Anomalous out-of-phase magnetic ac response in superconducting wires

Leonid Burlachkov [*] and Nikita Fuzailov [†]
Department of Physics, Bar-Ilan University, Ramat-Gan 5290002, Israel

(Received 1 February 2021; revised 30 June 2021; accepted 16 August 2021; published 26 August 2021)

We describe and explain two competing regimes of ac magnetic response in current-carrying type-II superconducting wires which were observed experimentally. In the usual regime, voltage $V(t)$, induced by vortex motion across the wire, is "in phase" with the external magnetic field $H(t) \propto \sin(\omega t)$. However, as frequency ω grows up or transport current I decreases, an anomalous, "out-of-phase" peak in $V(t)$ appears. If these two regimes coexist, then two peaks in voltage are observed per half period of $H(t)$ in both experiment and numerical simulations. At certain combinations of ω, I and the amplitude of the external field, the out-of-phase mechanism even overwhelms the usual, in-phase one. It is shown that the out-of-phase maximum in $V(t)$ is due to the inhibition effect of zero-field (annihilation) lines on flux motion. Such lines, if present in the sample, significantly decelerate magnetic relaxation and dramatically affect the induced voltage. A phase diagram enabling one to distinguish between the in-phase and out-of-phase regimes is constructed.

DOI: 10.1103/PhysRevB.104.064512

I. INTRODUCTION

This theoretical work was initiated by experimental studies [1,2] of the magnetic ac response in industrial current-carrying $YBa_2Cu_3O_{7-x}$ (YBCO) and $Bi_2Sr_2Ca_2Cu_3O_{10+x}$ (BSCCO) wires (tapes). These tapes were embedded at $T = 77$ K into an oscillating magnetic field $H(t) = H_{max} \sin(\omega t)$ directed perpendicular to the tape's surface, and voltage $V(t)$, which results from the magnetic flux motion across the tape, was measured along the direction of dc transport current I. Depending on the combination of three parameters, i.e., H_{max}, ω, and I, two different regimes in the magnetic response were observed [1,2]. In the first regime, the maxima of voltage $V(t)$ are in phase with $H(t)$. Such a behavior is easily understandable since the voltage is expected to be proportional to the number of moving vortices, which, in turn, should increase along with H. Contrariwise, in the out-of-phase regime, found at moderate ω and/or low enough I, a second maximum in $V(t)$ surprisingly appears, competes with the in-phase one, and even overwhelms it. This indicates the presence of a different, competing mechanism for flux motion and energy dissipation.

The description of the ac magnetic response in type-II superconductors is commonly based on the Bean model [3] for the critical state or its modifications, such as the Kim model [4]; see, also, [5–8]. All these descriptions imply infinitely fast flux flow at $j > j_c$ and extremely slow flux creep at $j < j_c$, where $j_c(B)$ is the critical density of the magnetization current and B is the mean (averaged over scales much greater than intervortex distance) magnetic field in the sample. Within such an approach, the magnetization current j is considered to be equal (by absolute value) to $j_c(B)$ everywhere. However, starting from the pioneer work on giant flux creep [9], it became clear that the applicability of critical state models is rather limited in high-temperature superconductors (HTSC). Except for the region of low temperatures, which varies from compound to compound, say, $T \lesssim 40$ K in YBCO and $T \lesssim 20$ K in BSCCO, the activation energy U associated with flux motion is low (of the order of kT, where k is the Boltzmann constant) or just vanishing. Therefore, vortices are almost unpinned, not ordered in a regular lattice due to thermal fluctuations, and, correspondingly, relaxation of the magnetization current is very fast [9]. As a result, the experimentally measured j appears to be a function of the sweeping rate of the external magnetic field, experimental "time window," history of the sample (field cooled or zero-field cooled), and other parameters. Usually, j turns out to be significantly less than j_c and even less than the depinning current j_{depin}; see Refs. [10,11] as reviews. The temperature $T = 77$ K used in experiments [1,2] is high enough to rule out the applicability of Bean-like critical state models.

