltonian:

$$\hat{H}_T = \hat{H}_S + \sum_{j=1}^{M}\sum_{\alpha} g_{j,\alpha}\hat{\mathcal{O}}_j^\dagger \hat{b}_\alpha + g_{j,\alpha}^* \hat{\mathcal{O}}_j \hat{b}_\alpha^\dagger + \sum_\alpha \omega_\alpha \hat{b}_\alpha^\dagger \hat{b}_\alpha, \tag{3}$$

where the bath is considered as a reservoir of harmonic oscillators in thermal equilibrium and $\hat{b}_\alpha$ ($\hat{b}_\alpha^\dagger$) is the annihilation (creation) operator of the bath for α mode with bosonic commutation relation $[\hat{b}_\alpha, \hat{b}_\beta^\dagger] = \delta_{\alpha\beta}$ and the bath is considered as the reservoir of harmonic oscillators $\hat{H}_B = \sum_\alpha \omega_\alpha \hat{b}_\alpha^\dagger \hat{b}_\alpha$.

Here we focus on the scenario in which the system Hamiltonian $\hat{H}_S$ possesses the symmetry in question exhibiting degeneracy, and the coupling $\hat{H}_{SB}$ and the bath $\hat{H}_B$ respect the same symmetry. That is to say the symmetry considered here is represented by a group $\mathcal{G}$, and $\hat{\mathcal{O}}$, $\hat{b}_\alpha$ are all invariant under the symmetry operations in $\mathcal{G}$.

Under this circumstance, a significant practical question is whether symmetry protected properties such as degeneracy in system are fundamentally stable against perturbations of the environment. To explore this, we consider the one-channel case $M = 1$ for simplicity.

Following the standard procedure (see Appendix A) to integrate out the bath under Markovian approximation and Born approximation, one can derive the Lindblad master equation as

$$\frac{d\hat{\rho}(t)}{dt} \equiv \mathscr{L}\hat{\rho}(t) = -i[\hat{H}_S, \hat{\rho}(t)] - \gamma\{\hat{\rho}(t), \hat{\mathcal{O}}^\dagger\hat{\mathcal{O}}\} + 2\gamma\hat{\mathcal{O}}\hat{\rho}(t)\hat{\mathcal{O}}^\dagger, \tag{4}$$

where $\mathscr{L}$ is the Liouvillian superoperator and $\{\hat{A}, \hat{B}\} \equiv \hat{A}\hat{B} + \hat{B}\hat{A}$ denotes the anticommutator. This equation dominates the dynamics of the density matrix.

In this way, the symmetry protected properties in degenerate space are connected with the dynamics governed by the Lindblad master equation. Here we focus on the von Neumann entropy in degenerate subspace to characterize the decoherent process, and the corresponding response of entropy is given by

$$\delta S(t) = S(t) - S_0(t), \tag{5}$$

where $S_0(t)$ is the unperturbed entropy with coherent evolution determined by $\hat{H}_S$. Specifically, we will study von Neumann entropy $S_{\rm v}(t) = -{\rm Tr}[\hat{\rho}(t)\ln\hat{\rho}(t)]$, which can be obtained once the master equation is solved. The decoherence dynamics in degenerate subspace and entropy growth can be derived by non-Hermitian linear response theory (see Appendix B).

Moreover, the break of degeneracy is reflected in the matrix representation of Liouvillian $\mathscr{L}$. Using the Choi-Jamiołkowski isomorphism [27,28], Eq. (4) can be mapped to the following equation:

$$\frac{d}{dt}|\rho\rangle = \hat{\mathscr{L}}|\rho\rangle, \tag{6}$$

with vectorized density matrix $|\rho\rangle$,

$$|\rho\rangle = \sum_{i,j}\rho_{i,j}|i\rangle \otimes |j\rangle, \tag{7}$$

where $\rho_{i,j} = \langle i|\hat{\rho}|j\rangle$ is the matrix element of $\hat{\rho}$. And $\hat{\mathscr{L}}$ denotes the matrix representation of the Liouvillian, which is written as

$$\hat{\mathscr{L}} = -i(\hat{H}_S \otimes \hat{I} - \hat{I} \otimes \hat{H}_S^{\rm T}) + \gamma[2\hat{\mathcal{O}} \otimes \hat{\mathcal{O}}^* - \hat{\mathcal{O}}^\dagger\hat{\mathcal{O}} \otimes \hat{I} - \hat{I} \otimes (\hat{\mathcal{O}}^\dagger\hat{\mathcal{O}})^{\rm T}]. \tag{8}$$

The Liouvillian superoperator can share the symmetry associated with the system [29], which determines Eq. (4). The fate of coherence in degenerate subspace depends on whether $\hat{\mathscr{L}}$ is proportional to the identity matrix in degenerate subspace.

III. COHERENCE ANALYSIS FOR DIFFERENT SYMMETRY COMBINATIONS

In this section, we classify the coherent dynamics in degenerate subspace regarding different combinations of symmetries respected by $\hat{H}_S$ and $\hat{O}$. Suppose the system respects unitary or antiunitary symmetry characterized by group $\mathcal{G}$. This means the matrix representation of $\hat{H}_S$ satisfies $[\hat{H}_S, \hat{U}_\mathcal{G}] = 0$ for any group element $\hat{U}_\mathcal{G} \in \mathcal{G}$. If $|\psi\rangle$ is one of the eigenstates of $\hat{H}_S$, i.e., $\hat{H}_S|\psi\rangle = E|\psi\rangle$, then $\hat{U}_\mathcal{G}|\psi\rangle$ would also be the eigenstate of $\hat{H}_S$. Thus $\{\hat{U}_\mathcal{G}|\psi\rangle\}$ spans a degenerate subspace when $\hat{U}_\mathcal{G} \in \mathcal{G}$ satisfying $\hat{U}_\mathcal{G}|\psi\rangle \neq |\psi\rangle$ exists, which is also an irreducible representation subspace of $\mathcal{G}$. We call this symmetry protected degeneracy. A notable example in a half-odd integer spin system is Kramers' degeneracy when $\mathcal{G}$ represents the time-reversal symmetry group.

