

Probing the superfluid-insulator phase transition by a non-Hermitian external fieldX. Z. Zhang¹ and Z. Song^{2,*}¹*College of Physics and Materials Science, Tianjin Normal University, Tianjin 300387, China*²*School of Physics, Nankai University, Tianjin 300071, China*

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We study the response of a thermal state of the Hubbard-like system to either global or local non-Hermitian perturbation, which coalesces the degenerate ground state within the $U(1)$ symmetry breaking phase. We show that the dynamical response of the system is strongly sensitive to the underlying quantum phase transition (QPT) from a Mott insulator to a superfluid state. The Uhlmann fidelity in the superfluid phase decays to a steady value determined by the order of the exceptional point (EP) within the subspace spanned by the degenerate ground states, but remains almost unchanged in the Mott insulating phase. It demonstrates that the phase diagram at zero temperature is preserved even though a local probing field is applied. Specifically, two celebrated models, including the Bose-Hubbard model and the Jaynes-Cummings-Hubbard model, are employed to demonstrate this property in the finite-size system, wherein fluctuations of the boson and polariton number are observed based on EP dynamics. This work presents an alternative approach to probe the superfluid-insulator QPT at nonzero temperature.

DOI: [10.1103/PhysRevB.104.094301](https://doi.org/10.1103/PhysRevB.104.094301)**I. INTRODUCTION**

In equilibrium and at zero temperature, the quantum phase transition (QPT) serving as one of the central issues in condensed matter physics can usually be described by a phenomenological order parameter according to the Landau-Ginzburg theory [1]. Therefore, a system experiences a symmetry breaking from one phase with a nonzero order parameter to another with a vanishing order parameter. The underlying mechanism is the degeneracy of the ground states. Thanks to the incredible advance in quantum simulation, especially in the context of quantum optics and atomic physics, a wide range of condensed matter systems has been theoretically investigated and many proposals for probing the QPT have been proposed [2–4]. QPTs might still be observed at sufficiently low temperatures, where the quantum fluctuations dominate and thermal fluctuations are not significant enough to excite the system from its ground state. At higher temperatures, thermal fluctuations conceal the quantum criticality. As a consequence, it leaves no residuals of the quantum phase diagram at absolute zero temperature.

Dissipation is ubiquitous in nature and plays an essential role in quantum systems such as inducing decoherence of quantum states. Recently, a promising research direction has been to investigate the effect of the non-Hermiticity on the QPT [5–11] and hence discover novel quantum matter. On the other hand, much attention has been paid to an intriguing possibility of dissipation as an efficient tool for the preparation and manipulation of quantum states [12–16]. In this new area, the understanding and controlling of the nonequilibrium dynamics of correlated quantum many-body systems

with dissipation is an urgent issue in diverse fields of physics, ranging from ultracold gases [17], Bose-Einstein condensates (BECs) placed in optical cavities [18], trapped ions [19,20], exciton-polariton BEC [21], and microcavity arrays coupled with superconducting qubits [22,23]. Given the above two fruitful topics, we naturally ask the following questions: Can we establish a non-Hermitian dynamic detection scheme to capture the phase of the Hermitian system and can we accurately predict the phase boundary?

The QPT and the corresponding critical phenomena can be understood with the concepts from quantum information, i.e., the quantum entanglement [24,25], the quantum fidelity [26–28], and the Loschmidt echo (LE) [29–36]. This provides a method for detecting QPT based on the response of the ground state under a perturbation. The recent development of the non-Hermitian Hamiltonian shows that it exhibits exclusive effects never before observed in a Hermitian system [15,37–39]. One of the most interesting phenomena is the critical dynamics based on the exceptional point (EP) [8,10,40]. Addressing the proposed question may shed light on this. In this paper, we propose a scheme to detect the Mott insulator-superfluid QPT based on the EP dynamics. In its essence, if there can exist a non-Hermitian perturbation relating the degenerate states with each other so as to form a Jordan block, then the order of the EP can be arbitrarily modulated according to the degeneracy of the involved states. The EP drives the system to evolve towards the corresponding coalescent state. Unlike the Hermitian system, the system evolution shows directional rather than periodic oscillations even though an initial thermal state is prepared. Based on this mechanism, we examine the response of two celebrated Hubbard-like systems to the external critical non-Hermitian field. It demonstrates that when the system is in the superfluid phase, the non-Hermitian external field forces the degenerate ground state

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to coalesce and thereby leads to a decay of the Loschmidt echoes. In the thermodynamic limit, it converges to zero but stays around 1 in the Mott insulating phase. This dynamical property holds at a low-temperature limit and is insensitive to whether the external field is localized and whether the external field is isotropic. Therefore, it provides a reliable scheme for detecting the Mott insulator-superfluid phase transition in a real physical system.

