Stochastic thermal feedback in switching measurements of a superconducting nanobridge caused by overheated electrons and phonons

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We study correlated switchings of a superconducting nanobridge probed with a train of current pulses. For pulses with a low repetition rate each pulse transits the superconducting bridge to the normal state with probability P independent of the outcomes in the preceding pulses. We show that with the reduction of the time interval between pulses long-range correlation between pulses occurs: stochastic switching in a single pulse raises the temperature of the bridge and affects the outcome of the probing for the next pulses. As a result, an artificial intricate stochastic process with an adjustable strength of correlation is produced. We identify the regime where apparent switching probability exhibits the thermal hysteresis with discontinuity at a critical current amplitude of the probing pulse. This engineered stochastic process can be viewed as an artificial phase transition and provides an interesting framework for studying correlated systems. The process resembles the familiar transition from the superconducting to normal state in the current-bias nanowire, proceeding through a phase slip avalanche. Due to its extreme sensitivity to the control parameter, i.e., electric current, temperature, or magnetic field, it offers the opportunity for ultrasensitive detection.

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I. INTRODUCTION

There are processes in nature whose understanding can be facilitated by constructing artificial systems which exhibit very similar, often the same, dynamics, expressed in analogous equations. This is the case for the mechanical model of the rf superconducting quantum interference device (SQUID) operation [1] or in studying quantum chaos by using microwave networks [2]. A great deal of understanding of the actual physical system can be acquired from such studies. In the present work we present the construction and validation of the artificial process which mimics multiphase slip or the phase diffusion behavior responsible for the switching of superconducting wires and junctions, respectively. Alternatively, one may envisage it as a useful tool to get some insight into processes exhibiting an abrupt transition between states due to unlikely strongly correlated fluctuation. Such a transition may, for example, lie at the heart of biological evolution, leading to the appearance of new species on Earth [3].

Measurements of Josephson junction (JJ) switching are widely used to study the decay of metastable states when the junction transits from a superconducting to finite voltage state [4–8]. They allow for the experimental determination of the current-phase relation [9] and find application in sensing tiny magnetic moments if two junctions are connected to form a SQUID loop [10]. They are used for reading out superconducting qubits [11] and detecting electromagnetic noise [12] or single photons [13]. Measurement of the switching probability as a function of the biasing current yields an S-shaped curve. Its exact shape bears information about the fluctuation mechanism underlying the escape process. For underdamped junctions the escape is triggered by a single thermal fluctuation or macroscopic quantum tunneling event that leads to the running of the superconducting phase down the tilted washboard potential [6,7]. However, for a junction in the moderately damped regime the transition from a superconducting to normal state cannot be explained by the assumption of a single fluctuation and instead involves phase diffusion, eventually leading to escape [14–17]. For wires at temperatures sufficiently close to T_c a similar intricate stochastic process is predicted: the switching to a normal state is the result of consecutive phase slip events, with each phase slip increasing the probability for the next phase slip to occur [18–21]. It is the experimental emulation of this last process that we present in our work: we consider the process of consecutive switching events in a regime where each switching event increases the probability for the next switching event to occur. We need to stress that our presentation does not investigate the switching mechanism of the bridge tested with current pulses. Whatever this mechanism is, we will use the fact that it transits the bridge to the normal state with a local electron temperature larger than T_c.

The independent switching experiments manifest themselves in the binomial distribution of switchings for a fixed value of the probing current: JJ behaves like a coin for which the head and tail experiment is performed [23]. By virtue of independence the switching probability is not affected by the outcome of the probing in any other pulse. An intriguing situation arises if we introduce correlation between switching events by reducing the time interval between probing pulses. In such a case, after a successful switching event the

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subsequent thermal relaxation is not complete when the next testing pulse arrives. Thus, the switching probability is enhanced. As we report below, it leads to the emergence of the correlated stochastic process with a tunable strength of correlation. We demonstrate hysteretic switching current dependencies exhibiting metastable behavior for a critical value of the probing current. We argue that this behavior is desired for threshold detection of various physical signals and may emulate natural stochastic processes involving the phase slip avalanche, which is possibly responsible for the superconducting-normal-state transition in nanowires [18].

II. SWITCHING THERMOMETRY

We fabricate two superconducting nanobridges, A and B (width $\approx$ 70 nm), interrupting the middle of a long nanowire (width = 600 nm, length = 100 μm) connected to large-area contact pads on both sides [see the inset in Fig. 1(a)]. The structure is prepared by means of conventional one-step electron-beam lithography followed by evaporation of 30 nm thick aluminum on the 300 μm thick silicon substrate. A typical I-V curve of the bridge reveals pronounced thermal hysteresis [Fig. 1(a)]: the bridge switches to the normal state at a specific current value known as the switching current I_{sw} and returns to the superconducting state only after the applied current I_A is sufficiently reduced. Such behavior is explained by self-heating of the sample. During switching, the nanobridge forms a hot spot, from which the normal region expands towards the contact pads: after a few tens of nanoseconds the bridge and nanowire are all above T_c. While the nanobridge resistance is only 2 Ω, the normal branch of the I-V curves, measured for bath temperatures $T_0 < T_c$, shows the normal state resistance of the whole sample, i.e., the nanobridge with connecting nanowires. It is noteworthy that the bath temperature T_0, as read out with a resistive RuO_x thermometer, remains completely unaltered, but the local temperature of the nanobridge and nanowire rises above T_c. Only lowering the applied current I_A leads to a reduction of both the Joule heating and the related electron temperature T_e and allows the sample to enter the superconducting state again when $I_A < I_{sw}(T_e)$. Our interpretation is in line with Ref. [24], which proves the overheating of electrons in the proximity junctions after switching. Also, for MoGe superconducting nanowires the hysteresis in the I-V curve is explained by self-heating [25] and not by the dynamics of the phase motion in a tilted washboard potential, as often assumed. In our case, the switching current is 1–2 orders of magnitude higher than in the mentioned references, and thus, heating effects are expected to be much more pronounced. Recently, correlations in switching events for the Nb-Al-Nb proximity junction were reported and attributed to thermal dynamics of electrons in the junction [26].

