


Dynamics and nonmonotonic drag for individually driven skyrmionsC. Reichhardt and C. J. O. Reichhardt *Theoretical Division and Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

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We examine the motion of an individual skyrmion driven through an assembly of other skyrmions in the absence of quenched disorder. The skyrmion behavior is determined by the ratio of the damping and Magnus terms, known as the intrinsic skyrmion Hall angle. For fixed drive in the damping dominated regime, the effective viscosity decreases monotonically with increasing skyrmion density. In contrast, in the Magnus dominated regime, the velocity varies nonmonotonically with density, and there is a regime in which the skyrmion moves faster with increasing density, as well as a pronounced speed-up effect in which a skyrmion traveling through a dense medium moves more rapidly than for low densities or the single-particle limit. The velocity-force curves in the Magnus-dominated regime exhibit an overshoot effect and other differences from the damping-dominated regime. We find a finite threshold force for skyrmion motion which increases with density as well as a drive-dependent skyrmion Hall angle. We map dynamic phase diagrams showing the threshold for motion, nonlinear flow, speed-up, and saturation regimes. In some cases, increasing the density can reduce the skyrmion Hall angle while producing a velocity boost, which could be valuable for applications.

DOI: [10.1103/PhysRevB.104.064441](https://doi.org/10.1103/PhysRevB.104.064441)**I. INTRODUCTION**

Skyrmions are particlelike magnetic textures [1–5] that have been identified in a growing number of materials, including numerous systems in which they are stable at room temperature [6–8]. Skyrmions can be set into motion with applied currents [3,4,6,8–11], magnetic field gradients [12], and thermal gradients [13]. Due to their size scale, room temperature stability, and mobility, skyrmions are promising candidates for various applications including memory [14–16] and novel computing architectures [17–19]. In terms of basic science, skyrmions represent a new class of systems that exhibit collective dynamics under an applied drive [20,21], similar to vortices in superconductors [22,23], colloids [24], Wigner crystals [25], frictional systems [26], and other soft matter systems [27]. A unique feature which distinguishes skyrmions from these other systems is the strong gyroscopic or Magnus force component of their dynamics, which generates velocities perpendicular to the net force experienced by the skyrmion [3,4,9,11,28]. One consequence of this is that a skyrmion moves at an angle, known as the skyrmion Hall angle, with respect to an applied driving force [3,28–32]. The Magnus term also produces many other effects, including accelerations or speed-ups of skyrmions interacting with barriers or walls [33–37], ratchet effects [38–41], and spiraling motion in a confined potential [5,42,43] or after a quench [44]. Such effects can be strongly modified by pinning or skyrmion-skyrmion interactions [5].

Beyond moving skyrmions with currents, other methods that can be used to manipulate individual skyrmions include scanning tunneling microscope tips [45], local magnetic fields [46,47], and optical traps [48]. Extensive experimental and theoretical studies have been performed in systems such as

colloids [49–53], granular matter [54–58], active matter [59], superconducting vortices [60–65], various soft matter systems [66–69], and metallic glasses [70] examining the dynamics of individually driven particles moving through an assembly of other particles under either a constant force or an increasing force. Measurements of the velocity-force relations or the drag on the driven particle provide information about how the effective viscosity of the system changes across density-induced transitions such as solid to liquid [71,72], liquid to glass [50,52], or unjammed to jammed [54,55,58]. For example, the viscosity can increase strongly at the onset of a glass or solid phase produced by an increase in the density. When the probe particle is driven with an increasing force, there can be a force threshold below which the particle is unable to move [49,50,73,74], as well as distinct features in the velocity-force curves [75,76] which change as the density increases.

In soft matter systems, driving a single probe particle through a background of other particles is known as active rheology [52,58,59,68,74]. Similar probes have been applied in type-II superconductors using individually dragged vortices [60–65]. Existing work on active rheology has involved overdamped systems, where the viscosity generally increases with increasing density. It is not clear how the addition of a Magnus term would impact active rheology. For example, it could change the threshold force for motion, increase or decrease the drag, modify the Hall angle, or alter the way in which the probe particle moves with respect to the background particles. Beyond skyrmions, there are many other chiral systems in which Magnus or gyroscopic effects can be important for active rheology, including chiral soft matter fluids and solids, active chiral systems [77–81], charged particles in a magnetic field [82,83], magnetic colloids in oscillating fields

[84], spinning particles in fluids [85,86], fluid vortices [87,88], and fracton systems [89].

In this work, we study active rheology for skyrmion systems in the absence of quenched disorder using a particle-based model. We apply an external driving force to one skyrmion and measure the velocity components and drag both parallel and perpendicular to the driving direction. For a constant drive, the velocity decreases monotonically with increasing skyrmion density in the damping-dominated regime. This is similar to what has been found for the active rheology of overdamped systems, where it becomes more difficult for the particle to pass through a denser medium due to the increased interactions with other particles. When the Magnus force is dominant, however, the driven skyrmion velocity has a strongly nonmonotonic density dependence and *increases* with increasing density. In this regime, the velocity can be boosted or accelerated beyond the expected velocity for an isolated driven skyrmion in the absence of surrounding skyrmions. As the density increases, the skyrmion Hall angle decreases and the velocity reaches a peak value before decreasing at higher densities. The velocity boost effect arises due to the creation of a locally asymmetric density profile around the driven skyrmion oriented perpendicular to the driving direction. This creates an additional repulsive force on the driven skyrmion which does not cancel out due to the lack of symmetry and is converted by the Magnus term into a velocity component aligned with the driving direction. The behavior is similar to the acceleration effect found for skyrmions interacting with walls and barriers [33–37].

For the case of a driving force that is increased from zero, the driven skyrmion exhibits a critical threshold for the onset of motion. The skyrmion Hall angle is zero at the threshold and increases with increasing velocity until it saturates to a constant value at high drives, similar to the behavior found for skyrmions driven over quenched disorder [5,30–32,90–95]. In the Magnus-dominated regime, the boost effect is nonmonotonic, with reduced boost at low and high drives and maximum boost at intermediate drives. This drive dependence originates from the reaction of the surrounding skyrmions to the motion of the driven skyrmion. Local density fluctuations of the background skyrmions have time to relax when the driven skyrmion velocity is low, but do not have time to form when the velocity is high. In each case, the boost effect is reduced. In contrast, for intermediate driven skyrmion velocities, density fluctuations in the surrounding skyrmions form and relax on the same timescale as the motion of the driven skyrmion.

In the strongly damped regime, the velocity-force curves are similar to those observed for the active rheology of overdamped systems, while in the Magnus-dominated regime, negative differential conductivity can appear when part of the velocity in the driving direction gets transferred into the direction perpendicular to the drive. We also find that under constant driving, there are regimes in which an increase in density produces a velocity boost with a reduction of the skyrmion Hall angle. This indicates that harnessing skyrmion-skyrmion interactions may be a viable method for reducing the skyrmion Hall angle without reducing the skyrmion speed, which could be important for applications. We map dynamic phase diagrams as a function of skyrmion density, driving

force, and the ratio of the damping term to the Magnus force. We discuss possible experimental realizations and the application of our results to the broader class of systems with gyroscopic forces.

II. SIMULATION

We consider an assembly of N skyrmions in a two-dimensional system with periodic boundary conditions in the x and y directions. A single skyrmion is coupled to an external drive and is driven through the other skyrmions in the absence of quenched disorder. The skyrmion dynamics are obtained using a particle-based model from the modified Thiele equation [5,29,30,44,96]. The equation of motion for skyrmion i is given by

$$\alpha_d \mathbf{v}_i + \alpha_m \hat{z} \times \mathbf{v}_i = \mathbf{F}_i^{ss} + \mathbf{F}_i^D, \quad (1)$$

where $\mathbf{v}_i = d\mathbf{r}_i/dt$ is the skyrmion velocity, α_d is the damping coefficient, and α_m is the coefficient for the Magnus force. The damping term aligns the velocities in the direction of the net external forces, while the Magnus term creates a perpendicular velocity component. The skyrmion-skyrmion interaction force is $\mathbf{F}_i^{ss} = \sum_{j=1}^N K_1(r_{ij}) \hat{\mathbf{r}}_{ij}$, where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between skyrmions i and j , and K_1 is the modified Bessel function. The driving force $\mathbf{F}_i^D = F^D \hat{\mathbf{x}}$ is applied only to the driven skyrmion and is zero for all other skyrmions.

In the absence of other skyrmions, the driven skyrmion would move in the direction of the intrinsic Hall angle, $\theta_{sk}^{\text{int}} = -\arctan(\alpha_m/\alpha_d)$. In the overdamped limit where $\alpha_m = 0$, $\theta_{sk}^{\text{int}} = 0^\circ$. When skyrmion-skyrmion interactions occur, the skyrmion Hall angle is reduced below its intrinsic value and is defined as $\theta_{sk} = \arctan(\langle V_\perp \rangle / \langle V_\parallel \rangle)$, where $\langle V_\parallel \rangle$ and $\langle V_\perp \rangle$ are the average velocities of the driven skyrmion parallel and perpendicular to the direction of the drive, respectively. In this work, each individual simulation is performed at a constant value of F_D and lasts 3×10^5 to 1×10^6 simulation time steps to avoid transient effects. For drives very near the critical threshold for motion, much larger time intervals are needed to reach a steady state [97]. The density of the system is given by $\rho = N/L^2$, where the system size $L = 36$.

In the particle-based or Thiele equation approach that we employ, our model includes the assumption that an external magnetic field is present which generates the skyrmions, but it does not explicitly include a magnetization term. There is also no skyrmion nucleation or annihilation; instead, the number of skyrmions is assumed to remain fixed. The initial skyrmion positions are obtained by simulated annealing from a high temperature to $T = 0$, a process which produces a triangular skyrmion lattice in the absence of pinning. Unless otherwise noted, we neglect thermal fluctuations. Our modified Thiele equation approach has been extensively employed [29,30,35,39,44,90,92,96,98] to study collective skyrmion dynamics with and without pinning, and has been shown to capture experimentally relevant features including depinning thresholds, the velocity dependence of the skyrmion Hall angle, and the skyrmion flow patterns. Many of these works also focused on the $T = 0$ case; however, in Sec. VI, we consider the impact of thermal fluctuations on our results.

Based on our previous work [30], we can fit our dimensionless simulation parameters to match skyrmions in MnSi. Here,

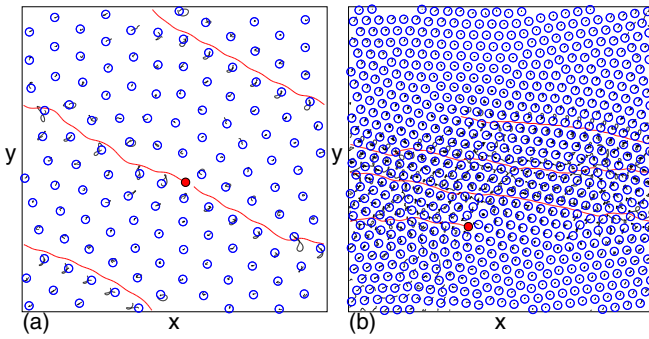


FIG. 1. Driven skyrmion and trajectory (red) along with the surrounding skyrmions (blue) and their trajectories (black) in a sample with $\alpha_m/\alpha_d = 0.98$ and an intrinsic skyrmion Hall angle of $\theta_{sk}^{\text{int}} = -44.4^\circ$ at a driving force of $F_D = 0.5$. (a) At $\rho = 0.1$, the driven skyrmion moves at an angle of $\theta_{sk} \approx -29^\circ$. (b) At $\rho = 0.5$, the magnitude of the skyrmion Hall angle θ_{sk} is reduced due to the increasing frequency of collisions with the other skyrmions.

skyrmion lattice constants of 10 to 30 nm with an exchange energy $J = 3$ meV/ a and a Dzyaloshinskii-Moriya interaction term $D = 0.3$ meV/ a^2 give skyrmion-skyrmion repulsive forces at distances of the lattice constant a of magnitude $F = 10^{-5}$ N/m. The forces acting on the driven skyrmion would be of the order of 5×10^{-7} to 1×10^{-5} N/m. These forces are well within the regime of what can be applied using local drives or local fields. Since we examine a range of parameters, our results can be generalized to most skyrmion systems in which both Magnus and damping terms are important.