Our paper is structured as follows: In Sec. II, we give the fundamental mechanism of the proposed non-Hermitian detecting scheme through a simple two-level system. In Secs. III and IV, we apply the proposal to examine the Mott insulator-superfluid QPT in two celebrated correlated many-body systems, namely, the Jaynes-Cummings-Hubbard (JCH) model and Bose-Hubbard (BH) model. We conclude and discuss our results in Sec. V.

II. INSIGHT INTO THE NON-HERMITIAN DETECTION

We first demonstrate the underlying mechanism of the considered proposal through a simple 2×2 matrix. The starting point is a Hermitian two-level system, with the eigenenergies E_1 and E_2 . In the energy representation $\{|\psi_1\rangle, |\psi_2\rangle\}$, the matrix form can be given as

$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, \quad (1)$$

where the two energy levels are arranged in ascending order such that $E_1 < E_2$. We focus on the dynamics of an initial thermal state with density matrix $\rho(0) = e^{-\beta H_0} / \text{Tr}(e^{-\beta H_0})$ at temperature $T = 1/\beta$ with $\hbar = 1$. Evidently, when the two energy levels are near degenerate and the low-temperature limit is assumed, the density matrix $\rho(0)$ is reduced to $\rho_I(0) = I/2$, where I is the identity matrix. On the contrary, if a gap $\delta = E_2 - E_1$ exists between the two involved energy levels, then the initial density matrix of the system is reduced to $\rho_{II}(0) = (I + \sigma_z)/2$. Note in passing that the first type of initial state is a maximally mixed state demonstrated by $\text{Tr}[\rho_I^2(0)] = 1/2$ and the second type of initial thermal state is a pure state characterized by $\text{Tr}[\rho_{II}(0)]^2 = 1$. The interplay between δ and β determines the constituents of each eigenstate in the mixed state. For instance, the larger β is required to involve the information of the excited state in the initial thermal state when δ is large. These evidences play the key role to understand the quench dynamics. After a non-Hermitian quench, the postquench Hamiltonian can be given as $H = H_0 + H'$, wherein $H' = \lambda|\psi_1\rangle\langle\psi_2|$. The corresponding matrix form is

$$H = \begin{pmatrix} E_1 & \lambda \\ 0 & E_2 \end{pmatrix}, \quad (2)$$

where λ is a real number and denotes the non-Hermitian coupling between two such energies. When the two energies of the prequench Hamiltonian are degenerate, $E_1 = E_2$, the postquench Hamiltonian H is in a Jordan block form such that the two eigenstates $|\varphi_1\rangle$ and $|\varphi_2\rangle$ of H coalesce. For any given pure initial state, the quenched Hamiltonian drives it to the coalescent state (see the Appendix for details). However, when $\delta \gg 1$, the quenched Hamiltonian H shares the same spectrum with H_0 ; the eigenstate $|\varphi_1\rangle$ is unchanged even though a nonzero perturbation λ presents, and the eigenstate

$|\varphi_2\rangle$ of H is in a superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$, that is, $|\varphi_2\rangle = (\lambda/\delta)|\varphi_1\rangle + |\varphi_2\rangle$. It is conceivable that the dynamics of the two initial thermal states will exhibit distinct behaviors. To give such differences, we first investigate the time evolution of the density matrix $\rho(t)$. It should obey the following equation:

$$i \frac{\partial \rho(t)}{\partial t} = H \rho(t) - \rho(t) H^\dagger, \quad (3)$$

which admits the formal solution

$$\rho(t) = e^{-iHt} \rho(0) e^{iH^\dagger t}. \quad (4)$$

Due to the non-Hermiticity nature, the time evolution of the density matrix is no longer unitary. Hence, in the subsequent analysis, we normalize $\rho(t)$ by taking

$$\rho(t) = e^{-iHt} \rho(0) e^{iH^\dagger t} / \text{Tr}[e^{-iHt} \rho(0) e^{iH^\dagger t}]. \quad (5)$$

The degree of distinguishability between the initial states $\rho(0)$ and $\rho(t)$ can be identified by the so-called Uhlmann fidelity [41,42],

$$L(t) = \{\text{Tr}[\rho^{1/2}(0)\rho(t)\rho^{1/2}(0)]^{1/2}\}^2, \quad (6)$$

also known as the Loschmidt echo (LE). For the first type of initial state $\rho_I(0)$, the straightforward algebra shows that

$$\rho_I(t) = \frac{1}{\Omega_I(t)} \begin{pmatrix} t^2 \lambda^2 + 1 & -it\lambda \\ it\lambda & 1 \end{pmatrix}, \quad (7)$$

where $\Omega_I(t) = \lambda^2 t^2 + 2$. Substituting $\rho_I(t)$ into Eq. (6), one can immediately obtain

$$L(t) = \left[\frac{1}{2\Omega_I^{1/2}(t)} \left(\sqrt{\Omega_I(t) + \lambda t[\Omega_I(t) + 2]^{1/2}} + \sqrt{\Omega_I(t) - \lambda t[\Omega_I(t) + 2]^{1/2}} \right) \right]^2. \quad (8)$$