We test the bridge with a train of N current pulses. In response to each pulse, depending on the probing current amplitude and fluctuations, the bridge may remain in the superconducting state or transit to a normal state in a process known as switching. For low probing currents the bridge never switches; for high current it always switches. At intermediate values of current the bridge switches on average n times. The ratio n/N is an estimator for the switching probability

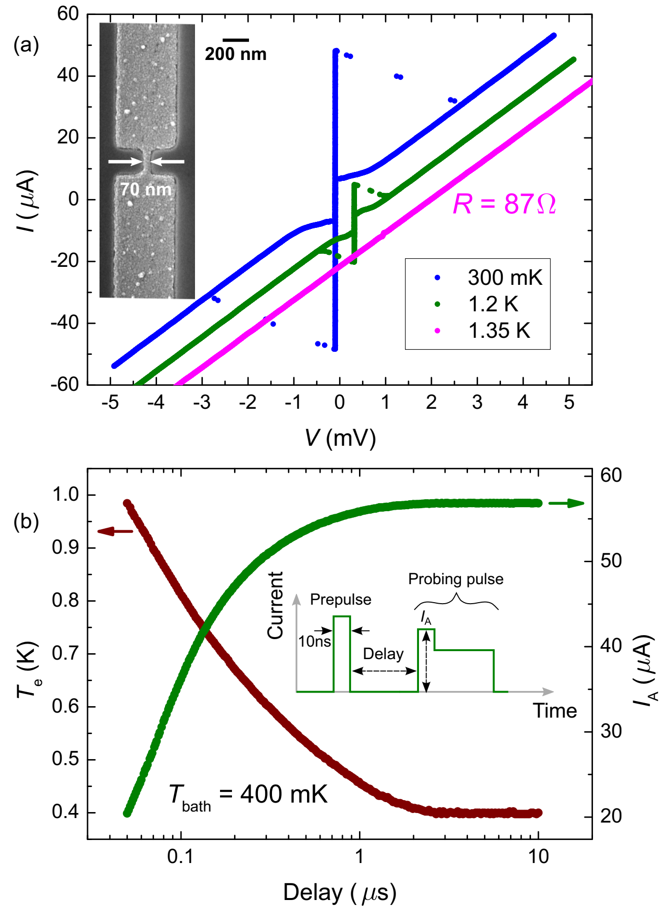

FIG. 1. The thermal dynamics of Dayem nanobridge A. (a) The current-voltage characteristics of a bridge, revealing switching behavior at a threshold current. The I-V curves are obtained at three temperatures: 300 mK, 1.2 K, and 1.35 K (from left to right, offset horizontally for clarity). $R = 87\,\Omega$; that is, the inverse slope of all I-V curves is the same as the normal state resistance of the sample (nanobridge + long nanowire) measured above $T_c = 1.3$ K. The scanning electron microscope image of the aluminum nanobridge is shown on the left-hand side. (b) Relaxation of the switching current after overheating the bridge with a heating pulse above T_c (right y axis) and the corresponding temporal variation of the electron temperature T_e (left y axis). The definition of the testing sequence consists of a 10 ns heating prepulse followed by the probing pulse (inset). The probing pulse consists of a short part of higher amplitude intended to test the junction (so-called *testing pulse*) and a much longer sustain part enabling the readout of the state of the bridge with a low-pass-filtered line. For details see Fig. 3 of Ref. [22].

P and renders the familiar S-shaped curve as the probing current increases (Fig. 2). The switching current dependence on temperature (see the inset in Fig. 2) makes the bridge a fast and sensitive thermometer [22], which is evident from the mutual shift of S curves measured at different temperatures.

To measure the thermal relaxation of the bridge we send a heating prepulse with the amplitude exceeding the switching threshold. This forced switching transits the bridge to the normal state with an electron temperature larger than T_c. The subsequent relaxation is probed with the delayed probing pulse. The sequence is repeated $N = 10\,000$ times with a

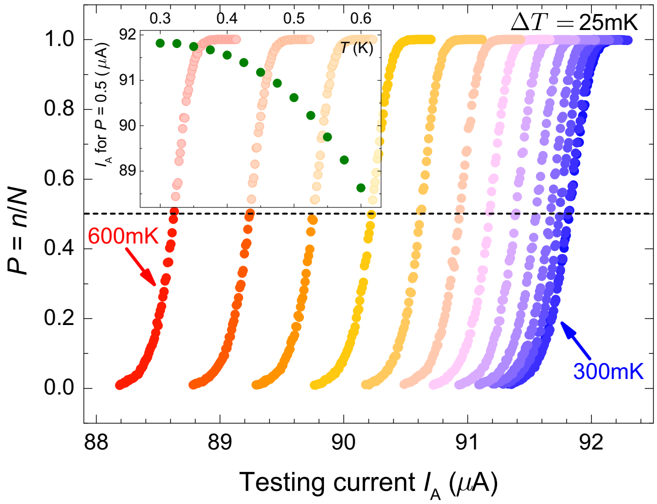

FIG. 2. JJ switching probabilities (S curves) measured at different bath temperatures for bridge B used to extract the switching current dependence $I_A(T)$ on the temperature presented in the inset. The dashed line corresponds to $P = 0.5$. The testing part of the probing pulse is 1 μs long.

period long enough to provide thermalization of the bridge at the bath temperature T_0 after each sequence. The current amplitude of the probing pulse is adjusted to yield $P = 0.5$ for each delay, and it is uniquely related to the temperature via a calibration curve $I_A(T)$, which, as an example, is presented for bridge B in the inset of Fig. 2. The typical relaxation profile of the switching current and the resulting temperature evolution are presented in Fig. 1(b). For a more detailed description of the method refer to our earlier works [22,23,27,28].

III. TWO THERMAL RELAXATION TIMES

We use the switching thermometry to analyze the influence of M heating prepulses on the temperature of the bridge and define dynamic thermal processes involved in the relaxation of the electron temperature towards the bath temperature T_0. The experimental data presented in this section are widely described in Sec. V of the Ref. [23]. For completeness of our presentation we analyze these data once again in the current paper, introducing as a new element quantification of the phonon overheating due to a single switching event.

We intentionally heat the bridge with a set of M prepulses preceding the actual testing pulse and vary the time between the last prepulse and the probing pulse [Fig. 3(a)]. The current amplitude of the prepulses is set to be constant, at a level exceeding the switching threshold, yielding forced switching in each prepulse. After a single prepulse ($M = 1$), the electron temperature of the nanobridge exceeds T_C and relaxes to the local phonon temperature on the timescale of a few microseconds [Fig. 3(c)]. This relaxation is governed by electron-phonon coupling and by hot-electron diffusion [22,23]. Both energy-relaxation channels bring electrons into thermal equilibrium with phonons. As we increase the number of prepulses M, local phonons become overheated. This overheating relaxes at a rate approximately 1000 times slower than it takes for the hot electrons to achieve the local phonon temperature [Fig. 3(b)]. Since the two processes exhibit such different dynamics, they can be described in the linear regime using the sum of two exponential decays, yielding the two relaxation times: $\tau_e = 1.26\,\mu$s for electrons and $\tau_{ph} = 1.26$ ms for phonons.