Experimentally, our system could be realized by using a local tip such as a magnetic force microscope to drive one of the skyrmions. Another possibility would be to consider a system containing a mixture of skyrmions that are mostly of one species or size with a small number of skyrmions of a different species or size [99,100]. If the different species have different mobilities and couple differently to a globally applied current or field, this will create a force differential which, in a moving reference frame, would be the same as skyrmions with no drive being penetrated by a smaller number of skyrmions responding to a drive. Similar experiments and simulations have been performed for colloidal assemblies [101–103], dusty plasmas [104], and active matter [105] for systems containing different species of particles that have different relative motion to one another. If the ratio of the two species is close to 50 percent, interactions between the driven particles become important and collective effects appear [100,101]; however, if the ratio of driven particles is small, the dynamics should be close to what we study in this work. We also note there are some experiments showing that localized laser spots can move individual magnetic bubbles, and a similar approach could be used for skyrmions [106].

III. RESULTS

In Fig. 1(a), we show an example of our system for a collection of skyrmions at a density of $\rho = 0.1$. The red particle is being driven at a constant force $F_D = 0.5$, while the blue particles are the nondriven or bath skyrmions. If the driven skyrmion did not collide with any other skyrmions, it

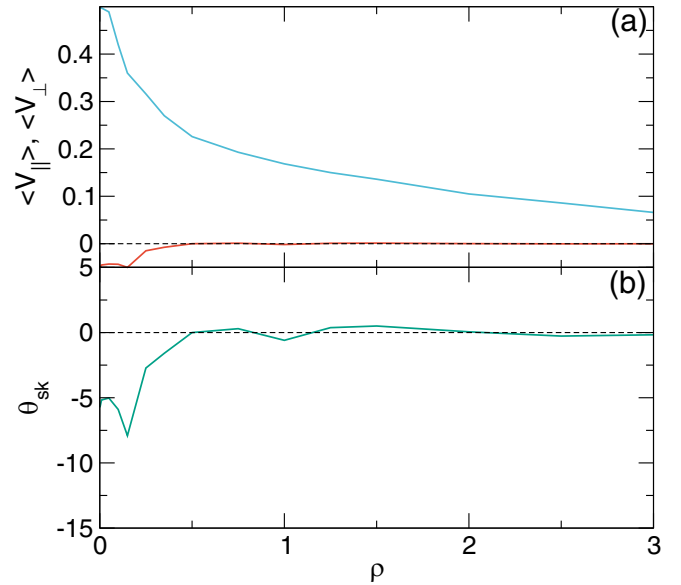


FIG. 2. (a) $\langle V_{\parallel} \rangle$ (blue) and $\langle V_{\perp} \rangle$ (red) vs ρ for a system with $\alpha_m/\alpha_d = 0.1$, $\alpha_d^2 + \alpha_m^2 = 1.0$, $\theta_{sk}^{\text{int}} = -5.739^\circ$, and $F_D = 0.5$. In this damping-dominated regime, $\langle V_{\parallel} \rangle$ decreases monotonically with increasing ρ . For all values of ρ , $\langle V_{\parallel} \rangle \leq V_0 = 0.5$. (b) The corresponding $\theta_{sk} = \arctan(\langle V_{\perp} \rangle / \langle V_{\parallel} \rangle)$.

would move at the intrinsic skyrmion Hall angle of $\theta_{sk}^{\text{int}} = -\arctan(\alpha_m/\alpha_d)$ with an average absolute velocity of $|V| = F_D/\sqrt{\alpha_d^2 + \alpha_m^2}$. We first consider systems in which we constrain $\alpha_d^2 + \alpha_m^2 = 1.0$. Here we can define the velocity in the single-particle limit to be $V_0 = |V|_{N=1} = F_D$. For the system in Fig. 1, $\theta_{sk}^{\text{int}} = -44.4^\circ$, so the ratio between the damping and Magnus forces is close to one. The driven skyrmion moves, on average, at an angle of $\theta_{sk} = -29^\circ$. This is smaller in magnitude than the intrinsic skyrmion Hall angle, indicating that collisions with the bath skyrmions are reducing θ_{sk} . As we show in greater detail below, the magnitude of θ_{sk} increases for higher values of F_D but decreases with increasing ρ . Figure 1(b) illustrates the same system at a higher density of $\rho = 0.5$, where the driven skyrmion moves at an even smaller θ_{sk} . In this work, we demonstrate that the skyrmion Hall angle and net velocity strongly depend on the skyrmion density, the driving force, and the ratio of the Magnus force to the damping term.

To quantify the behavior that we observe, we measure the velocity of the driven particle. In overdamped systems of collectively interacting particles under fixed driving, we expect to find a velocity smaller than the single-particle limit, $|V| < V_0$. The velocity should decrease with increasing ρ as the frequency of interactions between the driven and background particles increases, and there should be a critical density ρ_c above which $|V| = 0$. In Fig. 2(a), we plot $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ versus ρ for a system in the damping-dominated regime with $\alpha_m/\alpha_d = 0.1$, $\theta_{sk}^{\text{int}} = -5.739^\circ$, and $F_D = 0.5$. As ρ increases, $\langle V_{\perp} \rangle$ and θ_{sk} both decrease in magnitude and approach zero for $\rho > 0.5$, consistent with our expectations. In Fig. 3, we plot $|V| = \sqrt{\langle V_{\parallel} \rangle^2 + \langle V_{\perp} \rangle^2}$ for the same system, where the dashed line indicates that $|V| \leq V_0$ for all values of ρ . Here, $|V|$ monotonically decreases with increasing ρ , similar to

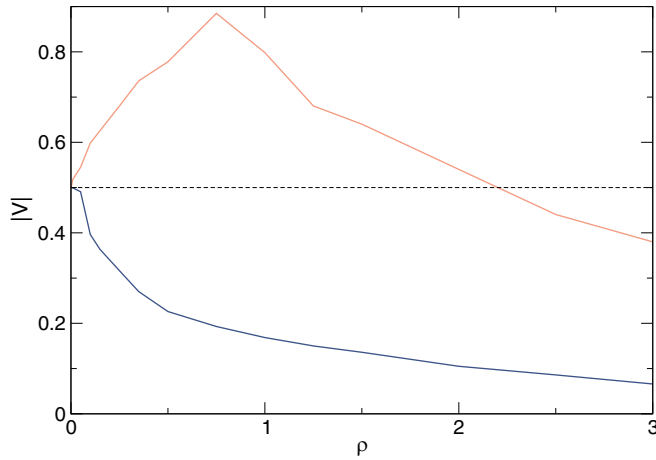


FIG. 3. $|V| = \sqrt{\langle V_{\parallel} \rangle^2 + \langle V_{\perp} \rangle^2}$ vs ρ for the system in Fig. 2 (blue) in the damping-dominated regime with $\alpha_m/\alpha_d = 0.1$, $\alpha_m^2 + \alpha_d^2 = 1.0$, $\theta_{sk}^{\text{int}} = -5.739^\circ$, and $F_D = 0.5$, and for the system in Fig. 4 (orange) in the Magnus-dominated regime with $\alpha_m/\alpha_d = 9.95$ and $\theta_{sk}^{\text{int}} = -84.26^\circ$. The dashed line indicates the expected response for an isolated particle with $|V| = V_0 = F_D = 0.5$. In the Magnus-dominated regime, there is a range of densities over which $|V|$ increases with increasing ρ and is higher than V_0 , indicating the existence of a velocity boost.

the behavior found in fully overdamped systems. When $\alpha_m = 0$, $\langle V_{\perp} \rangle = 0$, $\theta_{sk} = 0$, and $\langle V_{\parallel} \rangle$ has the same shape shown in Fig. 2 but its magnitude is slightly reduced. These results match what has been found in previous studies of active rheology in the overdamped regime.

Next we turn to the case of a system with a finite Magnus term. In Fig. 4, we plot $\langle V_{\parallel} \rangle$, $\langle V_{\perp} \rangle$, and θ_{sk} versus ρ for a sample in the Magnus-dominated regime with $\alpha_m/\alpha_d = 9.95$ and $\theta_{sk}^{\text{int}} = -84.26^\circ$ at $F_D = 0.5$. Here the velocities are highly nonmonotonic. $\langle V_{\parallel} \rangle$ initially increases with increasing ρ , reaches a maximum of $\langle V_{\parallel} \rangle = 0.78$ near $\rho = 1.25$, and then decreases again, while $\langle V_{\perp} \rangle$ has a small initial increase in magnitude followed by a decrease in magnitude to a value of zero near $\rho = 1.6$. The Hall angle has the value $\theta_{sk} = -84.26^\circ$ at low ρ , decreases linearly in magnitude with increasing ρ , and reaches zero for $\rho > 1.25$. The maximum value of $\langle V_{\parallel} \rangle = 0.78$ is higher than $V_0 = 0.5$, indicating that the velocity in the driving direction is being boosted as the skyrmion density increases. There is also a small boost in $|\langle V_{\perp} \rangle|$ near $\rho = 0.3$, where $|\langle V_{\perp} \rangle| \gtrsim V_0$. In Fig. 3, the plot of $|V|$ versus ρ for the system in Fig. 4 indicates that $|V|$ reaches a maximum value close to 0.9. The dashed line represents $V_0 = 0.5$, so that a velocity boost is occurring whenever $|V| > V_0$. We observe velocity boosting up to $\rho = 2.0$, while for $\rho > 2.0$, the net velocity decreases, indicating an increase in the effective damping.

The velocity boost or speed-up effect arises due to the Magnus force. In systems where a single driven skyrmion interacts with a wall or barrier which is parallel to the x direction, an applied drive parallel to the x direction generates a skyrmion Hall component which deflects the driven particle in the y direction. This increases the skyrmion-wall interaction force in the y direction, which results in a perpendicular skyrmion velocity component along the x direction.

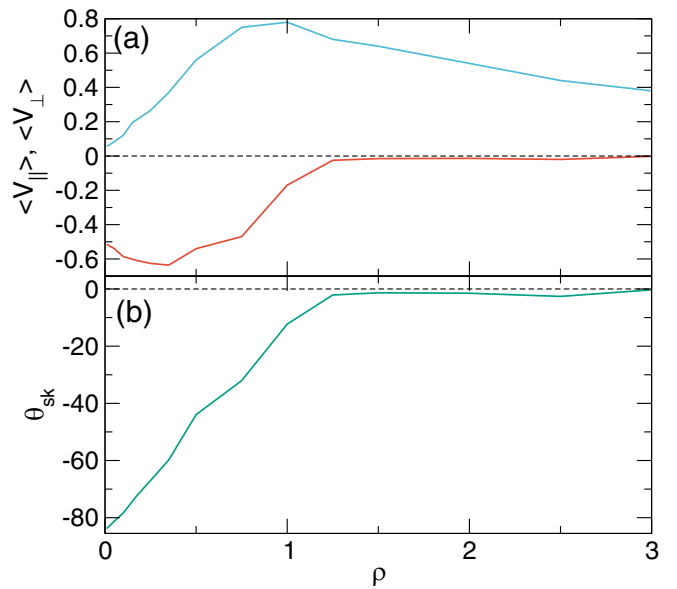


FIG. 4. (a) $\langle V_{\parallel} \rangle$ (blue) and $\langle V_{\perp} \rangle$ (red) vs ρ for the same system in Fig. 2 but in a Magnus-dominated regime, where $\alpha_m/\alpha_d = 9.95$, $\alpha_d^2 + \alpha_m^2 = 1.0$, $\theta_{sk}^{\text{int}} = -84.26^\circ$, and $F_D = 0.5$. There is a range of ρ over which $\langle V_{\parallel} \rangle > V_0 = 0.5$, indicating a velocity boost in the driving direction. (b) The corresponding $\theta_{sk} = \arctan(\langle V_{\perp} \rangle / \langle V_{\parallel} \rangle)$, showing a linear decrease in magnitude with increasing ρ followed by a saturation regime for $\rho > 1.25$.