To describe the coherent dynamics in degenerate space, we assume that the $\{U_G|\psi\rangle\}$ contains two orthogonal bases which are denoted by $|\phi_+\rangle$ and $|\phi_-\rangle$ corresponding to twofold degenerate subspace. The initial state is prepared to be a pure state $\hat{\rho}(0) = |\psi(0)\rangle\langle\psi(0)|$ with $|\psi(0)\rangle = \alpha|\phi_+\rangle + \beta|\phi_-\rangle$ and then the dynamics of the density matrix will be dominated by Lindblad master equation (4). We focus on the density matrix in the subspace $\hat{\rho}_G(t) = \hat{\Pi}_G\hat{\rho}(t)\hat{\Pi}_G$, where the ground-state projective operator is defined by $\hat{\Pi}_G = |\phi_+\rangle\langle\phi_+| + |\phi_-\rangle\langle\phi_-|$.

For a general total Hamiltonian, the bath and system-bath coupling do not respect the related symmetry of $\hat{H}_S$, in which case $\hat{\rho}_G(t)$ evolves into a mixed state where the decoherence happens. Nevertheless, what we are concerned about here is whether the complete information of the initial state, or at least the coherence, can be maintained when both the operators $\{\hat{a}_\alpha\}$ and $\{\hat{O}_j\}$ respect the same symmetry as the system Hamiltonian $\hat{H}_S$. As stated above, the fate of coherence is determined by unitarity or antiunitarity of the symmetry. This conclusion had been pointed out in Refs. [22,24] and here we provide a brief explanation. For the unitary symmetry, $\hat{\Pi}_G\hat{O}^\dagger\hat{O}\hat{\Pi}_G \propto \hat{\Pi}_G$, $\hat{\Pi}_G\hat{O}^\dagger\hat{\Pi}_G \propto \hat{\Pi}_G$, and $\hat{\Pi}_G\hat{O}\hat{\Pi}_G \propto \hat{\Pi}_G$ are guaranteed by Schur's lemma [30] as long as $\hat{O}^\dagger$, $\hat{O}$ respect the same symmetry as $\hat{H}_S$. In this case, Lindblad superoperator $\mathscr{L}$ only acts on the subspace density matrix as $\hat{\Pi}_G\mathscr{L}[\hat{\rho}(t)]\hat{\Pi}_G \propto \rho_G(0)$. Accordingly, if we define

$$\rho_G(t) = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \begin{pmatrix} \langle\phi_+|\hat{\rho}|\phi_+\rangle & \langle\phi_+|\hat{\rho}|\phi_-\rangle \\ \langle\phi_-|\hat{\rho}|\phi_+\rangle & \langle\phi_-|\hat{\rho}|\phi_-\rangle \end{pmatrix}, \quad (9)$$

then all of the matrix elements synchronously decay at the same rate. That is to say, the renormalized density matrix in subspace $\tilde{\rho}_G = \rho_G(t)/\mathrm{tr}[\rho_G(t)]$ is always equal to $\rho_G(0)$ under the time evolution of the Lindblad equation, which means the initial coherence is maintained. In addition, the matrix representation of $\hat{\mathscr{L}}$ in subspace is proportional to identity, namely,

$$\hat{\mathscr{L}}_G \propto \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (10)$$

which acts trivially on the state of subspace $\hat{\mathscr{L}}_G[\rho_{+,+}(t), \rho_{+,-}(t), \rho_{-,+}(t), \rho_{-,-}(t)]^\mathrm{T} \propto [\rho_{+,+}(0), \rho_{+,-}(0), \rho_{-,+}(0), \rho_{-,-}(0)]^\mathrm{T}$.

For the case of antiunitary symmetry, according to Schur's Lemma [25], if $\hat{O}$ is a Hermitian operator obeying this symmetry, it will be proportional to the identity matrix in degenerate subspace, i.e., $\hat{\Pi}_G\hat{O}^\dagger\hat{\Pi}_G = \hat{\Pi}_G\hat{O}\hat{\Pi}_G \propto \hat{I}_G$, and the condition (10) is also true, which means the density matrix will maintain its initial coherence. But if $\hat{O}$ is a non-Hermitian operator, then its matrix representation is no longer proportional to identity, i.e., $\hat{\Pi}_G\hat{O}\hat{\Pi}_G \not\propto \hat{I}_G$. In this case, one can find immediately $\tilde{\rho}_G = \rho_G(t)/\mathrm{tr}[\rho_G(t)] \neq \rho_G(0)$ from the master equation (4) and decoherence occurs. Meanwhile, $\hat{\mathscr{L}}_G$ is no longer proportional to identity:

$$\hat{\mathscr{L}}_G \not\propto \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (11)$$

The above statements for unitary or antiunitary symmetric systems can be naturally generalized to those with various combinations of symmetries. We summarize different kinds of symmetry combinations and the corresponding fate of coherence in Table I. We take the time-reversal symmetric group and quaternion group as candidates of the antiunitary and unitary group, respectively. Some representative examples will be illustrated in the next section.

IV. NUMERICAL RESULTS WITH SPIN-3/2 MODELS

In this section, we take spin-3/2 models as a concrete example to elucidate and verify the symmetry classification shown in Table I. This kind of high-spin system coupled to environment was extensively studied in nitrogen-vacancy (NV) centers [31,32]. The matrix representation of spin-3/2 angular momentum is chosen as

$$S_x = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix},$$

$$S_y = \frac{i}{2}\begin{pmatrix} 0 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & -2 & 0 \\ 0 & 2 & 0 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix},$$

$$S_z = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}. \quad (12)$$

The unitary and antiunitary symmetries will be discussed respectively.

A. Unitary symmetry

We first study the system with unitary symmetry. In order to illustrate the symmetry protected coherence, for simplicity, we choose the quaternion group $\mathcal{Q}$ as a candidate of the unitary group, which is a non-Abelian unitary group whose irreducible representation can be two-dimensional, spanning a twofold degenerate subspace. The matrix representation of $\mathcal{Q}$ is displayed explicitly in Appendix C. We then

TABLE I. Classification table for the fate of initial coherence protected by three kinds of symmetries in $\hat{H}_S$ and coupling operator $\hat{\mathcal{O}}$ with different types of symmetry combinations. Here $[\hat{H}_S, \mathcal{Q}] = 0$ ($[\hat{H}_S, \mathcal{T}] = 0$) means the system respects ($\mathcal{Q}$-symmetry) time-reversal symmetry. $\hat{I}_G$ represents the identity matrix in ground-state subspace.