Such dynamic investigations of the thermal processes of nanostructures are rare in the literature and limited to the lowest temperatures, where they are relatively slow. Some authors reported direct measurements of thermal relaxation times of copper ($\tau_{Cu} = 10\,\mu$s) and silver ($\tau_{Ag} = 500$ ns) at $T_0 = 200$ mK [29]. Similar measurements for gold yield $\tau_{Au} = 1.6\,\mu$s at $T_0 = 300$ mK [30]. The relaxation time for electrons in our nanostructure, measured at $T_0 = 500$ mK, is expected to be significantly shorter than $\tau_{Al} = 30\,\mu$s reported in Ref. [31] for $T_0 = 300$ mK, owing to strong suppression of electron-phonon coupling at low temperatures.

It is instructive to measure the increase in temperature of the local phonons with the number of prepulses after the fast electron-phonon relaxation is over and the slow phonon-bath relaxation has not yet started. This can be accomplished by adjusting the relaxation time to 10 μs [see point A in Fig. 3(b)], and the result is presented in Fig. 3(d). Recalculation of the switching current in terms of temperature with the aid of a calibration curve (see the inset of Fig. 1 in Ref. [23]) yields a $T_{ph}(N)$ dependence, showing a slow, but monotonous, increase with a tendency for saturation [Fig. 3(e)]. Obviously, the local substrate (phonon) temperature does not relax to the bath temperature before the arrival of the next testing pulse. Each switching event deposits a bit of energy in the substrate and gives rise to a δT_{ph} increase in the phonon temperature as defined 10 μs after the switching event. The contribution from each switching event relaxes exponentially over much longer times, $\tau_{ph} = 1.26$ ms, as determined from the fit in Fig. 3(b). We describe the excess phonon temperature "seen" by the $(n+1)$th probing pulse with the following recursive relation:

$$\Delta T_{ph}(n+1) = [\Delta T_{ph}(n) + \delta T_{ph}]\exp\left(-\frac{\Delta t}{\tau_{ph}}\right),$$
$$\Delta T_{ph}(1) = 0. \tag{1}$$

The formula applied to $T_{ph}(N)$ dependence in Fig. 3(e) with $\Delta t = 8\,\mu$s and $\tau_{ph} = 1.26$ ms yields $\delta T_{ph} = 0.50$ mK.

The above model is constructed in analogy to the effusion process, in which a gas escapes from a leaking volume through a pinhole. To good approximation, the amount of gas remaining in the container decreases with time exponentially. We assume that the temperature of the overheated phonons behaves in the same way. We thus consider the linear regime, where the departure of the phonon temperature ΔT_{ph} from the bath temperature is small and the cooling power of the phonons is proportional to ΔT_{ph}. However, in our "effusion process" the fixed amount of "gas," i.e., the δT_{ph} increment in temperature, is added at well-defined discrete time intervals.

IV. HYSTERETIC S CURVES AND METASTABLE STATES

In the experimental investigations presented in this section we intentionally correlate switching events using the time

