Polaron formation in a spin chain by measurement-induced imaginary Zeeman field

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We present a high-rate projective measurement-based approach for controlling nonunitary evolution of a quantum chain of interacting spins. In this approach, we demonstrate that local measurement of a single external spin coupled to the chain can produce a spin polaron, which remains stable after the end of the measurement. This stability results from the fact that the Hilbert space of the chain contains a subspace of *nondecaying* states, stable during the nonunitary evolution. These states determine the resulting final state of the chain and long-term shape of the polaron. In addition to formation of the spin polarons, the presented measurement protocol can be used for distillation of nondecaying states from an initial superposition or mixture.

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I. INTRODUCTION

Control of evolution of single-particle and many-body quantum systems is an important branch of modern physics [1–3] and applied mathematics [4,5] One of the most interesting realizations of such control is given by the quantum Zeno effect [6], where evolution of a quantum system undergoing high-rate repeated measurements slows down as a result of the feedback of the measurement on the measured system. The Zeno effect can provide efficient protocols for controlling spin 1/2 in various kinds of measurements and interactions, including direct coupling to the environment (e.g., Refs. [[7–11]]) for electrons and spin-orbit coupling [12] for cold atoms. In addition, the Zeno effect can lead to slow driven spin dynamics in quantum dots [13] and edge magnetization in graphene [14]. The Zeno effect plays an important role for electrons coupled to the nuclear bath [15–17]. In these systems, the finite rate measurements can be used for producing highly polarized states of arrays of nuclear spins [18]. Recently, the Zeno effect has been studied in the quantum cavity structures considered as a prospective element for quantum technologies utilizing light-solid interfaces [19].

Since interacting quantum spin systems demonstrate a rich variety of phenomena, the understanding of their measurement-based control can provide protocols useful both for handling quantum information and for understanding fundamental aspects of their physics [20–23]. Here, we study the physics of repeated projective measurements on a probe in a system of interacting quantum spins, termed the *Zeno-like* effect in Refs. [24,25]. We consider a single spin coupled to this chain as the probe and show that the selective projective measurements on the probe can be used as an instrument for producing special, almost stationary, states of the entire quantum system. These states correspond to spin polarons, systems of broad interest for understanding the physical properties in various quantum materials [26–30]. The effect of these frequent measurements on the probe amounts to evolution of the spin chain described by a *non-Hermitian* Hamiltonian with a *local* imaginary Zeeman field. The Hilbert space of the system thus consists of two subspaces, corresponding to the states, decaying and nondecaying, when undergoing the dynamics of the *non-Hermitian Hamiltonian*. These nondecaying states form a variety of spin polarons that remain stable after the termination of the measurement.

This paper is organized as follows. In Sec. II we describe the measured system and the measurement protocol, derive the corresponding non-Hermitian Hamiltonian, and find the properties and dimensionalities of the decaying and nondecaying subspaces. In Sec. III we present numerical results of the system evolution during the measurement and demonstrate the resulting spin polaron formation. In addition, in Sec. III we demonstrate that the repeated-measurement protocol can be used for distillation of the states in the Hilbert space of interest. Discussions of the results and conclusions are presented in Sec. IV.

II. NONUNITARY EVOLUTION OF A SPIN CHAIN

A. Total Hamiltonian and selective measurements of a probe

We consider an antiferromagnetic chain with N spins 1/2 described by the Hamiltonian

$$H_{\rm ch} = \sum_{n=1}^{N} (X_n X_{n+1} + Y_n Y_{n+1}), \tag{1}$$

where the coupling strength is set as 1 and X_n, Y_n are the Pauli matrices for the nth spin. We use the periodic boundary conditions with $X_{N+n} = X_n$ and $Y_{N+n} = Y_n$.

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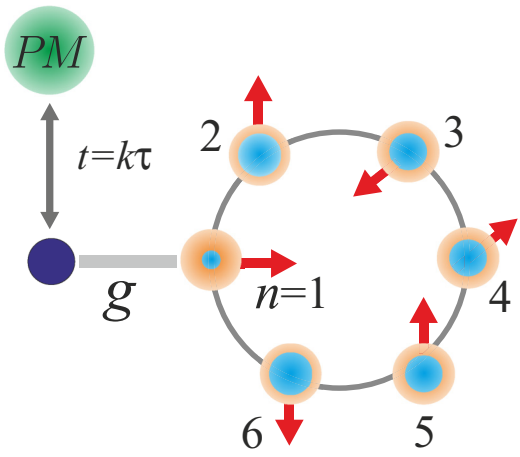

FIG. 1. Schematic picture of a spin chain with locally connected probe (dark blue circle) being under frequent selective measurements. The measurements are performed at times $t = k\tau$, where k is an integer. Red arrows correspond to the in-plane components of the spins, and the sizes of the blue circles correspond to their Z components.

Now we locally couple an additional probe spin to one of the spins ($n = 1$) in the chain, as shown in Fig. 1, such that the total Hamiltonian of the chain + probe reads

$$H = H_{\rm ch} + g(XX_1 + YY_1), \tag{2}$$

where X, Y without indices refer to the probe and g is a coupling constant.