Symmetry of $\hat{H}_S$	Hermiticity of $\hat{\mathcal{O}}$	Symmetry of $\hat{\mathcal{O}}$	Coherence/Decoherence	$\mathscr{L}_G \propto \hat{I}_G \otimes \hat{I}_G$ (Yes/No)
$[\hat{H}_S, \mathcal{Q}] = 0$	Hermitian	$[\hat{\mathcal{O}}, \mathcal{Q}] = 0$	Coherence	Yes
		$[\hat{\mathcal{O}}, \mathcal{Q}] \neq 0$	Decoherence	No
	Non-Hermitian	$[\hat{\mathcal{O}}, \mathcal{Q}] = 0$	Coherence	Yes
		$[\hat{\mathcal{O}}, \mathcal{Q}] \neq 0$	Decoherence	No
$[\hat{H}_S, \mathcal{T}] = 0$	Hermitian	$[\hat{\mathcal{O}}, \mathcal{T}] = 0$	Coherence	Yes
		$[\hat{\mathcal{O}}, \mathcal{T}] \neq 0$	Decoherence	No
	Non-Hermitian	$[\hat{\mathcal{O}}, \mathcal{T}] = 0$	Decoherence	No
		$[\hat{\mathcal{O}}, \mathcal{T}] \neq 0$	Decoherence	No
$[\hat{H}_S, \mathcal{T}] = 0$, $[\hat{H}_S, \mathcal{Q}] = 0$	Hermitian	$[\hat{\mathcal{O}}, \mathcal{T}] = 0$, $[\hat{\mathcal{O}}, \mathcal{Q}] = 0$	Coherence	Yes
		$[\hat{\mathcal{O}}, \mathcal{T}] = 0$, $[\hat{\mathcal{O}}, \mathcal{Q}] \neq 0$	Coherence	Yes
		$[\hat{\mathcal{O}}, \mathcal{T}] \neq 0$, $[\hat{\mathcal{O}}, \mathcal{Q}] = 0$	Coherence	Yes
		$[\hat{\mathcal{O}}, \mathcal{T}] \neq 0$, $[\hat{\mathcal{O}}, \mathcal{Q}] \neq 0$	Decoherence	No
	Non-Hermitian	$[\hat{\mathcal{O}}, \mathcal{T}] = 0$, $[\hat{\mathcal{O}}, \mathcal{Q}] = 0$	Coherence	Yes
		$[\hat{\mathcal{O}}, \mathcal{T}] = 0$, $[\hat{\mathcal{O}}, \mathcal{Q}] \neq 0$	Decoherence	No
		$[\hat{\mathcal{O}}, \mathcal{T}] \neq 0$, $[\hat{\mathcal{O}}, \mathcal{Q}] = 0$	Coherence	Yes
		$[\hat{\mathcal{O}}, \mathcal{T}] \neq 0$, $[\hat{\mathcal{O}}, \mathcal{Q}] \neq 0$	Decoherence	No

construct a $\mathcal{Q}$-symmetric Hamiltonian $\hat{H}_S = E_g(\hat{S}_x\hat{S}_y\hat{S}_z + \hat{S}_z\hat{S}_y\hat{S}_x)$ with twofold degenerate ground states which forms a two-dimensional irreducible representation space of group $\mathcal{Q}$. For Hermitian operator $\hat{\mathcal{O}}$ coupled to a system respecting $\mathcal{Q}$ symmetry such as $\hat{\mathcal{O}} = \hat{S}_y^2$, the coherence in ground-state subspace maintains all the time but decoherence occurs when this symmetry is broken, say, $\hat{\mathcal{O}} = \hat{S}_x\hat{S}_y + \hat{S}_y\hat{S}_x$ as shown in Fig. 1(a). The same situation happens to the non-Hermitian coupling operators that the symmetric and asymmetric $\hat{\mathcal{O}}$ are chosen as $\hat{\mathcal{O}} = \hat{S}_x\hat{S}_y\hat{S}_z$ and $\hat{\mathcal{O}} = \hat{S}_y\hat{S}_z$, respectively [see Fig. 1(b)]. The increasing entropy reflects a fact that the subspace density matrix evolves into a mixed state. The unchanged von Neumann entropy indicates that the density matrix in subspace is always a pure state, which means the coherence is maintained. This is a reasonable consequence that the decoherence appears only when the system is perturbed by symmetry-broken coupling. As mentioned above, Schur's lemma tells $\hat{\Pi}_G\hat{\mathcal{O}}\hat{\Pi}_G \propto \hat{I}_G$ if $\hat{\mathcal{O}}$ respects $\mathcal{Q}$ symmetry, which leads to $\mathscr{L}_G \propto \hat{I}_G \otimes \hat{I}_G$ according to expression (8). In contrast, $\mathscr{L}_G \not\propto \hat{I}_G \otimes \hat{I}_G$ if $\hat{\Pi}_G\hat{\mathcal{O}}\hat{\Pi}_G \not\propto \hat{I}_G$, since $\hat{\mathcal{O}}$ breaks $\mathcal{Q}$ symmetry which destroys the coherence, and the corresponding entropy of decoherent dynamics reaches the steady-state value $S_v(\infty) = \ln 2 \approx 0.693$, which is nothing but the maximum entropy in doublet space.

B. Antiunitary symmetry

From now on, we focus on the systems respecting time-reversal symmetry. Since time-reversal operation inverts the angular momentum operators, i.e., $\hat{T}\hat{S}_{x,y,z}\hat{T}^{-1} = -\hat{S}_{x,y,z}$, one can construct a time-reversal invariant Hamiltonian as $\hat{H}_S = E_g\{\hat{S}_x, \hat{S}_z\}$. The ground-state subspace of $\hat{H}_S$ is twofold degenerate due to Kramers' theorem and we denote two degenerate states as $|\phi_\pm\rangle$. Similar to the discussion of the unitary

