Letter

# Zeeman coupling and Dzyaloshinskii-Moriya interaction driven by electric current vorticity

Junji Fujimoto [1,*] Wataru Koshibae,[2] Mamoru Matsuo [1,2,3,4] and Sadamichi Maekawa[2,1]
[1] *Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China*
[2] *RIKEN Center for Emergent Matter Science, Wako, Saitama 351-0198, Japan*
[3] *Advanced Science Research Center, Japan Atomic Energy Agency, Tokai 319-1195, Japan*
[4] *CAS Center for Excellence in Topological Quantum Computation, University of Chinese Academy of Sciences, Beijing 100190, China*

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The Dzyaloshinskii-Moriya interaction, i.e., antisymmetric exchange interaction, combined with a Zeeman magnetic field gives rise to various magnetic states such as chiral, helical, and skyrmionic states. This interaction conventionally originates from spin-orbit coupling and thus needs somewhat heavy elements. In contrast, we here show a Dzyaloshinskii-Moriya interaction which is driven by electric current vorticity. We also find that the vorticity acts on localized spins as a Zeeman field, which may explain a recent experiment on current-driven magnetic skyrmion creation and annihilation without an external magnetic field in a device with a notch structure in FeGe. theory explains the control of skyrmion creation and annihilation by current direction and opens different possibilities for studies of magnetic textures by using structural settings.

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*Introduction.* Vortices are universal structures that appear across wide physical scales, such as rotating galaxies, typhoons, the bathtub vortex [1], the Abrikosov vortex in superconductors [2], and the optical vortex [3]. Recently, in quark matters, a vortical structure by nuclear collisions has been observed [4]. These vortical structures given above are characterized by vorticity and arise in a fluid, or an ensemble of interacting particles. Such vorticity has attracted much attention in the community of spin-related physics [5–7], since the discovery of spin current generation by the vorticity of a liquid metal [8].

Vortical structures are accompanied by a mechanical angular momentum, which may couple to the electron spin angular momentum through total angular momentum conservation [9] (Fig. 1). Spin-vorticity coupling (SVC) is derived from the local Poincaré symmetry in the relativistic quantum mechanics in curved spaces [10], as in a similar way to derive the coupling of U(1) current to the electromagnetic potential from the local U(1) gauge symmetry. We emphasize that SVC is a coupling between the electron spin and mechanical angular momentum and relies on neither magnetic moments nor conventional spin-orbit coupling.

Recently, spin current generation through SVC has been accomplished with a variety of vortical structures. In liquid metals [8,10], the vorticity originates from the velocity distribution of liquid metal particles. For vorticity induced by a surface acoustic wave [11,12], the spatial gradients of the lattice vibration are responsible for the vorticity. These vorticities arise from atomic motions, i.e., the spatial dependences of the flow and vibration of the atoms. In the case of naturally oxidized copper [13], the mobility of the conduction electrons depends on the spatial profile of the oxidization, and then the vorticity of the electrons arises when applying an electric field. These experiments have been performed in order to prove the existence of SVC, and it has been shown that the electron spin actually couples to various vorticities of atoms and electrons.

In addition to naturally oxidized copper, an electronic vorticity may arise in the hydrodynamic regime of electrons in graphene [14–17], which leads to collaborations between spintronics and the electron hydrodynamics [18]. Moreover, vorticity may appear in a device with a notch structure [19]. The electronic flow has a spatial dependence near the corner of the notch structure when applying an electric current, which yields a nonzero vorticity, which we focus on in this Letter.

The present Letter is originally motivated by a recent experiment on magnetic skyrmion creation and annihilation by the electric current in a device with a notch structure of FeGe [19]. The experiment reveals the following facts: (i) Magnetic skyrmions are created when an electric current is applied in the absence of a magnetic field, and the skyrmions are annihilated when the applied current direction is reversed; (ii) when applying an electric current without a notch structure, no skyrmion creation/annihilation occurs. Fact (ii) indicates that the notch structure plays an essential role in the creation and annihilation.