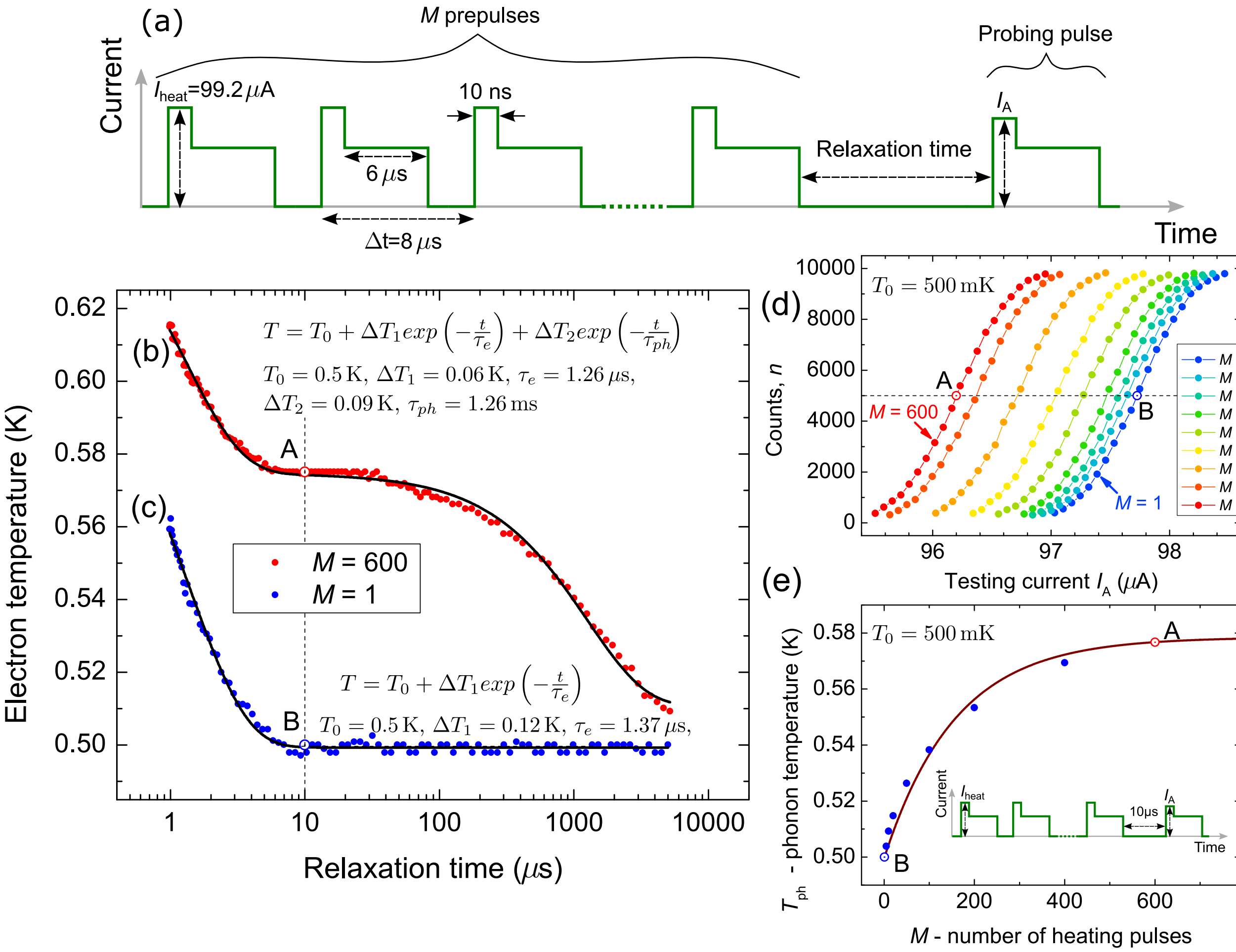

FIG. 3. The thermal dynamics of nanobridge B. (a) The definition of the testing sequence. The single sequence consists of M heating prepulses followed by the testing pulse. The sequence is repeated N times to measure the switching number n. The repetition period is 10 ms to allow for proper equilibration after each sequence. The testing time is 10 ns. (b) Relaxation of the nanobridge after $M = 600$ forced switchings. Two relaxation mechanisms are visible: the first, fast process is the same as in (c), and the second one, approximately 1000 times slower, is attributed to relaxation of the local phonon temperature (substrate) toward the bath temperature. The line is a fit to the sum of two exponential functions. (c) Relaxation of the excess hot-electron energy toward equilibrium with local phonons after a single forced switching ($M = 1$): the local phonons are at bath temperature. The line is a fit to the single exponential function. (d) S curves measured for different numbers of heating prepulses 10 μs after the end of the last prepulse, when the electrons are already thermalized at the local phonon temperature. (e) Dots present the experimentally measured temperature rise of the local phonons as extracted from the S curves presented in (d) along the dashed line $n = 5\,000$, with the aid of the calibration curve provided in Ref. [23]. The line is a single-parameter fit to Eq. (1) with $\Delta t = 8\,\mu$s and $\tau_{ph} = 1.26$ ms, yielding $\delta T_{ph} = 0.50$ mK.

interval between pulses as a knob for controlling the strength of correlation. For pulse periods $\Delta t = 100\,\mu$s no correlation is observed: switching events can be considered to be independent, for an increase of Δt above 100 μs changes neither the shape nor the position of the S curves. With the reduction of the period, switching in a single pulse starts to influence the measuring result in subsequent pulses (Fig. 4), leading to steepening of the S curve. The further reduction of the period destroys the familiar picture of the S curve: the number of switchings n corresponding to a fixed probing current amplitude seems to be completely random, as revealed by the scattered curve presented in Fig. 4 ($\Delta\tau = 0.55\,\mu$s). In fact these "noisy" data are well understood in terms of the panic switching that we studied in Ref. [23]: the first switching appears at random, but once it happens, switching events for all successive pulses become certain. We reserve the notion of probability for independent events, while, generally, we will use the switching number n/N to specify the number of switching events in the total number N of probing pulses.

As demonstrated in Sec. III, sending M pulses in a row, each with a high enough amplitude to make the bridge switch, increases the local phonon temperature. A similar effect is expected if M pulses switch the bridge only with some probability. In further studies we modify the probing protocol accordingly: the pulse train involves the thermalization pulses, intended to raise the phonon temperature in the bridge to its asymptotic value for a given probing pulse amplitude, which are followed by $N = 10\,000$ measuring pulses (Fig. 5).

Applying this protocol, we obtain the switching dependence presented in Fig. 6. It is visible there that the switching

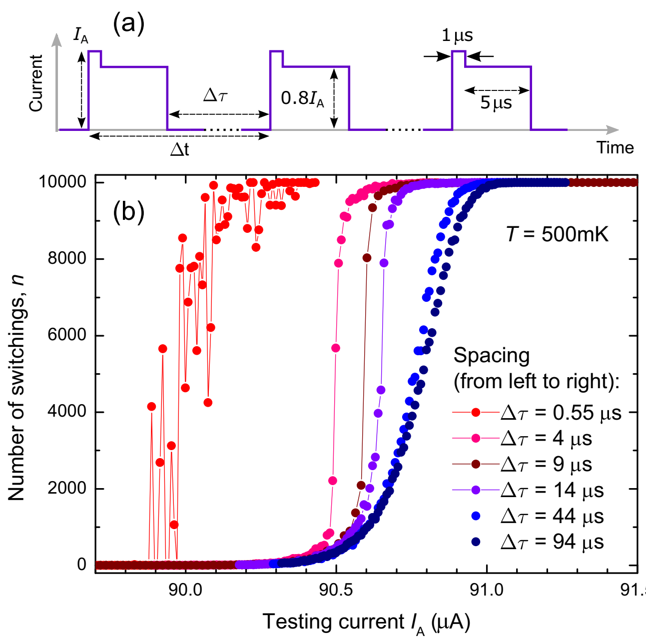

FIG. 4. (a) Train of N current pulses used to probe bridge B with different $\Delta\tau$. (b) Current dependencies of the switching number n for different spacings $\Delta\tau$ between probing pulses. Each point corresponds to a finite-length train measurement ($N = 10\,000$).

number $n(I_A)$ exhibits an abrupt transition from state $n(I_A = I_C) = n_C$ to state $n(I_A) = N$. We call this phenomenon *ensemble switching*, for it involves all testing pulses (Fig. 6). Below we present a qualitative explanation for the observed correlated dependency, leaving a quantitative rigorous treatment for the next section. The larger $n(I_A)$ in the total number of testing pulses N is, the higher the average electron temperature in the bridge is. As a result, points of higher apparent switching probability $P = n(I_A)/N$ belong to S curves measured at higher temperatures in the independent regime (shifted towards smaller values of the testing current; see Fig. 4). For moderate phonon overheating the S curve becomes steeper, but at shorter testing periods, the increase in $n(I_A)$ above a critical value n_C is accompanied by a temperature increase corresponding to the smaller value of the testing current I_A and back bending of the $n(I_A)$ dependence (Fig. 7, black dotted curve). Instead, since I_A is increased monotonically, $n(I_A)$ exhibits an abrupt transition at this point. The phenomenon has an avalanche character and relies on a thermal feedback mechanism: upon exceeding a threshold for the testing current I_C, a small increase in the independent switching probability increases the number of switchings. It, in turn, leads to increased phonon temperature, thus further increasing switching probabilities. The new stable state is the one with every pulse switching the wire to the normal state. Unlike for an independent switching experiment with uncorrelated pulses, where all values of $n(I_A)$ are stable, in the correlated regime $n(I_A)$ can exhibit only values up to n_C or $n = N$. Once the ensemble switching happens, the local phonon temperature becomes significantly elevated above the bath temperature. Starting from the state with $n = N$ and decreasing current, first, we observe a small suppression in the switching number n. It, in turn, leads to a reduction of the temperature, thus further reducing the switching probability and transition to the state with a small value of n. We call this transition the *ensemble retrapping*.