Our primary interest here is the steady value of the LE, $L(t)$ ($t \rightarrow \infty$), after a sufficiently long period, which can be given by setting $\lambda t_f \gg 1$. Within this condition, $\Omega_I(t_f) \approx \lambda^2 t_f^2$, and hence $L(t_f) \approx 1/2$. The physical picture is clear: The initial mixed state $\rho_I(0)$ contains components of two parities. When the non-Hermitian coupling λ is switched on, the postquench non-Hermitian Hamiltonian H contains only one coalescent state $|\varphi_c\rangle = |\psi_1\rangle$. Therefore, all the possible initial states will be driven towards this coalescent state. This indicates that the component with a certain parity ($|\psi_1\rangle$) of the thermal state $\rho_I(0)$ is dominant due to the EP dynamics. From this perspective, $\rho_I(t)$ loses half of the information regarding the $|\psi_2\rangle$, which results in $L(t_f) \approx 1/2$. These features do not occur when the Hermitian field H' is applied since $L(t)$ is always 1 as time t goes by. These conclusions still hold for the dynamical detection scheme of an N -fold degenerate system. In that setup, the non-Hermitian detection field is given as $H' = \lambda \sum_{j=1}^{N-1} |\psi_j\rangle\langle\psi_{j+1}|$ and postquench Hamiltonian H possesses only one eigenvalue whose geometric multiplicity is 1. Hence, the high-order EP point up to N -level coalescence is created. Any given arbitrary initial state will evolve towards the coalescent state $|\psi_1\rangle$ after a sufficiently long time. At this time, $\rho_I(t)$ tends to $|\psi_1\rangle\langle\psi_1|$, leading to $L(t_f) \approx 1/N$. It also demonstrates that the order of the EP determines the steady value of LE, $L(t)$.

For the second type of initial state $\rho_{\Pi}(0)$, the time evolution of the density matrix can be readily obtained as $\rho_{\Pi}(t) = \rho_{\Pi}(0) = (I + \sigma_z)/2$. The LE $L(t)$ can be given directly as $L(t) \approx L(0) = 1$. This denotes that a non-Hermitian detection does not substantially affect the dynamics due to the presence of gap δ . It is worth pointing out that if the Hermitian detection field $H' = \lambda|\psi_1\rangle\langle\psi_2| + \text{H.c.}$ is turned on, then LE $L(t) = 1 - \lambda^2 \sin^2(\omega t)/\omega^2$, wherein $\omega = \sqrt{\delta^2/4 + \lambda^2}$. Evidently, it is a periodic function. In the weak coupling limit $\lambda \ll 1$, $L(t_f)$ stays near 1, which is similar to that of the non-Hermitian detection scheme. In the following, we will demonstrate that the considered scheme can be applied to examine the QPTs, which are usually associated with the spontaneous symmetry breaking of the system. When the system enters from one phase to another, the system energy will undergo a transition from gap to gapless, wherein the phase transition point corresponds to the gap closing point. Hence, the system will exhibit distinct dynamic behaviors when it is in the different phases. Such difference can be detected by the current proposed scheme.

III. JAYNES-CUMMINGS-HUBBARD MODEL

The first celebrated QPT model is the JCH model that has emerged as a fundamental model at the interface of quantum optics and condensed matter physics [2,43–48]. It describes strongly correlated photons in a coupled qubit-cavity array and predicts a superfluid–Mott-insulator transition of polaritons. The corresponding Hamiltonian reads

$$H_0 = \sum_i H_i^{\text{JC}} + \sum_{\langle i,j \rangle} \kappa_{ij} (a_i^\dagger a_j + \text{H.c.}) - \sum_i \mu_i N_i, \quad (9)$$

with

$$H_i^{\text{JC}} = \omega_c a_i^\dagger a_i + \omega_a \sigma_i^+ \sigma_i^- + g(a_i \sigma_i^+ + a_i^\dagger \sigma_i^-), \quad (10)$$

$$N_i = \sigma_i^+ \sigma_i^- + a_i^\dagger a_i, \quad (11)$$

where $\sigma_i^+ = |e_i\rangle\langle g_i|$ and $\sigma_i^- = |g_i\rangle\langle e_i|$ (a_i^\dagger, a_i) correspond to the atomic (photonic) raising and lowering operators, respectively. $|g_i\rangle$ and $|e_i\rangle$ are the ground and excited states of the two-level system. The transition energy of the atomic system is ω_a , the cavity resonance is ω_c , and the cavity mediated atom-photon coupling is g , which is assumed to be real for our purposes. The whole system is given by a combination of the Jaynes-Cummings Hamiltonian H_i^{JC} with photon hopping between cavities κ_{ij} ($\langle i, j \rangle$ represent nearest-neighbor pairs) and the chemical potential term μ_i . Here, $N = \sum_i N_i$ is the total number of atomic and photonic excitations, which is a conserved quantity, i.e., $[N, H] = 0$. This is also called U(1) symmetry conserving the number of polaritons.