The wall-induced velocity contribution adds to the x direction velocity from the driving, generating a velocity boost effect. The magnitude of the boost velocity depends on the nature of the pairwise interaction between the wall and the skyrmion. If the interaction is of the power law form $V(r) \approx 1/r^\alpha$, then, as the skyrmion approaches the wall more closely, r decreases and the resulting velocity in the x direction increases [33–37]. The boost effect can also occur for skyrmions moving over a two-dimensional periodic substrate [29] or over random pinning [107], since the effect arises whenever the component of the substrate force which is perpendicular to the direction of drive is not balanced by a compensating substrate force in the other direction. This is due to the skyrmion Hall angle, which pushes the skyrmion to one side of the pinning sites.

For our single driven skyrmion, there is no pinning or barrier wall; however, a velocity boost effect can still occur due to the creation of a local density gradient in the surrounding medium. The mechanism of this effect is outlined in the schematic of Fig. 5(a), where a driven skyrmion is moving under a finite driving force F_D . For intermediate drives, the skyrmion tries to translate along θ_{sk}^{int} , which is at a negative angle with respect to the x axis. In the process, it displaces some of the surrounding skyrmions in the negative y direction, creating a local density imbalance $\Delta\rho$ such that the density is higher below the driven skyrmion and lower above it. Due to the repulsive pairwise interactions with the surrounding skyrmions, there is an unbalanced force in the positive y direction on the driven skyrmion, as indicated by the black arrow in the schematic. The Magnus term transforms this repulsive force into a velocity in the positive x direction, parallel to the drive, as indicated by the green arrow. Thus, the boost velocity

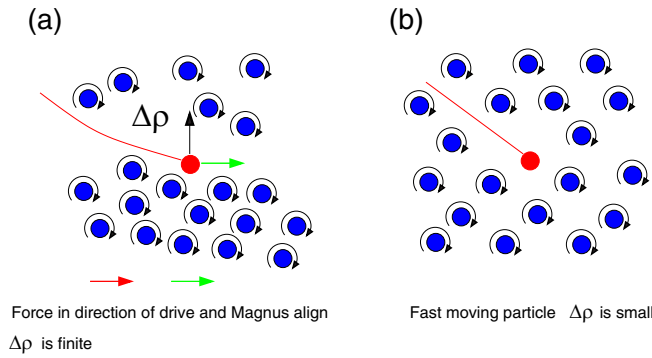


FIG. 5. (a) A schematic showing the mechanism creating the velocity boost. The driven skyrmion (red) is driven in the positive x direction through a background assembly of skyrmions (blue) and moves at a negative angle with respect to the x direction due to the skyrmion Hall effect. The motion creates a local density gradient $\Delta\rho$ which generates a repulsive force on the driven skyrmion in the positive y direction (black arrow). The Magnus term converts this force into a velocity in the positive x direction (green arrow), parallel to the drive. (b) The same at higher F_D , where the surrounding skyrmions do not have time to respond to the driven skyrmion. Here, $\Delta\rho$ and hence the boost velocity are reduced.

$V_{\text{boost}} = |V| - V_0 \propto \Delta\rho$. Here the x -direction velocity from the damping term is aligned with, but opposite to, the velocity induced by the Magnus force. In contrast, when F_D is larger, the surrounding skyrmions do not have time to respond to the rapidly moving driven skyrmion, producing a small $\Delta\rho$ and reducing V_{boost} , as shown schematically in Fig. 5(b). For drives that are only slightly above the threshold for motion, the driven skyrmion is moving sufficiently slowly that the surrounding skyrmions have enough time to completely relax any density perturbations, making $\Delta\rho$ small. The maximum boost velocity changes as a function of F_D and ρ due to the different relaxation times.

In Fig. 6(a), we plot $|V|$ vs ρ for the systems in Figs. 2 and 4 at $\alpha_m/\alpha_d = 0.1, 0.57, 0.98, 2.065, 4.924, 9.95$, and 19.97 . The images in Fig. 1 are taken from the sample with $\alpha_m/\alpha_d = 0.98$. For $\alpha_m/\alpha_d \leq 1.0$, there is no overshoot or velocity boost in $|V|$ and we always find $|V| < V_0$. When $\alpha_m/\alpha_d > 1.0$, an overshoot or velocity boost emerges with a peak velocity which increases and shifts to larger ρ with increasing Magnus force. In general, at a fixed drive, $|V|$ has a damping dependence of $|V_{\text{damp}}| \propto 1/\rho$ and a boost velocity contribution of $|V_{\text{boost}}| \propto (\alpha_m/\alpha_d)\Delta\rho$, where $\Delta\rho$ is a function of both the unperturbed density and the drive. For increasing ρ , larger values of $\Delta\rho$ can appear but the threshold driving force for the formation of density inhomogeneities also increases. For small ρ , $\Delta\rho$ is reduced and the boosting effect is diminished. The boosting effect occurs for any finite value of the Magnus term, but in general only when $\alpha_m/\alpha_d > 1.0$ do we find a regime in which the net velocity is higher than the single-particle limit. Figure 6(b) shows the corresponding θ_{sk} versus ρ . In every case, θ_{sk} starts at its intrinsic value for small ρ , decreases in magnitude with increasing ρ , and reaches a saturation at high ρ .

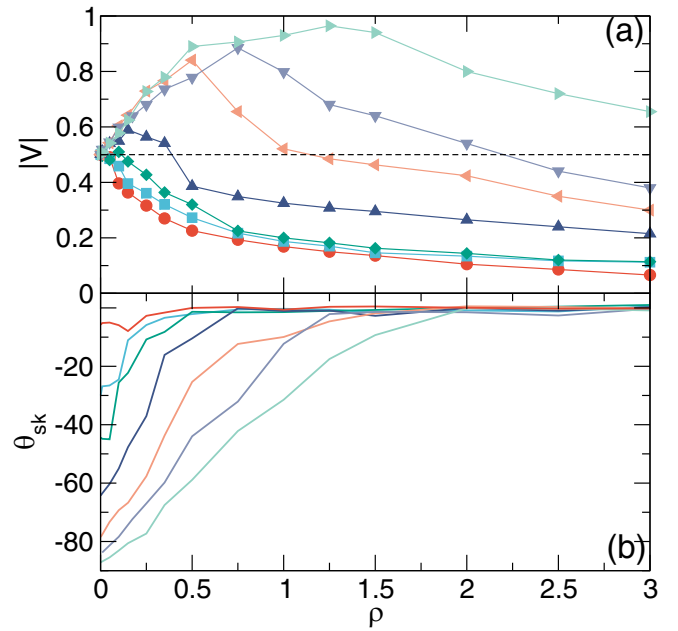


FIG. 6. (a) $|V|$ vs ρ for the systems in Figs. 2 and 4 with $\alpha_d^2 + \alpha_m^2 = 1.0$ and $F_D = 0.5$ at $\alpha_m/\alpha_d = 0.1$ (red circles), 0.57 (light-blue squares), 0.98 (dark-green diamonds), 2.065 (dark-blue up-triangles), 4.924 (purple down-triangles), 9.95 (orange left triangles), and 19.97 (light-green right triangles), from bottom to top. The dashed line corresponds to $V_0 = 0.5$. For $\alpha_m/\alpha_d > 1.0$, there is an overshoot or velocity boost regime with $|V| > V_0$. (b) The corresponding values of θ_{sk} vs ρ , from top to bottom. In each case, θ_{sk} saturates to zero for large enough values of ρ .

In Fig. 7, we map the regions where a velocity boost is present and absent as a function of ρ versus θ_{sk}^{int} for the system in Fig. 6. As θ_{sk}^{int} increases, the upper edge of the velocity boost window shifts to higher values of ρ . The red squares indicate

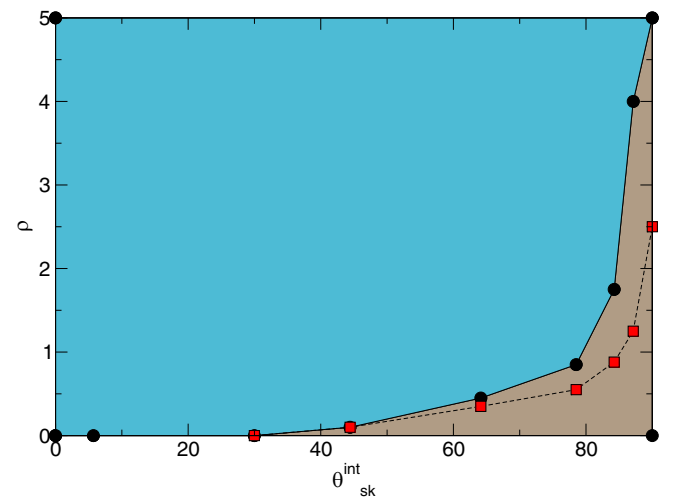


FIG. 7. Dynamic phase diagram as a function of ρ vs intrinsic skyrmion Hall angle θ_{sk}^{int} for the system in Fig. 6 with $\alpha_d^2 + \alpha_m^2 = 1.0$ and $F_D = 0.5$. The velocity boost is present in the brown region and absent in the blue region. The red squares indicate the points at which the boost of $|V|$ is maximized.

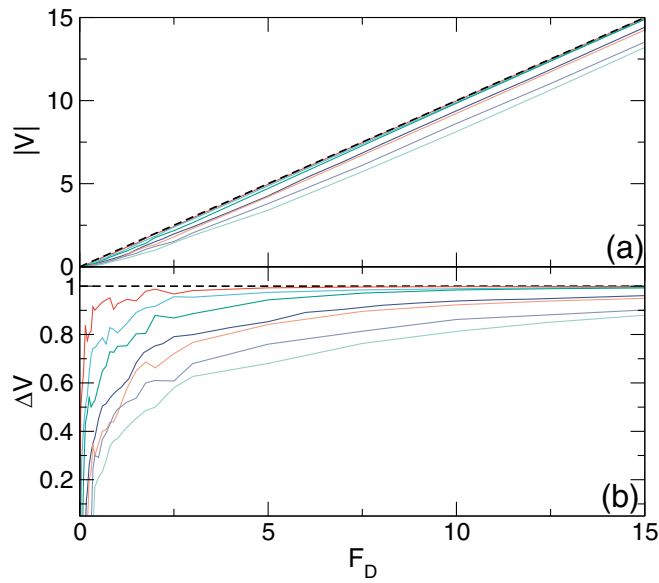


FIG. 8. (a) $|V|$ vs F_D for the system in Fig. 2 with $\alpha_m/\alpha_d = 0.1$, $\alpha_d^2 + \alpha_m^2 = 1.0$, and $\theta_{sk}^{\text{int}} = -5.739^\circ$ at $\rho = 0.05, 0.1, 0.25, 0.5, 1.0, 1.5$, and 2.0 , from top to bottom. The dashed line is the expected curve for the single-particle limit. Here, $|V|$ is always lower than the single-particle limit. (b) $\Delta V = |V|/V_0$ vs F_D , where V_0 is the expected velocity in the single-particle limit at a given drive. Here, $\Delta V < 1.0$, indicating increased damping compared to the single-particle limit. At higher drives, ΔV approaches the single-particle limit.

the values of ρ at which the boost of $|V|$ takes its maximum value for each choice of θ_{sk}^{int} . The boost disappears for all ρ when $\theta_{sk}^{\text{int}} \leq 44^\circ$.