Ramping current amplitude up and down while using continuous pulse generation renders hysteretic loops with hysteresis more pronounced for more strongly correlated pulses (Fig. 8). When the probing current amplitude is increased, we see an abrupt transition at n_C, with n_C lower for shorter intervals between probing pulses. In the fully correlated regime, when the single switching triggers the ensemble switching, $n_C = 0$, and $n(I_A)$ shows only two stable values: $n = 0$ or $n = N$. The corresponding trace defines a rectangular hysteresis with the ensemble switching and retrapping occurring at random current values. We call such an ensemble switching in the fully correlated regime *panic switching* [23].

It is important to emphasize that for the hysteretic loop to be observable, the continuous pulse train must be used. If a finite-length pulse train were applied (i.e., the train with N pulses of definite I_A amplitude), the bridge would relax to the bath temperature prior to the arrival of the next train, and the retrapping part of the hysteretic loop would not be accessible.

V. MODELING OF THE STOCHASTIC THERMAL FEEDBACK

In this section we develop a mathematical description of the switching correlation in our system, providing a proper account of the abrupt transitions observed in Figs. 6 and 8. As we saw in Sec. III, there are two thermal relaxation scales for the bridge. The first one is set by the electron-phonon relaxation time. It tells how long it takes for electrons to equilibrate with the local phonons. The second timescale is set by the relaxation of the local phonon temperature with the temperature of the thermal environment, i.e., bath temperature T_0. The slowly relaxing local phonon overheating raises the thermal asymptote for the relaxation of the electron temperature, thus influencing the switching probability over many probing pulses. The general model should assume the switching probabilities dependent on the history of the system.

We analyze a pulse train with $\Delta\tau = 2\,\mu s$ ($\Delta t = 8\,\mu s$) applied to our sample at $T_0 = 500$ mK [see the corresponding experimental curve in Fig. 8 and definition of the pulse train in Fig. 4(a)]. The relaxation time of the electron temperature is $\tau = 1.3\,\mu s$, as inferred from Figs. 3(b) and 3(c). It corresponds to overheating of electrons equal to $\Delta T_e \cong 20$ mK, which is present at $\Delta\tau = 2\,\mu s$ after switching off the M-pulse sequence. If there is another pulse sent at this moment, it “sees” electrons at a temperature elevated ΔT_e above the phonon temperature. But if a pulse is sent $\Delta t + \Delta\tau = 10\,\mu s$ later, electrons are already thermalized with phonons. It is noteworthy that overheating of electrons with respect to local phonons, which occurs in the given switching event, affects only the next probing pulse.

The second necessary ingredient of the model takes into account the slow phonon relaxation demonstrated in Fig. 3(b). It assumes that the phonon temperature remains elevated due to many switchings and each switching event gives rise to overheating of phonons, which we estimate to account for a $\delta T_{ph} = 0.50$ mK increase in phonon temperature for $t = 10\,\mu s$ after

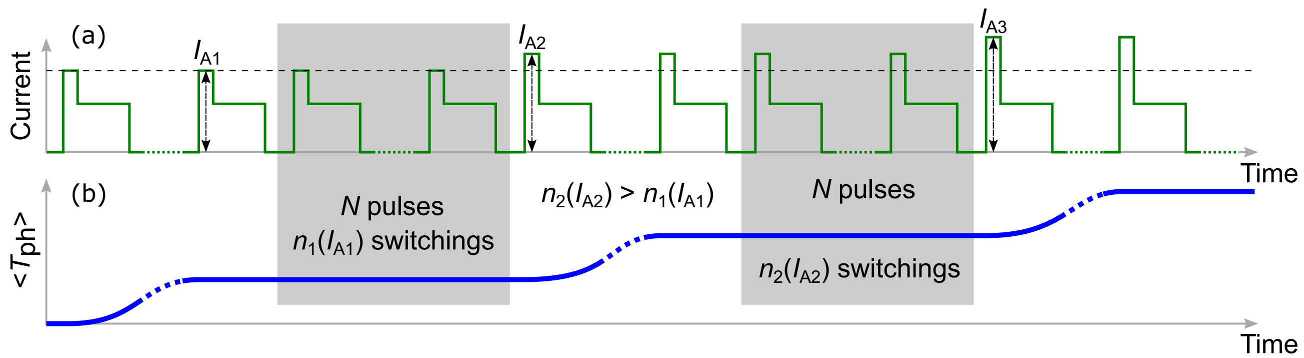

FIG. 5. (a) The continuous train of current pulses used to probe bridge B. The testing current amplitudes $I_{A1}, I_{A2}, I_{A3}, \ldots$ are changed between points in a steplike fashion, preserving the continuity of the train. (b) Schematic variation of the average local phonon temperature $\langle T_{ph} \rangle$ for the continuous train presented in (a). When I_A is increased, $\langle T_{ph} \rangle$ grows slightly after every switching event, reaching an elevated value after many switchings. Then the switching number is measured over a temporal window (gray region) extending over N pulses. The testing part of the probing pulse is 1 μs long.

the switching event [as fitted with the model presented in the Eq. (1)]. The contribution from each switching event relaxes exponentially over much longer times, $\tau_{ph} = 1.26$ ms, as inferred from Fig. 3(b).

We define the switching array S as consisting of ordered ones and zeros corresponding to a switching event or its lack in successive pulses; for example, $S = [1, 0, 1]$ encodes switching for the first and third probing pulses and no switching in the second pulse. The corresponding switching probability P in the $(n+1)$th pulse can be found from the temperature dependence of the probability for a fixed probing current amplitude $P(T_e)$: $P(n+1) = P[T_e(n+1)]$. We obtain $P(T_e)$ curves for different values of the probing current amplitude (Fig. 9) by interpolating the data presented in Fig. 2. We compare this probability with the randomly drawn number spanning the range from 0 to 1 to populate the S array; that is, we insert 1 into the array if the drawn number is equal to or smaller than P; otherwise, we insert 0.

