# Kick-induced rectified current in a symmetric nanoelectromechanical shuttle

Pinquan Qin [1] and Hee Chul Park [2,3,*]

[1] Department of Physics, Wuhan University of Technology, Wuhan 430070, China

[2] Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Korea

[3] Basic Science Program, Korea University of Science and Technology (UST), Daejeon 34113, Korea

(Received 18 July 2020; revised 1 June 2021; accepted 23 July 2021; published 5 August 2021)

We have studied rectified current in a geometrically symmetric nanoelectromechanical shuttle with periodic kicks and sinusoidal ac bias voltages. The rectified current is exactly zero under the geometrical symmetry generated by electrons transferring from source to drain electrodes via the movable shuttle. We investigate nonzero rectified currents through the regular motion of the shuttle in which the time-translational symmetry is broken. The motion of the shuttle, moreover, becomes chaotic with the same mechanism as a kicked rotor and generates a scattered current with increasing kick strength. We point out that the time-translational symmetry breaking of the instantaneous current has an important role in the manipulation of the rectified current.

DOI: 10.1103/PhysRevB.104.064303

## I. INTRODUCTION

The rapid development of nanotechnology in recent years has created a new class of quantum electronic devices, called nanoelectromechanical systems [1,2], that are able to incorporate mechanical degrees of freedom. One particular example is the single-electron shuttle [3–10], a system in which a movable mesoscopic object, namely, the shuttle, begins to transport electrons one by one beyond a certain critical bias. In this case, the Coulomb blockade and temperature render Ohm's law for electrical conductance invalid because the electrical current is not necessarily proportional to the voltage drop across the device. Instead, current forms because electrons tunnel from a source to a drain electrode via the nanometer-sized shuttle. Since quantum-mechanical tunneling probability is exponentially sensitive to the tunneling distance between the shuttle and the electrodes, the position of the shuttle is crucial to defining the system's electrical properties. On the other hand, the Coulomb force that accompanies discrete nanoscale charge fluctuations drives the motion of the shuttle. Previous research [11] has reported that a geometrically symmetric shuttle with sinusoidal ac driven voltage has a rectified current of exactly zero. We want to treat the case of a symmetric shuttle with a periodic driven force beyond a simple sinusoidal ac voltage.

The kicked rotor model is a prototypical model for studying both classical and quantum chaos [12–14]. This model represents a single-particle system with a kicked driven potential where the strengths are discrete at every periodic time $T_k$. In classical theory, a particle with a strong kick strength shows chaotic motion with a positive Lyapunov exponent, whereas in quantum theory, two momentum-space phenomena arise, dynamic localization [13,15–17], and quantum resonance [18–20], which correspond to whether the driving period is an irrational or rational multiple of $2\pi$, respectively. It has been shown that there are rectified currents in a geometrically asymmetric nanoelectromechanical shuttle with time-periodic bias voltage [11,21]; such results not only advance our knowledge about the dynamics of self-excited oscillators onthe nanoscale, but also provide the means to optimize nanodevices to generate rectified current for practical purposes.

In this paper, we study a geometrically symmetric electron shuttle with an applied periodic bias voltage that is a combination of kicked and sinusoidal ac voltage. First, we find that the rectified current is nonzero with regular motion of the shuttle when we turn on the kick. The reason for this rectified current under regular motion is a breaking of the time-translational symmetry of the instantaneous current due to the interplay between the period of the self-oscillating shuttle and the period of the driven bias voltage with kicks. On the other hand, we also observe a chaotic motion of the shuttle with increasing kick strength, a phenomenon caused by the nonlinear force induced by the bias voltage at the kicks, even though the system is exactly geometrically symmetric. It is noted that the rectified current is generated by the time-translational-symmetry breaking of the instantaneous current due to the interplay between self-oscillation period and external kick period. Moreover, scattered rectified currents arise from the chaotic motion of the shuttle following the kicked harmonic-oscillator mechanism.

## II. MODEL OF A SYMMETRIC SHUTTLE

Let us consider a nanoelectromechanical shuttle as the combination of two metallic electrodes and a movable nanodot. The nanodot is initially located symmetrically in the center of the two electrodes, and the distance between the electrodes is large enough for sufficient nanodot (or nanoshuttle) oscillation without touching either electrode. We apply a symmetric time-dependent voltage to both electrodes that drives the oscillation of the shuttle between the two electrodes

*hcpark@ibs.re.kr

 

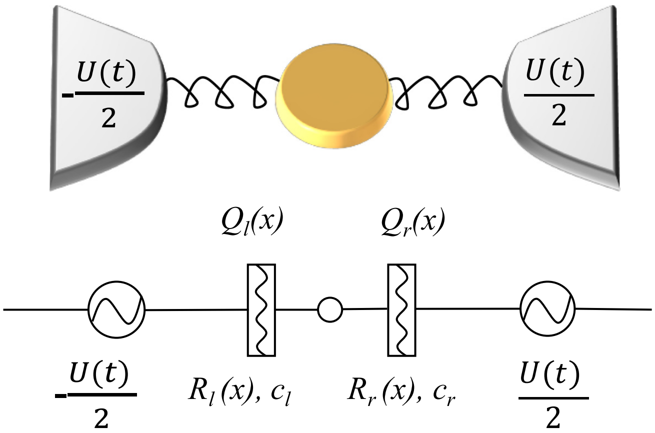


FIG. 1. Model schematic showing an electron shuttle with symmetric bias voltages. An equivalent circuit is drawn below. The rectangles with wavy lines in the circuit represent position-dependent tunnel junctions.