IV. DRIVE DEPENDENCE

The size of the velocity overshoot and the value of the skyrmion Hall angle at a given skyrmion density is also a function of the magnitude of the driving force. In Fig. 8(a), we plot $|V|$ versus F_D for the system in Fig. 2 with $\alpha_m/\alpha_d = 0.1$ in the damping-dominated limit at $\rho = 0.05, 0.1, 0.25, 0.5, 1.0, 1.5$, and 2.0 . The dashed lines indicate the expected behavior in the single-particle limit. For any given value of F_D , Fig. 8(a) shows that $|V|$ decreases monotonically with increasing ρ . In Fig. 8(b), we plot $\Delta V = |V|/V_0$ versus F_D , where V_0 is the velocity in the single-particle limit at each value of F_D . When $\Delta V = 1.0$, the driven skyrmion is moving with the same velocity it would have in the single-particle limit. $\Delta V < 1.0$ indicates increased damping, while $\Delta V > 1.0$ is a signature of a boosted velocity. Here we find $\Delta V < 1.0$ over the entire range of ρ , indicating that the driven skyrmion is moving slower than the free particle limit. The damping effect is strongest at low drives, and the overall damping increases with increasing ρ . There is also a threshold F_c below which the driven skyrmion does not move, as shown in Fig. 9(a) where we plot a blowup of the low drive region from Fig. 8(a). When $F_D < F_c$, $|V| = 0.0$. F_c starts from zero in the single-particle limit and increases with increasing ρ . The plot of ΔV versus F_D in Fig. 9(b) indicates that ΔV is zero below F_c and rises to a saturation value which approaches the single-particle limit

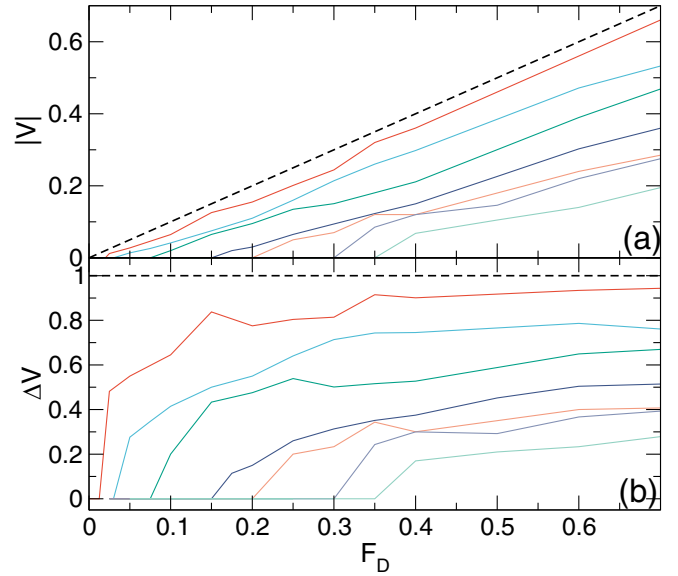


FIG. 9. A blowup of Fig. 8 at low values of F_D for the system in Fig. 2 with $\alpha_m/\alpha_d = 0.1$, $\alpha_d^2 + \alpha_m^2 = 1.0$, and $\theta_{sk}^{\text{int}} = -5.739^\circ$ at $\rho = 0.05, 0.1, 0.25, 0.5, 1.0, 1.5$, and 2.0 , from top to bottom. The dashed line is the expected curve for the single-particle limit. (a) $|V|$ vs F_D , showing that there is a threshold force F_c with $|V| = 0.0$ for $F_D < F_c$. F_c shifts to higher values of F_D with increasing ρ . (b) The corresponding ΔV vs F_D .

as ρ decreases. In general, ΔV increases towards $\Delta V = 1.0$ with increasing F_D since the surrounding skyrmions have less time to respond to a rapidly moving driven skyrmion. For overdamped systems with quenched disorder, the velocity of the probe particle has the power law form $V = (F_d - F_c)^\alpha$, so the particle moves the most slowly just above the threshold depinning force [5,20,21], while at high drives the velocity generally approaches the pin-free limit [20,21]. In our system, there is no quenched disorder but the background bath of skyrmions effectively serves as a type of deformable quenched disorder. At high velocities, where the background skyrmions do not have time to respond before the driven skyrmion has passed beyond them, the system looks more like a single skyrmion moving through an array of fixed defects or scattering sites.

In Fig. 10, we plot $|V|$ and ΔV versus F_D for varied ρ in the Magnus-dominated regime with $\alpha_m/\alpha_d = 9.95$ and $\alpha_d^2 + \alpha_m^2 = 1.0$ for $\rho = 0.05, 0.1, 0.25, 0.5, 1.0, 1.5$, and 2.0 . The dashed line indicates the expected behavior in the single-particle limit. Here we find that $|V|$ is generally larger than the single-particle limit, indicating a velocity boost. In Fig. 10(b), ΔV increases with increasing ρ and has a peak at F_{max} which shifts to higher values of F_D as ρ becomes larger. For large F_D , ΔV decreases back to the single-particle limit. When $\rho = 2.0$, the peak value of ΔV is almost 2.5 times larger than the free particle limit. We still observe a threshold force F_c for motion which is almost the same as that found for the overdamped case. In general, we do not observe a significant difference in the threshold for motion as a function of the ratio of the Magnus term to the damping. In systems with quenched disorder, it has been shown that the depinning threshold often decreases with increasing Magnus force since the motion of

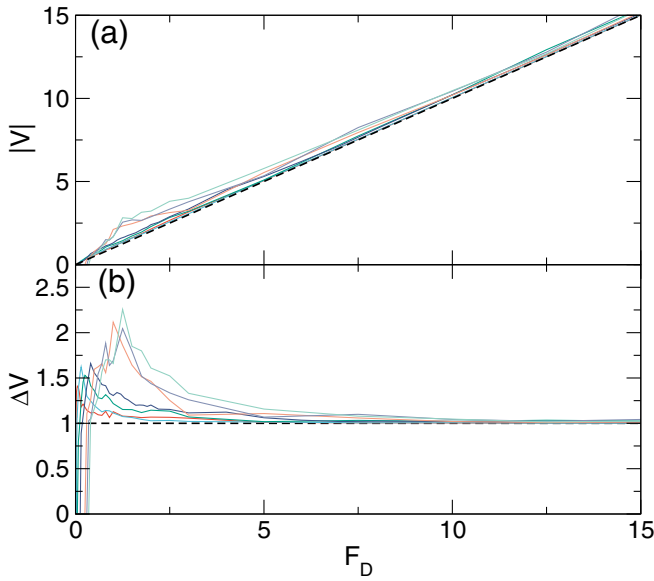


FIG. 10. (a) $|V|$ vs F_D for a system with $\alpha_m/\alpha_d = 9.95$ and $\alpha_d^2 + \alpha_m^2 = 1.0$ at $\rho = 0.05, 0.1, 0.25, 0.5, 1.0, 1.5,$ and 2.0 , from bottom to top. $|V|$ is generally above the dashed line, which is the expected behavior in the single-particle limit. (b) The corresponding $\Delta V = |V|/V_0$ vs F_D , where V_0 is the expected velocity for the single-particle limit at a given drive. Here, $\Delta V > 1.0$ indicates the occurrence of velocity boosting, with ΔV approaching the single-particle limit at higher drives.

the skyrmions requires a longer time to relax when the Magnus force is larger [5,44]. This increases the time required in the presence of disorder for the spiraling motion of the skyrmions to settle into equilibrium after a finite applied drive is increased [5,29,30,108]. In our work, we apply a constant drive and wait for the system to settle into a steady state. The transient time to reach that state increases near the threshold, suggesting that the observation of a reduced pinning threshold in other studies with quenched disorder could be the result of the increased relaxation time for larger Magnus forces. In studies where the drive is continuously increased from zero to a finite value, the sweeping rate above which transient effects become important should depend on both the damping and Magnus force. Such rate effects will be the subject of a future work.

In Fig. 10(b), we find that ΔV decreases with increasing drive for $F_D > F_{\max}$ and approaches $\Delta V = 1.0$ at high drives. The decrease follows the form $\Delta V = [A/(F_D - F_{\max})] + 1.0$. In Fig. 11, we plot ΔV versus F_D for the specific case of $\rho = 2.0$ from the system in Fig. 10(b), along with a fit using $A = 0.65$ and $F_{\max} = 0.85$. A similar fit can be performed for the other ΔV versus F_D curves. This type of $1/F_D$ dependence on fluctuations or dynamic disorder is also often observed in driven systems with quenched disorder [5,29,30].

Another effect of the emergence of a density inhomogeneity $\Delta\rho$ is a reduction in the skyrmion Hall angle at lower drives, since the skyrmions that accumulate below the driven particle partially block the motion in the $-y$ direction. In Fig. 12, we plot θ_{sk} versus F_D for the system in Fig. 10 with $\alpha_m/\alpha_d = 9.95$ for $\rho = 0.05, 0.1, 0.25, 0.5, 1.0, 1.5,$ and

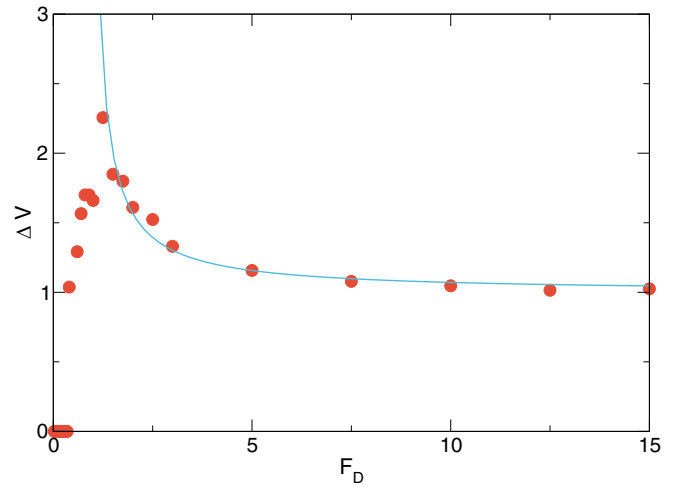


FIG. 11. Circles: ΔV vs F_D from the system with $\rho = 2.0$, $\alpha_m/\alpha_d = 9.95$, and $\alpha_d^2 + \alpha_m^2 = 1.0$ in Fig. 10. The line indicates a fit to $\Delta V = [A/(F_D - F_{\max})] + 1.0$ with $A = 0.65$ and $F_{\max} = 0.85$.

2.0. The dashed line indicates the single-particle limit, where $\theta_{sk} = -\arctan(\alpha_m/\alpha_d)$ for all values of F_D . We find a finite interval of F_D over which the magnitude of θ_{sk} increases with increasing F_D , followed by a saturation close to the intrinsic value at higher drives. As the driven skyrmion approaches the saturation regime, it is moving fast enough that the surrounding skyrmions cannot respond to its presence, giving a small $\Delta\rho$ and a reduced boost effect, as shown in Fig. 5(b). The increase of θ_{sk} with increasing F_D has also been observed for skyrmion assemblies driven over random disorder, where at higher drives the skyrmions are only weakly perturbed by the pinning [29–32,90–95].

From the features in $\langle V_{\perp} \rangle$, $\langle V_{\parallel} \rangle$, $|V|$, and θ_{sk} , we can construct a dynamic phase diagram as a function of F_D versus ρ for the strongly damped and Magnus-dominated limits. In Fig. 13, we show the phase diagram for the damping-dominated system with $\alpha_m/\alpha_d = 0.1$ in Figs. 8 and 9. Red indicates the pinned regime where $F_D < F_c$. In the green

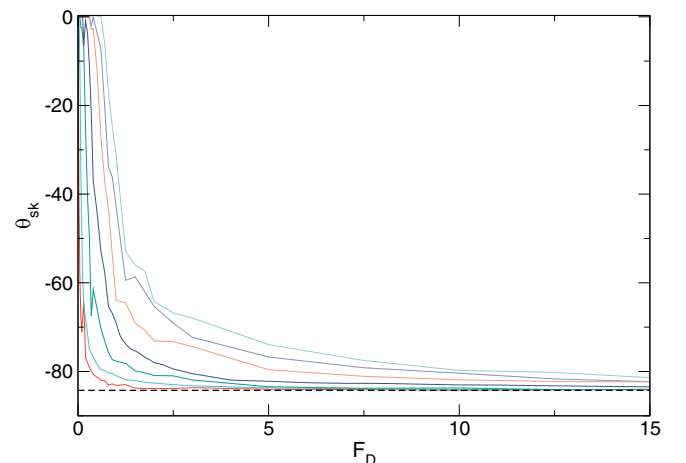


FIG. 12. θ_{sk} vs F_D for the system in Fig. 10 with $\alpha_m/\alpha_d = 9.95$ and $\alpha_d^2 + \alpha_m^2 = 1.0$ at $\rho = 0.05, 0.1, 0.25, 0.5, 1.0, 1.5,$ and 2.0 , from bottom to top. The dashed line is the single-particle limit.