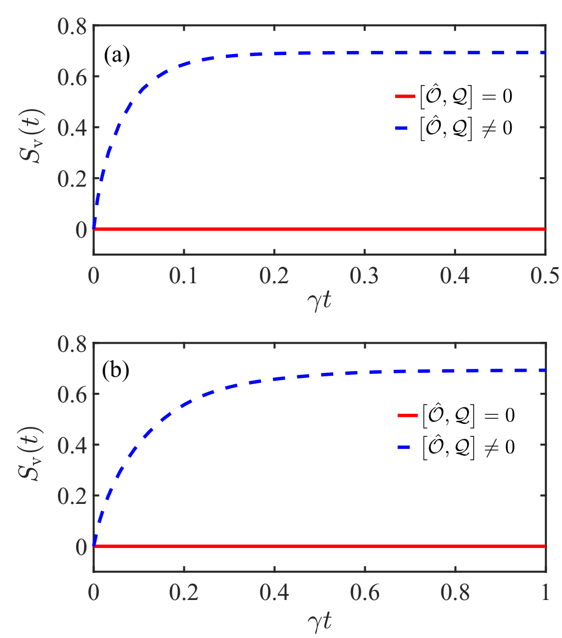

FIG. 1. Time evolution of von Neumann entropy in degenerate subspace protected by unitary symmetry ($\mathcal{Q}$ symmetry). (a) von Neumann entropy as a function of time for Hermitian coupling operator: with $\mathcal{Q}$ symmetry $\hat{Q} = \hat{S}_y^2$ (red solid line) and without $\mathcal{Q}$ symmetry $\hat{Q} = \hat{S}_x\hat{S}_y + \hat{S}_y\hat{S}_x$ (blue dashed line). (b) von Neumann entropy as a function of time for non-Hermitian coupling operator: with $\mathcal{Q}$ symmetry $\hat{Q} = \hat{S}_x\hat{S}_y\hat{S}_z$ (red solid line) and without $\mathcal{Q}$ symmetry $\hat{Q} = \hat{S}_x\hat{S}_y\hat{S}_z$ (blue dashed line).

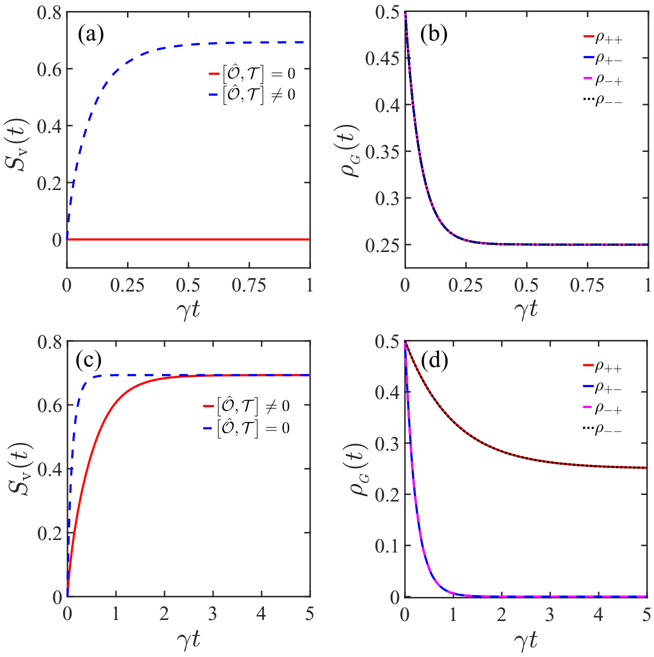

FIG. 2. Time evolution of von Neumann entropy and density matrix in degenerate subspace protected by time-reversal symmetry. (a) von Neumann entropy as a function of time for Hermitian coupling operator: with time-reversal symmetry $\hat{Q} = S_x^2$ (red solid line) and without time-reversal symmetry $\hat{Q} = S_z$ (blue dashed line). (b) Matrix elements of density matrix associated with red solid line in (a) evolve over time. (c) von Neumann entropy as a function of time for non-Hermitian coupling operator: without time-reversal symmetry $\hat{Q} = S_xS_yS_z$ (red solid line) and with time-reversal symmetry $\hat{Q} = iS_z$ (blue dashed line). (d) Matrix elements of density matrix associated with the blue dashed line in (c) evolve over time. The initial density matrix is chosen as $|\psi(0)\rangle\langle\psi(0)|$ with $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\phi_+\rangle + |\phi_-\rangle)$.

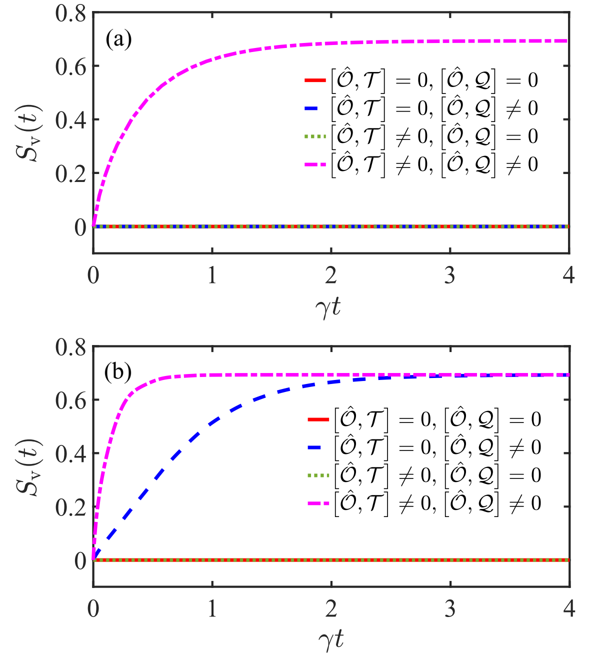

FIG. 3. von Neumann entropy as a function of time with the system Hamiltonian respecting both $\mathcal{Q}$ symmetry and time-reversal symmetry. The Hermitian and non-Hermitian coupling operators are shown in (a) and (b), respectively. For the Hermitian case, the four kinds of coupling $\hat{\mathcal{O}}$ are chosen as $\hat{S}_x^2$ (red solid line), $\hat{S}_x\hat{S}_y + \hat{S}_y\hat{S}_x$ (blue dashed line), $\hat{S}_x\hat{S}_y\hat{S}_z + \hat{S}_z\hat{S}_y\hat{S}_x$ (green dotted line), and $\hat{S}_x$ (dot-dashed line). For the non-Hermitian case, the corresponding choice is $i(\hat{S}_x\hat{S}_y\hat{S}_z - \hat{S}_z\hat{S}_y\hat{S}_x)$, $\hat{S}_x\hat{S}_y$, $\hat{S}_x\hat{S}_y\hat{S}_z$, and $\hat{S}_x^2\hat{S}_z$. The initial value of density matrix is chosen as $|\psi(0)\rangle\langle\psi(0)|$ with $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\phi_+\rangle + |\phi_-\rangle)$.

symmetric system, here the Hermitian and non-Hermitian coupling $\hat{\mathcal{O}}$ will be investigated, respectively. In order to verify the case of $[\hat{H}_S, \mathcal{T}] = 0$ in Table I, we prepare the initial state on $|\psi(0)\rangle = \alpha|\phi_+\rangle + \beta|\phi_-\rangle$, and then calculate the time evolution of von Neumann entropy in the ground-state subspace by means of the Lindblad equation shown in Fig. 2.