(see Fig. 1) where the force is a function of $x$ which indicates the displacement of the shuttle. In this system, we neglect nonequilibrium contributions to the current-induced force and pumping current [22–24].

In the adiabatic limit, namely, when the electronic relaxation is much faster than the mechanical motion, a classical circuit, such as the one shown in Fig. 1 can be used to analyze the electronic properties of the shuttle [25]. Based on Kirchhoff's law for a circuit structure, we obtain the following equations:

$$\frac{Q_l}{R_l c_l} - \frac{Q_r}{R_r c_r} = 0, \tag{1}$$

$$\frac{Q_l}{c_l} + \frac{Q_r}{c_r} = U(t), \tag{2}$$

where $Q_j$ is the induced charge, $R_j$ is the resistance, $c_j$ is the capacitance at the $j$th junction, and $U$ is the applied time-dependent voltage. The position-dependent resistances at each junction are $R_l = R_l^0 e^{(d+x)/\lambda}$ and $R_r = R_r^0 e^{(d-x)/\lambda}$ where $\lambda$ is the phenomenological tunneling length consideration of the electron tunneling process between the electrodes and the shuttle. The capacitances depending on the position are $c_l = c_{l0}/(1 + x/d)$ and $c_r = c_{r0}/(1 - x/d)$ with a geometrically simplified form, and $d$ is the half distance between the two electrodes. With geometric symmetry, i.e., $R_l^0 e^{d/\lambda} = R_r^0 e^{d/\lambda} = R_0$ and $c_{l0} = c_{r0} = c$, the position-dependent resistances of the left and right electrodes are $R_l = R_0 e^{x/\lambda}$ and $R_r = R_0 e^{-x/\lambda}$. We point out that the assumption of constant capacitance can quantitatively affect the results but not the qualitative observations. The total charge on the movable nanoshuttle is as follows:

$$Q_s(x,t) = \frac{R_l(x)c_l(x) - R_r(x)c_r(x)}{R_l(x) + R_r(x)} U(t), \tag{3}$$

where under the condition $d \gg \lambda$ the charge can be reduced by $Q_s \approx \tanh(x/\lambda) c U(t)$.

The time-dependent bias voltage between the two leads is a combination of normal sinusoidal ac voltage and time-periodic kicks. This pulsed voltage is given by $U(t) = \bar{U}(t) + \xi D_\varepsilon(t/T_k)$, where $\bar{U}(t)$ is the normal sinusoidal ac voltage, $\bar{U}(t) = \alpha \sin \omega t$, and $\alpha$ and $\xi$ are the strengths of the sinusoidal ac voltage and the kicks, respectively. Discrete kicks only affect the shuttle during $\varepsilon$ as follows:

$$D_\varepsilon(t/T_k) = \begin{cases} 0, & t/T_k \in [n-1, n-\varepsilon), \\ 1, & t/T_k \in [n-\varepsilon, n), \end{cases} \tag{4}$$

where $T_k$ is the kick period and $\varepsilon$ is the kick duration with $\xi\varepsilon$ kept very small. The electric energy of the oscillating shuttle is $E(x,t) = Q_s(x,t)U_s(x,t)$ where the voltage drop between the electrodes is $U_s(x,t) = \frac{1}{2}U(t)(R_l - R_r)/(R_l + R_r)$ based on Eqs. (1) and (2) as well as the symmetric voltage distribution on the electrodes. Correspondingly, the electric force exerted on the shuttle is

$$F = -\frac{\partial E(x,t)}{\partial x} = -\frac{cU^2(t)}{\lambda}\mathcal{F}(x), \tag{5}$$

where the derivative of a function is following:

$$\begin{aligned}\mathcal{F}(x) = -\frac{1}{2}\Bigg\{ & \frac{\frac{x}{d}}{1-\left(\frac{x}{d}\right)^2} - \frac{\left(2-\frac{\lambda}{d}\right)\tanh\frac{x}{\lambda}}{1-\left(\frac{x}{d}\right)^2} + \frac{2\tanh^3\frac{x}{\lambda}}{1-\left(\frac{x}{d}\right)^2} \\ & - \frac{\frac{x}{d}\tanh^2\frac{x}{\lambda}}{1-\left(\frac{x}{d}\right)^2} - \frac{2\frac{\lambda}{d}\frac{x}{d}\tanh^2\frac{x}{\lambda}}{\left[1-\left(\frac{x}{d}\right)^2\right]^2} + \frac{2\frac{\lambda}{d}\left(\frac{x}{d}\right)^2\tanh\frac{x}{\lambda}}{\left[1-\left(\frac{x}{d}\right)^2\right]^2}\Bigg\}.\end{aligned} \tag{6}$$