In order to study flux dynamics in HTSC at high temperatures, one has to rule out Bean-like models and analyze the vortex diffusion equation [12–15], which results directly from Maxwell equations and laws of field transformation. The only assumption concerns the particular type of $U(B, j)$ dependence. Our previous studies [15,16] based on this approach demonstrate that vortex dynamics is strongly decelerated if zero-field lines ($B = 0$) are present at the sample boundary or in its vicinity. For instance, the "field-on" magnetic relaxation, where $H \neq 0$ at the boundary, is exponentially fast [16] in the flux-flow ($U = 0$) regime: $|m| \propto \exp(-t/\tau)$. Here, m is the magnetic moment and τ is the characteristic time of the flux flow in the sample which depends on its size and viscous friction for flux motion [18]. Contrariwise, relaxation in the remanent state ($H = 0$) is dramatically slower (in the same flux-flow regime) and obeys the power-law dependence: $m \propto (\tau/t)$ [13,16]. In addition, it was shown [16] that $B = 0$ lines

[*]leonid@biu.ac.il
[†]fuzailovnikita@gmail.com

strongly affect the relaxation rate dm/dt and cause anomalies in the $m(t)$ dependence in experiments where H is swept on at a constant rate: $H \propto t$.

In an infinite slab geometry, $B = 0$ lines can also be called annihilation lines since they divide vortices of different polarities, which approach and annihilate each other. The dynamics of vortices around an annihilation line is quite curious; see, for instance, Ref. [17]. In this article, we show that annihilation lines significantly affect vortex dynamics in oscillating external field $H(t)$, giving rise to the appearance of the out-of-phase regime in a magnetic response which competes with the usual, in-phase regime, and even overwhelms it at certain conditions. In the "coexistence " region between these two regimes, two maxima in voltage $V(t)$ per half period of $H(t)$ are found in our theoretical analysis, in complete agreement with the experimental results [1,2]. Reducing three parameters, i.e., $H_{\max}$, ω, and I, to two dimensionless ones, we build up a phase diagram which allows one to predict where the out-of-phase regime should be expected. This is important for the design and application of current-carrying HTSC wires. We also show that in the case where $H(t)$ does not change sign and, correspondingly, $B \neq 0$ everywhere in the sample, the out-of-phase regime never appears.

The experiments we refer to (see Refs. [1,2]) were carried out in industrial superconducting tapes (YBCO and BSCCO) produced by the American Superconductor Corporation, approximately 4 mm wide and of thickness 0.2 mm. These tapes are thin, and therefore magnetic field $\vec{B}$ is strongly curved around them. Nevertheless, here we confine ourselves by considering a simpler problem of an infinite (in the field direction) slab. We show that even in this simplified and easily solvable case, one gets two competing regimes in the ac magnetic response very similar to those observed experimentally.

II. FLUX DYNAMICS IN A CURRENT-CARRYING SUPERCONDUCTING SLAB EMBEDDED IN AN AC MAGNETIC FIELD

Consider an infinite (along the y and z directions) superconducting slab with $-d/2 \leqslant x \leqslant d/2$. The external magnetic field $H(t) = H_{\max}\sin(\omega t)$ is applied along the z axis, and transport current I flows in the y direction, as shown in Fig. 1. Of course, I is defined as the current per unit "height" of the slab (in the z direction) and is measured in A/m, whereas $\partial I/\partial x = j$ is a true current density. In such a geometry, the spatial and temporal dependence of $B(x,t)$ on x is determined by the one-dimensional flux-diffusion equation [12–15]:

$$\frac{\partial B}{\partial t} = -c\frac{\partial E}{\partial x} = \frac{\partial}{\partial x}\left[\mathcal{A}\frac{\phi_0}{c\eta}Bj\exp(-U/kT)\right], \tag{1}$$

where $E = Bv/c$ is the induced electric field, v is the vortex velocity, ϕ_0 is the unit flux, η is the Bardeen-Stephen drag (viscous friction) coefficient for flux flow [18], c is the speed of light, and $\mathcal{A} \simeq 1$ is a numerical factor [14]. Note that Bv is the flow of vortices. Magnetic field B and current density j are related by a Maxwell equation,

$$j = \frac{\partial I}{\partial x} = \frac{c}{4\pi}\frac{\partial B}{\partial x}. \tag{2}$$