We consider the case where the pure initial state of the total system is $|\Phi_0\rangle = |\uparrow\rangle_{\rm pr} \otimes |\psi_0\rangle$. Here, $|\uparrow\rangle_{\rm pr}$ is the state of probe, and $|\psi_0\rangle$ is a state of the chain. The total system evolution is governed by Hamiltonian (2). After time τ we make a projective measurement on the system $|\uparrow\rangle_{\rm pr}\langle\uparrow|_{\rm pr} \otimes \mathbb{I}_{[N]}$, where $\mathbb{I}_{[N]}$ is the identity operator acting in the Hilbert space of the spin chain. If the measurement is successful, the state of the system becomes $|\uparrow\rangle_{\rm pr} \otimes |\psi_1\rangle$, where $|\psi_1\rangle$ is the evolution-produced state of the chain, and then the next step of evolution and measurement can be made. In the opposite case the evolution is discarded. After M repetitions the state of the chain becomes [31]

$$|\psi_0\rangle \to |\psi_M\rangle = \frac{V^M|\psi_0\rangle}{\sqrt{\langle\psi_0|V^{\dagger M}V^M|\psi_0\rangle}}, \tag{3}$$

where effective nonunitary operator V acting in the Hilbert space of the spin chain defined as $V \equiv \langle\uparrow|_{\rm pr}\exp(-iH\tau)|\uparrow\rangle_{\rm pr}$ is obtained by taking the corresponding matrix element solely in the probe-related subspace. The probability of the outcome $|\uparrow\rangle_{\rm pr}$ of the measurement at a single step with the number $j > 0$ is

$$p_j = \langle\psi_{j-1}|V^\dagger V|\psi_{j-1}\rangle\,. \tag{4}$$

The survival probability of getting M successful sequential probe measurements is

$$P_M = p_1 p_2 \cdots p_M = \langle\psi_0|V^{\dagger M}V^M|\psi_0\rangle\,. \tag{5}$$

We are interested in the high-rate measurement-produced dynamics; thus further we assume $\tau \to 0$. The usual feature of such a nonunitary process is a saturation dynamics [18,32], which means that, on the one hand, $p_k \to 1$ for $k \to \infty$ (or $P_k \to \text{const} > 0$) and, on the other hand, the mean value of some operator or a fidelity related to a state of the chain remains a constant after a large number of iterations. For example, selective measurements of a coupled probe qubit can cool a mechanical oscillator to its ground state [32].

B. Effective Hamiltonian of the spin chain

In order to qualitatively understand the dynamics of the spin chain and characterize the properties of a quantum state after applying an infinite number of iterations, we introduce the *effective* measurement-induced Hamiltonian $H_{\rm M}$ which acts in the Hilbert space of the spin chain [33] such that the nonunitary evolution has the form

$$V = e^{-iH_{\rm M}\tau}. \tag{6}$$

Assuming $\tau \to 0$ (and $g\tau \to 0$), we write $H_{\rm M}$ as a small-τ expansion with $H_{\rm M}(\tau) = A + \tau B$, where operators A and B are usually Hermitian and non-Hermitian, respectively. Such a non-Hermiticity has been noticed in earlier publications, for instance, Eq. (21) in Ref. [25]. In order to find $H_{\rm M}$ in (6) by matching it with the definition of V following (3), we write

$$\langle\uparrow|_{\rm pr}\mathbb{I} - i\tau H - \frac{\tau^2}{2}H^2|\uparrow\rangle_{\rm pr} + O(\tau^3) = e^{-iA\tau - iB\tau^2}, \tag{7}$$

where $\mathbb{I}$ is the identity operator acting in the total chain-probe Hilbert space. Then, by using the explicit equation (2) we transform [with an accuracy of $O(\tau^3)$] the left-hand side of (7) into

$$\begin{aligned}
&\mathbb{I}_{[N]} - i\tau H_{\rm ch} - \frac{\tau^2}{2}H_{\rm ch}^2 - \tau^2 g^2(\mathbb{I}_{[N]} - Z_1 \otimes \mathbb{I}_{[N-1]}) \\
&\quad = e^{-iH_{\rm ch}\tau} - \tau^2 g^2(\mathbb{I}_{[N]} - Z_1 \otimes \mathbb{I}_{[N-1]}) \\
&\quad = e^{-iH_{\rm ch}\tau}e^{-g^2(\mathbb{I}_{[N]} - Z_1\otimes\mathbb{I}_{[N-1]})\tau^2} \\
&\quad = e^{-iH_{\rm ch}\tau - i[-ig^2(\mathbb{I}_{[N]} - Z_1\otimes\mathbb{I}_{[N-1]})]\tau^2},
\end{aligned} \tag{8}$$

where Z_n is the Pauli operator σ_z acting on the nth spin in the chain and $\mathbb{I}_{[N-1]}$ is the identity operator for all the spins except that with $n = 1$. By comparing (8) with the right-hand side of (7) we found $A = H_{\rm ch}$ and $B = -ig^2(\mathbb{I}_{[N]} - Z_1 \otimes \mathbb{I}_{[N-1]})$. Thus, finally, we have

$$H_{\rm M} = H_{\rm ch} + ig^2\tau Z_1 \otimes \mathbb{I}_{[N-1]} - ig^2\tau\mathbb{I}_{[N]}. \tag{9}$$