The function is reduced by $\mathcal{F}(x) \approx \frac{\sinh(x/\lambda)}{\cosh^3(x/\lambda)}$ under the large gap assumption, $d \gg \lambda$, whereas $\mathcal{F}(x) \approx -\frac{x/d}{[1-(x/d)^2]}$ under the additional small displacement assumption, $x \ll \lambda$, which means we can ignore the interplay between dynamics and charge transfer. Here, we concentrate on the tunneling effect without geometric capacitance under the proper approximation. We carefully consider the electric force by the derivative of the total electric energy of the oscillating shuttle of which the electric charge depends on the displacement due to electron tunneling. We write $F = F_c + F_k$ as the combination of smooth and kicked forces: $F_c = -\frac{c\alpha^2}{\lambda}\sin^2\omega t\,\mathcal{F}(x)$ as induced by the sinusoidal ac voltage and the discrete kicked force $F_k = -c\xi^2 D_\varepsilon(t/T_k)\mathcal{F}(x)/\lambda$ as induced by the kicks. In order to investigate the influence of $F_k$, we maintain a weak coupling and general kick strength as $\alpha \ll \xi$ and a finite $\xi^2\varepsilon$.

The equation of motion for the nanoshuttle is governed by Newton's equation considering the shuttle as a damped harmonic oscillator,

$$m\ddot{x} + m\gamma\dot{x} + m\omega_0^2 x = F_c + F_k, \tag{7}$$

where $\gamma$ is the dissipation coefficient. This governing equation respects the parity symmetry under $(x \to -x)$. It is satisfied by both the continuous force and the discrete kicked force.

The instantaneous current between the two electrodes is defined by $I(t) = Q_l(x,t)/cR_l(x) = U(t)/2R_0 \cosh\ x/\lambda$ through the charge distribution on the shuttle. Given time-interval $[0, \tau]$, the average current is calculated as

$$I_{dc} = \frac{1}{2R_0\tau}\int_0^\tau \frac{\bar{U}(t) + \xi D_\varepsilon(t/T_k)}{\cosh x_t/\lambda} dt. \tag{8}$$

If the instantaneous current has a period of $T_I$, the rectified current is the average current with $\tau = T_I$. Under the small $\xi\varepsilon$

condition, we find $\int_0^{T_I} \xi D_\varepsilon(t/T_k)/\cosh(x_t/\lambda)dt \sim 0$. Then the rectified current can be rewritten as follows:

$$I_{dc} = \frac{\alpha}{2R_0 T_I} \int_0^{T_I} \frac{\sin\,\omega t}{\cosh\,x_t/\lambda} dt. \quad (9)$$

For simplification, we use a dimensionless equation of motion as $\ddot{\tilde{x}} + \tilde{\gamma}\dot{\tilde{x}} + \tilde{x} = \tilde{F}_c + \tilde{F}_k$, where $\tilde{F}_c = -\tilde{\alpha}^2 \sin^2 \tilde{\omega}\tilde{t}\mathcal{F}(\tilde{x})$ and $\tilde{F}_k = -\tilde{\xi}^2 \tilde{D}_\varepsilon(\tilde{t})\mathcal{F}(\tilde{x})$ with $\mathcal{F}(\tilde{x}) = \sinh\tilde{x}/\cosh^3(\tilde{x})$. Here, $\tilde{x} = x/\lambda$, $\tilde{t} = t\omega_0$, $\tilde{\gamma} = \gamma/\omega_0$, $\tilde{\alpha} = \alpha\sqrt{c}/\sqrt{m}\lambda\omega_0$, $\tilde{\xi} = \xi\sqrt{c}/\sqrt{m}\lambda\omega_0$, and $\tilde{\omega} = \omega/\omega_0$. $\tilde{D}_\varepsilon(\tilde{t}) \equiv D_\varepsilon(\tilde{t}/\tilde{T}_k)$ with period $\tilde{T}_k = T_k\omega_0$. The corresponding rectified current is calculated as

$I_{dc} = (I_0\tilde{\alpha}/\tilde{\tau}) \int_0^{\tilde{\tau}/\omega_0} \sin\tilde{\omega}\tilde{t}/\cosh\tilde{x}d\tilde{t}$, where $I_0 = \lambda\omega_0\sqrt{m}/2\sqrt{c}R_0$ is the magnitude of the current. In the following discussion, we omit all tilde(˜) symbols.

## III. DRIVEN NANOSHUTTLE WITHOUT KICKS

Let us start by introducing a driven nanoshuttle without kicks to investigate the simplest case of zero rectified current at a fixed point and limit cycles. The system is invariant under parity transformation with only sinusoidal ac voltage $\ddot{x} + \gamma\dot{x} + x = F_c$ since the time-dependent part of $F_c$ has a period of $\pi/\omega$, which gives two equivalent solutions $\pm x$ under geometrical symmetry. The period of the shuttle is $\pi/\omega$ or $2\pi/\omega$ following Floquet theory and the equation of motion as a second-order differential equation. In the case of an odd shuttle period $\pi/\omega$, any position of the shuttle $x(t)$ returns to the same position after time-shift $t \to t + \pi/\omega$. On the other hand, for an even period, $2\pi/\omega$ has two possibilities: $x(t + \pi/\omega) = \pm x(t)$. For both cases, the rectified current is exactly zero because the position dependence of the rectified current is an even function whereas its time dependence is an odd function, $\sin\,\omega(t + \pi/\omega) = -\sin\,\omega t$ based on Eq. (9). In this situation, the instantaneous current has time-translational symmetry as $I_{t'} = -I_t$ with two time points $t$ and $t' = t + \pi/\omega$. Therefore, the rectified current in a shuttle with geometrical symmetry and sinusoidal ac-driven voltage is always zero under periodic regular motion with any $\alpha$ and $\omega$. In the following section, we point out that the a breaking of the time-translational symmetry can generate a finite rectified current.