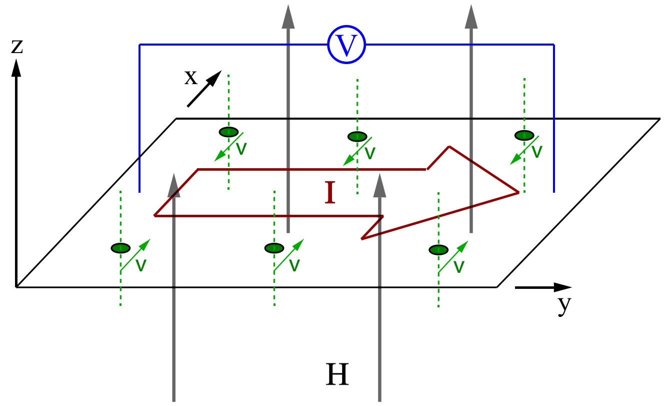

FIG. 1. Geometry of experiments [1,2] and of our theoretical analysis. Transport current I (red arrow) flows in the y direction, and voltage V is measured along the same axis. The external ac magnetic field H is applied in the z direction. Vortices (shown as green dashed lines) move along the x axis.

The displacement current (proportional to $\partial E/\partial t$) is smaller by a factor of $\omega v d/c^2$ and should be neglected in Eq. (2).

Experiments [1,2] where an out-of-phase peak in voltage was observed were performed at liquid nitrogen temperature $T = 77$ K where vortices in both compounds, YBCO and BSCCO, move in the regime of nonactivated flux flow [10,11] with $U = 0$. Along with that, as shown in Ref. [16], the decelerating effect of annihilation lines is most pronounced also in the flux-flow regime. Therefore, assuming $U = 0$ in Eq. (1), we get the flux-diffusion equation for the case of flux flow,

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\phi_0}{4\pi\eta}|B|\frac{\partial B}{\partial x}\right), \tag{3}$$

with the boundary conditions

$$B(x = \pm d/2, t) = H_{\max}\sin(\omega t) \mp 2\pi I/c. \tag{4}$$

As was shown in Ref. [15], the characteristic time of the relaxation of magnetization currents in this case is

$$\tau = \frac{\pi}{2}\frac{\eta d^2}{\phi_0 H_{\max}}. \tag{5}$$

Choosing dimensionless variables, i.e., $b = B/H_{\max}$, $\xi = x/d$, $\tilde{t} = t/\tau$, $\tilde{\omega} = \omega\tau$, $h = H/H_{\max} = \sin(\omega t)$, and $\tilde{I} = 4\pi I/H_{\max}c$, we reduce Eq. (3) to the following form:

$$\frac{\partial b}{\partial \tilde{t}} = \frac{1}{8}\frac{\partial}{\partial \xi}\left(|b|\frac{\partial b}{\partial \xi}\right), \tag{6}$$

where $-1/2 \leqslant \xi \leqslant 1/2$ and the boundary conditions are

$$b(\xi = \pm 1/2, \tilde{t}) = h(\tilde{t}) \mp \tilde{I}/2 = \sin(\omega t) \mp \tilde{I}/2. \tag{7}$$

Note that $\tilde{\omega}\tilde{t} = \omega t$. Equations (6) and (7) contain two dimensionless parameters: $\tilde{\omega}$ and $\tilde{I}$. It should be mentioned that at $\tilde{I} > 2$ (high currents), we have $b(\xi = -1/2, \tilde{t}) > 0$ at all $\tilde{t}$ and, correspondingly, $b(\xi = -1/2, \tilde{t}) < 0$; see Eq. (7). Therefore, at $\tilde{I} > 2$, there always exists at least one $b = 0$ line inside the sample. At low currents, $\tilde{I} < 2$, annihilation lines enter into or exit from the sample at $b(\xi = \pm 1/2, \tilde{t}) = 0$, as shown in Fig. 2. Regarding frequency, the case $\tilde{\omega} \ll 1$ or,