For simplicity, we assume that the homogeneous inter-cavity hopping $\kappa_{ij} = \kappa \delta_{i,j+1}$ occurs for nearest neighbors and zero disorder, $\mu_i = \mu$, for all sites. Because of the photonic repulsion arising from g , the system supports two phases, that is, the Mott insulating and superfluid phases. Such phases can be determined by employing a mean-field approximation. Although the mean-field theory as an approximation theory is not particularly accurate, it can give the basic property of the ground state of two phases. To capture such property, we

first give the dressed states $|\pm, n\rangle$ of H^{JC} (the subscript i is omitted), where n is the number of excitations in the cavity. The concrete forms of such states can be given as

$$|\pm, n\rangle = \frac{g\sqrt{n}|g, n\rangle + [-\Delta/2 \pm \chi(n)]|e, n-1\rangle}{\sqrt{2\chi^2(n) \pm \chi(n)\Delta}} \quad \forall n \geq 1, \quad (12)$$

and the corresponding eigenenergies are

$$E_{\pm, n} = n\omega_c \pm \chi(n) - \Delta/2, \quad (13)$$

where detuning $\Delta = \omega_c - \omega_a$ and $\chi(n) = \sqrt{ng^2 + \Delta^2/4}$. The ground state for the dressed state system is defined as $|g, 0\rangle$ with eigenenergy $E_g = 0$. Taking the decoupling approximation $a_i^\dagger a_j = \psi^* a_j + \psi a_i^\dagger - |\psi|^2$ with $\psi = \langle a_i \rangle$, we can demonstrate that when the system is in the Mott insulating phase $\psi = 0$, the ground state of the system has a fixed number of polaritonic excitation on each site, which is determined by the system parameters. There must be a gap between the ground state and the first excited state of the system. As a comparison, the system is in the superfluid phase when $\psi \neq 0$. At this time, the system is gapless and the ground state at each site corresponds to a coherent state of excitations over the $|-, n\rangle$ branch. Note that the condition of $E_{-, n} < E_{+, n}$ is assumed. These properties allow us to dynamically identify two such phases by employing the non-Hermitian probing field that can be given as $H' = \lambda \sum_i a_i$ in this scenario. After a quench, one can expect that $L(t)$ of the initial thermal state with a low-temperature limit will not decay due to the protection of the gap. On the contrary, when the system is tuned to the superfluid phase, the ground state is forced to be degenerate to break the U(1) symmetry. The degenerate ground states possessing the different excitation numbers can be related to each other through H' such that a Jordan block form appears. The degeneracy of the ground state determines the order of the EP of H . In the thermodynamic limit, the steady value of $L(t)$ quickly approaches 0 according to the EP dynamics of $\rho_I(t)$ in the aforementioned section. In the finite-size system, the change of the ground state symmetry accords with that predicted by the mean-field theory, but the exact phase boundary cannot be determined by that approximation. As a benchmark, the purity $\text{Tr}[\rho^2(0)]$ is employed to identify whether the ground state is degenerate in the low-temperature limit. Evidently, $\text{Tr}[\rho^2(0)] = 1$ when the ground state is not degenerate. On the other hand, the presence of the degenerate ground states makes the purity tend to $1/N_c$, with N_c denoting the degeneracy. Here, two points should be noted: First, the system is initialized in the ground state and no other excited states are involved when the zero-temperature limit is assumed. All the aforementioned statements about the two types of initial density matrices are strictly true. The so-called low-temperature limit means that thermal fluctuations are not significant enough to excite the system from its ground state. Second, there can exist a gap in the finite-size system even though the system is in the deep superfluid regime ($\kappa \gg g$) characterized by the constant number of the correlation function $\langle a_i^\dagger a_j \rangle$. However, it will vanish as the system dimension increases. This property does not affect the validity of the currently proposed non-Hermitian scheme to detect the QPT boundary at which the excitation spectrum is gapless.