## IV. DRIVEN NANOSHUTTLE WITH KICKS

Now let us turn on the kicks through a discrete pulsed bias voltage. The discrete pulses induce a nonsmooth function and break the time-translational symmetry of the instantaneous current whereas the system and applied bias voltage maintain their geometrical and temporal symmetries.

Figure 2 shows the deviation of the oscillating shuttle $\Delta x$ as a function of the strength and frequency of the sinusoidal ac voltage with a given kick period and strength. Here, $\Delta x = x_{\max} - x_{\min}$ where $x_{\max}$ is the maximum shuttle position and $x_{\min}$ is the minimum shuttle position. The quantity $\Delta x$ can tell us whether the shuttle trajectory decays to a balanced zero position or has a finite motion region. In Fig. 2(a), the motion of the shuttle shows Arnold tongues, indicating unstable limit circles when the kick strength is weak. Each tongue signifies

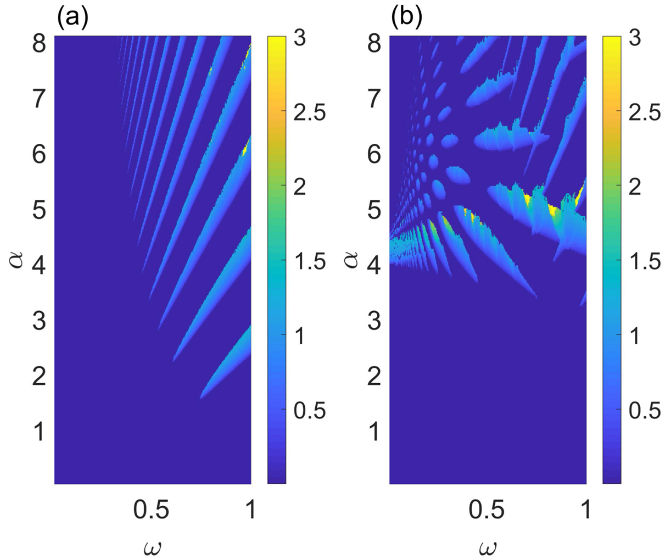


FIG. 2. Deviation of motion $\Delta x$ as a function of sinusoidal ac voltage strength $\alpha$ and frequency $\omega$ with a kick period of $T_k = 2\pi/9$. The given parameters are a dissipation coefficient of $\gamma = 0.1$ and a kick duration of $\varepsilon = 10^{-6}$. (a) Weak kicks show Arnold tongues, which indicate symmetry breaking $\xi = 300$. (b) Strong kicks develop new instabilities in the dynamics $\xi = 1300$.

distinguished oscillations corresponding to frequency, and the kicks generate self-oscillations by perturbing the stable fixed points; such features reflect a symmetry breaking the same as the results in Ref. [25]. When the kick strength is strong, more unstable features appear as shown in Fig. 2(b). Strong kicks generate new tongues and lift up the original tongues; this main feature indicates time-translational symmetry breaking as well as chaotic behavior.

In order to elucidate the chaotic motion resulting from the kicks, we introduce the Lyapunov exponent as an important quantity to characterize the chaotic or regular motion of a dynamic system. The maximal Lyapunov exponent can be defined as follows:

$$\lambda = \lim_{N\to\infty} \lim_{\delta Z(0)\to 0} \frac{1}{N} \ln \frac{|\delta Z(N)|}{|\delta Z(0)|}, \quad (10)$$

where $\delta Z(0)$ is the initial separation of two trajectories in phase space and $\delta Z(N)$ is the separation of two corresponding trajectories after kick period $N$. With the given parameters, we can see that the motion of the shuttle can develop from fixed points or regular motions into chaotic motions with increasing kick strength as in Fig. 3(a). Figure 3(b) shows finite rectified currents in the chaotic regions. In addition to these scattered rectified currents caused by the chaotic motions, there are several current tongues caused by regular motion in (b) that are absent in (a). We point out through these two figures that the shuttle exhibits fixed points, regular motions, and chaotic motions. Furthermore, the shuttle system generates corresponding nonzero rectified current in both regular and chaotic regions due to the symmetry breaking.

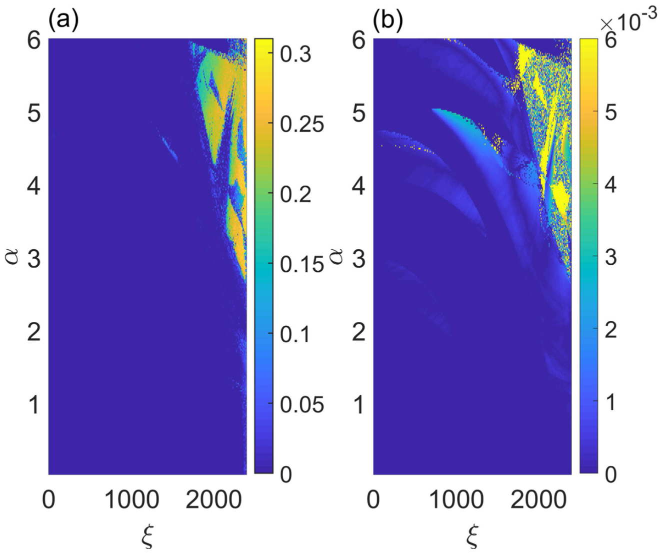


FIG. 3. (a) Lyapunov exponent $\lambda$ and (b) rectified current $I_{dc}$ as a function of the strengths of sinusoidal ac voltage $\alpha$ and kicks $\xi$. The parameters are the same as those in Fig. 2 with a given frequency of sinusoidal ac voltage $\omega = 1$. Finite currents appear under both regular and chaotic motion.