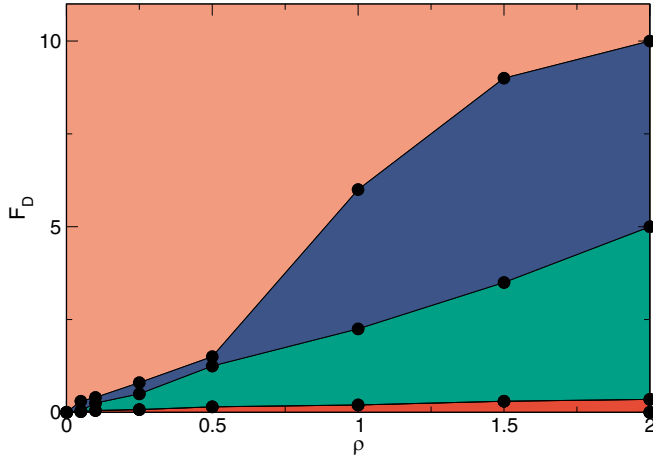


FIG. 13. Dynamic phase diagram as a function of F_D vs ρ for a damping-dominated system with $\alpha_m/\alpha_d = 0.1$ and $\alpha_m^2 + \alpha_d^2 = 1.0$. Red: pinned. Green: the driven skyrmion moves at $\theta_{sk} = 0.0^\circ$. Blue: the skyrmion Hall angle is increasing in magnitude with drive. Orange: the saturation regime.

region, the driven skyrmion is moving but its velocity is strictly along the driving direction so that $\theta_{sk} = 0.0^\circ$. In the blue region, there is a finite but growing θ_{sk} , while in the orange region, the skyrmion Hall angle has saturated. The thresholds for motion parallel and perpendicular to the driving direction both increase with increasing ρ . Previous work in systems with quenched disorder also showed that there can be separate thresholds for the onset of motion parallel and perpendicular to the drive, with a region above the first depinning threshold where the skyrmions can flow at $\theta_{sk} = 0^\circ$ for small intrinsic skyrmion Hall angles [31,98,109]. As the ratio of the Magnus force to the damping term increases, the threshold for transverse motion shifts to smaller F_D . In the saturation regime, the response resembles the single-particle limit and θ_{sk} approaches the intrinsic skyrmion Hall value of $\theta_{sk}^{\text{int}} = 5.7^\circ$.

In Fig. 14, we show the dynamic phase diagram as a function of F_D versus ρ for the Magnus-dominated system with $\alpha_m/\alpha_d = 9.95$. There is still a pinned regime, but the region in which there is finite motion with $\theta_{sk} = 0.0^\circ$ is absent or too small to detect. In the blue region, the velocity is significantly boosted beyond the single-particle limit and the skyrmion Hall angle increases with increasing drive. In the orange region, both θ_{sk} and the velocity approach the single-particle limit. Between the damping-dominated and Magnus-dominated limits, the dynamic phase diagram includes a combination of the features of the phase diagrams in Fig. 13 and Fig. 14.

V. VARIED MAGNUS TO DAMPING RATIO AND VELOCITY-FORCE CURVES

We next fix the density ρ and vary the ratio of the Magnus and damping terms while maintaining the normalization relation $\alpha_d^2 + \alpha_m^2 = 1.0$. In Fig. 15(a), we plot $\langle V_{\parallel} \rangle$ versus F_D for systems with $\rho = 0.5$ at $\alpha_m/\alpha_d = 9.95$ and 0.1. Figures 15(b) and 15(c) show the corresponding values of $\langle V_{\perp} \rangle$ and θ_{sk} versus F_D , respectively. For the Magnus-dominated case of

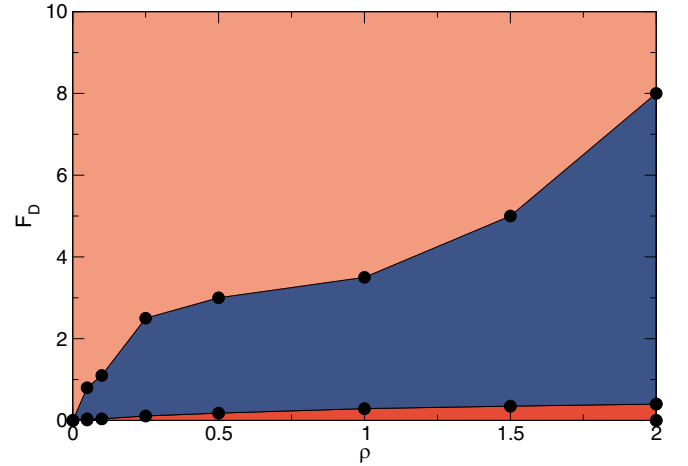


FIG. 14. Dynamic phase diagram as a function of F_D vs ρ for the Magnus-dominated system with $\alpha_m/\alpha_d = 9.95$ and $\alpha_m^2 + \alpha_d^2 = 1.0$. Red: pinned. Blue: the skyrmion Hall angle is increasing in magnitude with drive and a velocity boost occurs. Orange: the saturation regime.

$\alpha_m/\alpha_d = 9.95$, $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ become finite at almost the same value of F_D , which also corresponds to the appearance of a nonzero θ_{sk} . For the damping-dominated sample with $\alpha_m/\alpha_d = 0.1$, $\langle V_{\perp} \rangle$ and θ_{sk} remain zero up to $F_D \approx 1.0$.

The ratio of the Magnus to the damping term determines the shape of the velocity-force curves. For $\alpha_m/\alpha_d = 0.1$, both $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ increase monotonically with F_D according to the linear behavior $V \propto (F_D - F_c)$, where F_c is the threshold for motion in either the parallel or the perpendicular direction. When $\alpha_m/\alpha_d = 9.95$, $\langle V_{\perp} \rangle$ still increases linearly with increasing F_D above the depinning threshold; however,

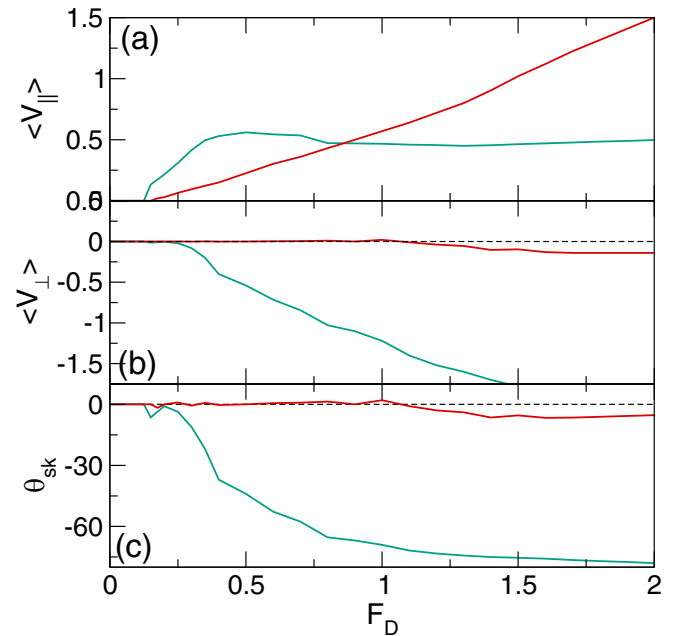


FIG. 15. (a) $\langle V_{\parallel} \rangle$ vs F_D for systems with $\rho = 0.5$ and $\alpha_m^2 + \alpha_d^2 = 1.0$ at $\alpha_m/\alpha_d = 9.95$ (green) and 0.1 (red). (b) The corresponding $\langle V_{\perp} \rangle$ vs F_D . (c) The corresponding θ_{sk} vs F_D .

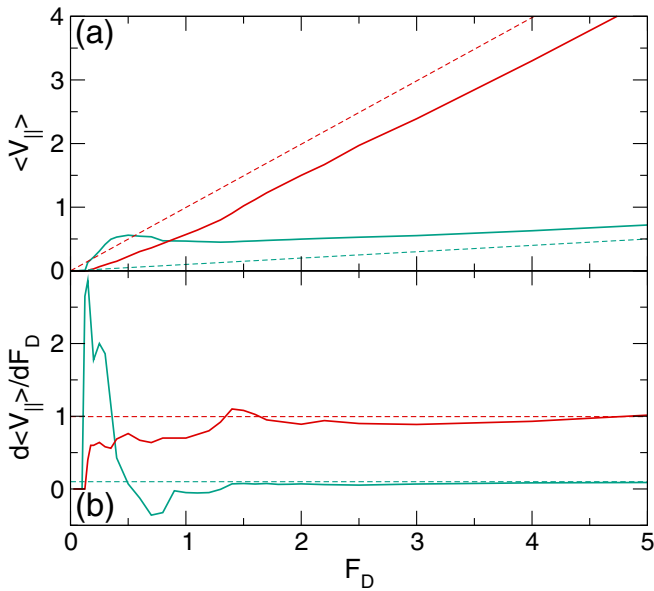


FIG. 16. (a) $\langle V_{\parallel} \rangle$ vs F_D for the system in Fig. 15 with $\rho = 0.5$ and $\alpha_m^2 + \alpha_d^2 = 1.0$ at $\alpha_m/\alpha_d = 9.95$ (green) and 0.1 (red). The dashed lines are the expected velocity-force curves in the single-particle limit for these two cases. (b) The corresponding $d\langle V_{\parallel} \rangle/dF_D$ vs F_D , where there is a region of negative differential conductivity with $d\langle V_{\parallel} \rangle/dF_D < 0.0$ for the $\alpha_m/\alpha_d = 9.95$ sample.

$\langle V_{\parallel} \rangle$ exhibits a nonmonotonic behavior in which it initially rises rapidly with increasing F_D but then decreases again. In this regime, the velocity decreases even though the drive is increasing, giving $d\langle V_{\parallel} \rangle/dF_D < 0$, which is known as negative differential conductivity [21]. In Fig. 16(a), we plot $\langle V_{\parallel} \rangle$ versus F_D for the samples from Fig. 15. The green dashed line shows the expected value for $\langle V_{\parallel} \rangle$ in the single-particle limit at $\alpha_m/\alpha_d = 9.95$, which increases linearly according to $\langle V_{\parallel} \rangle = 0.995F_D$. For the $\alpha_m/\alpha_d = 0.1$ sample, the red dashed line is the single-particle limit, $\langle V_{\parallel} \rangle = 0.1F_D$. In the damping-dominated system with $\alpha_m/\alpha_d = 0.1$, $\langle V_{\parallel} \rangle$ is below the single-particle limit, while in the Magnus-dominated system with $\alpha_m/\alpha_d = 9.95$, $\langle V_{\parallel} \rangle$ is higher than the single-particle limit due to the boosting effect. In Fig. 16(b), we show the corresponding values of $d\langle V_{\parallel} \rangle/dF_D$, with the single-particle limits marked by dashed lines. For the $\alpha_m/\alpha_d = 9.95$ system, the initial peak in $d\langle V_{\parallel} \rangle/dF_D$ corresponding to the depinning transition is followed by a region of negative differential conductivity where $d\langle V_{\parallel} \rangle/dF_D < 0.0$. At higher drives, $d\langle V_{\parallel} \rangle/dF_D$ saturates to the value expected in the single-particle limit. For the damping-dominated system with $\alpha_m/\alpha_d = 0.1$, $d\langle V_{\parallel} \rangle/dF_D$ is initially below the single-particle value due to the increased damping from the surrounding skyrmions, while at higher drives, it approaches the single-particle limit. The negative differential conductivity in the Magnus-dominated system is affected by the density, and for low ρ , the shape of the velocity-force curve approaches that found in the single-particle limit.