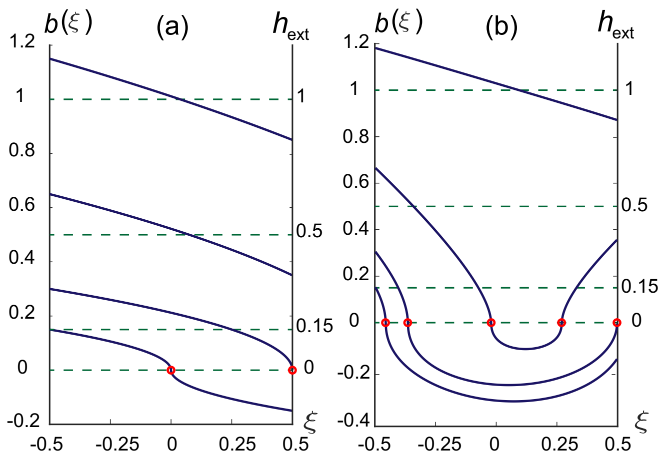

FIG. 2. Magnetic field profiles $b(\xi)$ at $\tilde{I} = 0.3$. Red dots show the position of the annihilation ($b = 0$) lines. (a) Quasistatic case $\tilde{\omega} \ll 1$; see Eqs. (8) and (9). (b) Numerical solution of Eq. (6) at moderate frequency, $\tilde{\omega} = 0.1$.

the same, $\tau \ll 1/\omega$ corresponds to the adiabatic (quasistatic) limit $\partial b/\partial\tilde{t} \ll 1$, where flux relaxation is very fast and $b(\xi)$ acquires its "static" (fully relaxed) shape at given $H(t)$. In this case, the solution of Eq. (6) becomes

$$b(\xi) = \sqrt{h^2 - 2\tilde{I}h\xi + \frac{\tilde{I}^2}{4}}, \tag{8}$$

provided $\tilde{I}/2 < |h(\tilde{t})| = |\sin(\omega\tau)|$, which means that $b \neq 0$ everywhere in the sample. In the case $\tilde{I}/2 > |h(\tilde{t})|$, the annihilation line is inevitably present, and for $\tilde{I} > 2$, this condition is fulfilled at all $\tilde{t}$. If we denote the position of the $b = 0$ line as ξ_0, then the quasistatic solution of Eqs. (6) and (7) is

$$b(\xi) = \sqrt{\left(2h^2 + \frac{\tilde{I}^2}{2}\right) \cdot |\xi - \xi_0|}. \tag{9}$$

The adiabatic solutions described by Eqs. (8) and (9) are shown in Fig. 2(a). The upper three curves are described by Eq. (8), whereas the lower curve corresponds to Eq. (9). The positions of the annihilation lines are shown by red dots.

On the contrary to the adiabatic (quasistatic) case, in the limit $\tilde{\omega} \gtrsim 1$ or, the same, $\tau \gtrsim 1/\omega$, the dynamics of the vortices is slow if compared with changes in $H(t)$ and, correspondingly, $b(\xi)$ profiles are far from the "static" ones described by Eqs. (8) and (9). Instead, they are strongly curved [see Fig. 2(b)] and magnetization current $j \propto \partial b/\partial\xi$ changes sign as a function of both time and coordinate. The value of τ, as follows from Eq. (5), depends on d, H_{max}, and viscous constant η, which, in turn, is a function of temperature. Using the experimental data obtained in Ref. [19], we can estimate $\eta \approx 2 \times 10^{-6}$ g cm^{-1} s^{-1} at $T = 77$ K. According to Refs. [1,2], we take $d = 4$ mm and $H_{max} \simeq 300$ G, and get $\tau \approx 8 \times 10^{-3}$ s. Then the characteristic frequency ω_0, which "separates" the adiabatic regime and the high-frequency one, is $\omega_c = 1/\tau \simeq 125$ Hz.