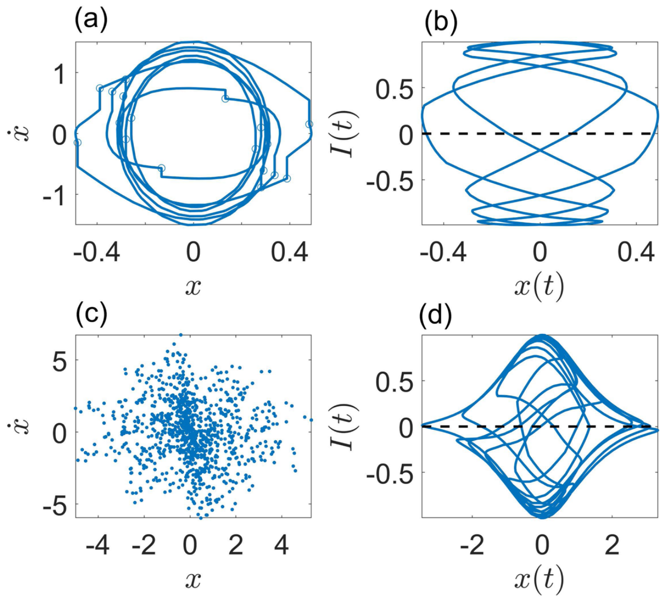


FIG. 4. (a) and (c) Trajectories in phase-space $[x(t), \dot{x}(t)]$ and (b) and (d) corresponding instantaneous current as a function of position $[x(t), I(t)]$. In (a) and (b), $(\xi, \alpha) = (1050,\ 4.9)$, and in (c) and (d), $(\xi, \alpha) = (2000,\ 5.08)$. The parameters are the same as those in Fig. 3. Regular motion is seen in (a), whereas instantaneous current with broken time-translational symmetry is shown in (b). Chaotic motion appears in (c), and aperiodic instantaneous currents are seen in (d).

## V. MOTION OF THE DRIVEN NANOSHUTTLE WITH KICKS

The trajectories in the phase space and corresponding instantaneous currents as a function of position contain fruitful information about the finite rectified currents on the Arnold tongues $[x(t), \dot{x}(t)]$ and $[x(t), I(t)]$, respectively. Let us examine more closely the special points corresponding to the regular motion and chaotic motion in Fig. 3. In Figs. 4(a) and 4(b), the phase-space trajectory shows regular motion and the instantaneous current exhibits time-translational symmetry breaking in one period so that the rectified current is finite. In Figs. 4(c) and 4(d), meanwhile, the disordered trajectory in the phase space shows chaotic motion from which the instantaneous current exhibits irregular patterns. We can see that the kicks generate a rectified current in the geometrically symmetric electron shuttle in both regular and chaotic regions.

The parts of the corresponding time-dependent forces $F_c \sim \sin^2 \omega t$ and $F_k \sim D_\varepsilon(t)$ have periods of $\pi$ and $2\pi/9$ in the calculation, respectively. The period for the equation of motion, Eq. (7), is determined by the lowest common multiple of the periods of the two forces and spatial symmetry. The position of the shuttle has a period of $2\pi$ with $x(t + 2\pi) = x(t)$ or a $4\pi$ period with $x(t + 2\pi) = -x(t)$. The instantaneous current is an odd function of time and an even function of position as shown in Fig. 4(b). Therefore, the rectified current is finite under regular motion. For the current, the term $\sin \omega t$ keeps the same value under the time-shift $t \to t + 2\pi$ [Eq. (9)], so there is no time-translational symmetry cancellation during integration within one period of the instantaneous current. The rectified current is determined by the properties of the motion within the time interval $[0, 2\pi]$. As the shuttle now no longer has parity symmetry $(x \to -x)$ with the time-shift $t \to t + \pi$, the time-translational symmetry of the instantaneous current is broken, and the rectified current takes a nonzero value.