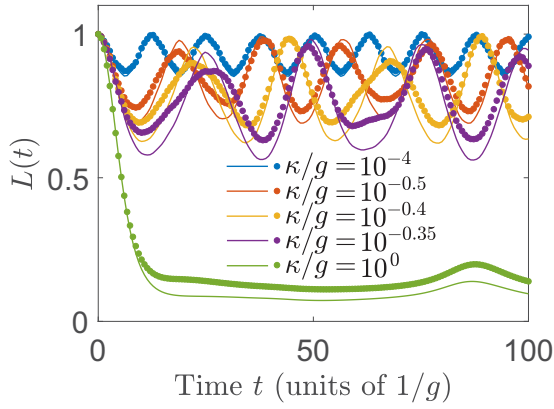


FIG. 1. Time evolution of LEs for different κ with a given chemical potential μ . The lines and dots denote the LEs for $\beta = 5$ and $\beta = 10$, respectively. The system consists of two cavities, where the photon number of each cavity is truncated at a finite value $n = 10$. The other system parameters are $g = 1$, $\omega_c = 5g$, $\Delta = 0$, $\lambda = 0.1g$, and $\mu = -6.5g$. The profiles of the LEs in the two regions are distinct, independent of the temperature of the initial thermal states, and converge to 1.0 and 0.2, respectively.

To verify the above conclusion, we perform the numerical simulations for $L(t)$ of the initial state $\rho(0)$ at different phases in the finite system. In Fig. 1, the non-Hermitian quenched

Hamiltonian that drives the system exhibits two distinct behaviors of $L(t)$: In the Mott insulating phase characterized by a fixed number of excitations per site with no fluctuations, $L(t)$ will stay at 1 as time t goes on; in a superfluid phase, $L(t)$ tends towards a steady value depending on the purity of the initial thermal state. These results agree with our prediction and demonstrate that LEs are insensitive to temperature and tend towards different values in different phases. To compare with the phase diagram obtained by the mean-field theory and the purity, we introduce an average LE in the time interval $[0, T]$ that is defined as

$$\bar{L} = \frac{1}{T} \int_0^T L(t) dt, \quad (14)$$

where $T \gg 1$. The average LE as a function of the values of parameters κ and μ with given $\Delta = 0$ is plotted in Figs. 2(c) and 2(d). Comparing to the order parameter ψ obtained by the mean-field approximation in the thermodynamic limit, it indicates that the average LE can be used to identify the quantum phase diagram at nonzero temperatures even in small-size systems.

Next we will examine how a local external field affects the $L(t)$. Consider the postquench Hamiltonian with the form