III. VOLTAGE AND PHASE DIAGRAM FOR AC MAGNETIC RESPONSE

The value measured in experiments [1,2] was voltage V in the y direction (along the dc current I); see Fig. 1. According to the Kirchhoff rule, the voltage (per unit length) is

$$V = E(x) - j(x)\rho(x), \tag{10}$$

where $E(x)$ is the induced electric field determined by Eq. (1) and $\rho = \phi_0|B|/\eta c^2$ is the resistivity associated with flux flow [18,20]. Though E, j, and ρ depend on x, the voltage V is, of course, independent of x: One can easily check that $\partial V/\partial x = 0$ immediately follows from Eq. (3). The value of V can be derived by integration of Eq. (3),

$$E_{x=d/2} - E_{x=-d/2} = -\int_{-d/2}^{d/2} \frac{\partial B}{\partial t} dx = -\frac{\phi_0}{4\pi c\eta}\left(|B|\frac{\partial B}{\partial x}\right)_{x=-d/2}^{x=d/2}. \tag{11}$$

Taking into account that $|E(x = d/2)| = |E(x = -d/2)|$ due to symmetry requirements and using Eqs. (10) and (11), we get dimensionless voltage (per unit length in the y direction):

$$\tilde{V} = \frac{8c\tau V}{H_{max}d} = \left(|b|\frac{\partial b}{\partial\xi}\right)_{\xi=1/2} + \left(|b|\frac{\partial b}{\partial\xi}\right)_{\xi=-1/2}. \tag{12}$$

In Fig. 3, we plot $\tilde{V}$ as a function of ωt, at $\tilde{I} = 0.3$ and different frequencies $\tilde{\omega}$. As $\tilde{\omega}$ grows up, the shape of the $\tilde{V}(\omega t)$ curves changes dramatically. At small $\tilde{\omega} \ll 1$, the time dependence of $\tilde{V}$ is almost in-phase with $h(\tilde{t}) = \sin(\omega t)$, i.e., the voltage reaches a maximum at $|h| = 1$ and becomes minimal at $h = 0$; see Fig. 3(a). This agrees, of course, with the expression for voltage,

$$\tilde{V}(\tilde{t}) = \begin{cases} 2\sin^2(\omega t) + \tilde{I}^2/2 & \text{if } b = 0 \text{ line exists} \\ 2\tilde{I} \cdot |\sin(\omega t)| & \text{if } b \neq 0 \text{ everywhere}, \end{cases} \tag{13}$$

which results from the adiabatic solutions [see Eqs. (8) and (9), combined with Eq. (12)]. Nonetheless, a small kink (out-of-phase maximum) appears in $\tilde{V}(\omega t)$, as we see in the left part of Fig. 3(a). As $\tilde{\omega}$ increases, the out-of-phase peak starts to grow up; see Fig. 3(b). At moderate frequencies, this second peak becomes larger and overwhelms the in-phase maximum; see Figs. 3(c) and 3(d). Our numerical results are in qualitative accordance with the experimental data presented in Refs. [1,2], in spite of the fact that in our theoretical analysis, we consider an infinite slab whereas the experiment was carried out in thin tapes. For low currents $\tilde{I} < 0.5$, the in-phase maximum disappears completely at $\tilde{\omega} > 0.6$; see Fig. 3(e). It is quite curious that $\tilde{V}$ becomes negative in a certain range of phase ωt, as shown in Figs. 3(b)–3(e). As $\tilde{\omega}$ increases, this effect becomes more pronounced. It means that the superconducting sample periodically stores and releases energy, and this effect requires further investigation.

In order to explain the appearance of an out-of-phase peak in $V(t)$ and its domination at high frequencies, let us note that as follows from Eq. (12) and was mentioned in Ref. [7], voltage is determined by the amount of vortices which cross the sample boundaries at $\xi = \pm 1/2$. As shown in Ref. [16], if $b = 0$ at the boundary, it appears to be effectively "locked