On the other hand, the chaotic motion, the origin of which is the periodic kicks, generates a chaotic current that remains an arbitrary amount of the rectified current after averaging. After rescaling $t' = 2\omega t$, the equation of motion of the nanoshuttle is rewritten as (the prime is omitted, and the equation is dimensionless),

$$\ddot{x} + \bar{\gamma}\dot{x} + \delta^2 x + \mathcal{F}(x)[A - A\cos(t) + B\mathcal{D}_\varepsilon(t)] = 0, \quad (11)$$

where $\bar{\gamma} = \gamma/2\omega$, $\delta = 1/2\omega$, $A = \alpha^2/8\omega^2$, $B = \xi^2/4\omega^2$, $\mathcal{D}_\varepsilon(t) \equiv D_\varepsilon(t/\mathcal{T}_k)$, and $\mathcal{T}_k = 2\omega T_k$. Supposing $A$ is a very small constant, we could use two variable expansion methods [26] in which we define two time variables as $\kappa = t$ and $\eta = At$. The time derivative of $x$ can be expressed as the derivative of $\kappa$ and $\eta$. By expanding $x = x_0 + Ax_1 + \cdots$, $\delta = \delta_0 + A\delta_1 + \cdots$, and $\mathcal{F}(x) = \mathcal{F}(x_0) + A\mathcal{F}'(x_0)x_1 + \cdots$, we can get the following equations after collecting the terms with the same order,

$$\frac{\partial^2 x_0}{\partial \kappa^2} + \delta_0^2 x_0 + \mathcal{F}(x_0)B\mathcal{D}_\varepsilon(\kappa) = 0, \quad (12)$$

$$\frac{\partial^2 x_1}{\partial \kappa^2} + \delta_0^2 x_1 + \mathcal{F}'(x_0)x_1 B\mathcal{D}_\varepsilon(\kappa) = -\mathcal{F}(x_0)(1 - \cos\kappa) - 2\frac{\partial^2 x_0}{\partial \kappa \partial \eta} - \mu\frac{\partial x_0}{\partial \kappa} - 2\delta_0\delta_1 x_0, \quad (13)$$

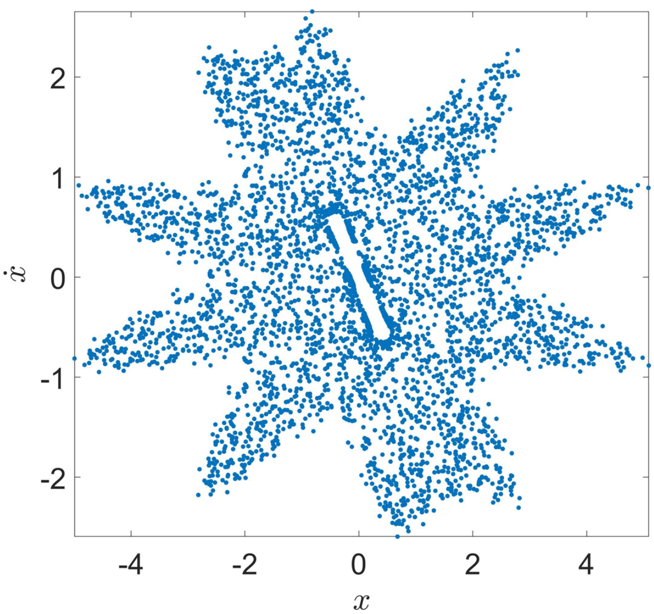


FIG. 5. Trajectory of the shuttle in phase-space $[x(t), \dot{x}(t)]$. The figure is calculated from Eq. (12) with the parameters $\mathcal{T}_k = 4\pi/9$, $\delta_0 = 0.49$, and $B\varepsilon = 3$.

where $\mu = \bar{\gamma}/A$. Let us expand $\mathcal{F}(x_0) \sim x_0$ with small $x_0$. Then Eqs. (12) and (13) can be rewritten as follows:

$$\frac{\partial^2 x_0}{\partial \kappa^2} + \delta_0^2 x_0 + x_0 B\mathcal{D}_\varepsilon(\kappa) = 0 \tag{14}$$

$$\frac{\partial^2 x_1}{\partial \kappa^2} + \delta_0^2 x_1 + x_1 B\mathcal{D}_\varepsilon(\kappa) = -x_0(1-\cos\kappa) - 2\frac{\partial^2 x_0}{\partial\kappa\,\partial\eta} - \mu\frac{\partial x_0}{\partial \kappa} - 2\delta_0\delta_1 x_0. \tag{15}$$

Equation (14) is the equation of motion for a kicked harmonic oscillator. The condition for a trajectory with a positive Lyapunov exponent is derived as

$$|\cos\,\delta_0\mathcal{T}_k - B\varepsilon\,\sin(\delta_0\mathcal{T}_k)/2\delta_0| > 1, \tag{16}$$

where $\mathcal{T}_k$ is the period of $\mathcal{D}_\varepsilon(t)$ in Eq. (11). In this condition, $x_0$ quickly reaches a huge number after several time evolution steps and destroys the validity of the expansion $\mathcal{F}(x_0) \sim x_0$. However, the nonlinearity of $\mathcal{F}(x_0)$ in Eq. (12) guarantees that the trajectory is confined and induces the chaotic motion of $x_0$. We can demonstrate such chaotic motion via numerical calculations. Figure 5 shows a chaotic phase-space diagram under the condition of a positive Lyapunov exponent with parameters $\mathcal{T}_k = 4\pi/9$, $\delta_0 = 0.49$, and $B\varepsilon = 3$. These parameters are close to those given in Fig. 4(c), and the condition for a positive Lyapunov exponent, which is 1.16, is satisfied. As a result of the chaotic motion, the time-translational symmetry of the instantaneous current is broken, and the rectified current is finite and quasirandomized, even though the system preserves its geometrical symmetry.

