Letter

Insulating regime of an underdamped current-biased Josephson junction supporting $\mathbb{Z}_3$ and $\mathbb{Z}_4$ parafermions

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We study analytically a current-biased topological Josephson junction supporting $\mathbb{Z}_n$ parafermions. First, we show that in an infinite-size system a pair of parafermions on the junction can be in n different states; the $2\pi n$ periodicity of the phase potential of the junction results in a significant suppression of the maximum current I_m for an insulating regime of the underdamped junction. Second, we study the behavior of a realistic finite-size system with avoided level crossings characterized by splitting δ. We consider two limiting cases: when the phase evolution may be considered adiabatic, which results in the 2π periodicity of the effective potential, and the opposite case, when Landau-Zener transitions restore the $2\pi n$ periodicity of the phase potential. We also study the case with time-reversal symmetry and show that breaking this symmetry gives different phase periodicity reductions. resulting current I_m is exponentially different in the opposite limits, which allows us to propose another detection method to establish the appearance of parafermions in the system experimentally, based on measuring I_m at different values of the splitting δ.

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Introduction. Topological superconducting systems have recently attracted much attention both from a fundamental point of view and as possible platforms of a quantum computer [1–3]. One of the effects, which may indicate the topological properties of the system, is the fractional Josephson effect [1,4–6]. In a trivial Josephson junction (JJ) the low-energy properties (such as the Josephson current) are determined by a 2π-periodic phase potential. The best studied fractional Josephson effect is in junctions formed by topological superconductors supporting Majorana bound states (MBSs) [7–11]. In this case, the phase potential of the system is 4π periodic due to the possibility of the coherent transfer of a single electron as a result of the coupling of the MBSs on the sides of the junction. However, it is known that quasiparticle poisoning can spoil the 4π periodicity, which is a potential problem for all systems hosting MBSs [5,12–15]. Moreover, MBSs have Ising-type braiding statistics, which is not sufficient for universal quantum computation [3,16].

More exotic effects are predicted for systems with $\mathbb{Z}_n$ symmetries ($n > 2$), where the domain walls between topological and trivial phases host $\mathbb{Z}_n$ parafermions (PFs) [17–34] with more complex braiding statistics, which allows one to perform an entangling gate and makes PFs computationally more powerful than MBSs [35,36]. The effective state formed by a pair of PFs carries a fractional charge $2e/n$, which is robust against extrinsic quasiparticles (integer-charge quasiparticles cannot induce transitions between the n possible states of the system). In general, the emergence of PFs is predicted for systems with strong electron-electron interactions; a pair of PFs on the junction sides enables the tunneling of $2e/n$ fractional quasiparticles and, therefore, results in a $2\pi n$ periodicity in the phase [18,20,23,29,35,37–40]. The Hamiltonian of such a system takes the form

$$H = \frac{q^2}{2C} + U(\phi), \quad [\phi, q] = 2ei, \tag{1}$$

where the first term corresponds to the charging energy: C is the capacitance of the junction, and q is the charge on the junction; $U(\phi)$ is the $2\pi n$-periodic phase potential.

An experimental demonstration of parafermion edge states presents a complex problem. However, recent experiments on induced superconductivity in the edge states of systems with a fractional quantum Hall effect (FQHE) seem to be promising for this purpose [41–44]. A crossed Andreev pairing gap Δ_c across the superconductor separating two counterpropagating edge states has been reported [42,44], which is supposed to be sufficient for the formation of PFs [35].

We propose that for the direct observation of PFs one needs to combine two such setups into an effective JJ (see Fig. 1), so that the fractional Josephson effect can be observed. Moreover, we discuss a general experimental method to distinguish topological JJs hosting PFs from nontopological ones, based on the properties of an underdamped JJ, which depends crucially on the periodicity of $U(\phi)$. Compared to JJs hosting MBSs [45], systems hosting PFs are much more complex, which, however, opens up ways to reduce phase periodicity and detect such exotic bound states. Moreover, the observation of these effects can be even more feasible than for MBSs due to the intrinsic insensitivity to quasiparticle poisoning.

We start with a general model of a JJ hosting $\mathbb{Z}_3$ or $\mathbb{Z}_4$ PFs on the junction sides. We discuss the voltage peak $V_m = RI_m$ in the I-V characteristics of such a device, shunted by a large resistance R and biased by a current. This peak corresponds to a transition from an effectively insulating to a conducting state [46–48]; its magnitude depends on the tunneling amplitude between the minima of the phase potential, therefore, the $2\pi n$ periodicity plays a crucial role. Moreover, if one can control the transitions between the n possible states of a PF pair on the junction (i.e., tuning the splitting δ at avoided crossings by changing the applied magnetic field [49]), one can effectively change the periodicity of the potential and, as a result, control the value of V_m. In our work, we consider the temperature to be low enough, i.e., $T \ll \omega_0$, with ω_0 being the level spacing in the minima of the phase potential $U(\phi)$, to ignore thermal fluctuations, which, in general, would result in smoothening of the voltage peaks.