The following recursive stochastic relation predicts the electron temperature seen by the $(n+1)$th probing pulse:

$$T_e(n+1) = T_0 + S(n)\,\Delta T_e + \Delta T_{ph}(n+1) \tag{2}$$

$$\Delta T_{ph}(n+1) = [\Delta T_{ph}(n) + S(n)\delta T_{ph}] \exp\left(-\frac{\Delta t}{\tau_{ph}}\right). \tag{3}$$

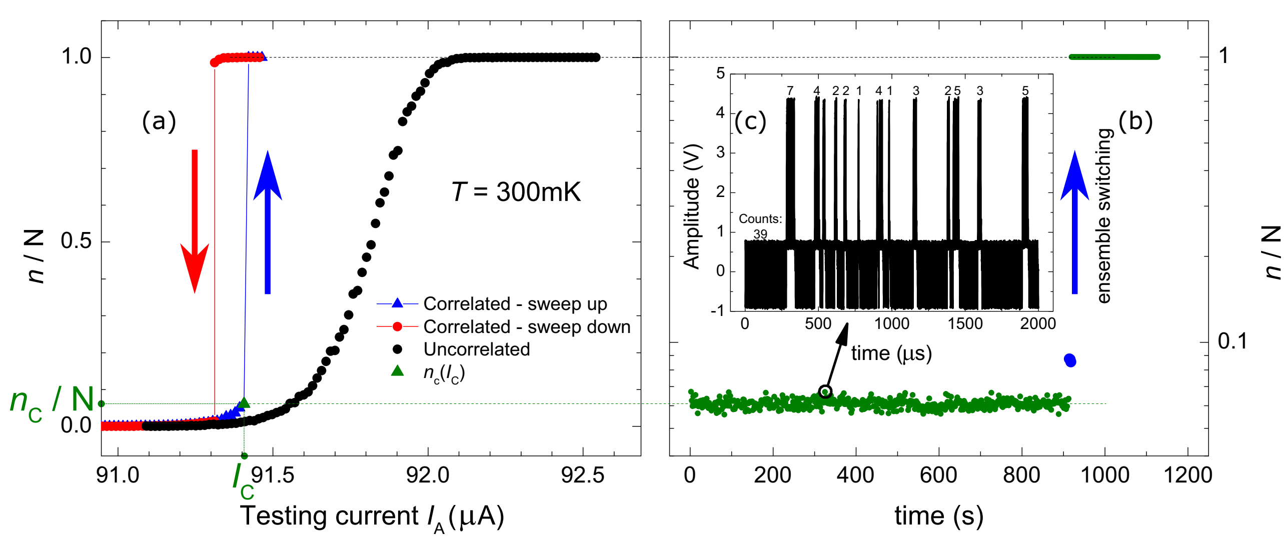

FIG. 6. Metastable state close to the transition point. (a) n/N dependence on the testing current I_A for bridge B with an abrupt transition at $I_A = I_C$ recorded for correlated pulses in the continuous train measurement. (b) The time trace of the apparent switching probability (switching number n/N) recorded in the neighborhood of I_C. The transition corresponds to the increment of the probing current by ~15 nA (forced transition), but it may also happen spontaneously for fixed current. Blue points correspond to a transient between two stable states, when ensemble switching happens. Each point is the result of a measurement extending over the time window covering $N = 125\,000$ pulses with a single pulse period of $\Delta t = 8\,\mu$s and $\Delta\tau = 2\,\mu$s. The pulse train duration is $\Delta t_{train} = N\Delta t = 1$ s. (c) The discrete stochastic trajectory: an oscilloscope screen shot of 250 pulses chosen from 125 000 pulses used to obtain a single point in (b), revealing bunching of switching events in a metastable state recorded while waiting for the spontaneous ensemble switching close to I_C. The number of switched pulses in a row is indicated above each bunch.

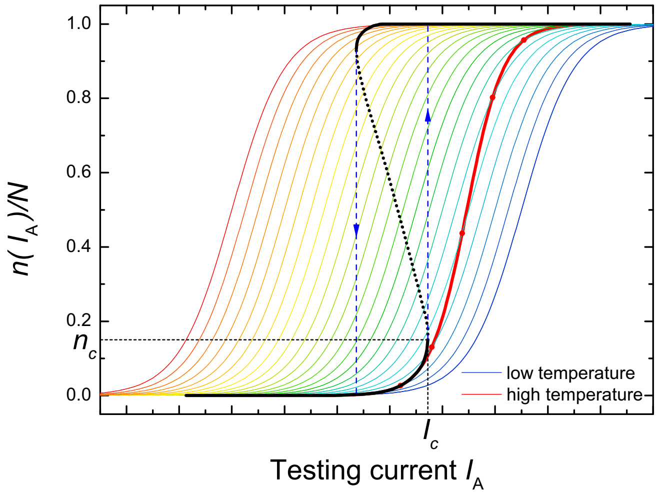

FIG. 7. Back-bent S curve: schematic presentation of S curves for different bath temperatures in the independent switching regime (see Fig. 2). Moderate correlation leads to steepening of the S curve like that observed in Fig. 4 (thick red line). On reducing the probing period, the discontinuous transition in the switching number n at $n = n_C$ is expected, giving rise to thermally stabilized hysteresis (black curve, with the dotted part not accessible to our experiment).

The postulated equation (3) for $\Delta T_{ph}(n+1)$ involves modification of the effusion process introduced in Sec. III [Eq. (1)] by assuming a random, but constant, increment of the phonon temperature at well-defined time intervals.

We are in the position to simulate the ensemble switching. We start with the $P(T_e)$ dependence interpolated for 90.18 μA. Iterating the model for $N = 40\,000$ pulses does not lead to $P = 1$ as we progress with the simulation (Fig. 10). The situation is different for $I_A = 90.2\,\mu$A. Here, P goes to 1 after

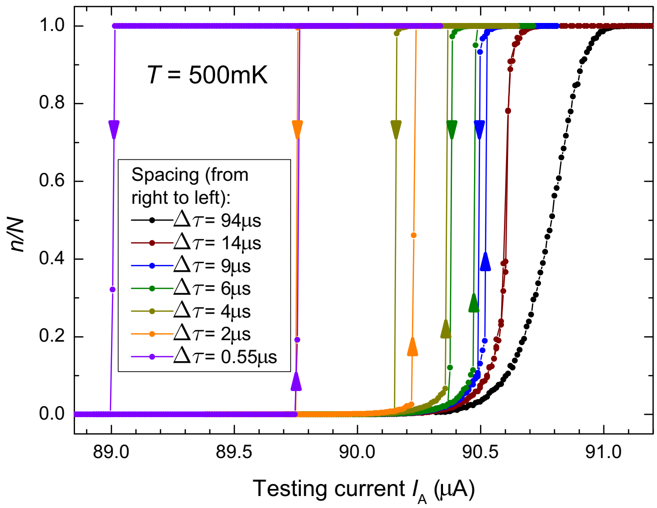

FIG. 8. Experimental hysteresis loops for bridge B: probing current dependencies of the switching number n/N for different temporal spacings $\Delta\tau$ between probing pulses obtained for the continuous train measurement. The rightmost curve is acquired in the independent regime. The leftmost hysteretic loop corresponds to the fully correlated regime, i.e., the panic switching [23].