In Fig. 17(a), we plot $\langle V_{\perp} \rangle$ versus F_D for the system in Fig. 15 at $\rho = 0.5$ with $\alpha_m/\alpha_d = 9.95$ and 0.1 . The dashed lines are the expected values for the single-particle limit, which obey $\langle V_{\perp} \rangle = -0.1F_D$ for the $\alpha_m/\alpha_d = 0.1$ system

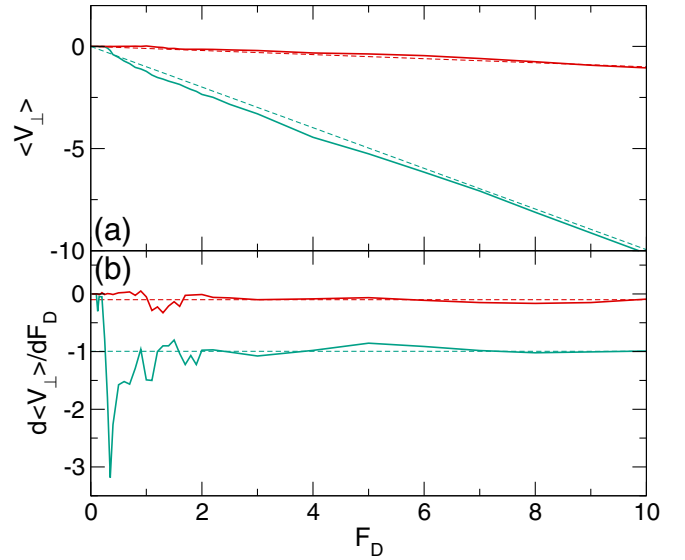


FIG. 17. (a) $\langle V_{\perp} \rangle$ vs F_D for the system in Fig. 15 with $\rho = 0.5$ and $\alpha_m^2 + \alpha_d^2 = 1.0$ at $\alpha_m/\alpha_d = 9.95$ (green) and 0.1 (red). The dashed lines are the expected velocity-force curves in the single-particle limit for these two cases. (b) The corresponding $d\langle V_{\perp} \rangle/dF_D$ vs F_D .

and $\langle V_{\perp} \rangle = -0.995F_D$ for the $\alpha_m/\alpha_d = 9.95$ system. In the Magnus-dominated regime, $|\langle V_{\perp} \rangle|$ is slightly larger than it would be in the single-particle limit, while in the damping-dominated regime it is slightly lower. Most of the velocity boost in the Magnus-dominated regime is parallel to the driving direction since the induced density gradient is perpendicular to the drive. In Fig. 17(b), we show the corresponding $d\langle V_{\perp} \rangle/dF_D$ versus F_D curves where, unlike the parallel velocity plotted in Fig. 16, there is no regime of negative differential mobility. There is still a peak in $|d\langle V_{\perp} \rangle/dF_D|$ near the depinning threshold, while at high drives, the curves approach the single-particle limit.

In Fig. 18(a), we plot $\langle V_{\parallel} \rangle$ versus F_D for the system in Fig. 15 with fixed $\rho = 0.5$ and $\alpha_m^2 + \alpha_d^2 = 1.0$ at $\alpha_m = 0, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.97, 0.995$, and 0.998 . Figures 18(b) and 18(c) show the corresponding $\langle V_{\perp} \rangle$ and θ_{sk} , respectively, versus F_D . We find negative differential conductivity for $\alpha_m > 0.9$ or $\alpha_m/\alpha_d > 5.0$, indicating that this effect appears only when the Magnus force is sufficiently large. There is no negative differential conductivity in the $\langle V_{\perp} \rangle$ versus F_D curves, while the plots of θ_{sk} versus F_D indicate that as α_m decreases, the threshold drive above which the skyrmion Hall angle becomes finite increases.

From the features in Fig. 18, we can construct a dynamic phase diagram as a function of F_D versus α_m , as shown in Fig. 19. Here we outline the pinned phase, the flowing regime with $\theta_{sk} = 0.0^\circ$, the region in which θ_{sk} increases with increasing F_D , and the saturation regime where there is little change in θ_{sk} . For drives above the dashed line, the velocities are boosted compared to the single-particle limit. The boost is strongly reduced in the saturation regime. Figure 19 indicates that the depinning threshold is approximately constant as a function of increasing α_m , while the regime in which the velocity is locked in the driving direction grows in extent with

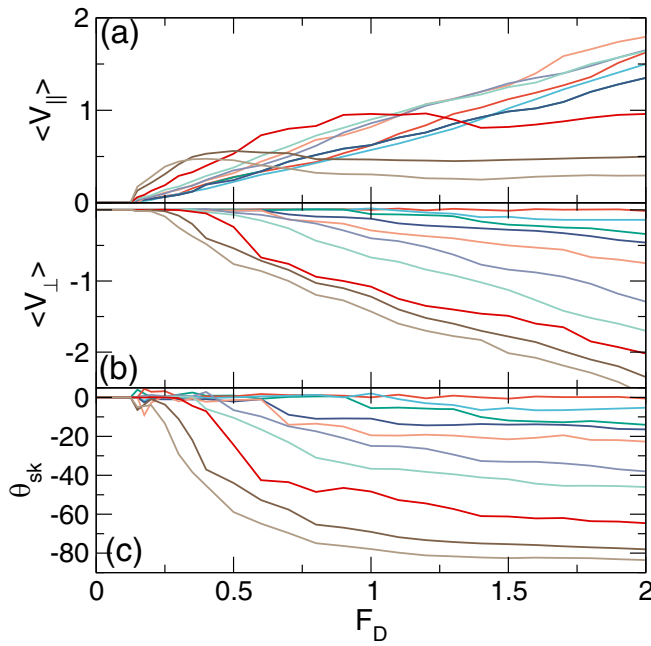


FIG. 18. (a) $\langle V_{\parallel} \rangle$ vs F_D for the system in Fig. 15 with $\rho = 0.5$ and $\alpha_m^2 + \alpha_d^2 = 1.0$ at $\alpha_m = 0.0, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.97, 0.995$, and 0.998 , from bottom to top. (b) The corresponding $\langle V_{\perp} \rangle$ vs F_D . (c) The corresponding θ_{sk} vs F_D .

decreasing α_m . At $\alpha_m = 0$, the onset of saturation coincides with the point where the velocity-force curves start to grow linearly with F_D . The transition demarcating the onset of a velocity boost shifts to lower α_m as ρ increases.

We next relax the constraint of $\alpha_m^2 + \alpha_d^2 = 1.0$ and instead hold either the Magnus or damping term constant while varying the other quantity. For a fixed drive, this means that in the single-particle limit, $|V|$ obeys $|V| \propto F_D / \sqrt{\alpha_m^2 + \alpha_d^2}$. In

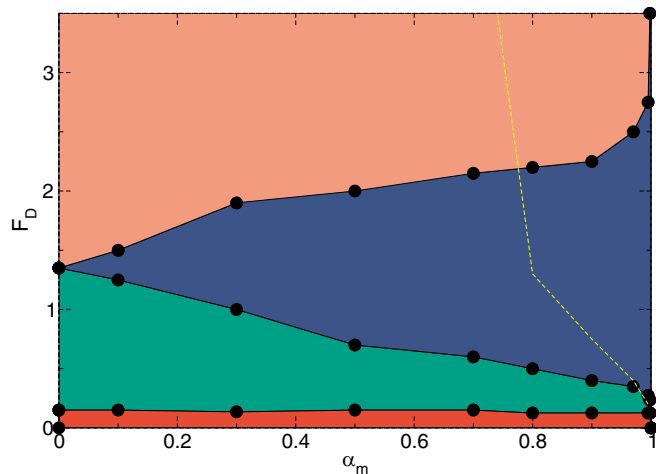


FIG. 19. Dynamic phase diagram as a function of F_D vs α_m for the system in Fig. 18 with $\rho = 0.5$ and $\alpha_m^2 + \alpha_d^2 = 1.0$. Red: pinned. Green: the driven skyrmion moves at $\theta_{sk} = 0.0^\circ$. Blue: the skyrmion Hall angle is increasing in magnitude with drive. Orange: the saturation regime. For F_D above the dashed line, there is a velocity boost compared to the single-particle limit.

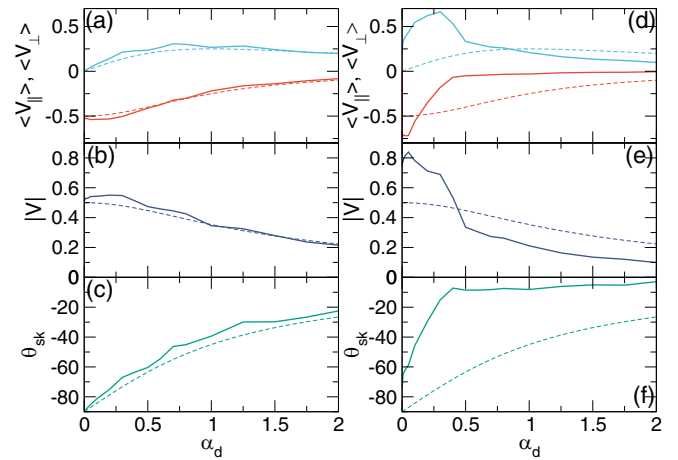


FIG. 20. (a) $\langle V_{\parallel} \rangle$ (blue) and $\langle V_{\perp} \rangle$ (red) vs α_d for a system with $\rho = 0.05$, $F_D = 0.5$ and $\alpha_m = 1.0$. The blue dashed line is the expected single-particle behavior which goes as $\langle V_{\parallel} \rangle = F_D \alpha_d / (1 + \alpha_d^2)$ and the red dashed line is the single-particle behavior of $\langle V_{\perp} \rangle = -F_D / (1 + \alpha_d^2)$. (b) The corresponding $|V|$ vs α_d , where the dashed line is the single-particle limit of $|V| = F_D / (1 + \alpha_d^2)^{1/2}$. (c) The corresponding skyrmion Hall angle θ_{sk} vs α_d . The dashed line is the single-particle limit of $\theta_{sk} = -F_D \arctan(1/\alpha_d)$. (d) $\langle V_{\parallel} \rangle$ (blue) and $\langle V_{\perp} \rangle$ (red) vs α_d for the same system but with $\rho = 0.5$. (e) $|V|$ vs α_d for the system in (d). (f) θ_{sk} vs α_d for the system in (d). The denser system exhibits a speed-up effect.

Fig. 20(a), we plot $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ versus α_d for a system with $\rho = 0.05$, $F_D = 0.5$, and $\alpha_m = 1.0$. Here, $\langle V_{\parallel} \rangle$ is zero for $\alpha_d = 0.0$, increases to a maximum value near $\alpha_d = 1.0$, and then decreases with increasing α_d , while $\langle V_{\perp} \rangle$ gradually approaches zero as α_d increases. In the single-particle limit, $\langle V_{\parallel} \rangle = F_D \alpha_d / (\alpha_m^2 + \alpha_d^2)$ and $\langle V_{\perp} \rangle = -F_D \alpha_m / (\alpha_m^2 + \alpha_d^2)$, so for a fixed $\alpha_m = 1.0$, $\langle V_{\parallel} \rangle = F_D \alpha_d / (1 + \alpha_d^2)$, plotted as a dashed line. In this case, $\langle V_{\parallel} \rangle = 0.0$ when $\alpha_d = 0.0$, and the parallel velocity also approaches zero at large α_d . Similarly, in the single-particle limit, $\langle V_{\perp} \rangle = -F_D / (1 + \alpha_d^2)$, so that when $\alpha_d = 0.0$, $\langle V_{\perp} \rangle = -F_D = -0.5$. For $\rho = 0.05$, the density is low enough that the behavior is close to the single-particle limit. In Fig. 20(b), we plot $|V|$ versus α_d for the system in Fig. 20(a), where the dashed line is the single-particle limit of $|V| = F_D / (1 + \alpha_d^2)^{1/2}$. There is a small boost in $|V|$ for $\alpha_d < 1.0$, while $|V|$ drops below the free particle limit for $\alpha_d > 1.5$. Figure 20(c) shows the corresponding θ_{sk} versus α_d for the $\rho = 0.05$ system, where the magnitude of θ_{sk} gradually decreases with increasing α_d . The dashed line is the single-particle limit, $\theta_{sk} = -F_D \arctan(1/\alpha_d)$. The measured skyrmion Hall angle is smaller in magnitude than the single-particle value due to the collisions with the background skyrmions. In Fig. 21(a), we show the positions and trajectories of the driven skyrmion and the background skyrmions for the system in Fig. 20(a) at $\rho = 0.05$ and $\alpha_d = 0.3$. The driven skyrmion is moving at a skyrmion Hall angle close to $\theta_{sk} = -65^\circ$. Although there are some collisions with the background skyrmions, there is only a small boost in the velocity.