Magnetic skyrmions appear in a magnet with the Dzyaloshinskii-Moriya (DM) interaction [20,21] when applying a magnetic field. The effect of an electric current on the skyrmion *in an external magnetic field* was studied by numerical simulations [22–24]. However, the experiment was performed in the *absence* of an external magnetic field; hence, no justification could be provided by the simulations. The creation and annihilation without external fields has a significant advantage on applications, thus deserving of presenting a reasonable theory.

*Corresponding author: fujimoto.junji@gmail.com; present address: Department of Physics, University of Tokyo, Bunkyo, Tokyo 113-0033, Japan.

 

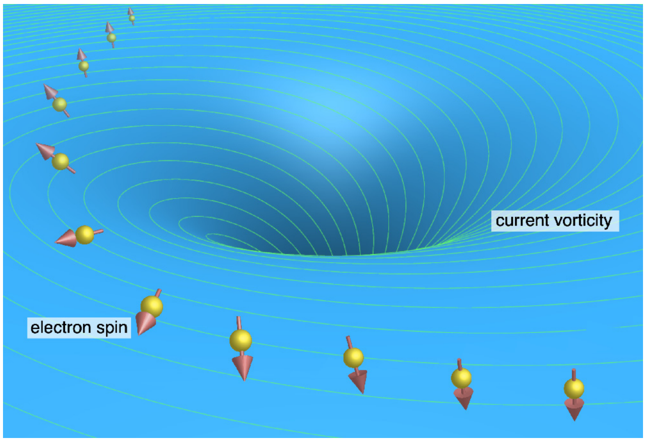


FIG. 1. Schematic figure of spin-vorticity coupling. The red arrow with a yellow sphere denotes the conduction electron spin, which is scattered by the current vorticity due to spin-vorticity coupling.

In this Letter, we consider that the electric current near the notch structure has a nonzero vorticity, which is given by $\boldsymbol{\Omega} = \nabla \times \boldsymbol{v}_{\rm drift}$, where $\boldsymbol{v}_{\rm drift}$ is the drift velocity, and show that the vorticity acts as an effective magnetic field through the exchange interaction between the conduction and localized spins [Fig. 2(a)]. We find that the effective Zeeman field is proportional to the current vorticity, so the Zeeman field reverses its direction when the current direction is reversed. Our results explain the above experimental facts. We also show that the vorticity induces an additional DM interaction between the localized spins [Fig. 2(b)]. The $D$ vector is parallel to the vorticity, which suggests that the DM interaction is designed by geometrical configurations of samples such as the design of current flow. We also discuss briefly that although magnetic

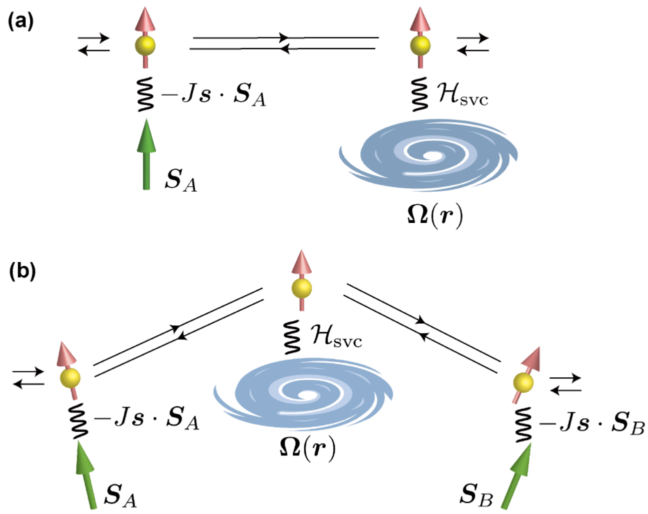


FIG. 2. Schematic descriptions of our theory. (a) The Zeeman field acting on the localized spin due to vorticity. (b) The DM interaction between the localized spins. The two green arrows represent the two localized spins $\boldsymbol{S}_A$ and $\boldsymbol{S}_B$, and the red arrows with yellow spheres denote the conduction electron spins $\boldsymbol{s}$, which interact through the exchange interaction $J$. The conduction electron spin also couples the vorticity $\boldsymbol{\Omega}(\boldsymbol{r})$ through SVC.

skyrmions are created near the edge of the notch structure, where the current vorticity is large, the spin current induced by the spatial gradient of the vorticity assists the skyrmions to spread to the entire area in the sample.