The form of the second term (9) shows that our process can be mimicked by introducing an *imaginary Zeeman field* [34–36]. Here, we should note the following: (i) Our setting produces a *local* imaginary magnetic field instead of the uniform one [37] simulated by atomic ensembles with real radiation decay, and (ii) in the limit $\tau \to 0$ (and $g^2\tau \to 0$) we come to the bare spin chain Hamiltonian. The magnitude of this field is proportional to the squared coupling constant between the probe and the spin from the chain and linearly proportional to the time interval between two measurements. The third term in (9) ensures that the norm of a state vector after acting on the evolution governed by (9) will not increase.

To clarify the effect of the imaginary magnetic field, we rewrite (9) in the following form:

$$H_{\mathrm{M}} = H_{\mathrm{ch}} - 2ig^2\tau\,\Pi_1, \quad \Pi_1 = |\downarrow\rangle_1\langle\downarrow|_1 \otimes \mathbb{I}_{[N-1]}, \tag{10}$$

with operator Π_1 acting as a projector $|\downarrow\rangle\langle\downarrow|$ on spin number 1 and as a unity operator to other spins in the chain.

C. Decaying and nondecaying subspaces

Dynamical evolution of the initial state $|\psi_0\rangle$ governed by non-Hermitian H_{M} can be described as (see Ref. [38])

$$|\psi(t)\rangle = \sum_{j=1}^{2^N} \langle\beta_j|\psi_0\rangle\, e^{-i\lambda_j t}\,|\alpha_j\rangle\,, \tag{11}$$

where

$$H_{\mathrm{M}}\,|\alpha_j\rangle = \lambda_j\,|\alpha_j\rangle\,, \quad H_{\mathrm{M}}^{\dagger}\,|\beta_j\rangle = \lambda_j^{*}\,|\beta_j\rangle\,. \tag{12}$$

The operator Π_1 in Hamiltonian (10) is positive semidefinite, and thus we have $\mathrm{Im}\lambda_j \leqslant 0$, which leads to exponential suppression of all eigenstates of H_{M} with $\mathrm{Im}\lambda_j < 0$. In contrast, the states with $\mathrm{Im}\lambda_j = 0$ will not decay.

The Hilbert space $\mathcal{H}$ of the chain can be divided into two subspaces: decaying $\mathcal{H}_{\mathrm{D}}$ and nondecaying $\mathcal{H}_{\mathrm{ND}}$, with $\mathcal{H} = \mathcal{H}_{\mathrm{D}} \oplus \mathcal{H}_{\mathrm{ND}}$. As a result, any initial state of the chain can be decomposed into a superposition of two orthogonal vectors belonging to these subspaces: $|\psi_0\rangle = a\,|\mathrm{D}\rangle + b\,|\mathrm{ND}\rangle$, where $|\mathrm{D}\rangle$ ($|\mathrm{ND}\rangle$) belongs to the decaying (nondecaying) subspace. The survival probability P_∞ of the infinite process is given by $P_\infty = |b|^2$. The average survival probability for a random pure initial state as well as for a thermal mixed state with infinite temperature [but not for pure uniform superposition of eigenstates (1)] is

$$\overline{P}_\infty = \frac{\dim\mathcal{H}_{\mathrm{ND}}}{\dim\mathcal{H}}. \tag{13}$$

Here, we define the average survival probability as $\overline{P}_\infty = 2^{-N}\sum_{m=1}^{2^N} P_\infty(m)$, where $P_\infty(m)$ is the survival probability of the infinite process with an initial state equal to the mth vector of some fixed basis in $\mathcal{H}$.

Let us define basis $\{|\phi_j\rangle\}$ with $j = 1\cdots K$ in $\mathcal{H}$, where $|\phi_j\rangle$ are the eigenstates of H_{ch} that satisfy

$$\Pi_1\,|\phi_j\rangle = 0. \tag{14}$$

Note that in the case of degeneracy of H_{ch} we have the freedom to choose $\{|\phi_j\rangle\}$ to satisfy (14), and thus we can define

$$\dim\mathcal{H}_{\mathrm{ND}} = \max(K). \tag{15}$$

Thus from (14) and (15) we conclude that $\dim\mathcal{H}_{\mathrm{ND}}$ does not depend on the value of $g^2\tau$ in (10). We find numerically the following expression for nondecaying subspace dimension:

$$\dim\mathcal{H}_{\mathrm{ND}} = \begin{cases} 2^{(N-1)/2}, & \text{odd } N \\ 3\times\,2^{(N-4)/2}, & \text{even } N. \end{cases} \tag{16}$$

As follows from (13) and (16) we have exponential decay of $\overline{P}_\infty$ with increasing spin chain size N: $\overline{P}_\infty \propto 2^{-N/2}$. In Fig. 2(a) we show the numerical results for $\log_2\overline{P}_\infty$ as a function of N.