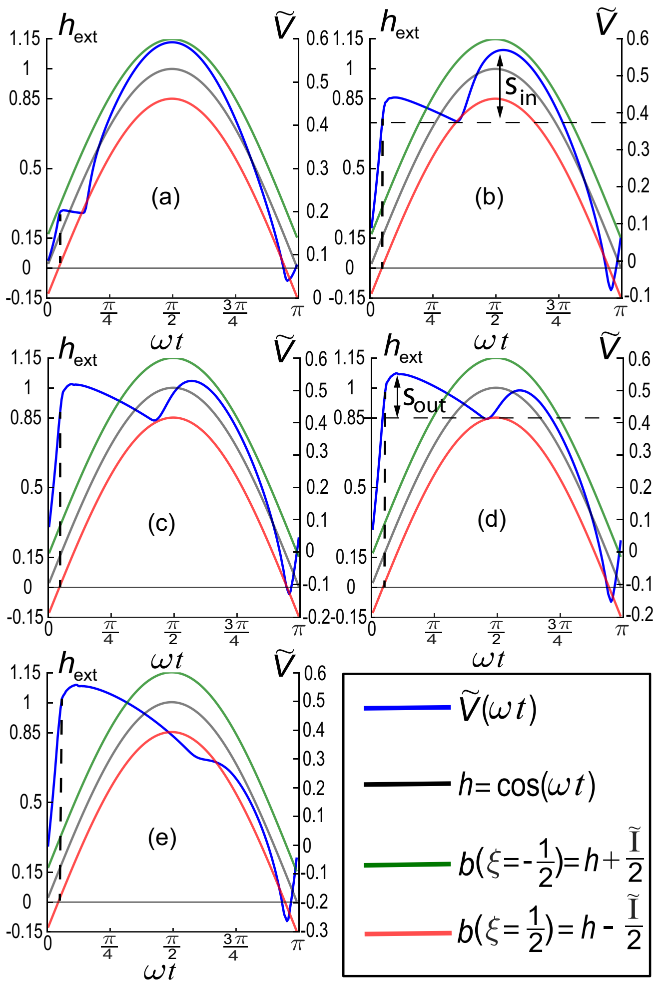

FIG. 3. Voltage $\tilde{V}$ induced by the motion of vortices at $\tilde{I} = 0.3$ and various frequencies: (a) $\tilde{\omega} = 0.05$, (b) $\tilde{\omega} = 0.25$, (c) $\tilde{\omega} = 0.35$, (d) $\tilde{\omega} = 0.4$, and (e) $\tilde{\omega} = 0.6$.

up" for the exit of vortices. Therefore, when annihilation line appears at a boundary, i.e., $b_{\xi=-1/2} = 0$ or $b_{\xi=1/2} = 0$, the corresponding term in Eq. (12) gets suppressed. It should be clearly emphasized that the condition $b = 0$ does not mean that the flow of vortices, $Bv \propto b\,\partial b/\partial\xi$, vanishes at the same point. As shown in Ref. [16], it remains finite due to diverging $\partial b/\partial\xi$. However, the flux dynamics in the vicinity of the sample boundary gets strongly inhibited. In the quasistatic regime [see Fig. 2(a)], this effect is not prominent on the background of very slow changes in $H(t)$. As $\tilde{\omega}$ grows up, a dramatic slow down of flux motion results in significant changes in the magnetic response and, in turn, in $V(t)$. At moderate $\tilde{\omega}$, the current density $j(\xi) \propto \partial b/\partial\xi$ is of different signs at $\xi = \pm 1/2$; see the field profiles with red dots in Fig. 2(b). So two contributions to $\tilde{V}$ [see Eq. (12)] become large by absolute value, but of different sign. Suppression of the negative term [out of two in the right-hand side of Eq. (12)] results in a dramatic increase of V. Therefore, at low currents $\tilde{I} < 2$, the second peak in $\tilde{V}(\tilde{t})$ appears exactly where the $b = 0$ line enters the sample; see the black dashed lines in Fig. 3.