$$H = H_0 + \lambda a_i, \quad (15)$$

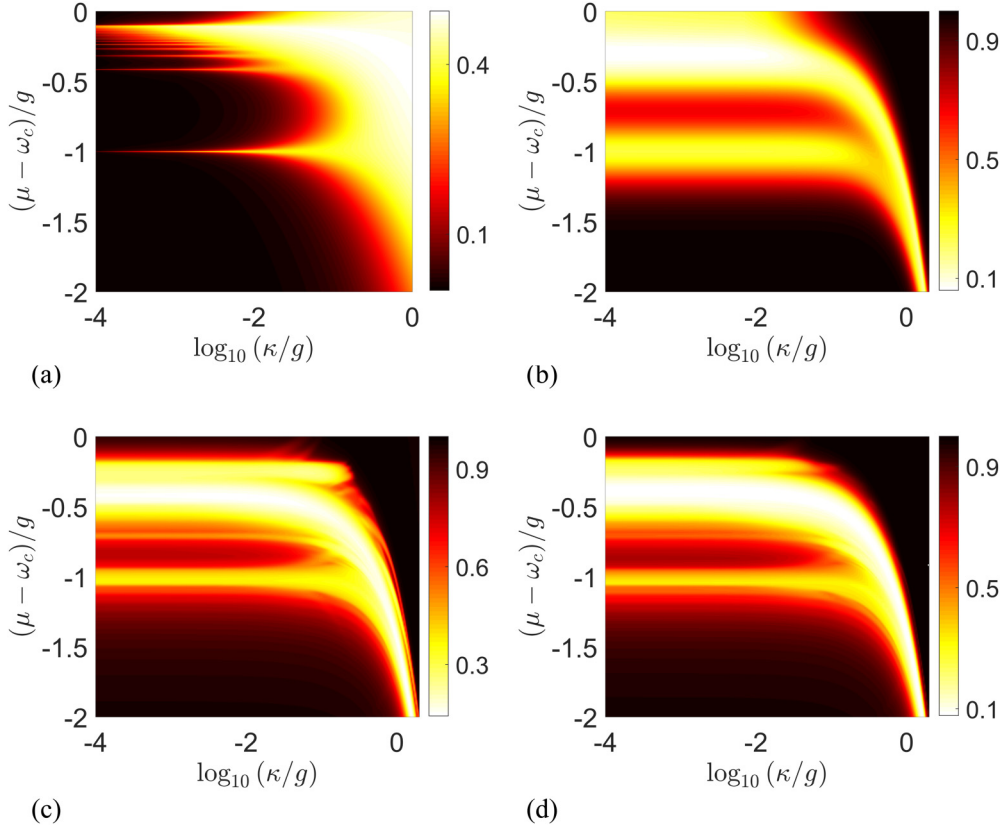


FIG. 2. Comparison of phase diagrams obtained by the mean-field approximation, purity, and average LEs. Here we set $T = 200$. Other parameters: $g = 1$, $\lambda = 0.1$, (c) $\beta = 3$, and (d) $\beta = 20$. The behaviors of \bar{L} and the purity accord with each other and demonstrate the phase boundary even in the finite-size system. It also presents a clear manifestation of the Mott insulator to superfluid quantum phase transition at nonzero temperature. Note that in the upper right part of (b)–(d), the system possesses a gap due to the finite-size effect, which can be expected to vanish as N increases to infinite. Note that the color bar of (a) is reversed as opposed to that of (b)–(d).

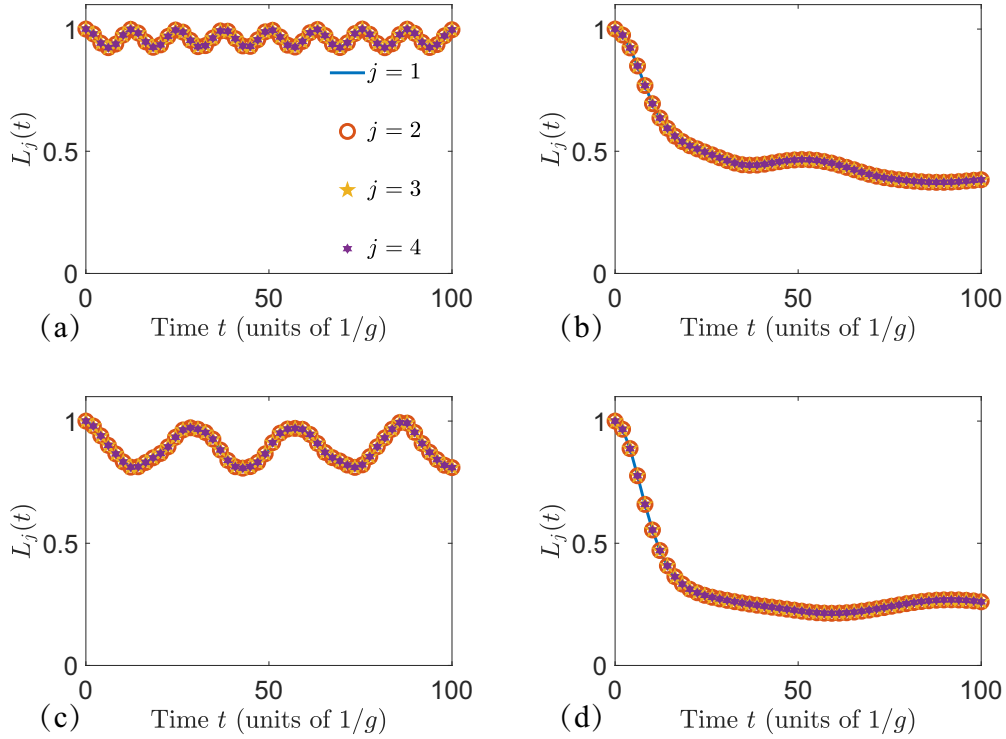


FIG. 3. Numerical simulation for LEs under the postquench Hamiltonian (15) for different i values. (a), (c) LEs in the Mott insulating phase when $\kappa/g = 10^{-4}$ and $\kappa/g = 10^{-0.08}$. (b), (d) LEs in the superfluid phase when $\kappa/g = 10^{-0.3}$ and $\kappa/g = 10^0$. The system is composed of four cavities and the other system parameters are $\mu = -6.5g$ and $\beta = 10$. The LE decays rapidly to 0.25, whereas it remains at one in the Mott insulating phase. This evidence manifests that a local dissipation can lead to a significant change of $\rho(t)$, and thereby serves as a dynamical signature to identify different phases of matter.

where λa_i is the component of operator $\lambda \sum_i a_i$. In this case, the LE is denoted by $L_j(t)$. In the superfluid phase, a local external field can indeed make the degenerate ground states coalesce, and thereby the long-term behavior of $L_j(t)$ is expected to be similar to that of the postquench Hamiltonian in the presence of the global non-Hermitian field. We perform the numerical simulation in Fig. 3. We can see that $L_j(t)$ decay to a steady value in the superfluid phase, but remain 1 in the Mott insulating phase after a sufficiently long time. This accords with our prediction. This evidence manifests that a local dissipation qualitatively affects the dynamics of the initial state through the EP and hence provides an alternative mechanism to probe the QPT from the Mott insulator to superfluid state.