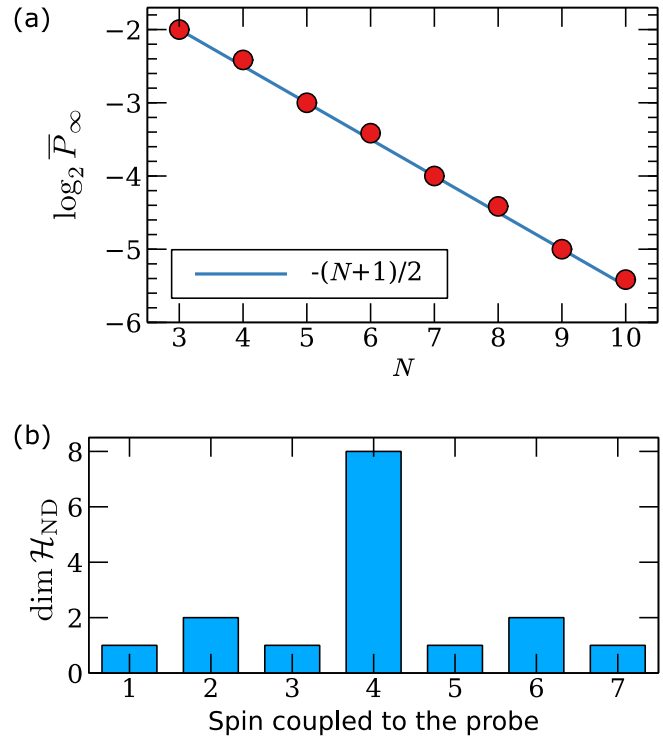

FIG. 2. (a) Numerically estimated survival probability $\overline{P}_\infty$ as a function of spin number N. (b) Dimension of nondecaying subspace vs the number of spin which probe coupled with for an open chain with $N = 7$.

It is interesting to note the following. On the one hand, $\dim\mathcal{H}_{\mathrm{ND}}$ is equal to the number of real eigenvalues of (10), and on the other hand, we can make random unitary transformations of Hamiltonian (10) to obtain a set of random non-Hermitian matrices. It is known [39] that the expected number R_L of real eigenvalues of an $L\times L$ random non-Hermitian matrix for $L\to\infty$ is $R_L \approx \sqrt{2L/\pi} \propto L^{1/2}$. In our case the dimension of random matrices is $L = 2^N$; thus $R_L \propto 2^{N/2} \propto \dim\mathcal{H}_{\mathrm{ND}}$, which coincides with (16).

For completeness, we mention that for a spin chain with open boundary conditions, our expression (10) is the same, except that the value of $\dim\mathcal{H}_{\mathrm{ND}}$ depends on the position of the spin connected to the probe. Our numerical estimation shows that $\dim\mathcal{H}_{\mathrm{ND}} = 1$ for any position of the probe for chains with even N, and $\dim\mathcal{H}_{\mathrm{ND}}$ has the maximum value for the probe connected to the central spin for chains with odd N [see Fig. 2(b) for an example]. As can be seen from (10) and (14) the general form of the state vector which is the result of the process after a large number of iterations can be written as

$$|\psi_M\rangle \approx e^{-iH_{\mathrm{M}}M\tau} \approx |\uparrow\rangle_1 \otimes |\bar{\psi}_M\rangle\,, \quad M\to\infty, \tag{17}$$

where $|\bar{\psi}_M\rangle$ is the state of all spins in the chain except the first one. Expression (17) can be seen as a highly localized spin polaron [26–30]. Note that all states in $\mathcal{H}_{\mathrm{ND}}$ have the form (17). Moreover, the state $|\uparrow\rangle_{\mathrm{pr}} \otimes |\mathrm{ND}\rangle$ is an eigenstate of the total Hamiltonian (2), and this is why it remains stable during the process with the survival probability of each step being equal to 1. In other words, the probe remains disentangled from the chain being in the $|\mathrm{ND}\rangle$ state under the evolution

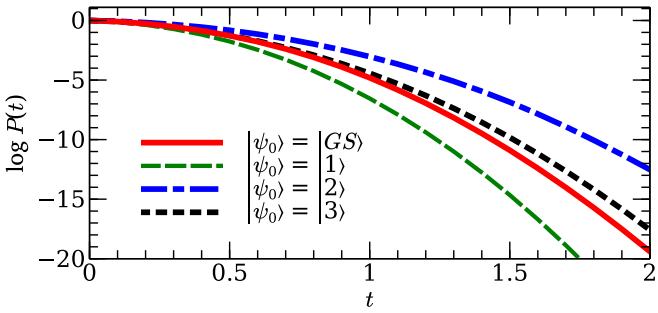

FIG. 3. Survival probability P as a function of time for the first four eigenstates as initial states of the spin chain with $N = 6$, $\tau = 0.03$, and $g = 4$.

governed by (2). This situation may be comparable to the chain state being in the decoherence-free subspace [40].