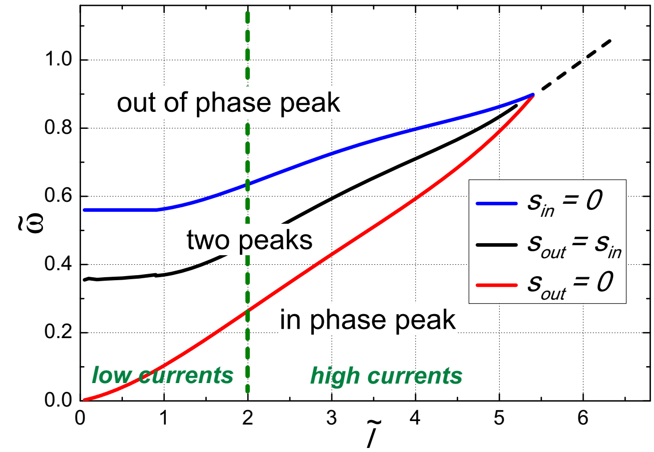

FIG. 4. Phase diagram for the type of magnetic ac response. Two peaks (in-phase and out-of-phase) coexist in the area between the red and blue lines. The "median" line where $s_{out} = s_{in}$ (see Fig. 3) is shown in black.

Let us define the height of in-phase and out-of-phase voltage peaks, s_{in} and s_{out}, as shown in Figs. 3(b) and 3(d), and summarize our results in a "phase diagram"; see Fig. 4. The red and blue lines confine the area where two peaks in $V(t)$ are observed, and the black line correspond to the "median" where $s_{in} = s_{out}$. The two peak area is finite and does not stretch over $\tilde{I} > 5.5$ or $\tilde{\omega} > 0.9$. This fact is in line with our explanation for the origin of the out-of-phase peak. At high currents $\tilde{I} > 2$, no $b = 0$ lines appear at the sample boundaries. But as such a line, which always exists at high currents since $b_{\xi=-1/2} = 0$ and $b_{\xi=1/2} = 0$ are of opposite signs, approaches a boundary, the lock-up effect partially takes place. Therefore, at $\tilde{I} \gtrsim 2$, the two peak area gets narrow; see Fig. 4. As $\tilde{I}$ grows up further, the two peak area vanishes and disappears completely at $\tilde{I} \approx 5.4$, since now the annihilation line stays "deep" inside the sample and does not approach its boundaries; correspondingly, the lock-up effect

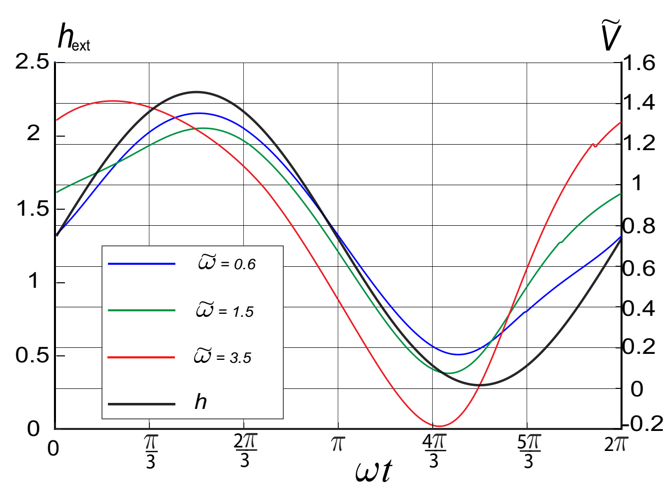

FIG. 5. Voltage $\tilde{V}(t)$ during one cycle of external dc+ac magnetic field h determined by Eq. (14), which is shown as a black line. No annihilation lines are present and no two peak phase is observed.

fades out. However, if we "bend around" the two peak area in the way from the in-phase area under the red curve towards the out-of-phase area above the blue curve (see Fig. 4), we find a single maximum in $\tilde{V}(\omega t)$ all the way, but it will gradually drift away from $\omega t = \pi/2$. The black dotted line, which is a natural extrapolation of the solid black median, shows where the maximum has passed one-half distance between its in-phase and out-of-phase positions.