For the Hermitian case, as shown in Fig. 2(a), the initial coherence will maintain (vanish) if the operator $\hat{\mathcal{O}}$ coupled by the system respects (breaks) time-reversal symmetry. However, there will be a dramatic difference when this time-reversal symmetric system couples to non-Hermitian operator $\hat{\mathcal{O}}$. Figure 2(c) plots the von Neumann entropy, from which one can see clearly that the coherence is always destroyed despite whether or not $\hat{\mathcal{O}}$ respects time-reversal symmetry. Hence we demonstrate that coherence could survive only if the time-reversal symmetric coupling operator is also Hermitian. This is consistent with Schur's Lemma for antiunitary groups that an antiunitary symmetric and Hermitian operator is proportional to identity in degenerate subspace, i.e., $\hat{\Pi}_G\hat{\mathcal{O}}\hat{\Pi}_G \propto \hat{I}_G$, which results in the relation $\hat{\mathscr{L}}_G \propto \hat{I}_G \otimes \hat{I}_G$. When the situation that any of the symmetry and Hermiticity is not satisfied happens, it results in $\hat{\Pi}_G\hat{\mathcal{O}}\hat{\Pi}_G \not\propto \hat{I}_G$, which causes the decoherence and, meanwhile, $\hat{\mathscr{L}}_G \not\propto \hat{I}_G \otimes \hat{I}_G$.

With regard to density matrix $\hat{\rho}_G(t)$ in ground-state subspace, for coherent evolution, all matrix elements decay with the same rate during the whole region of time as shown in Fig. 2(b). For the decoherent process, the nondiagonal elements could decay to zero, but its diagonal elements keep finite, which means all coherence is lost and the system reaches maximum entanglement in ground-state subspace [see Fig. 2(d)].

C. Both unitary and antiunitary symmetry

There is another case in which the Hamiltonian respects both time-reversal symmetry and $\mathcal{Q}$ symmetry, such as $\hat{H}_S = E_g\hat{S}_z^2$, whose degeneracy of ground states is protected by both symmetries. This highly symmetric system allows for more types of symmetry combinations which contain four circumstances for both Hermitian and non-Hermitian couplings as shown in the third row of Table I. The corresponding entropy dynamics associated with different symmetries are shown in Fig. 3. For the Hermitian case, the initial coherence in ground-state subspace is always maintained whichever symmetry the coupling operator $\hat{\mathcal{O}}$ has. This is consistent with the conclusions in subsections IV A and IV B that $\hat{\Pi}_G\hat{\mathcal{O}}\hat{\Pi}_G \propto \hat{I}_G$ regardless of unitarity or antiunitarity, which signifies the decoherence occurs only when the coupling operator breaks both symmetries, as shown in Fig. 3(a). For the non-Hermitian case, the coherence will be destroyed once the $\mathcal{Q}$ symmetry is broken by coupling operator $\hat{O}$ as shown in Fig. 3(b). This reflects the fact that the coherence protected only by antiunitary symmetry is fragile under non-Hermitian perturbation. One point worth emphasizing is that the classification in Table I is applicable to all groups containing two- or higher-dimensional irreducible representations.

V. SUMMARY AND OUTLOOK

We study the fate of coherence in degenerate subspace, which is protected by unitary symmetry or antiunitary symmetry. With symmetries lying in the system Hamiltonian, and the interaction between system and environment, we have demonstrated that the coherence could be fragile when the symmetry is antiunitary. We elaborate on the classification of various symmetry combinations and analyze the stability of coherence detailedly and confirm it by means of several spin-3/2 models. Moreover, the conclusion is extended to interacting topological phases in many-body systems where we investigated entropy dynamics in the symmetry protected ground-state manifold for the AKLT model (Appendix D). These results could be applied to the investigation of robustness in time-reversal invariant topological edge states.

Given that the recent experimental progress in controlling and manipulating dissipation in ultracold atoms provides an unprecedented opportunity for understanding the dynamics of open quantum systems [33–39], we expect our work will guide the preparation for stable time-reversal symmetry protected topological states in the laboratory. There is another intriguing issue about the open quantum systems beyond the Born-Markov approximation. In this case, the coherence protected by antiunitary symmetry could also be fragile even when the system couples Hermitian operators with the same symmetry [40]. The fate of coherence that we have identified could assist in experimentally preparing stable time-reversal symmetry protected topological phases.

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APPENDIX A: DERIVATION OF THE LINDBLAD MASTER EQUATION

Here we provide a derivation of the Lindblad master equation (4) from the total Hamiltonian (3). We start from the general total Hamiltonian (1)

$$\hat{H}_T = \hat{H}_S + \hat{H}_B + \hat{H}_{SB}, \tag{A1}$$

and the Liouville–von Neumann equation of total density matrix $\hat{\rho}_T$ is given by

$$\frac{d}{dt}\hat{\rho}_T = -i[\hat{H}_T, \hat{\rho}_T]. \tag{A2}$$

Introducing the interaction picture of $\hat{H}_S + \hat{H}_B$,

$$\hat{\rho}^{\mathrm{I}}(t) = e^{i(\hat{H}_S+\hat{H}_B)t}\hat{\rho}_T(t)e^{-i(\hat{H}_S+\hat{H}_B)t}, \tag{A3}$$

then the equation of motion for $\hat{\rho}^{\mathrm{I}}(t)$ will be

$$\frac{d}{dt}\hat{\rho}^{\mathrm{I}}(t) = -i\big[\hat{H}_{SB}^{\mathrm{I}}(t), \hat{\rho}^{\mathrm{I}}(t)\big], \tag{A4}$$

where $\hat{H}_{SB}^{\mathrm{I}}(t) = e^{i(\hat{H}_S+\hat{H}_B)t}\hat{H}_{SB}e^{-i(\hat{H}_S+\hat{H}_B)t}$. Integrating Eq. (A4) from 0 to t yields

$$\hat{\rho}^{\mathrm{I}}(t) = \hat{\rho}^{\mathrm{I}}(0) - i\int_0^t \big[\hat{H}_{SB}^{\mathrm{I}}(t'), \hat{\rho}^{\mathrm{I}}(t')\big]dt'. \tag{A5}$$

Substituting Eq. (A5) into the right-hand side of Eq. (A4), we have

$$\frac{d}{dt}\hat{\rho}^{\mathrm{I}}(t) = -i[\hat{H}_{SB}^{\mathrm{I}}(t), \hat{\rho}^{\mathrm{I}}(0)] - \left[\hat{H}_{SB}^{\mathrm{I}}(t), \int_0^t \big[\hat{H}_{SB}^{\mathrm{I}}(t'), \hat{\rho}^{\mathrm{I}}(t')\big]dt'\right]. \tag{A6}$$