III. NUMERICAL EVIDENCE FOR SPIN POLARON FORMATION

To numerically illustrate the above statements on the decaying and nondecaying subspaces and demonstrate the evolution of the measured system, we use an $N = 6$ spin chain, with coupling to probe with strength $g = 4$, and intervals between measurements are $\tau = 0.03$. We numerically found that for the chosen spin chain, the first eigenstates are lying in the $\mathcal{H}_{\rm D}$ subspace. In Fig. 3 we show how survival probability decreases with time for initial states from the $\mathcal{H}_{\rm D}$ subspace.

As an example of dynamical spin polaron formation we use initial state $|\psi_0\rangle = \sum_{l=0}^{16} |l\rangle / \sqrt{17}$, which is the one of possible uniform superposition of eigenstates of the first four energy levels of the chain with $N = 6$, $g = 4$, and $\tau = 0.03$. In this setting the ground state is nondegenerate, the first excited level has degeneracy 2, the second level has degeneracy 6, and the third one has degeneracy 8. In Fig. 4(a) we show magnetization $\langle\psi_k|Z_n|\psi_k\rangle$ of each spin in the chain, where k is the number of the step. For convenience we use time $t = k\tau$ instead of number k in the plot. Interaction and measurements with the probe are switched off at $t = 12$ (corresponding to $M = 400$ steps), and after this, spin chain evolution is governed by the Hamiltonian (1). The survival probability of such a process (for our choosing of eigenstates in degenerate subspaces) is $P_{400} \approx 0.02$. For comparison, in Fig. 4(b) we let $g = 0.5$, and the nonunitary process is not interrupted. The survival probability of this process $P_\infty \approx 0.017$, which coincides with the squared amplitude of the initial state in the $\mathcal{H}_{\rm ND}$. Note that, as expected, $\mathcal{H}_{\rm ND}$ is the same for both $g = 4$ and $g = 0.5$. Figure 4 shows that the process ends up with the formation of a localized spin polaron at spin 1 of the chain. As can be seen, the total magnetization of the chain increases during the nonunitary process and, after switching off the probe, remains constant due to the fact that $[\sum_n Z_n, H_{\rm ch}] = 0$.

To study averaged dynamics, we use 100 randomly chosen initial states $|\psi_0\rangle$ of the chain and plot the average magnetization for this ensemble in Fig. 5. In this simulation we apply $M = 300$ evolution-measurement cycles; that is, after time $T = 300\tau = 9$ (vertical line in Fig. 5) the evolution of the spin chain is governed by the Hamiltonian (1), and the probe

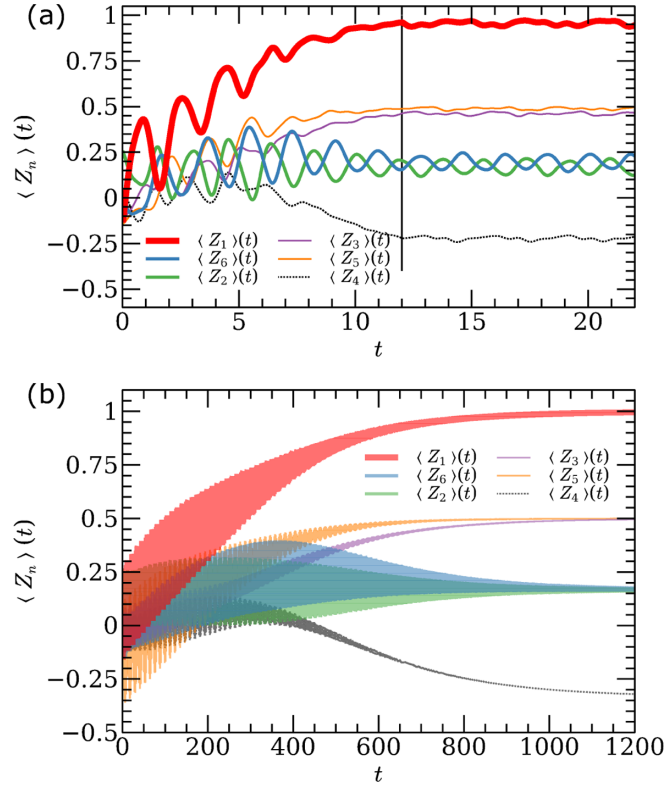

FIG. 4. Example of magnetization dynamics for the spin chain with $N = 6$, $\tau = 0.03$. (a) $g = 4$, and the probe spin is disconnected at $t = 12$ (vertical line). (b) $g = 0.5$, and the probe is not disconnected. Numeration of spins corresponds to Fig. 1.

is disconnected from the chain. As can be seen in Fig. 5, the quantum state of each chain in the ensemble approximately satisfies (17) as a result of the nonunitary process. Relation (17) also holds after switching off the probe. This is due to our special choice of couplings between the chain and probe: The first term on the right-hand side of (10) coincides with the bare spin chain Hamiltonian (1). Calculations including the ZZ coupling to the probe-chain interaction show that in