$\mathbb{Z}_3$ case. A pair of $\mathbb{Z}_3$ PFs coupled via a JJ allows the transport of $2e/3$ fractional quasiparticles through the junction. As a result, the phase potential of the junction takes the form [20,21,26]

$$U(\phi) = -E_J \cos\phi - E_{2e/3} \cos\left(\frac{\phi - 2\pi m}{3}\right), \tag{2}$$

where $E_{2e/3}$ is the PF coupling amplitude, which governs fractional quasiparticles tunneling; $m \in \{0, 1, 2\}$ corresponds to one of the three states of the tunnel-coupled PF pair; and E_J corresponds to Cooper-pair tunneling through the junction. We consider the regime of a well-defined phase, i.e., $E_c = \frac{e^2}{2C} \ll E_J$, and we assume the trivial Josephson tunneling to be dominant, $E_J \gg E_{2e/3}$. The lowest-energy band dispersion takes the form [46,47,50]

$$E^{(0)}(k) = \frac{\omega_0}{2} - 2\nu_{6\pi} \cos(6\pi k), \tag{3}$$

where $\omega_0 \approx \sqrt{8E_J E_c}(1 + \frac{1}{18}\frac{E_{2e/3}}{E_J})$ is the harmonic frequency for the low-energy bands, and $\nu_{6\pi}$ is the amplitude for 6π tunneling between the ground states in the absolute minima of $U(\phi)$. The tunneling amplitude for the topological junction is given by [50]

$$\nu_{6\pi} = \sqrt{3}\frac{4E_c}{\sqrt{\pi}}\left(\frac{2E_J}{E_c}\right)^{3/4} e^{-S_{6\pi}}, \tag{4}$$

where

$$S_{6\pi} = 3S_0\left(1 + \left[1 + \ln\frac{16E_J}{3E_{2e/3}}\right]\frac{E_{2e/3}}{8E_J}\right). \tag{5}$$

It is convenient to compare it to the amplitude of 2π tunneling in a trivial junction [46,51]:

$$\nu_0 = \frac{4E_c}{\sqrt{\pi}}\left(\frac{2E_J}{E_c}\right)^{3/4} e^{-S_0}, \quad S_0 = \sqrt{8E_J/E_c}. \tag{6}$$

The tunneling amplitude $\nu_{6\pi}$ is sufficiently smaller than ν_0 due to a factor of 3 in the exponent ($S_0 \gg 1$).

If we now consider a system consisting of such a junction with a large shunting resistance (underdamped junction), $R > R_Q = 2\pi/(2e)^2$, and apply a current, the junction would be in an effectively insulating regime up to some maximum value of the applied current I_m, determined by the dispersion of the lowest band [46,47], which can be seen as a sharp voltage peak $V_m = RI_m$. The value of this current depends on the bandwidth $4\nu_{6\pi}$ and is given by [50]

$$I_m^{6\pi} = e96\sqrt{3\pi}E_c\left(\frac{2E_J}{E_c}\right)^{3/4} e^{-S_{6\pi}}\frac{R_Q}{R}. \tag{7}$$

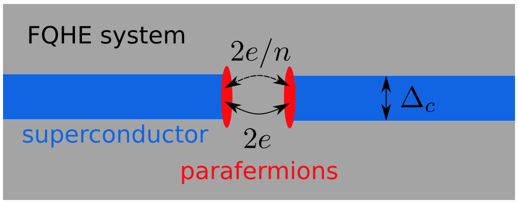

FIG. 1. Schematic representation of the FQHE stucture. Narrow superconducting strips (blue) induce pairing of amplitude Δ_c between counterpropagating FQHE edge states. Two strips placed close to each other form an effective JJ, and a pair of $\mathbb{Z}_n$ parafermions on the junction forms a channel for $2e/n$ fractional quasiparticles tunneling between the superconducting strips along with ordinary Cooper pairs of charge $2e$.

The result is modified in the case of nonzero population of the excited band [50], however, it still holds the exponential dependence on the reduced phase periodicity.