*Model.* To make the problem clear, we consider a simple model in which the conduction electrons interact with two localized spins $\boldsymbol{S}_A$ and $\boldsymbol{S}_B$ through an exchange interaction with strength $J$ and also couple with the vorticity through SVC. The Hamiltonian is given by

$$\mathcal{H} = \frac{\boldsymbol{p}^2}{2m_{\rm e}} - J\boldsymbol{S}_A(\boldsymbol{r})\cdot\boldsymbol{\sigma} - J\boldsymbol{S}_B(\boldsymbol{r})\cdot\boldsymbol{\sigma} - \frac{\hbar}{4}\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}(\boldsymbol{r}), \tag{1}$$

where $\boldsymbol{p}$ and $m_{\rm e}$ are the conduction electron momentum and mass, respectively, $\boldsymbol{S}_i(\boldsymbol{r}) = \boldsymbol{S}_i\delta(\boldsymbol{r}-\boldsymbol{R}_i)$, $i \in \{A, B\}$, denotes the localized spin divided by $\hbar$ with the position $\boldsymbol{R}_i$, and $\boldsymbol{\sigma}$ is the Pauli matrix that describes the conduction electron spin. The last term of the Hamiltonian represents SVC and is derived from the Dirac equation with a spin connection, which reproduces inertial effects on the electron spin in a conductive viscous fluid [10]. The Hamiltonian in relativistic quantum mechanics is obtained as $\mathcal{H}_{\rm svc} = -\frac{1}{2}\boldsymbol{\Sigma}\cdot\boldsymbol{\Omega}$ with the spin operator $\boldsymbol{\Sigma}$ of the $4\times4$ matrices. In this work, we take the nonrelativistic limit and obtain the Hamiltonian in which the spin operator is replaced by $\hbar\boldsymbol{\sigma}/2$. Here, we consider that the current velocity may be expanded by a Fourier series and focus on the lowest and second lowest components [see Supplemental Material (SM) [25]]. Then, the vorticity has the following spatial dependence, $\boldsymbol{\Omega}(\boldsymbol{r}) = \boldsymbol{\Omega}_1(\boldsymbol{r}) + \boldsymbol{\Omega}_2(\boldsymbol{r})$,

$$\boldsymbol{\Omega}_1(\boldsymbol{r}) = \boldsymbol{\Omega}_0\frac{e^{-\kappa|\boldsymbol{r}-\boldsymbol{R}_V|}}{\kappa|\boldsymbol{r}-\boldsymbol{R}_V|}, \quad \boldsymbol{\Omega}_2(\boldsymbol{r}) = 2\boldsymbol{\Omega}_0\frac{e^{-\kappa|\boldsymbol{r}-\boldsymbol{R}_V|}}{\kappa|\boldsymbol{r}-\boldsymbol{R}_V|}\cos\varphi, \tag{2}$$

where $\kappa$ is a specific length scale of the vorticity, $\boldsymbol{R}_V$ is the center position, $\boldsymbol{\Omega}_0 = \kappa v_{\rm drift}\hat{z}$, and $\varphi$ is the angle of $\boldsymbol{r}$.

Here, we emphasize a property of vorticity. The $(\boldsymbol{k}, \boldsymbol{k}')$ matrix element of the vorticity with an incoming wave vector $\boldsymbol{k}$ and outgoing wave vector $\boldsymbol{k}'$ is expressed as

$$\boldsymbol{\Omega}(\boldsymbol{k}, \boldsymbol{k}') = \boldsymbol{\Omega}_1(\boldsymbol{k}, \boldsymbol{k}') - 2i\boldsymbol{\Omega}_2(\boldsymbol{k}, \boldsymbol{k}'), \tag{3}$$

and by interchanging $\boldsymbol{k} \leftrightarrow \boldsymbol{k}'$, the matrix elements obey the relations

$$\boldsymbol{\Omega}_1(\boldsymbol{k}, \boldsymbol{k}') = \boldsymbol{\Omega}_1(\boldsymbol{k}', \boldsymbol{k}), \quad \boldsymbol{\Omega}_2(\boldsymbol{k}, \boldsymbol{k}') = -\boldsymbol{\Omega}_2(\boldsymbol{k}', \boldsymbol{k}). \tag{4}$$

(The details of this property are shown in SM [25].) We find below that, from the property of $\boldsymbol{\Omega}_n$, the $n = 1$ component is crucial for the Zeeman coupling and the $n = 2$ component is essential for the DM interaction.