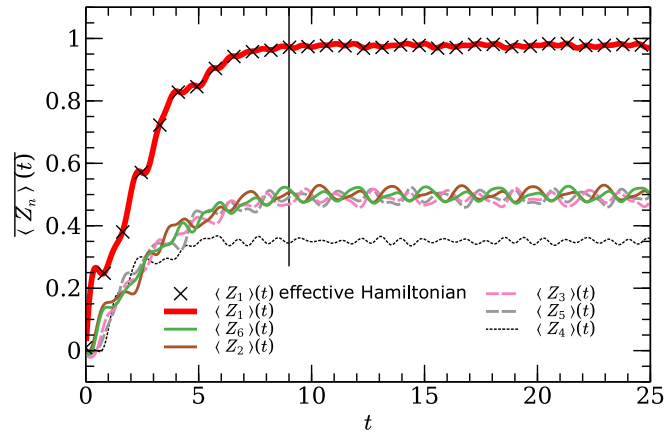

FIG. 5. Averaged local magnetization from the ensemble of 100 random initial states. The probe spin disconnected at $t = 9$ (vertical line), and after this time, spin chain evolution is governed by Hamiltonian (1).

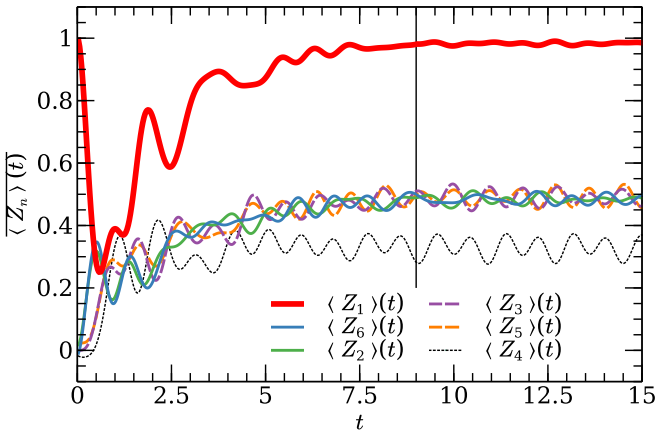

FIG. 6. Averaged local magnetization from the ensemble of 100 random initial states in the form $|\psi_0\rangle = |\uparrow\rangle_1 \otimes |\text{Random state}\rangle$.

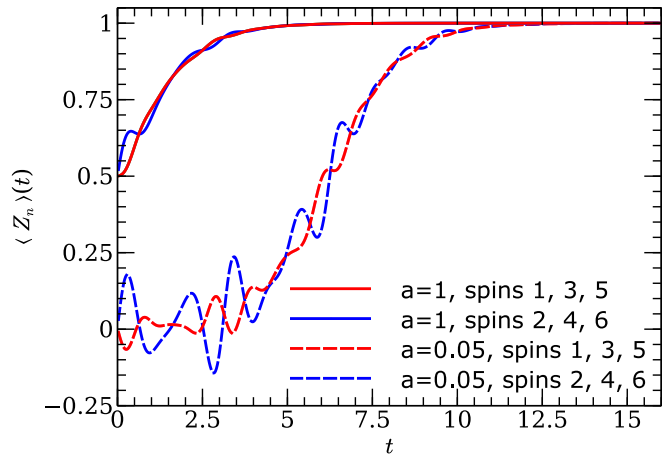

FIG. 7. Distillation of a nondecaying state $|\Uparrow\rangle$ from initial superposition $|\psi_0\rangle = (|\text{GS}\rangle + a\,|\Uparrow\rangle)(1 + |a|^2)^{-1/2}$.

this case the polaron becomes unstable after switching off the probe. This results from the fact that the Hermitian part of the Hamiltonian in Eq. (10) does not coincide with H_{ch} anymore. On the other hand, adding a ZZ-coupling term to the chain Hamiltonian without changing the probe-chain interaction does not change our results dramatically producing only a difference in the values of $\dim \mathcal{H}_{\text{ND}}$. For example, at $N = 6$ one obtains $\dim \mathcal{H}_{\text{ND}} = 6$ for the XY spin chain Hamiltonian (1), while one obtains $\dim \mathcal{H}_{\text{ND}} = 3$ for the XXX coupling [41]. Also we can see that for our choice of τ and g, the result from a direct simulation of our discrete algorithm coincides with the result obtained from continuous evolution governed by non-Hermitian Hamiltonian (10). The average survival probability of the whole process is $P_{300} \approx 0.097$, which is little higher than the $6/64 \approx 0.94$ predicted by expression (13) with $\dim \mathcal{H}_{\text{ND}} = 6$ and $\dim \mathcal{H} = 64$ due to $P_\infty < P_{300}$.

The relation (17) is necessary but not sufficient for a state $|\psi\rangle$ to belong to $\mathcal{H}_{\text{ND}}$. To illustrate this circumstance, we show in Fig. 6 the averaged result of 100 processes with initial state

$$|\psi_0\rangle = |\uparrow\rangle_1 \otimes |\text{Random state}\rangle\,. \tag{18}$$

As can be seen in Fig. 6, such a choice of initial state can be treated as an unstable polaron, which, in turn, can be transformed into a stable one via a nonunitary process. The average survival probability for such a special initial state in our simulation is $P_{300} \approx 0.19$, which is approximately a factor of 2 larger than that for totally random initial states (13) in the previous example. This is because when we choose the initial state in the form (18) we effectively truncate $\dim \mathcal{H}$ in expression (13) twice and do not change $\dim \mathcal{H}_{\text{ND}}$.