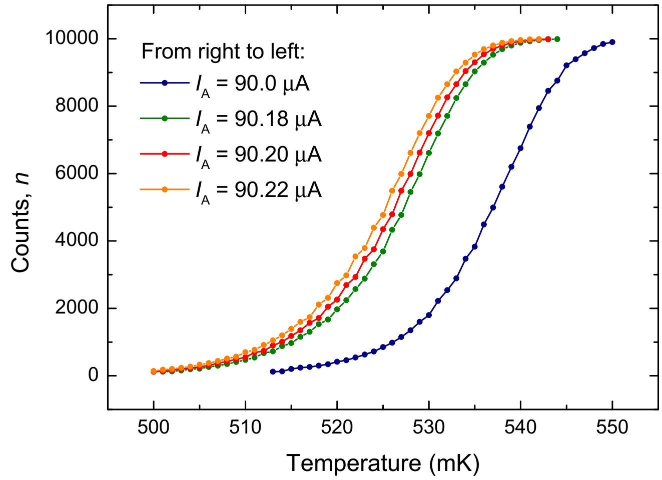

FIG. 9. Thermal S curves $P(T_e) = n(T_e)/N$ obtained for slightly different testing current amplitudes, as interpolated from the data presented in Fig. 2.

a sequence of heating pulses for n around 32 000, indicating the appearance of the ensemble transition. In both cases, the switching probability for each next pulse takes on a random value governed by a corresponding pattern of stochastic thermal heating. The two branches of the trace in each plot indicate the stochastic influence of the electron overheating: in fact, the trace jumps between these two branches as we progress with the simulation. Once the ensemble transition happens, the electron temperature approaches its asymptotic value $T_e = 598$ mK, which indicates that electrons are overheated with respect to the bath temperature by 98 mK when the next probing pulse arrives i.e., 2 μs after the last switching event. One should remember that the electron temperature keeps changing all the time during the experiment: it goes above $T_c = 1.3$ K when the bridge is transferred to the normal state; then subsequently, it relaxes towards the phonon temperature. In the discussed case the phonon temperature is 578 mK after ensemble switching occurs [see also the asymptote in Fig. 3(e)].

Our data remain in quantitative agreement with the switching number transition observed for the curve $\Delta\tau = 2\,\mu$s (Fig. 8), marking the proper n_c/N ratio at the onset of the ensemble switching [0.0258 in experiment (Fig. 8), approximately 0.025 in simulation (Fig. 10)]. The repetition of the simulation with the same parameters reveals the stochastic moment of the ensemble switching, as expected. For $I_A = 90.20\,\mu$A switching for $N = 40\,000$ is unlikely. For $I_A = 90.22\,\mu$A it becomes almost certain. For $I_A = 90.18\,\mu$A it does not happen even for $N = 100\,000$; for $I_A = 90.24\,\mu$A it always appears. The range of currents over which ensemble switching happens agrees well with the experimental trace in Fig. 6(b), where the probing current amplitude is increased by 15 nA, leading to an abrupt transition. One can think about a generalized S curve, describing the probability of the ensemble transition as a function of the probing current amplitude. Apparently, the width of such a generalized S curve (approximately 15 nA) is much narrower than the width of the usual S curve describing the probability of switching of the bridge to the normal state (approximately 500 nA; see Fig. 2).