In Fig. 20(d), we show $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ versus α_d for the same system in Fig. 20(a), but at a higher density of $\rho = 0.5$. The dashed lines are the expected behavior in the single-particle

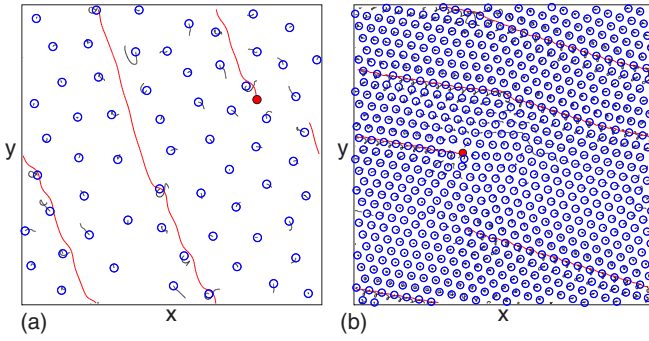


FIG. 21. Driven skyrmion and trajectory (red) along with the surrounding skyrmions (blue) and their trajectories (black) for the system in Fig. 20(a) with $\rho = 0.05$, $\alpha_d = 0.3$, and $\alpha_m = 1.0$, where the skyrmion Hall angle is large. (b) The same but at $\rho = 0.5$, where the skyrmion Hall angle is smaller and the velocity of the driven skyrmion is higher.

limit. Here there is a boost in $\langle V_{\parallel} \rangle$ for $\alpha_d < 0.75$, while $\langle V_{\parallel} \rangle$ drops below the single-particle limit at higher α_d . There is also a small boost in $\langle V_{\perp} \rangle$ for $\alpha_d < 0.3$. We note that unlike the low density case, $\langle V_{\parallel} \rangle$ is finite at $\alpha_d = 0.0$ due to collisions with the background skyrmions. Figure 20(e) shows the corresponding $|V|$ versus α_d and the expected single-particle limit, indicating that there is a net velocity boost for $\alpha_d < 0.5$ followed by increased damping for $\alpha_d > 0.5$. In Fig. 20(f), we plot the corresponding θ_{sk} versus α_d and the single-particle limit. The skyrmion Hall angle rapidly decreases in magnitude with increasing α_d , reaching a saturation for $\alpha_d > 0.5$. Figure 21(b) shows the positions and trajectories of the driven and bath skyrmions for the system in Fig. 20(d) at $\rho = 0.5$ and $\alpha_d = 0.3$, where the skyrmion Hall angle is much smaller than that found in the $\rho = 0.05$ system illustrated in Fig. 21(a); however, the velocity of the driven skyrmion is much larger for the $\rho = 0.5$ system.

In Fig. 22(a), we plot $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ versus α_m for a system with fixed $\alpha_d = 1.0$, $\rho = 0.05$, and $F_D = 0.5$. In the single-particle limit with $\alpha_d = 1.0$, $\langle V_{\parallel} \rangle = F_D/(1 + \alpha_m^2)$ and $\langle V_{\perp} \rangle = -F_D\alpha_m/(1 + \alpha_m^2)$. We find that $\langle V_{\parallel} \rangle$ monotonically decreases and changes from being slightly higher than the free particle limit for $\alpha_m < 5.0$ to being slightly lower than the free particle limit for $\alpha_m > 5.0$. $\langle V_{\perp} \rangle$ starts from zero at $\alpha_m = 0.0$, reaches a maximum value near $\alpha_m = 1.0$, and gradually drops back to zero with increasing α_m , closely following the single-particle limit. In Fig. 22(b), we show the corresponding $|V|$ versus α_m along with a dashed line indicating the single-particle limit of $|V| = F_D/(1 + \alpha_m^2)^{1/2}$, which decreases monotonically with increasing α_m . There is a small boost in the velocity due to the collisions with the background skyrmions. Figure 21(c) shows θ_{sk} versus α_m for the same system as well as the single-particle limit of $\theta_{sk} = -\arctan(\alpha_m)$. Here the measured skyrmion Hall angle is slightly smaller in magnitude than the single-particle limit due to the collisions with bath skyrmions.

In Fig. 22(d), we plot $\langle V_{\parallel} \rangle$, $\langle V_{\perp} \rangle$, and the single-particle limits for the same system from Fig. 22(a), but at a higher density of $\rho = 0.5$. Here, $\langle V_{\parallel} \rangle$ is considerably below the single-particle limit at $\alpha_m = 0.0$ due to the increased frequency of collisions; however, for $\alpha_m > 2.5$, it is considerably higher than the single-particle limit. $\langle V_{\perp} \rangle$ is close to zero

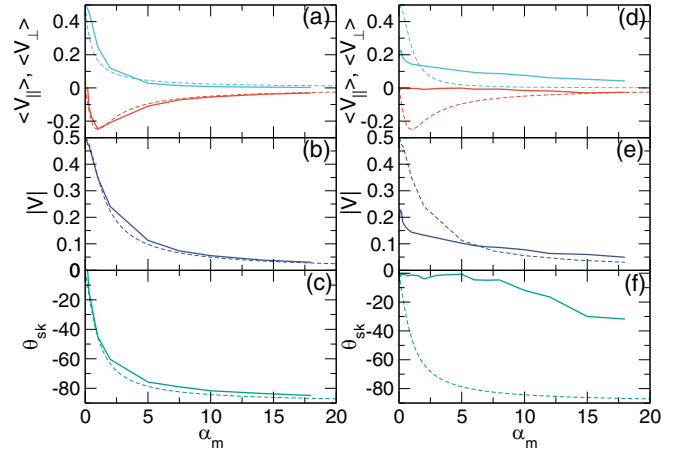


FIG. 22. (a) $\langle V_{\parallel} \rangle$ (blue) and $\langle V_{\perp} \rangle$ (red) vs α_m for a system with $\rho = 0.05$, $F_D = 0.5$, and $\alpha_d = 1.0$. The blue dashed line is the expected single-particle behavior $\langle V_{\parallel} \rangle = F_D/(1 + \alpha_m^2)$ and the red dashed line is the single-particle behavior $\langle V_{\perp} \rangle = -F_D\alpha_m/(1 + \alpha_m^2)$. (b) $|V|$ vs α_d for the system in panel (a), where the dashed line is the single-particle limit of $|V| = F_D/(1 + \alpha_m^2)^{1/2}$. (c) The corresponding skyrmion Hall angle θ_{sk} vs α_m , where the dashed line is the single-particle limit of $\theta_{sk} = -\arctan(\alpha_m)$. (d) $\langle V_{\parallel} \rangle$ (blue) and $\langle V_{\perp} \rangle$ (red) vs α_d for the same system, but with $\rho = 0.5$. (e) The corresponding $|V|$ vs α_m and (f) the corresponding θ_{sk} vs α_m .

for $\alpha_m < 7.0$, but begins to increase at larger α_m . In Fig. 22(e), we show the corresponding $|V|$ versus α_m . $|V|$ falls below the dashed line representing the single-particle limit up to $\alpha_m = 7.0$ and exhibits a boost for higher α_m . Figure 22(f) illustrates the corresponding θ_{sk} versus α_m and the single-particle limit. θ_{sk} is close to zero for $\alpha_m < 7.0$, while for higher α_m , the magnitude of θ_{sk} increases and a velocity boost appears. For higher values of F_D , all of the quantities gradually approach the single-particle limit.

In Fig. 23(a), we plot $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ versus ρ for a system with fixed $\alpha_m = 1.0$, $\alpha_d = 0.3$, and $F_D = 0.5$. The

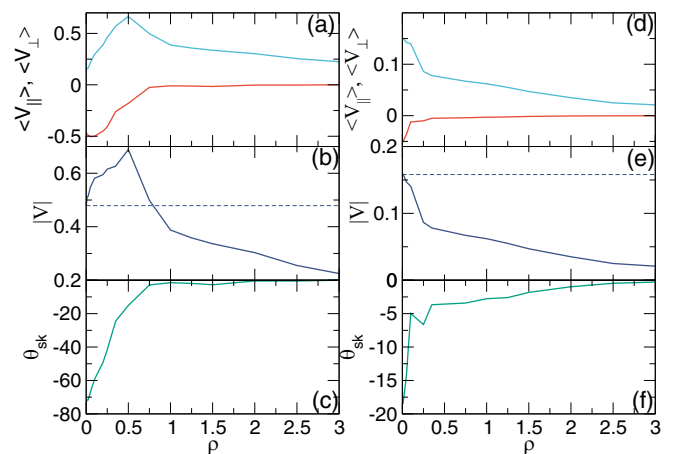


FIG. 23. (a) $\langle V_{\parallel} \rangle$ (blue) and $\langle V_{\perp} \rangle$ (red) vs ρ for a system with $\alpha_d = 0.3$, $\alpha_m = 1.0$, and $F_D = 0.5$. (b) The corresponding $|V|$ vs ρ . (c) The corresponding skyrmion Hall angle θ_{sk} vs ρ . (d) $\langle V_{\parallel} \rangle$ (blue) and $\langle V_{\perp} \rangle$ (red) vs ρ for the same system with a higher $\alpha_d = 3.0$. (e) The corresponding $|V|$ vs ρ . (f) The corresponding θ_{sk} vs ρ .

single-particle limit for these parameters gives $\langle V_{\parallel} \rangle = 0.14$ and $\langle V_{\perp} \rangle = -0.48$. $\langle V_{\parallel} \rangle$ initially increases with increasing ρ to a peak value of 0.65, a strong boost that is four times larger than the single-particle limit. As ρ increases further, $\langle V_{\parallel} \rangle$ gradually decreases; however, even at the high density of $\rho = 3.0$, $\langle V_{\parallel} \rangle$ is still almost twice as large as the single-particle limit, indicating the continuing effectiveness of the boost effect. $\langle V_{\perp} \rangle$ decreases monotonically in magnitude with increasing density, approaching a value close to zero for $\rho > 0.7$. In Fig. 23(b), we show the corresponding $|V|$ versus ρ , where the dashed line is the single-particle limit of $|V| = F_D/(\alpha_d^2 + \alpha_m^2)^{1/2}$ which, for these coefficients, is $|V| = 0.4789$. Here there is a boost in the velocity up to $\rho = 0.7$, while at higher densities, the velocities are much more strongly damped. This indicates that it is possible for the parallel velocity to be boosted while the overall velocity is not boosted. Figure 23(c) illustrates the corresponding θ_{sk} versus ρ . For low density, the skyrmion Hall angle is near the intrinsic value of $\theta_{sk}^{\text{int}} = -73.3^\circ$, and as ρ increases, θ_{sk} approaches zero once $\rho > 0.7$.