Now we take the Born-Markov approximation. By replacing $\hat{\rho}^{\mathrm{I}}(t')$ to $\hat{\rho}^{\mathrm{I}}(t)$ and setting $\hat{\rho}(t) = \hat{\rho}_S(t)\otimes\hat{\rho}_B$ where $\hat{\rho}_B$ is the density matrix of the bath in thermal equilibrium and taking partial trace of the bath, we arrive at

$$\frac{d}{dt}\hat{\rho}_S(t) = -i\ \mathrm{tr}_B\left\{\big[\hat{H}_{SB}^{\mathrm{I}}(t), \hat{\rho}_S(0)\otimes\hat{\rho}_B\big]\right\} - \int_0^t dt'\,\mathrm{tr}_B\left\{\big[\hat{H}_{SB}^{\mathrm{I}}(t), \big[\hat{H}_{SB}^{\mathrm{I}}(t'), \hat{\rho}_S(t)\otimes\hat{\rho}_B\big]\big]\right\}. \tag{A7}$$

The first term in the right hand side of Eq. (A7) can be equal to zero by setting $\mathrm{tr}_B(\hat{H}_{SB}) = 0$. This is because, for the case of $\mathrm{tr}_B(\hat{H}_{SB}) \neq 0$, we can always adopt a shift $\hat{H}_{SB}^{\mathrm{I}} \to \hat{H}_{SB}^{\mathrm{I}} - \mathrm{tr}_B(\hat{H}_{SB})$. Substituting t' by $t - t'$ and extending the upper limit to infinity in the integral, we have

$$\frac{d}{dt}\hat{\rho}_S(t) = -\int_0^\infty dt'\,\mathrm{tr}_B\left\{\big[\hat{H}_{SB}^{\mathrm{I}}(t), \big[\hat{H}_{SB}^{\mathrm{I}}(t-t'), \hat{\rho}_S(t)\otimes\hat{\rho}_B\big]\big]\right\}. \tag{A8}$$

Thus we obtain the Markovian master equation (A8). Here we should emphasize that the infinite upper limit is a permissible approximation if the integrand disappears sufficiently fast. This is valid if the system relaxation time is much larger than the time scale over which the bath correlation functions decay.

It is straightforward to obtain the Lindblad master equation [41]. Putting the total Hamiltonian (3) into Eq. (A8) and setting

$$\sum_\alpha\sum_\beta g_\alpha g_\beta^* \langle\hat{b}_\alpha(t)\hat{b}_\beta^\dagger(t')\rangle_{\mathrm{B}} = \gamma\delta(t-t'),$$

$$\sum_\alpha\sum_\beta g_\alpha g_\beta^* \langle\hat{b}_\alpha^\dagger(t)\hat{b}_\beta(t')\rangle_{\mathrm{B}} = 0,$$

$$\sum_\alpha\sum_\beta g_\alpha g_\beta^* \langle\hat{b}_\alpha(t)\hat{b}_\beta(t')\rangle_{\mathrm{B}} = 0,$$

where $\langle\ldots\rangle_{\mathrm{B}} = \mathrm{tr}_B(\ldots\hat{\rho}_B)$, and $\gamma = \pi|g|^2\rho$ with density of state ρ of the bath, then we can derive the Lindblad master equation (4) directly.

APPENDIX B: RESPONSE OF DENSITY MATRIX AND VON NEUMANN ENTROPY GROWTH

In this Appendix, we derive the response of density matrix and von Neumann entropy growth utilizing non-Hermitian linear response theory. As discussed in Ref. [23], the whole system coupling to a bath with white noise can be described by a non-Hermitian effective Hamiltonian

$$\hat{H}_{\mathrm{eff}} = \hat{H}_S + \hat{H}_{\mathrm{diss}}, \tag{B1}$$

with $\hat{H}_{\mathrm{diss}} = (-i\gamma\hat{O}^\dagger\hat{O} + \hat{O}^\dagger\hat{\xi} + \hat{\xi}^\dagger\hat{O})$. Here $\hat{\xi}(t)$, $\hat{\xi}^\dagger(t)$ present the Langevin noise operators which obey the following relations:

$$\begin{gathered}\langle\hat{\xi}(t)\hat{\xi}^\dagger(t_1)\rangle_{\mathrm{noise}} = 2\gamma\delta(t - t_1),\\ \langle\hat{\xi}(t)\hat{\xi}(t_1)\rangle_{\mathrm{noise}} = \langle\hat{\xi}^\dagger(t)\hat{\xi}(t_1)\rangle_{\mathrm{noise}} = \langle\hat{\xi}^\dagger(t)\hat{\xi}^\dagger(t_1)\rangle_{\mathrm{noise}} = 0,\end{gathered} \tag{B2}$$

where $\langle\ldots\rangle_{\mathrm{noise}}$ denotes the noise average [42]. This formalism is equivalent to the total Hamiltonian (3) with a white noise bath [23]. In the interaction picture, time evolution of density matrix can be expressed by

$$\hat{\rho}(t) = \hat{U}_{\mathrm{eff}}(t)\hat{\rho}\hat{U}^\dagger_{\mathrm{eff}}(t), \tag{B3}$$

where $\hat{U}_{\mathrm{eff}}(t) = \hat{\mathrm{T}}\,\exp\,(-i\int_0^t \hat{H}_{\mathrm{diss}}(t')dt')$ with time-ordered operator $\hat{\mathrm{T}}$. Taking $\hat{H}_{\mathrm{diss}}$ as perturbation and then averaging the noise, one can obtain the density matrix with the first-order correction of γ:

$$\begin{aligned}\langle\hat{\rho}(t)\rangle_{\mathrm{noise}} &= \langle U'_{\mathrm{eff}}(t)\hat{\rho}U'^\dagger_{\mathrm{eff}}(t)\rangle_{\mathrm{noise}}\\ &= \left\langle\left(1+\sum_{n=1}^{\infty}(-i)^n\int_{t_1<\cdots<t_n}\hat{H}_{\mathrm{diss}}(t_1)\ldots\hat{H}_{\mathrm{diss}}(t_n)dt_1\ldots dt_n\right)\hat{\rho}\left(1+\sum_{n=1}^{\infty}(i)^n\int_{t_1<\cdots<t_n}\hat{H}^\dagger_{\mathrm{diss}}(t_n)\ldots\hat{H}^\dagger_{\mathrm{diss}}(t_1)dt_1\ldots dt_n\right)\right\rangle_{\mathrm{noise}}\\ &\approx \hat{\rho} - \int_0^t dt'\gamma\{\hat{O}^\dagger(t')\hat{O}(t'),\hat{\rho}\} + 2\int_0^t dt'\gamma\hat{O}(t')\hat{\rho}\hat{O}^\dagger(t'),\end{aligned} \tag{B4}$$

where $\langle\ldots\rangle_{\mathrm{noise}}$ is the noise average and the correlation function Eq. (B2) has been applied. The linear response of density matrix is defined by

$$\delta\hat{\rho}(t) \equiv \rho(t) - \hat{\rho} = -\int_0^t dt'\gamma\{\hat{O}^\dagger(t')\hat{O}(t'),\hat{\rho}\} + 2\int_0^t dt'\gamma\hat{O}(t')\hat{\rho}\hat{O}^\dagger(t'). \tag{B5}$$