## VI. CONCLUSION

In this paper, we studied a geometrically symmetric electron shuttle with kicks and sinusoidal ac voltage and found the following. If the kicks are turned off, there is no rectified current due to the time-translational symmetry of the instantaneous current, even though the shuttle has finite motion displacement between the two electrodes. However, when the kicks are turned on, the period of the shuttle motion changes, which results in a shuttle motion period that is incommensurate with the driven bias voltage with kicks. This breaks the time-translational symmetry of the instantaneous current so that the rectified current is finite with regular shuttle motion. Moreover, the electron shuttle exhibits chaotic motion due to the nonlinear force caused by the kicks. Under chaotic motion, the time-translational symmetry of the instantaneous current is clearly broken, and the rectified current is nonzero even with a geometrically symmetric shuttle. Our results present some discrepancy with the general expectation for zero rectified current in a geometrically symmetric electron shuttle. As a number of properties of rectified currents relate to particular bias voltages, these findings inform applications using nanoelectromechanical systems.

## ACKNOWLEDGMENTS

This work was supported by Project Code No. IBS-R024-D1 and the Korea Institute for Advanced Study (KIAS) funded by the Korean Government (MSIT).

## APPENDIX: DERIVATION OF THE CHAOTIC REGION

The equation of motion for a kicked harmonic oscillator is as follows, also see Eq. (12):

$$\frac{\partial^2 x_0}{\partial \kappa^2} + \delta_0^2 x_0 + x_0 B\mathcal{D}_\varepsilon(\kappa) = 0. \tag{A1}$$

Suppose we know the position and momentum of the oscillator $(x_0^n, p_0^n)$ at the beginning of the time-interval $[n\mathcal{T}_k - \varepsilon, n\mathcal{T}_k)$, where $x_0^n = x_0(n\mathcal{T}_k)$ and $p_0^n = \dot{x}_0(n\mathcal{T}_k)$. Under small $\varepsilon$, the position remains unchanged at the end of this time period, but momentum changes to $p_0^n - B\varepsilon x_0^n$. This point in the phase space is the initial state for the next time-interval $[n\mathcal{T}_k, (n+1)\mathcal{T}_k - \varepsilon)$. In this period, the shuttle behaves exactly like a harmonic oscillator. The position and momentum are determined by

$$x_0^{n+1} = x_0^n \cos\,\delta_0\mathcal{T}_k + \frac{(p_0^n - B\varepsilon x_0^n)}{\delta_0}\sin\,\delta_0\mathcal{T}_k, \tag{A2}$$

$$p_0^{n+1} = (p_0^n - B\varepsilon x_0^n)\cos\,\delta_0\mathcal{T}_k - \delta_0 x_0^n \sin\,\delta_0\mathcal{T}_k. \tag{A3}$$

We can put these into matrix form as

$$\begin{bmatrix} x_0^{n+1} \\ p_0^{n+1} \end{bmatrix} = M \begin{bmatrix} x_0^n \\ p_0^n \end{bmatrix}, \tag{A4}$$

where

$$M = \begin{bmatrix} \cos\,\delta_0\mathcal{T}_k - \frac{B\varepsilon}{\delta_0}\sin\,\delta_0\mathcal{T}_k & \frac{1}{\delta_0}\sin\,\delta_0\mathcal{T}_k \\ -B\varepsilon\,\cos\,\delta_0\mathcal{T}_k - \delta_0\sin\,\delta_0\mathcal{T}_k & \cos\,\delta_0\mathcal{T}_k \end{bmatrix}. \tag{A5}$$

It is easy to find that $\det M = 1$, which means that the multiple of two eigenvalues of matrix $M$ will be 1. Due to the simple form of matrix $M$, we can solve the eigenvalue as

$$\lambda_{\pm} = \cos\,\delta_0\mathcal{T}_k - \mathcal{B}\,\sin\,\delta_0\mathcal{T}_k \pm \sqrt{(\cos\,\delta_0\mathcal{T}_k - \mathcal{B}\,\sin\,\delta_0\mathcal{T}_k)^2 - 1}, \tag{A6}$$

where $\mathcal{B} = B\varepsilon/2\delta_0$.

The eigenmatrix of $M$ can be written as $U$ giving

$$MU = U\lambda, \tag{A7}$$

where $\lambda$ is the eigenvalue matrix of $M$. We then get

$$U^{-1}\begin{bmatrix} x_0^n \\ p_0^n \end{bmatrix} = \lambda^n U^{-1}\begin{bmatrix} x_0^0 \\ p_0^0 \end{bmatrix}. \tag{A8}$$

Suppose the matrix $U^{-1}$ has the form

$$U^{-1} = \begin{bmatrix} \mathcal{U}_a & \mathcal{U}_b \\ \mathcal{U}_c & \mathcal{U}_d \end{bmatrix}, \tag{A9}$$

and then we have

$$\mathcal{U}_a x_0^n + \mathcal{U}_b p_0^n = (\lambda_+)^n\big(\mathcal{U}_a x_0^0 + \mathcal{U}_b p_0^0\big), \tag{A10}$$

$$\mathcal{U}_c x_0^n + \mathcal{U}_d p_0^n = (\lambda_-)^n\big(\mathcal{U}_c x_0^0 + \mathcal{U}_d p_0^0\big), \tag{A11}$$

where $\lambda_{\pm}$ are the corresponding eigenvalues. From this equation set we can solve the following:

$$x_0^n = \frac{1}{\mathcal{U}_a\mathcal{U}_d - \mathcal{U}_b\mathcal{U}_c}\big[(\lambda_+)^n\mathcal{U}_d\big(\mathcal{U}_a x_0^0 + \mathcal{U}_b p_0^0\big) - (\lambda_-)^n\mathcal{U}_b\big(\mathcal{U}_c x_0^0 + \mathcal{U}_d p_0^0\big)\big]. \tag{A12}$$