In Fig. 23(d), we plot $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ versus ρ for a system with a larger $\alpha_d = 3.0$ at $\alpha_m = 1.0$ and $F_D = 0.5$. For these parameters, the single-particle limit gives $\langle V_{\parallel} \rangle = 0.15$ and $\langle V_{\perp} \rangle = -0.05$. We find that both $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ monotonically decrease in magnitude with increasing ρ . In Fig. 23(e), we show the corresponding $|V|$ versus ρ , where the dashed line is the single-particle limit of $|V| = 0.158$. Here there is no boost and $|V|$ drops off rapidly with increasing ρ , showing a change in slope near $\rho = 0.3$ to a slower decline. Figure 23(f) illustrates the corresponding θ_{sk} versus ρ , which starts off near $\theta_{sk} = -18.4^\circ$ and approaches zero for $\rho > 0.3$. The change in the skyrmion Hall angle and velocity across $\rho = 0.3$ occurs because the density is low enough for $\rho < 0.3$ that the bath skyrmions act like a fluid which is not strongly coupled to the driven skyrmion, while when $\rho > 0.3$, the bath skyrmions act more like a solid, increasing the drag on the driven skyrmion.

VI. THERMAL EFFECTS

Throughout this work, we have neglected thermal effects; however, it is known that thermal creep effects can appear near depinning in skyrmion systems [95,98,110]. In this section, we consider the effects of adding Langevin thermal kicks F^T to the system with the properties $\langle F^T(t) \rangle = 0$ and $\langle F^T(t)F^T(t') \rangle = 2k_B\alpha_d T \delta(t - t')$. A similar approach for adding temperature to a particle-based skyrmion model was studied in Refs. [44,98]. We fix $F_D = 0.5$ and $\rho = 0.5$ for a Magnus-dominated system similar to the one in Fig. 15 with $\alpha_m/\alpha_d = 9.95$, and for an overdamped system with $\alpha_m/\alpha_d = 0.0$. Here we impose the constraint $\alpha_m^2 + \alpha_d^2 = 1.0$ such that in the single-particle limit, the net velocity of the driven particle is equal to $F_D = 0.5$ for both the Magnus-dominated and overdamped systems.

In Fig. 24(a), we plot $\langle V_{\parallel} \rangle$ and $\langle V_{\perp} \rangle$ versus F^T for a sample with $\alpha_m/\alpha_d = 9.95$ in the Magnus-dominated limit. Here, $\langle V_{\parallel} \rangle$ decreases gradually as F^T increases, while $\langle V_{\perp} \rangle$ shows only small changes. In Fig. 24(b), we show the net velocity $|V|$ versus F^T for the Magnus-dominated system from Fig. 24(a) and for an overdamped system with $\alpha_m/\alpha_d = 0.0$. The dashed line is the single-particle limit of $V_0 = 0.5$. For the

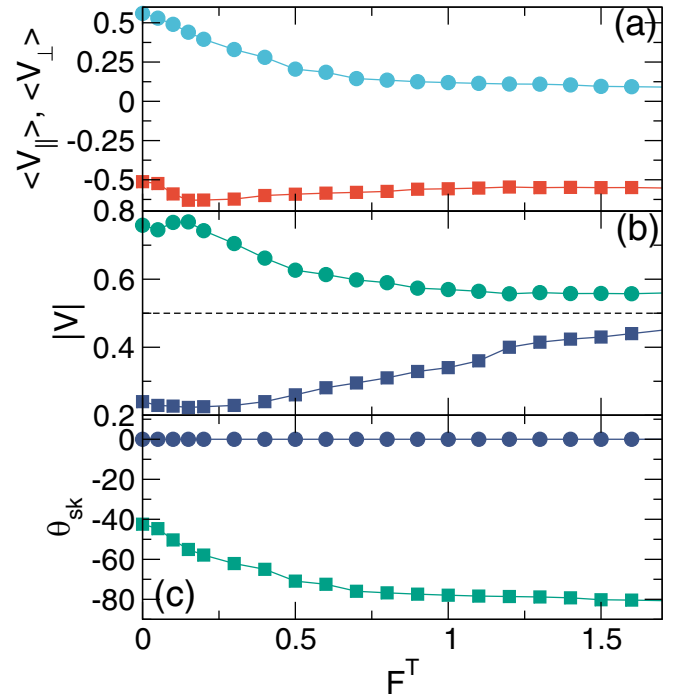


FIG. 24. The effects of thermal fluctuations for system at a constant $F_D = 0.5$ with $\rho = 0.5$. (a) The velocities in the perpendicular direction ($\langle V_{\perp} \rangle$, red squares) and parallel direction ($\langle V_{\parallel} \rangle$, blue circles) vs F^T for a system with $\alpha_m/\alpha_d = 9.95$. $\langle V_{\parallel} \rangle$ decreases with increasing thermal fluctuations. (b) The net skyrmion velocity $|V|$ for the Magnus-dominated system from (a) (green circles) and for an overdamped system with $\alpha_m/\alpha_d = 0$ (blue squares). The dashed line indicates the velocity in the single-particle limit for both cases. Here, with increasing F^T , the velocity for the Magnus-dominated system decreases from its boosted value, while the velocity in the overdamped system increases. (c) The skyrmion Hall angle θ_{sk} vs F^T for the Magnus-dominated system from (a) (green squares) and for the overdamped system (blue circles).

Magnus-dominated case, the velocity at $F^T = 0.0$ is close to $|V| = 0.75$, indicating a boost effect as discussed previously. As the thermal forces increase, the boost effect is reduced. The boost effect is generated by the formation of a local density gradient which, in the presence of a Magnus term, creates a velocity component perpendicular to the gradient, which adds to the velocity of the skyrmion in the direction of drive. Under increasing thermal fluctuations, the surrounding skyrmions begin to thermalize and can no longer support a density gradient over finite times. In Fig. 24(c), we plot the corresponding Hall angle θ_{sk} versus F^T for the Magnus and overdamped samples. In the Magnus-dominated system, θ_{sk} starts off near $\theta_{sk} = -40^\circ$ at $F^T = 0.0$ and gradually approaches the intrinsic value of $\theta_{sk}^{\text{int}} = -84^\circ$ at higher thermal forces. Here, despite the fact that the net velocity is decreasing with increasing F^T , the skyrmion Hall angle is also decreasing, which is opposite to the behavior found for skyrmions driven over pinning in the presence of thermal fluctuations. This behavior is, however, consistent with the reduction of the boost effect at higher drives where the gradient cannot form. In the case of finite F^T , it is the thermal effects which cause the density gradients to relax away rapidly, destroying the boost effect.

In Fig. 24(b), the net velocity $|V|$ for the overdamped system is small at $F^T = 0.0$ and gradually increases with increasing thermal force. In this case, there is no velocity boost; instead, the interactions between the driven and bath particles create a trapping or pinning effect which slows down the driven particle. As the thermal forces increase, the driven particle can more easily escape from the trapping potentials created by the other particles. At higher temperatures, the behavior approaches that of the single-particle limit and the velocities in both the Magnus and overdamped systems approach the expected single-particle limit of $|V| = V_0 = 0.5$. These results show that thermal effects can have opposite effects on the velocity of the driven skyrmion depending on whether the system is overdamped or Magnus dominated.

VII. DISCUSSION

In our work, the driven skyrmion is free to move in any direction. In certain chiral soft matter systems, a similar probe particle could be implemented using a magnetic or charged particle coupled to a uniform magnetic or electric field which does not couple to the remaining particles, allowing the probe particle to move at any angle. In skyrmion systems, it is possible to have samples containing multiple species of skyrmions, some of which could couple more strongly than others to an externally imposed field. The closest experimental realization of our system for skyrmions would be to drag a single skyrmion using some form of localized trap. Such an arrangement would constrain the driven skyrmion to move only in the direction the trap is being translated, and would not allow the driven skyrmion to move at a speed greater than that of the trap. In this case, changes in the effective viscosity could be deduced by measuring the force or the fluctuations exerted by the skyrmion on the trap. The case of a trap moving at a constant velocity will be studied in another work; however, the results of the present study can be used as a guide to understand which different velocity regimes could arise.

In our work, we have only considered a particle-based model, which neglects internal degrees of freedom and shape changes of the skyrmion. Such modes could increase or decrease the damping experienced by the driven skyrmion or change the nature of the skyrmion motion. We have also assumed a simple pairwise repulsion between skyrmions, but it is possible for skyrmions to have more complex interactions, such as competing interactions at different length scales. This could produce additional coupling/decoupling or depinning transitions.

In constant velocity experiments, the force the skyrmion experiences could have a periodic signature if the motion occurs through a skyrmion solid, or a broad band noise signature if the skyrmion is moving through a glass or liquid state. Numerical work [90,92] and experiments [111,112] on collectively moving skyrmions have shown the presence of both broad and narrow band noise, so it would be interesting to study the fluctuations exerted on a single skyrmion as it moves through a bath of other skyrmions. We can also compare our results to active rheology in overdamped chiral granular systems, where a disk with short range repulsive interactions is pushed through an assembly of spinning grains [58]. The granular system generally does not exhibit any velocity boost

due to its overdamped nature. The probe particle in the granular system has no intrinsic Hall angle, but as a function of driving force shows a finite Hall angle at intermediate drives, with no Hall angle at low or high drives. The finite Hall angle arises as a result of collisions between the probe particle and the spinning disks, which create a deflection of the probe particle perpendicular to the driving direction. This deflection decreases in magnitude as the velocity of the probe particle increases.

VIII. SUMMARY

We have numerically examined the active rheology of a single skyrmion driven through a bath of other skyrmions in the absence of quenched disorder. Active rheology has been used to study the changes in drag on driven probe particles in various soft matter and superconducting vortex systems where the dynamics is overdamped. In those systems, the velocity of the probe particle under a constant driving force rapidly decreases with increasing bath particle density due to an increased dragging effect. For skyrmions, which have a strong Magnus force, we find that the behavior differs strongly from what is observed in the damping-dominated limit. The driven skyrmion velocity in the driving direction is highly nonmonotonic as a function of density, and can increase rather than decrease when the density is increased. This effect appears as a boost in the net velocity. At higher densities, the velocity decreases with increasing density. The skyrmion Hall angle also decreases as the bath density increases. The magnitude of the velocity boost depends on the system density, the strength of the Magnus term, and the applied drive. For a fixed density, as we increase the driving force we find a critical threshold force below which the driven skyrmion does not move, a regime in which the magnitude of the skyrmion Hall angle increases with drive, and a regime at higher drive where the skyrmion Hall angle saturates. The drive dependence of the skyrmion Hall angle is similar to that observed for skyrmions driven over quenched disorder. If the Magnus force is dominant, a velocity boost appears which is maximum for an intermediate drive and diminishes at higher drives. When the damping force is strong, the velocities are reduced but approach the single-particle limit at higher drives. The velocity-force curves in the damped regime have linear or monotonic behavior, while in the Magnus-dominated regime, the velocity in the driving direction can decrease with increasing drive, leading to negative differential conductivity. The maximum velocity boost shifts to higher drives with increased density. The velocity boost originates when the driven skyrmion moves at a finite skyrmion Hall angle and creates a localized density inhomogeneity in the background skyrmions, which generate an unbalanced pairwise repulsive force on the driven skyrmion perpendicular to the driving direction. The Magnus force then converts this force into an additional velocity component in the direction of drive. At low bath densities, the localized density fluctuation relaxes quickly and the velocity boost is small, while at high drives, the driven skyrmion moves too quickly past the bath skyrmions for a localized density fluctuation to form, so the velocity boost is again reduced. We find regimes in which the skyrmion Hall angle decreases with a simultaneous increase in the skyrmion velocity,

suggesting that skyrmion-skyrmion interactions can be useful for producing effects that are of value for use in devices. For a constant driving force, we find that in the Magnus-dominated regime, the net skyrmion velocity decreases from its boosted value with increasing thermal fluctuations due to a thermally induced reduction in the magnitude of the density gradient around the driven particle. In contrast, for the overdamped system, thermal effects increase the net velocity by reducing the trapping effectiveness of the background skyrmions. We discuss possible experimental realizations of this system where a single skyrmion could be driven with some form of tip or optical trap while deflection forces on the skyrmion are measured. Beyond skyrmions, our results should be relevant to any kind of active rheology in systems with gyroscopic

forces, such as active chiral matter, fluid vortices, electrons in a magnetic field, fractons, and other gyroscopic systems.

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