From the response of density matrix, one can calculate the von Neumann entropy which characterizes the loss of coherence. Here we focus on the density matrix in symmetry protected subspace such as Kramers' degenerate space $\hat{\rho}_{\mathrm{K}}(t) = \Pi_{\mathrm{K}}\hat{\rho}(t)\Pi_{\mathrm{K}}$ ($\hat{\rho}_{0,\mathrm{K}} = \Pi_{\mathrm{K}}\hat{\rho}\Pi_{\mathrm{K}}$) and the corresponding response of von Neumann entropy is given by

$$\begin{aligned}\delta S_{\mathrm{v}}(t) &= S_{\mathrm{v}}(t) - S_{0,\mathrm{v}}(t)\\ &= -\,\mathrm{Tr}\left[\frac{\hat{\rho}_{0,\mathrm{K}} + \delta\hat{\rho}_{\mathrm{K}}(t)}{\mathrm{Tr}\,[\hat{\rho}_{0,\mathrm{K}} + \delta\hat{\rho}_{\mathrm{K}}(t)]}\ln\left(\frac{\hat{\rho}_{0,\mathrm{K}} + \delta\hat{\rho}_{\mathrm{K}}(t)}{\mathrm{Tr}\,[\hat{\rho}_{0,\mathrm{K}} + \delta\hat{\rho}_{\mathrm{K}}(t)]}\right)\right] + \mathrm{Tr}\,[\hat{\rho}_{0,\mathrm{K}}\ln\hat{\rho}_{0,\mathrm{K}}]\\ &= \mathrm{Tr}\left[\hat{\rho}_{0,\mathrm{K}}\ln\hat{\rho}_{0,\mathrm{K}} - [\delta\hat{\rho}_{\mathrm{K}}(t)/\,\mathrm{Tr}\,\delta\hat{\rho}_{\mathrm{K}}(t)]\ln\hat{\rho}_{0,\mathrm{K}}\right]\mathrm{Tr}\,\delta\hat{\rho}_{\mathrm{K}}(t).\end{aligned} \tag{B6}$$

We initialize the density matrix in Kramers' degenerate space, i.e., $\mathrm{Tr}\hat{\rho}_{0,K} = 1$. It is clear that, when $\delta\hat{\rho}_{\mathrm{K}}(t) \propto \hat{\rho}_{0,\mathrm{K}}$ [namely, $\delta\hat{\rho}_{\mathrm{K}}(t) = \hat{\rho}_{0,\mathrm{K}}(t)\,\mathrm{tr}\,\delta\hat{\rho}_{\mathrm{K}}(t)$], we have $\delta S_{\mathrm{v}}(t) = 0$ and $\delta S_{\mathrm{R}}(t) = 0$, but $\delta S_{\mathrm{v}}(t) \neq 0$ if $\delta\hat{\rho}_{\mathrm{K}}(t) \neq \hat{\rho}_{0,\mathrm{K}}$. In other words, the time-reversal symmetry breaking discussed in the main text leads to the growth of entropy.

APPENDIX C: MATRIX REPRESENTATION OF QUATERNION GROUP

This Appendix provides a matrix representation of quaternion group $\mathcal{Q}$ discussed in the main text. The $\mathcal{Q}$ group $\{Q_j, j = 1,\ldots,8\}$ is a non-Abelian group that is isomorphic to subset $\{\mathrm{e}, \mathrm{i}, \mathrm{j}, \mathrm{k}, \bar{\mathrm{e}}, \bar{\mathrm{i}}, \bar{\mathrm{j}}, \bar{\mathrm{k}}\}$, whose multiplication table is displayed in Table II. The $\mathcal{Q}$ group contains a two-dimensional irreducible representation (see Table III), which can be described as a subgroup of the special linear group $\mathrm{SL}_2(\mathbb{C})$. We can construct the following four-dimensional reducible representation:

$$\begin{gathered}Q_1 = I\otimes I,\quad Q_2 = -Q_1,\quad Q_3 = -I\otimes i\sigma_z,\quad Q_4 = -Q_3,\\ Q_5 = -i\sigma_x\otimes\sigma_y,\quad Q_6 = -Q_5,\quad Q_7 = -i\sigma_x\otimes\sigma_x,\\ Q_8 = -Q_7,\end{gathered} \tag{C1}$$

TABLE II. Multiplication table (Cayley table) of quaternion group.

Element	e	$\bar{\mathrm{e}}$	i	$\bar{\mathrm{i}}$	j	$\bar{\mathrm{j}}$	k	$\bar{\mathrm{k}}$
e	e	$\bar{\mathrm{e}}$	i	$\bar{\mathrm{i}}$	j	$\bar{\mathrm{j}}$	k	$\bar{\mathrm{k}}$
$\bar{\mathrm{e}}$	$\bar{\mathrm{e}}$	e	$\bar{\mathrm{i}}$	i	$\bar{\mathrm{j}}$	j	$\bar{\mathrm{k}}$	k
i	i	$\bar{\mathrm{i}}$	$\bar{\mathrm{e}}$	e	k	$\bar{\mathrm{k}}$	$\bar{\mathrm{j}}$	j
$\bar{\mathrm{i}}$	$\bar{\mathrm{i}}$	i	e	$\bar{\mathrm{e}}$	$\bar{\mathrm{k}}$	k	j	$\bar{\mathrm{j}}$
j	j	$\bar{\mathrm{j}}$	$\bar{\mathrm{k}}$	k	$\bar{\mathrm{e}}$	e	i	$\bar{\mathrm{i}}$
$\bar{\mathrm{j}}$	$\bar{\mathrm{j}}$	j	k	$\bar{\mathrm{k}}$	e	$\bar{\mathrm{e}}$	$\bar{\mathrm{i}}$	i
k	k	$\bar{\mathrm{k}}$	j	$\bar{\mathrm{j}}$	$\bar{\mathrm{i}}$	i	$\bar{\mathrm{e}}$	e
$\bar{\mathrm{k}}$	$\bar{\mathrm{k}}$	k	$\bar{\mathrm{j}}$	j	i	$\bar{\mathrm{i}}$	e	$\bar{\mathrm{e}}$

TABLE III. Character table of quaternion group.