*Zeeman field due to the vorticity.* Now, we derive the Zeeman coupling of the localized spins to the vorticity. This derivation is equivalent to the calculation of the total energy of the system up to first order in both the exchange interaction and SVC. We use the approximation that the matrix elements are constant for $\boldsymbol{k}, \boldsymbol{k}'$; $\boldsymbol{\Omega}_n(\boldsymbol{k}, \boldsymbol{k}') = \bar{\boldsymbol{\Omega}}_n$, after using Eq. (4), since the obtained results do not change in the sense that $\boldsymbol{\Omega}_1$ contributes to the Zeeman coupling and $\boldsymbol{\Omega}_2$ does not contribute. The details of the calculation are shown in SM [25]. Then, we have

$$E^{(1)} = -C(R_{AV})\boldsymbol{S}_A\cdot\boldsymbol{\omega} - C(R_{BV})\boldsymbol{S}_B\cdot\boldsymbol{\omega}, \tag{5}$$

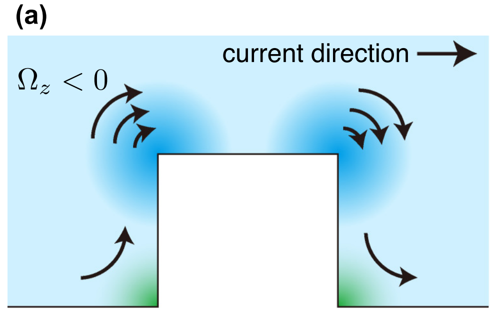


FIG. 3. Schematic description of the vorticity $\boldsymbol{\Omega}$ due to the electric current near the notch structure. (a) When the current is applied along the left-to-right direction, the vorticity is nonzero at the four corners of the notch; the blue-colored areas have the opposite vorticity direction to the green-colored areas. (b) The current direction is reversed from (a); the vorticity direction is also reversed. Note that the absolute value of the vorticity does not change between the left-to-right and right-to-left currents. Since the vorticity-induced Zeeman field is proportional to the vorticity, the direction of the Zeeman field depends on that of vorticity and thus the current.

where $R_{iV} = |\boldsymbol{R}_i - \boldsymbol{R}_V|$ with $i \in \{A, B\}$, $\boldsymbol{\omega} = \bar{\boldsymbol{\Omega}}_1/|\bar{\boldsymbol{\Omega}}_1|$, and the coefficient $C(r)$ is defined as

$$C(r) = \frac{m_e J k_F^2 |\bar{\boldsymbol{\Omega}}_1|}{8\pi\hbar\kappa^2 N} f(k_F r). \tag{6}$$

Here, $k_F$ is the Fermi wave number, $N$ is the lattice number per unit volume, and $f(x) = J_0(x)Y_0(x) + J_1(x)Y_1(x)$ with the Bessel function $J_n$ and the Neuman function $Y_n$. The spatial dependence is of the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction type [26–28] in two dimensions [29]. The difference from the usual RKKY interaction is that the vorticity is much larger than the atomic scale, $\kappa r_0 \ll 1$ while $k_F r_0 \sim 1$, with the atomic length scale of the distance being $R_{AV} \sim R_{BV} \sim r_0$. Hence, $E^{(1)}$ is rewritten as

$$E^{(1)} \simeq -g\mu_B \boldsymbol{B}_\Omega \cdot \sum_{i=A,B} \boldsymbol{S}_i, \tag{7}$$

where $\boldsymbol{B}_\Omega = C(r_0)\boldsymbol{\omega}/g\mu_B$, $g$ is the $g$-factor, and $\mu_B$ is the Bohr magneton. Equation (7) is regarded as the Zeeman coupling between the localized spins and the vorticity, which means that the vorticity acts on the localized spins as a magnetic field. The crucial point is that the Zeeman field is parallel to the vorticity, as shown below.