Exponential sensitivity to initial states is a characteristic feature [42,43] of nonunitary processes in quantum mechanics. The algorithm described can be used to extract states in the $\mathcal{H}_{\text{ND}}$ from the initial superposition even for cases of a small amplitude of such states. The "price" for this possibility is a proportionally small survival probability P_∞. As can be easily found the ground state $|\text{GS}\rangle$ of a spin chain (1) with $N = 6$ is a decaying state, while the state $|\Uparrow\rangle = |\uparrow\uparrow\cdots\uparrow\rangle$ with all spins being up is a nondecaying one.

Let us take an initial state in the form $|\psi_0\rangle = (|\text{GS}\rangle + a\,|\Uparrow\rangle)(1 + |a|^2)^{-1/2}$ and simulate the process with $g = 4$ and $\tau = 0.03$ for $a = 1$ and $a = 0.05$ (see Fig. 7). As expected, the survival probability is $P_\infty = |a|^2/(1 + |a|^2)$, and as a result we have a chain in the state $|\Uparrow\rangle$. Note that in this example we achieve not a polaron but a uniform magnetization of a chain by using local control. Exponential amplification of such small perturbations in the initial states also can be seen as a "butterfly effect."

Note that if the initial state of the spin chain is a mixture of the form

$$\rho_0 = p\,|\text{ND}\rangle\,\langle\text{ND}| + (1 - p)\,|\text{D}\rangle\,\langle\text{D}|\,, \tag{19}$$

our algorithm transforms it into a pure state $|\text{ND}\rangle$ with survival probability $P_\infty = p$. Thus our proposal can distill one many-body state from another via only local [44,45] control.

As mentioned in the above, the proposed process can also be used for quantum state discrimination [46]. For instance, if we have an *a priori* known set of states $\{|\psi_1\rangle = |\text{ND}\rangle\,,\ |\psi_2\rangle = a\,|\text{ND}\rangle + b\,|\text{D}\rangle\}$ and the initial state of the chain is one of them, we can distinguish these two possibilities via running our nonunitary process and check whether outcome $|\downarrow\rangle_{\text{pr}}$ happens. In the general case, $|\,\langle\psi_1|\psi_2\rangle\,| \geqslant 0$, and we need only one copy of a quantum state.

IV. DISCUSSION AND CONCLUSION

We have shown that the effect of high-rate projective measurements of a single spin coupled to an antiferromagnetic quantum spin-1/2 chain can be mimicked by the evolution of the chain caused by a non-Hermitian Hamiltonian. This Hamiltonian includes an imaginary Zeeman field determined by a short time interval between the measurements and coupling between the measured spin and the chain. The physical origin of this non-Hermiticity, not based on the decay of the states involved in the spin dynamics and measurement, is therefore qualitatively different from that presented in Ref. [37]. We obtained this non-Hermitian operator and identified the sets of states and corresponding Hilbert subspaces decaying and surviving under the action of this measurement-produced Hamiltonian. Thus the measurement process results in the distillation of the corresponding states. The surviving

states produce a stable ferromagnetic (with nonzero total spin) polaron with the expectation values of the spins oscillating around some stationary values after the end of the measurement. Since the produced polarons depend on the initial state of the quantum chain and the measurement protocol, these results can be applied to formation of quantum spin states on demand as well.

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[1] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, Cambridge, 2009).

[2] F. Poggiali, P. Cappellaro, and N. Fabbri, Phys. Rev. X **8**, 021059 (2018).

[3] C. P. Koch, M. Lemeshko, and D. Sugny, Rev. Mod. Phys. **91**, 035005 (2019).

[4] A. Borzi, G. Ciaramella, and M. Sprengel, *Formulation and Numerical Solution of Quantum Control Problems* (SIAM-Society for Industrial & Applied Mathematics, Philadelphia, 2017).

[5] V. Krotov, *Global Methods in Optimal Control Theory* (CRC Press, New York, 1995).

[6] W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990).

[7] D. F. Mundarain and J. Stephany, Phys. Rev. A **73**, 042113 (2006).

[8] V. V. Dobrovitski, A. E. Feiguin, D. D. Awschalom, and R. Hanson, Phys. Rev. B **77**, 245212 (2008).

[9] J. Wolters, M. Strauß, R. S. Schoenfeld, and O. Benson, Phys. Rev. A **88**, 020101(R) (2013).

[10] A. Luis, I. Gonzalo, and M. A. Porras, Phys. Rev. A **87**, 064102 (2013).

[11] S. He, Q.-H. Chen, and H. Zheng, Phys. Rev. A **95**, 062109 (2017).

[12] E. Y. Sherman and D. Sokolovski, New J. Phys. **16**, 015013 (2014).

[13] D. V. Khomitsky, L. V. Gulyaev, and E. Y. Sherman, Phys. Rev. B **85**, 125312 (2012).