Finite-size effects may lift the degeneracy between the ground states and result in transitions between the three possible states of the PF pair on the junction. In particular, the overlap with PFs localized on the outer sides of the topological system [49] plays a crucial role. A similar effect has been discussed before for a JJ hosting MBSs [52,53]. The resulting spectrum of a three-level system formed by a pair of PFs localized on the sides of the junction has avoided level crossings at πn: with energy splitting 2δ [at $\pi(2n+1)$] and $2\delta'$ (at $2\pi n$) (see Fig. 2). Away from the avoided level crossings each branch consists of one of the three states with energy $-E_{2e/3}\cos([\phi - 2\pi m]/3)$, where m labels the state; at avoided level crossings the state is given by a superposition of two states with different m's. If δ is small ($\delta \ll E_{2e/3}$) and can be treated perturbatively, the ground state energy is given by

$$E_g \approx \min_m\left\{-E_{2e/3}\cos\left(\frac{\phi - 2\pi m}{3}\right)\right\}. \tag{8}$$

As we consider $\delta \ll E_{2e/3}$, we have neglected the corrections to the energy at the avoided crossing points. In the adiabatic limit (discussed in detail below), the phase potential of the topological JJ is given by $U(\phi) \approx -E_J\cos\phi + E_g$, which is 2π periodic. We can calculate the instanton action for a 2π phase slip in a topological junction (it is different from the

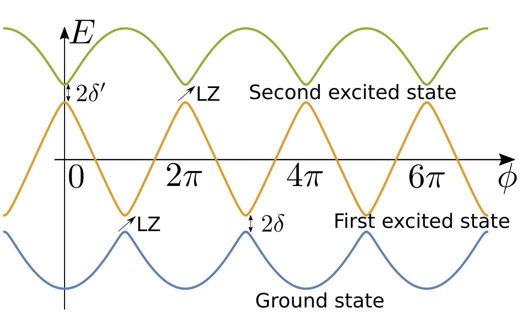

FIG. 2. The spectrum of a three-level system, formed by a pair of localized $\mathbb{Z}_3$ PFs with degeneracy lifting due to finite-size effects of overlapping PFs.

nontopological action S_0 due to the $E_{2e/3}$ term):

$$S_{2\pi} = S_0\left(1 + \frac{3}{8}\left[2\,\mathrm{arcoth}\sqrt{3} - \ln 3\right]\frac{E_{2e/3}}{E_J}\right). \quad (9)$$

The resulting tunneling amplitude takes the form

$$\nu_{2\pi} = \frac{4E_c}{\sqrt{\pi}}\left(\frac{2E_J}{E_c}\right)^{3/4} e^{-S_{2\pi}}. \quad (10)$$

And, finally, we can calculate the maximum value of the current for the insulating regime:

$$I_m^{2\pi} = e32\sqrt{\pi}E_c\left(\frac{2E_J}{E_c}\right)^{3/4} e^{-S_{2\pi}}\frac{R_Q}{R}. \quad (11)$$

As one can see from Fig. 2, the above analysis is valid, if the phase dynamics may be considered adiabatic in comparison to the dynamics of the state formed by a pair of localized PFs. That means that as long as we can neglect Landau-Zener transitions (LZTs) at $\phi = 2\pi(2n+1)$, the effective potential is determined by the ground state energy of the topological junction for the fixed phase $E_g - E_J\cos\phi$. The probability of LZT is given by

$$P_{\mathrm{LZ}} = \exp\left(-\frac{2\pi\delta^2}{\dot{\phi}E_{2e/3}}\right) \approx \exp\left(-\frac{\delta^2}{\nu_{2\pi}E_{2e/3}}\right). \quad (12)$$

Then, if $\delta \gg \delta_{2\pi} = \sqrt{\nu_{2\pi}E_{2e/3}}$, we can neglect LZTs and assume the potential to be effectively 2π periodic. In the limit $\delta \ll \delta_{6\pi} = \sqrt{3\nu_{6\pi}E_{2e/3}}$ (the factor $3\nu_{6\pi}$ arises from a new characteristic velocity for phase evolution due to 6π tunneling: $\dot{\phi} = 6\pi\nu_{6\pi}$), we come back to the 6π periodicity and to the result given by Eq. (7). In principal, δ' may be different from δ (see Fig. 2), however, the difference between them is not essential as long as δ is smaller than the characteristic energy scales, $\delta' = \delta[1 + O(\delta/E_{2e/3})]$. As a result, if one can control δ, one can switch the system from an effectively 6π to an effectively 2π state, which should be observable as a drop in the voltage peak $V_m = RI_m$ and indicate the presence of PFs in the system. Moreover, as it was shown in Ref. [49] and before for systems hosting MBSs [54–56], the splitting is oscillating around zero as a function of the chemical potential and the applied magnetic field. The latter is easy to control experimentally, while varying it changes other energy scales of the junction, such as E_J or E_c, very slowly and monotonically. As a result, the value of the peak $V_m = RI_m$ changes between two exponentially different values, given by Eqs. (7) and (11), if one varies the applied magnetic field. Strictly speaking, for any nonzero δ, there are all possible $2\pi l$ tunnelings with integer l. Moreover, for $l \geqslant 3$ there are several different tunneling processes, as there can be several different effective potentials, i.e., 6π tunneling can either be with four LZ transitions or only with two transitions (staying at the same branch three times at avoided crossings). However, the probability of each such tunneling decays exponentially with δ for any $l > 1$. Therefore, for $\delta \gg \delta_{6\pi}/2$ the 2π-periodic contribution dominates, which gives (see Fig. 3)

$$I_m \approx \left[1 - \exp\left(-\frac{\delta^2}{\nu_{2\pi}E_{2e/3}}\right)\right]I_m^{2\pi}. \quad (13)$$