One important point regarding the vorticity of the electric current near the notch structure is the dependence on the current direction (Fig. 3). Since the vorticity is defined by $\boldsymbol{\Omega} = \nabla \times \boldsymbol{v}_{\rm drift}$, the direction of the vorticity changes when the current direction is changed, as shown in Fig. 3. Accordingly, the effective magnetic field $\boldsymbol{B}_\Omega$ changes its direction when the vorticity direction is changed. This feature qualitatively explains the experimental fact (i), because the magnetic skyrmion creation/annihilation arises from the competition between the DM interaction and the magnetic field.

Now, we evaluate the magnitude of the magnetic field $\boldsymbol{B}_\Omega$. We denote $\Delta r$ as the specific length scale of the vorticity, and $\Delta r = \kappa^{-1}$. The $n = 1$ component of the vorticity is estimated as $|\bar{\boldsymbol{\Omega}}_1| \simeq |\boldsymbol{\Omega}_0| = \kappa v_{\rm drift}$. For FeGe with the resistivity $\rho \sim 10^{-6}\ \Omega\,$m [30] and the mobility $\mu_e \sim 10^{-2}$ m$^2$/V s [31], we consider the applied current density $j \sim 10^9$ A/m$^2$, which yields $v_{\rm drift} = \mu_e \rho j \simeq 10$ m/s. Then, we find that the magnitude of the vorticity is $|\boldsymbol{\Omega}_0| \simeq 10^9$ s$^{-1}$ with $\Delta r = \kappa^{-1} \sim 10^{-8}$ m. Assuming that the lattice number corresponds to the electron number, $N = N_e = k_F^2/2\pi$, the obtained Zeeman field is evaluated as

$$|\boldsymbol{B}_\Omega| = \frac{|C(r_0)|}{g\mu_B} \simeq \frac{|\boldsymbol{\Omega}_0|}{\gamma}\frac{J}{\epsilon_F}\left(\frac{k_F}{\kappa}\right)^2 |f(k_F r_0)| \simeq 0.7\ \text{T},$$

where $\gamma = 1.76 \times 10^{11}$ rad s$^{-1}$ T$^{-1}$ is the gyromagnetic ratio, and we have assumed $J/\epsilon_F \simeq 0.2$, $k_F r_0 \sim 1$, $\kappa r_0 \sim 0.01$, and $|f(k_F r_0)| \sim 0.5$. The Zeeman field is strong enough to create and annihilate the magnetic skyrmions [32]. Note that the above value describes the upper limit of the Zeeman field and might be more strongly suppressed far from the center of the vorticity.

It should also be noted that the obtained magnetic field due to the vorticity is different from the Ampère magnetic field. The Ampère field is given by $B_A = \mu_0 j\pi(\Delta r)^2/2\pi\Delta r = (\mu_0/2)j\Delta r \sim 10^{-5}$ T with the vacuum permeability $\mu_0$.

*Antisymmetric exchange interaction.* Here, we consider a further effect of SVC, which is a vorticity-driven DM interaction between the localized spins. Expanding the total energy with respect to the second order of the exchange interaction and the first order of SVC (see SM [25] and Ref. [33] therein), we have

$$E^{(2)} = -D\boldsymbol{\omega} \cdot (\boldsymbol{S}_A \times \boldsymbol{S}_B), \tag{8}$$

and the coefficient $D$ is given by

$$D = \frac{6\hbar J^2 |\bar{\boldsymbol{\Omega}}_2|}{\pi N^2 \kappa^2}\left(\frac{m_e k_F}{2\hbar^2}\right)^2 \bar{g}(k_F r_0), \tag{9}$$

where $\bar{g}(x) = \int_0^1 t J_0(xt)[Y_0(xt)]^2 dt$, and we have approximated as $R_{ij} = |\boldsymbol{R}_i - \boldsymbol{R}_j| \sim r_0$, $i, j \in \{A, B, V\}$, as calculated for the Zeeman coupling. We emphasize that the coefficient $D$ contains only the $n = 2$ component of $\boldsymbol{\Omega}_n$. Equations (7) and (8) are the central results of this work.