IV. THE ROLE OF ANNIHILATION LINES - DISCUSSION

In order to prove and emphasize the crucial role of annihilation ($B = 0$) lines in the appearance of the out-of-phase peak in $V(t)$, we analyzed the case where $H(t)$ has a dc component,

$$H(t) = H_0 + H_1 \sin(\omega t) \text{ where } H_0 > H_1. \qquad (14)$$

Here, $H(t)$ does not change sign and $B = 0$ lines are absent. In Fig. 5, we show the results of the numerical solution of Eq. (3) at $h_0 = 1.3$, $h_1 = 1$, $\tilde{I} = 0.3$, and various $\tilde{\omega}$. Note that H_1 substitutes H_{max} in the definitions of dimensionless h_0, $\tilde{I}$, and $\tilde{\omega}$. We found no trace of the second peak in contrast to "pure" ac $H(t)$ with $H_0 = 0$, whereas a clear coexistence of in-phase and out-of-phase maxima was found at the same values of $\tilde{I}$ and $\tilde{\omega}$; see Figs. 3 and 4. A single maximum in $V(t)$ was observed in the case of alternating, but not changing sign $H(t)$ at any frequencies and transport currents. This confirms that annihilation lines are particularly responsible for the appearance of the out-of-phase magnetic response regime. However, as $\tilde{\omega}$ grows up, we observe another interesting phenomenon: the maximum of $V(t)$ drifts away from the maximal value of $|H(t)|$; see Fig. 5. This effect requires further study.

To conclude, we described and explained the experimentally observed out-of-phase regime in the ac magnetic response of current-carrying type-II superconducting wires. It was proved that its appearance is conditioned by the presence of annihilation ($B = 0$) lines in the sample (wire). We found that in a wide area of the current-frequency phase diagram, the out-of-phase regime coexists with the usual in-phase one and even overwhelms the latter.

ACKNOWLEDGMENT

We thankfully acknowledge useful conversations with Y. Yeshurun, A. Shaulov, G. Lukovsky, B. Ya. Shapiro, and A. Friedman.

[1] G. Lukovsky, A. Friedman, Y. Wolfus, L. Burlachkov, and Y. Yeshurun, IEEE Trans. Appl. Supercond. **17**, 3137 (2007).
[2] G. Lukovsky, M.Sc. thesis, Bar-Ilan University, 2006.
[3] C. P. Bean, Phys. Rev. Lett. **8**, 250 (1962); Rev. Mod. Phys. **36**, 31 (1964).
[4] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **129**, 528 (1963).
[5] L. Ji, R. H. Sohn, G. C. Spalding, C. J. Lobb, and M. Tinkham, Phys. Rev. B **40**, 10936 (1989).
[6] S. Shatz, A. Shaulov, and Y. Yeshurun, Phys. Rev. B **48**, 13871 (1993).
[7] G. P. Mikitik and E. H. Brandt, Phys. Rev. B **64**, 092502 (2001).
[8] R. Prozorov, A. Shaulov, Y. Wolfus, and Y. Yeshurun, Phys. Rev. B **52**, 12541 (1995).
[9] Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988).
[10] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).
[11] Y. Yeshurun, A. P. Malozemoff, and A. Shaulov, Rev. Mod. Phys. **68**, 911 (1996).
[12] M. R. Beasley, R. Labusch, and W. W. Webb, Phys. Rev. **181**, 682 (1969).
[13] V. M. Vinokur, M. V. Feigel'man, and V. B. Geshkenbein, Phys. Rev. Lett. **67**, 915 (1991).
[14] Y. Abulafia, A. Shaulov, Y. Wolfus, R. Prozorov, L. Burlachkov, Y. Yeshurun, D. Majer, E. Zeldov, and V. M. Vinokur, Phys. Rev. Lett. **75**, 2404 (1995).
[15] L. Burlachkov, D. Giller, and R. Prozorov, Phys. Rev. B **58**, 15067 (1998).
[16] L. Burlachkov and S. Burov, Phys. Rev. B **103**, 024511 (2021).
[17] F. Bass, B. Ya. Shapiro, and M. Shvartser, Phys. Rev. Lett. **80**, 2441 (1998).
[18] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965); see, also, M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (McGraw-Hill, New York, 1996), p. 167.
[19] M. Golosovsky, M. Tsindlekht, H. Chayet, and D. Davidov, Phys. Rev. B **50**, 470 (1994).
[20] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965).