[14] M. Golor, S. Wessel, and M. J. Schmidt, Phys. Rev. Lett. **112**, 046601 (2014).

[15] D. Klauser, W. A. Coish, and D. Loss, Phys. Rev. B **78**, 205301 (2008).

[16] T. Nutz, P. Androvitsaneas, A. Young, R. Oulton, and D. P. S. McCutcheon, Phys. Rev. A **99**, 053853 (2019).

[17] D. S. Smirnov, E. A. Zhukov, D. R. Yakovlev, E. Kirstein, M. Bayer, and A. Greilich, Phys. Rev. B **102**, 235413 (2020).

[18] L.-A. Wu, J. Phys. A: Math. Theor. **44**, 325302 (2011).

[19] N. V. Leppenen, L. Lanco, and D. S. Smirnov, Phys. Rev. B **103**, 045413 (2021).

[20] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).

[21] G. Burkard, D. Loss, and D. P. DiVincenzo, Phys. Rev. B **59**, 2070 (1999).

[22] S. Bose, Phys. Rev. Lett. **91**, 207901 (2003).

[23] A. Das and B. K. Chakrabarti, Rev. Mod. Phys. **80**, 1061 (2008).

[24] H. Nakazato, T. Takazawa, and K. Yuasa, Phys. Rev. Lett. **90**, 060401 (2003).

[25] L.-A. Wu, D. A. Lidar, and S. Schneider, Phys. Rev. A **70**, 032322 (2004).

[26] L. I. Glazman and A. S. Ioselevich, JETP Lett. **47**, 547 (1988).

[27] F. Grusdt, M. Kánasz-Nagy, A. Bohrdt, C. S. Chiu, G. Ji, M. Greiner, D. Greif, and E. Demler, Phys. Rev. X **8**, 011046 (2018).

[28] J. E. Prilepsky, A. S. Kovalev, M. Johansson, and Y. S. Kivshar, Phys. Rev. B **74**, 132404 (2006).

[29] H. Nakano and Y. Takahashi, J. Phys. Soc. Jpn. **72**, 1191 (2003).

[30] R. Hayn, A. F. Barabanov, J. Schulenburg, and J. Richter, Phys. Rev. B **53**, 11714 (1996).

[31] Expression (3) can be derived by taking into account the obvious $M = 1$ case and then using the induction by assuming $|\psi_{M-1}\rangle = \frac{V^{M-1}|\psi_0\rangle}{\sqrt{\langle\psi_0|V^{\dagger M-1}V^{M-1}|\psi_0\rangle}}$ and $|\psi_M\rangle = \frac{V|\psi_{M-1}\rangle}{\sqrt{\langle\psi_{M-1}|V^{\dagger}V|\psi_{M-1}\rangle}}$.

[32] Y. Li, L.-A. Wu, Y.-D. Wang, and L.-P. Yang, Phys. Rev. B **84**, 094502 (2011).

[33] X. Turkeshi, A. Biella, R. Fazio, M. Dalmonte, and M. Schiró, Phys. Rev. B **103**, 224210 (2021).

[34] T. D. Lee and C. N. Yang, Phys. Rev. **87**, 410 (1952).

[35] B. M. McCoy and T. T. Wu, Phys. Rev. **155**, 438 (1967).

[36] V. Azcoiti, G. Di Carlo, E. Follana, and E. Royo-Amondarain, Phys. Rev. E **96**, 032114 (2017).

[37] T. E. Lee and C.-K. Chan, Phys. Rev. X **4**, 041001 (2014).

[38] G. Dattoli, A. Torre, and R. Mignani, Phys. Rev. A **42**, 1467 (1990).

[39] A. Edelman, E. Kostlan, and M. Shub, J. Am. Math. Soc. **7**, 247 (1994).

[40] L.-A. Wu, P. Zanardi, and D. A. Lidar, Phys. Rev. Lett. **95**, 130501 (2005).

[41] Note that instead of the XY or XXX model the bare spin chain Hamiltonian can be represented by any Hermitian matrix. As can be checked numerically, for most such random Hamiltonians we have $\dim \mathcal{H}_{\rm ND} = 0$, and thus our proposal becomes not applicable. The condition $\dim \mathcal{H}_{\rm ND} > 0$ is obviously related to the symmetries of $H_{\rm ch}$ and degeneracies in its spectrum.

[42] A. Gilyén, T. Kiss, and I. Jex, Sci. Rep. **6**, 20076 (2016).

[43] G. Zhu, O. Kálmán, K. Wang, L. Xiao, D. Qu, X. Zhan, Z. Bian, T. Kiss, and P. Xue, Phys. Rev. A **100**, 052307 (2019).

[44] P. V. Pyshkin, E. Y. Sherman, J. Q. You, and L.-A. Wu, New J. Phys. **20**, 105006 (2018).

[45] P. V. Pyshkin, E. Y. Sherman, and L.-A. Wu, Phys. Rev. A **100**, 063401 (2019).

[46] J. Bae and L.-C. Kwek, J. Phys. A: Math. Theor. **48**, 083001 (2015).