The obtained DM interaction is also proportional to the current vorticity and does not have any symmetry requirement on the local spins. Instead, the electric current direction is breaking the symmetry. This dependence suggests that the DM interaction might be designed by the physical settings, such as the change in the current direction. Moreover, other vorticities such as the vorticity induced by surface acoustic waves [12] and the vorticity in naturally oxidized copper [13] may also give rise to DM interactions.

A vorticity-driven DM interaction may contribute to the skyrmion states. We find that the DM interaction has a different $D$ vector from the intrinsic DM interaction of FeGe. As a result, the DM interaction distorts the magnetic skyrmion (see SM [25]). However, the effects of the vorticity-driven DM interaction require further investigation.

Now, we evaluate the coefficient of the vorticity-driven DM interaction. The $n = 2$ component of the vorticity is estimated as $|\bar{\mathbf{\Omega}}_2| \simeq |\mathbf{\Omega}_0| = \kappa v_{\rm drift}$. Then,

$$D \simeq \frac{3\pi\hbar|\mathbf{\Omega}_0|}{2}\left(\frac{J}{\epsilon_{\rm F}}\right)^2\left(\frac{k_{\rm F}}{\kappa}\right)^2\bar{g}(k_{\rm F}r_0) \simeq 0.2 \text{ meV},$$

by evaluating $\hbar|\mathbf{\Omega}_0| \simeq 6.58 \times 10^{-7}$ eV and setting $\bar{g}(k_{\rm F}r_0) \sim 0.2$. The density is estimated as $D/10$ Å$^2 \sim 0.3$ mJ/m$^2$ and is comparable to the intrinsic DM interaction $\sim$1 mJ/m$^2$ [34].

*Spin current originating from the vorticity.* Moreover, we refer to another effect of the vorticity, namely, the spin current originating from the spatial gradient of the vorticity. The SVC has the spatial dependence shown in Eq. (2), which generates the spin current $j^\alpha_{{\rm s},i}$ along the Hamiltonian given as $j^\alpha_{{\rm s},i}\partial_i\Omega^\alpha$ with the spatial index $i = x, y$ and the spin index $\alpha = x, y, z$. This spin current possibly contributes to the skyrmion creation and annihilation in the experiment [19], since the created skyrmions quickly spread over the entire region of the system, although the creation only occurs near the notch structure. We evaluate the magnitude of the spin current at $k_{\rm F}r_0 \sim 1$ (see SM [25] and Ref. [35] therein) and find that $(2e/\hbar)|j^z_{{\rm s},i}(r_0)| \sim 10^{-4}$ A/nm.

Before the conclusion, we comment on other possible mechanisms of the creation of skyrmions. One is the modulation of local magnetic anisotropy induced by a thermomagnetoelastic mechanism [36] and structural defects with a pronounced nonzero curvature near the notch. Through modulation, skyrmions can be created as predicted by Ref. [37]. Magnetic surfaces with nonzero curvature may generate a DM interaction [38,39], which may promote the formation of skyrmions. However, the experiment [19] shows the annihilation of skyrmions in addition to the creation just by reversing the direction of the applied electric field, which cannot be explained by the above mechanisms. Note that spin-orbit couplings in the vicinity of notches could also be enhanced and create a DM interaction or effective magnetic field, but it is not easy to estimate them.

*Conclusion.* We have considered that the electric current near the notch structure has a nonzero vorticity and shown the following two effects on the localized spins by means of the conduction electron spin. The first effect is the Zeeman magnetic field, which reverses when the applied current direction is reversed. According to this feature with a sufficient magnitude, our theory may explain the creation and annihilation of magnetic skyrmions in the experiment. The second effect is the vorticity-driven DM interaction. The $D$ vector of the DM interaction is proportional to the vorticity, which suggests that we can design the DM interaction by structural configurations. We also find that the DM interaction contributes to the distortion of the magnetic skyrmion. The present Letter explains the control of the creation and annihilation of skyrmions by current direction and opens different avenues for the exploration of the design of magnetic textures by structural or mechanical settings.

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