IV. BOSE-HUBBARD MODEL

The second celebrated model delineating a Mott-insulator-superfluid transition is the BH model. The corresponding Hamiltonian is

$$H_0 = - \sum_{(i,j)} \kappa_{ij} (b_i^\dagger b_j + \text{H.c.}) + \frac{U}{2} \sum_i n_i (n_i - 1) - \mu \sum_i n_i. \quad (16)$$

Here, b_i^\dagger and b_i are the bosonic creation and annihilation operators such that $n_i = b_i^\dagger b_i$ gives the number of particles on-site, i . κ_{ij} , μ , and U are tunable parameters of the BH model, corresponding to the tunneling, chemical potential, and interaction

strength, respectively. The system also respects the U(1) symmetry, that is, $[\sum_i n_i, H] = 0$, which conserves the number of

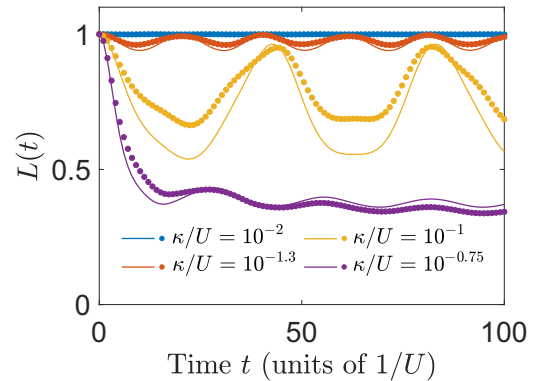


FIG. 4. Time evolution of LEs for different ratio of κ/U . The lines and dots denote the LEs for $\beta = 5$ and $\beta = 10$, respectively. The system consists of four sites with photon numbers truncated at a finite value $n = 15$. The chemical potential is assumed to be $\mu/U = 0.5$ such that each site's occupation number of the ground state is 1 within the Mott insulating phase. The strength of the non-Hermitian perturbation field is set to be $\lambda/U = 0.1$ to induce the Jordan block form in the superfluid phase. Again, the LEs exhibit different dynamical behaviors in two such phases: $L(t)$ tends towards a steady value 0.2 in the Mott insulating phase, whereas it remains 1 in the superfluid phase. The profiles of the LEs are independent of the temperature of the initial thermal states.