Now suppose we have $|\cos\delta_0\mathcal{T}_k - \mathcal{B}\,\sin\delta_0\mathcal{T}_k| > 1$ and $\lambda_- < 1$, and then $(\lambda_-)^n \to 0$. We get

$$x_0^n = \frac{\mathcal{U}_d(\lambda_+)^n\big(\mathcal{U}_a x_0^0 + \mathcal{U}_b p_0^0\big)}{\mathcal{U}_a\mathcal{U}_d - \mathcal{U}_b\mathcal{U}_c} = \mathcal{C}e^{n\,\ln\,\lambda_+}, \tag{A13}$$

where $\mathcal{C} = \mathcal{U}_d(\mathcal{U}_a x_0^0 + \mathcal{U}_b p_0^0)/(\mathcal{U}_a\mathcal{U}_d - \mathcal{U}_b\mathcal{U}_c)$ is a constant related to the initial condition $x_0^0$, $p_0^0$. Now we have a positive Lyapunov exponent $L_y = \ln\,\lambda_+$.


[1] A. N. Cleland, *Foundations of Nanomechanics: From Solid-State Theory to Device Applications* (Springer, Berlin/Heidelberg, 2013).
[2] S. E. Lyshevski, *MEMS and NEMS: Systems, Devices, and Structures* (CRC Press, Boca Raton, FL, 2018).
[3] L. Y. Gorelik, A. Isacsson, M. V. Voinova, B. Kasemo, R. I. Shekhter, and M. Jonson, Phys. Rev. Lett. **80**, 4526 (1998).
[4] J. H. Cho, M. Seo, T. Lee, J. H. Kim, J. S. Chan, J. Y. Min, K.-H. Ahn, R. H. Blick, H. C. Park, and C. Kim, New J. Phys. **19**, 033033 (2017).
[5] Z. Rong, C. Wei-Dong, D. Su-Qing, and Y. Ning, Chin. Phys. B **22**, 117305 (2013).
[6] M. J. Moeckel, D. R. Southworth, E. M. Weig, and F. Marquardt, New J. Phys. **16**, 043009 (2014).
[7] S. I. Kulinich, L. Y. Gorelik, A. N. Kalinenko, I. V. Krive, R. I. Shekhter, Y. W. Park, and M. Jonson, Phys. Rev. Lett. **112**, 117206 (2014).
[8] M. Zhao and R. H. Blick, Phys. Rev. E **93**, 063306 (2016).
[9] C. Kim, M. Prada, G. Platero, and R. H. Blick, Phys. Rev. Lett. **111**, 197202 (2013).
[10] C. Kim, M. Prada, H. Qin, H.-S. Kim, and R. H. Blick, Appl. Phys. Lett. **106**, 061909 (2015).
[11] F. Pistolesi and R. Fazio, Phys. Rev. Lett. **94**, 036806 (2005).
[12] B. V. Chirikov, Phys. Rep. **52**, 263 (1979).
[13] G. Casati, B. Chirikov, F. Izraelev, and J. Ford, in *Stochastic Behavior in Classical and Quantum Hamiltonian Systems* (Springer, Berlin, 1979), pp. 334–352.
[14] F. M. Izrailev, Phys. Rep. **196**, 299 (1990).
[15] J. E. Bayfield, G. Casati, I. Guarneri, and D. W. Sokol, Phys. Rev. Lett. **63**, 364 (1989).
[16] F. L. Moore, J. C. Robinson, C. Bharucha, P. E. Williams, and M. G. Raizen, Phys. Rev. Lett. **73**, 2974 (1994).
[17] M. Bitter and V. Milner, Phys. Rev. Lett. **117**, 144104 (2016).
[18] F. Izrailev and D. Shepelyanskii, Theor. Math. Phys. **43**, 553 (1980).
[19] M. Sadgrove, A. Hilliard, T. Mullins, S. Parkins, and R. Leonhardt, Phys. Rev. E **70**, 036217 (2004).
[20] I. Dana, V. Ramareddy, I. Talukdar, and G. S. Summy, Phys. Rev. Lett. **100**, 024103 (2008).
[21] P. Qin and H. C. Park, Physica E **117**, 113835 (2020).
[22] F. Reckermann, J. Splettstoesser, and M. R. Wegewijs, Phys. Rev. Lett. **104**, 226803 (2010).
[23] H. L. Calvo, L. Classen, J. Splettstoesser, and M. R. Wegewijs, Phys. Rev. B **86**, 245308 (2012).
[24] R. A. Bustos-Marún and H. L. Calvo, Entropy **21**, 824 (2019).
[25] K.-H. Ahn, H. C. Park, J. Wiersig, and J. Hong, Phys. Rev. Lett. **97**, 216804 (2006).
[26] J. Kevorkian, Astron. J. **71**, 878 (1966).