Representation/conjugacy class	{e}	{ē}	{i, ī}	{j, j̄}	{k, k̄}
Trivial representation	1	1	1	1	1
i-kernel	1	1	1	−1	−1
j-kernel	1	1	−1	1	−1
k-kernel	1	1	−1	−1	1
Two-dimensional representation	2	−2	0	0	0

where $\sigma_{x,y,z}$ denote Pauli matrices and I is the identity matrix. One can easily find that the matrix representation (C1) obeys the multiplication Table II and contains two-dimensional irreducible representation.

APPENDIX D: APPLICATION TO THE HALDANE PHASE IN AKLT MODEL

In this Appendix, we study the Affleck-Kennedy-Lieb-Tasaki (AKLT) model [43] which serves as an example to show that the conclusion in the main text can also be applied to interacting topological phases. The AKLT model exhibits the symmetry protected topological phase (Haldane phase), and its Hamiltonian is written as

$$\hat{H} = J\sum_{j=1}^{L-1}\left(\mathbf{S}_j\cdot\mathbf{S}_{j+1} + \frac{1}{3}(\mathbf{S}_j\cdot\mathbf{S}_{j+1})^2 + \frac{2}{3}\right), \tag{D1}$$

where $\mathbf{S}_j = (S_j^x, S_j^y, S_j^z)$ denotes the spin-1 vector operator, L represents the system size, and J is the coupling strength. One can rewrite the AKLT Hamiltonian (D1) as a sum of the local spin-2 projector of the nearest neighbor pair of sites $\hat{H} = J\sum_{j=1}^{L-1}\hat{P}^{S=2}_{j,j+1}$. Then the zero-energy ground states can be constructed by excluding spin-2 components on each nearest neighbor site. For an open boundary condition, the model contains four degenerate ground states which are effectively represented by two effective free spins-1/2 on each edge. These degenerate edge states can be expressed by the following matrix product states (MPS):

$$|\Psi_G\rangle = \sum_{\{\sigma_1\cdots\sigma_L\}} \mathrm{Tr}\left(b_A^{l^T} A^{[\sigma_1]}\cdots A^{[\sigma_L]} b_A^r\right)|\sigma_1\ldots\sigma_L\rangle, \tag{D2}$$

where $\sigma_j = -1, 0, 1$ denotes the component of S_z at site j. The matrix $A^{[\sigma_j]}$ takes the form

$$A^{[-1]} = \sqrt{\frac{2}{3}}\begin{pmatrix}0 & 0\\ -1 & 0\end{pmatrix},\quad A^{[0]} = \frac{1}{\sqrt{3}}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix},$$
$$A^{[1]} = \sqrt{\frac{2}{3}}\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \tag{D3}$$

and the left (right) boundary vector b_A^l (b_A^r) corresponds to free spin-1/2 on the left (right) edge. Specifically, they are set to

$$b_A^l = \begin{pmatrix}1\\0\end{pmatrix},\quad b_A^r = \begin{pmatrix}0\\1\end{pmatrix}.$$

if both edges are set to spin up. The degenerate edge states in the AKLT model are protected by both antiunitary symmetry (time-reversal symmetry) and unitary symmetry (dihedral symmetry D_2) [44]. The D_2 group is isomorphic to $Z_2\times Z_2$, which is represented by π rotation about two orthogonal axes. Hence we can investigate the fate of coherence in ground-state subspace of the AKLT model, as similarly discussed in Sec. IV C. The corresponding numerical results for both Hermitian and non-Hermitian couplings are shown in Fig. 4. We can see that the fate of coherence is the same as that in the noninteracting system, Fig. 3, which further confirms the classification table. The unstability of edge states in the AKLT chain is also found in recent works [24,45].

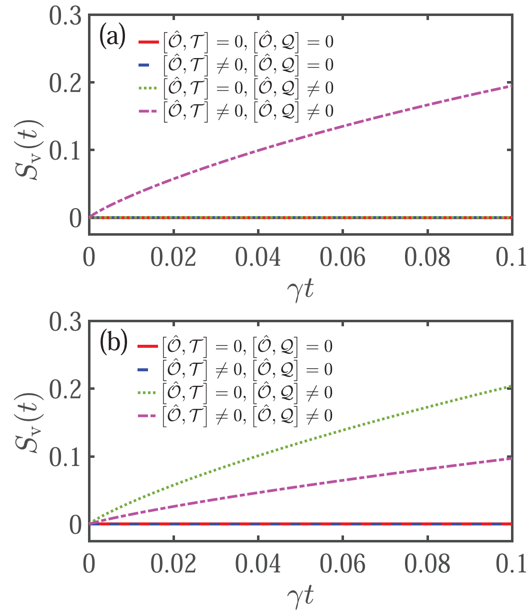

FIG. 4. von Neumann entropy in ground-state subspace of the AKLT model as a function of time with the system Hamiltonian respecting both D_2 symmetry and time-reversal symmetry with Hermitian (a) and non-Hermitian (b) couplings, respectively. For Hermitian coupling, the four different kinds of operators are chosen as $\hat{\mathcal{O}} = (\hat{S}_1^z)^2$ (red solid line), $\hat{\mathcal{O}} = i(\hat{S}_1^x)^2(\hat{S}_1^y)^2 + \mathrm{H.c.}$ (blue dashed line), $\hat{\mathcal{O}} = \hat{S}_1^x\hat{S}_1^y + \hat{S}_1^y\hat{S}_1^x$ (green dotted line), and $\hat{\mathcal{O}} = \hat{S}_1^z$ (dot-dashed line). For non-Hermitian couplings, the corresponding choices are $\hat{\mathcal{O}} = (\hat{S}_1^x)^2(\hat{S}_1^y)^2$, $\hat{\mathcal{O}} = i(\hat{S}_1^z)^2$, $\hat{\mathcal{O}} = i\hat{S}_1^z$, and $\hat{\mathcal{O}} = \hat{S}_1^x(\hat{S}_1^z)^2$. The model parameters are chosen as $L = 10$ and $\gamma/J = 0.01$.

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