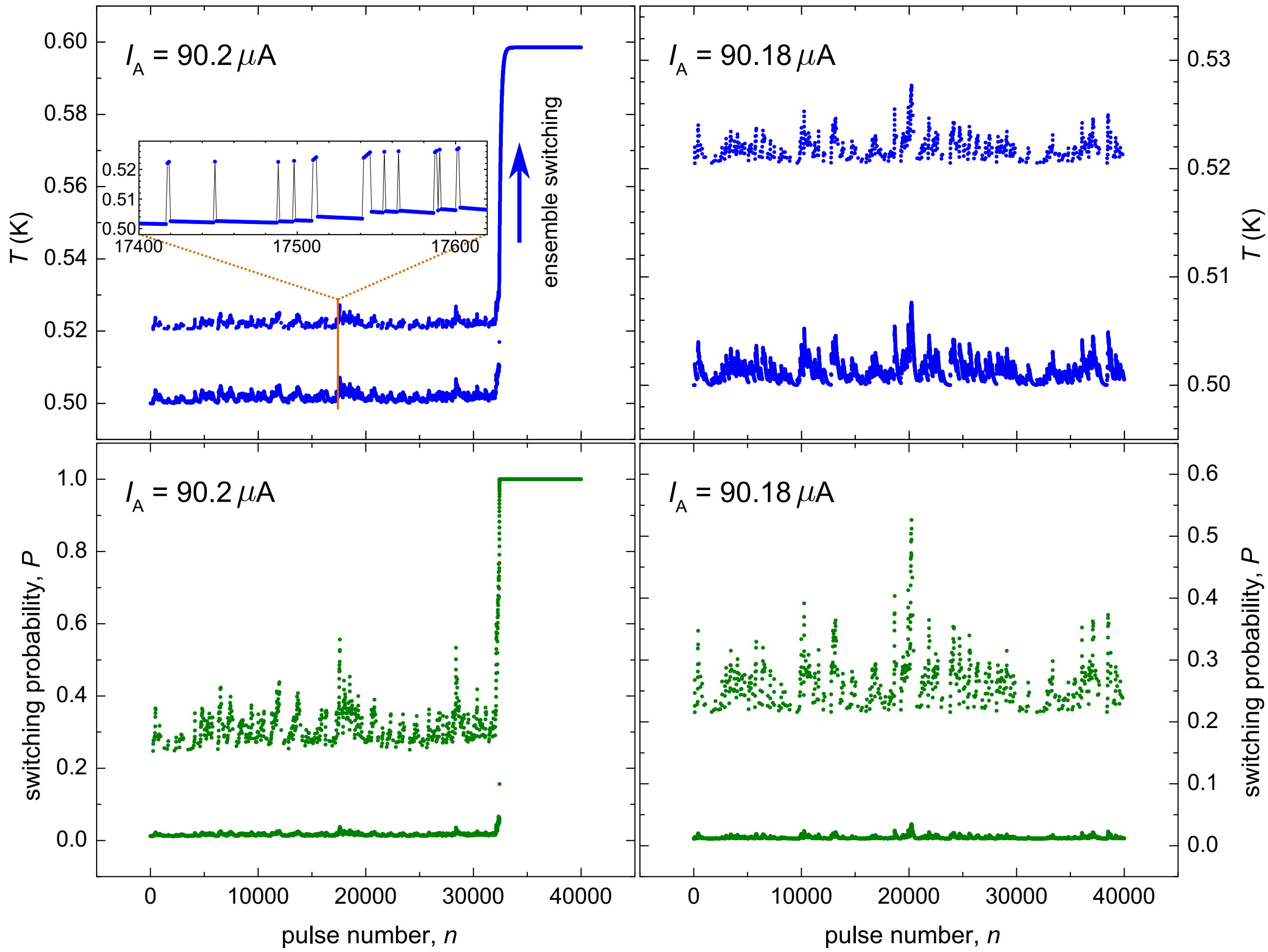

FIG. 10. Numerically calculated traces of electron temperature $T(n)$ [Eq. (2)] and probability $P(n)$ obtained with the aim of $P(T)$ dependence collected for $I_A = 90.2$ and $90.18\,\mu$A. $T_0 = 500$ mK. The calculation corresponds to the abrupt transition observed experimentally in the hysteretic switching dependence in Fig. 8 (curve $\Delta\tau = 2\,\mu$s). If there is a switching event for the $(n-1)$th testing pulse, the probability or temperature of the nth pulse adds a point to the upper branch of the corresponding trace. Otherwise, the evolution progresses through the lower branch, as seen in the enlarged image of the temperature trace collected for $I_A = 90.2\,\mu$s. It is noteworthy that the trace exhibits a similar bunching of switching events prior to the ensemble switching in comparison to the experimental stochastic trajectory in Fig. 6(c). Such a dichotomy of the traces comes from the random creation of the overheated electrons in the switching event, which affects only the nearest pulse (see the discussion in the text).

This observation supports our claim of high sensitivity of the discovered process on the control parameter, i.e., electric current, temperature, or magnetic field.

The model also properly accounts for steepening of S curves for moderately correlated pulses [Fig. 4(b)] when $\Delta\tau$ is long enough to allow electrons to thermalize with local phonons [$\Delta\tau > 10\,\mu$s; see point A in Fig. 3(b)]. In such a case consecutive switchings lead to an elevated phonon temperature and higher switching probability. The larger independent switching probability implicates more switching events and larger overheating of phonons. It is observed as the increase of the slope in the measured switching number dependencies.

VI. PERSPECTIVES FOR DETECTION

We have presented an abrupt transition in the switching number n/N at a critical value of the probing current amplitude and have coined it the ensemble switching. We also observed such switching upon increasing the bath temperature by a fraction of a millikelvin. Similarly, one may expect to see the transition for a small change in magnetic flux if, instead of the single junction, one uses a SQUID. The abrupt transition observed in the correlated measurements means that an extremely small change in the control parameter (i.e., probing current, temperature, or magnetic flux) changes the stochastic process dramatically: from a setting in which rare random bunched switchings are observed, the system is transformed to a fully deterministic configuration when each probing pulse drives the superconductor-normal-metal transition. Owning to inherent thermal feedback imposed by correlations, the transition exhibits a latching effect, as revealed by the hysteretic probing current dependencies.

One may argue that the above features, i.e., the hysteresis with discontinuity at a transition point, are desired for a detector; for example, it can be employed to detect magnetization reversals that produce tiny changes in magnetic flux [10]. Magnetic flux dependence of the switching probability $P(\Phi)$ in the independent regime is an S curve with a finite slope $dP/d\Phi$. In the correlated regime this slope becomes much steeper, thus making the detector very sensitive. We speculate that such a detector would be less sensitive to electromagnetic and thermal noise since the ensemble switching involves many single switching events and appears only in response to a permanent change in the control parameter: the slow response of the phonon temperature imposes a low-pass filtering on the ensemble transition. Unless the switching events are fully correlated, a single accidental switching event is not capable of driving the ensemble transition. It is in contrast to the transition edge sensors (TESs) for which obtaining a stable operation in the narrow superconducting transition may be challenging [32–34]. The fast dynamical response of the TESs may appear problematic in a noisy environment. The presented ensemble switching could be a detection scheme of choice when extraction of the permanent change in the monitored signal buried in the ambient noise is required. Another difference compared to the TESs is the opportunity to observe an ensemble transition in a wide range of temperatures; that is, it is possible to adjust the hysteresis for each temperature by tuning the duration between testing pulses. Having demonstrated experimentally the feasibility of the proposed scheme (Fig. 6), it remains for future studies to verify its sensitivity, but from the analysis presented in Sec. V it is clear that the width of a generalized S curve describing the probability of the ensemble transition is 1–2 orders of magnitude narrower than the width of the familiar S curve describing the probability of a single switching event.

VII. DISCUSSION

The artificial stochastic process we have created remains in strong analogy to the process which is responsible for switching superconducting wires due to a multiple-phase-slip mechanism [18]. In the former case switching in a single pulse increases the probability for switching during the next pulses; in the latter case a single phase slip occurrence increases the probability for the appearance of the next phase slips. In both cases accumulated heating leads, eventually, to a phase transition (we extend the meaning of phase transition to the artificial process which we describe). Multiphase slip escape is conventionally analyzed by means of the mean first passage time approach, which evaluates the lifetime of the metastable state [35]. The approach allows us to consider a continuous stochastic process with phase slips appearing at any time. It is in contrast to our artificial stochastic process, which is discrete, with single switching events appearing at well-defined moments, bounded by the duration of the testing pulse.

Our studies show that phonons in the current-probed nanostructures tend to be overheated with respect to the bath temperature. In our experiment we use an electric current at the level of 10–100 μA. It is a much larger value than typically used in testing tunnel junctions. However, at temperatures $\leqslant$ 100 mK, when heat capacities are vanishingly small, even very small currents may produce significant overheating of phonons, especially in experiments conducted in steady states. This possibility is often excluded in many works by the assumption of low Kapitza resistance. Even if it is correct, the assumption does not assert the thermalization of the substrate at the bath temperature.

VIII. CONCLUSION

Our work provides understanding of the unconventional behavior of nanoscale superconducting bridges arising from the thermal feedback in strongly correlated switching measurements. We engineered a discrete stochastic process with a controllable strength of correlation, which may facilitate the understanding of similar processes observed indirectly in nature, e.g., the multiphase slip driven superconductor-normal-metal transition. We developed a numerical recursive model incorporating stochastic heating and deterministic cooling. The model involves overheating of electrons, giving rise to the nearest-neighbor correlation in the switching measurements, and overheating of phonons, accounting for long-range correlations between pulses. Our artificially produced stochastic trajectories, like the one presented in Fig. 6, provide an interesting experimental framework for studying lifetimes of metastable states. The introduced switching protocol, using continuous pulse trains, can be used as a basis for a hysteretic detector of magnetic flux, current, and temperature when a vanishingly small, but permanent, change in these parameters is traced.

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