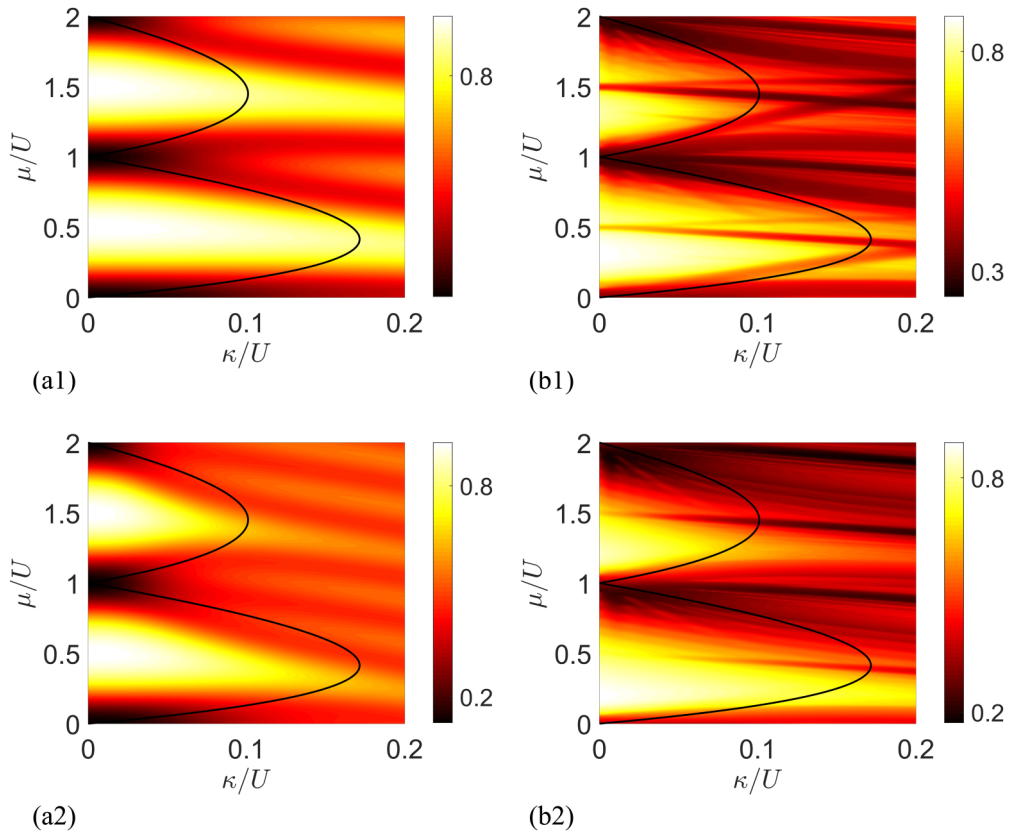


FIG. 5. Contour plot of average LEs, (a) \bar{L} and (b) \bar{L}_i . The Mott lobes denoted by the red and blue solid lines are obtained by the mean-field approximation. Here the local dissipation is applied to site 1 and $T = 200$ is set to evaluate the average LEs. Other system parameters are (a1) $\lambda/U = 0.1$ and $\beta = 3$, (a2) $\lambda/U = 0.1$ and $\beta = 20$, (b1) $\lambda/U = 0.2$ and $\beta = 3$, and (b2) $\lambda/U = 0.2$ and $\beta = 20$. It demonstrates that the response of the thermal state to a nonlocal and local non-Hermitian perturbation field is the same so that two such fields can be served as a signature to witness the QPT in a real experiment.

bosons instead of polaritons. The BH model is closely related to the Hubbard model which originated in solid-state physics as an approximate description of superconducting systems and the motion of electrons between the atoms of a crystalline solid. In the experiment of an ultracold atom loaded into the optical lattice, the considered model can be explored from the superfluid to Mott insulating phases [49–51] by addressing the laser field and manipulating the Feshbach resonance [52–54]. This model can also be used to describe physical systems such as bosonic atoms in an optical lattice, as well as certain magnetic insulators [55–57].

Again, the system exhibits two different phases of matter by tuning the ratio t/U . The Mott insulating phase is essentially a product of single-site states of bosons where there is a finite energy gap opposing the addition of a boson. The excitation spectrum of the superfluid is gapless in the sense that the sum of the energy cost needed to add and to remove one particle from the system is zero. The superfluid phase shows boson number fluctuations instead of polariton number fluctuations in the JCH model. According to the value of ψ , the mean-field phase boundary of the BH model is shown in Fig. 5. With the same procedure, we consider the quench dynamics of the initial thermal state $\rho(0)$. The non-Hermitian applied field is $H' = \lambda \sum_i b_i$. After a quench, we first evaluate the performance of $L(t)$ in two such different phases of matter. The evolved density matrix $\rho(t)$ is the same

as that in the JCH model, which can be shown in Fig. 4. In addition, we numerically compute the $\bar{L}(t)$ and $\bar{L}_i(t)$ in the finite-size system. Figure 5 shows that the Mott lobes can be determined by \bar{L} , indicating that the phase diagram can be preserved in the finite-size system. Note in passing that a local non-Hermitian quench field $H' = \lambda b_i^\dagger$ can also dynamically identify two such phases, which can be shown by comparing Figs. 5(a) and 5(b). The proposed detecting schemes are insensitive to the thermal fluctuation in the low-temperature limit. Hence, it paves the way to understanding the spontaneous symmetry breaking of matter at nonzero temperatures.

V. CONCLUSION

In conclusion, we have witnessed the Mott insulator to superfluid phase transition from zero to nonzero temperatures. The gapless excitation spectrum, which serves as the signature of the U(1) symmetry breaking, is crucial to achieving the conclusion. Such nonzero-temperature QPT can be probed through an inhomogeneous non-Hermitian external field. The evolved state with specific direction arising from the EP dynamics amplifies the difference between two phases of matter, which has no counterpart in the Hermitian regime and allows distinct responses in two such phases. We expect that the scheme proposed in this paper can be exploited to uncover as

yet unexplored Hubbard-like models in a variety of physical systems.

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APPENDIX

Consider that the eigenstates of the two-level system are degenerate. In the basis of $\{|\psi_1\rangle, |\psi_2\rangle\}$, the postquenched Hamiltonian is

$$H = \begin{pmatrix} E & \lambda \\ 0 & E \end{pmatrix}, \quad (\text{A1})$$

where $E = E_1 = E_2$ is supposed. It has a Jordan block structure such that the degenerates become coalesced with a coalescent state, $|\varphi_c\rangle = (1 \ 0)^T$. For an arbitrary initial state $|\Phi(0)\rangle = a|\psi_1\rangle + b|\psi_2\rangle$, its time evolution can be determined by the propagator $U(t)$ that has an explicit form

$$U(t) = e^{-iEt} \left[I_2 - i\lambda t \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right]. \quad (\text{A2})$$

Hence, the evolved state, by neglecting the overall phase, can be given as $|\Phi(t)\rangle = (a - ibt\lambda)|\psi_1\rangle + b|\psi_2\rangle$. After a sufficiently long time, the probability in $|\psi_1\rangle$ overwhelms that in $|\psi_2\rangle$, ensuring the final evolved state be the coalescent state $|\varphi_c\rangle$.

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