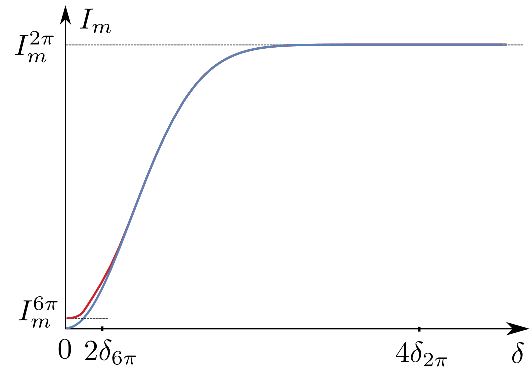

FIG. 3. The dependence of the maximum current I_m on the splitting δ. The blue line indicates the 2π-periodic contribution [see Eq. (13)], which is suppressed for small δ. The total I_m (red line, hand-drawn) starts at $I_m^{6\pi}$ and merges with $I_m^{2\pi}$ at $\delta \gg \delta_{6\pi}/2$. The parameters used: $\nu_{6\pi} = 0.01\nu_{2\pi}$.

$\mathbb{Z}_4$ *case.* The above analysis can also be performed for $\mathbb{Z}_4$ PFs. A pair of $\mathbb{Z}_4$ PFs localized on the sides of a junction results in the phase potential [18,37,38,40,57]

$$U = -E_J\cos\phi - \sum_{n=1}^{2} E_{e/n}\cos\left(\frac{\phi - 2\pi m}{2n}\right). \quad (14)$$

E_e represents single-electron tunneling, $E_{e/2}$ stands for the tunneling of $e/2$ fractional quasiparticles, and $m \in \{0, 1, 2, 3\}$ indicates one of the four possible states of the PF pair; E_J is a trivial Josephson energy. In several theoretical works [29,40], the Cooper-pair tunneling was predicted to be dominating, i.e., $E_J \gg E_e, E_{e/2}$. The harmonic frequency, determining the lowest-energy bands, is given by $\omega_0 \approx \sqrt{8E_JE_c}(1 + \frac{E_e}{8E_J} + \frac{E_{e/2}}{32E_J})$. With the assumptions taken above, we calculate the instanton action for tunneling between the lowest minima of the phase potential (expansion in E_e/E_J and $E_{e/2}/E_J$):

$$S_{8\pi} = 4S_0\left[1 + \frac{1}{8}\left(1 + \ln\frac{16E_J}{E_e}\right)\frac{E_e}{E_J} + \frac{1}{8}\left(1 + \ln\frac{2^{9/2}E_J}{E_{e/2}}\right)\frac{E_{e/2}}{E_J}\right]. \quad (15)$$

As a result, we can derive the current I_m at which the junction switches from an insulating to conducting state:

$$I_m^{8\pi} = e256\sqrt{\pi}E_c\left(\frac{2E_J}{E_c}\right)^{3/4} e^{-S_{8\pi}}\frac{R_Q}{R}. \quad (16)$$

Finite-size effects play exactly the same role as in the case of $\mathbb{Z}_3$ PFs. By varying an applied magnetic field, one would tune the overlap with PFs on the outer edges of the system [49], which can drive the system to an effectively 2π-periodic state with a result similar to Eq. (11) (with additional parametrically small corrections in the tunneling action). However, it is also possible to get a more sophisticated phase periodicity reduction. Some systems hosting $\mathbb{Z}_4$ PFs possess time-reversal symmetry (TRS) [18,40] (without a magnetic field). If one applies a local magnetic field, the TRS is broken, which results in the lifting of Kramers degeneracy. We can consider the splitting δ to be small in comparison to the energy

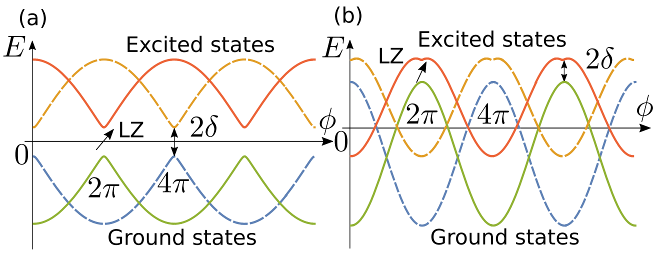

FIG. 4. The spectrum of a system with TRS formed by a pair of localized $\mathbb{Z}_4$ PFs: (a) with $E_e = 0$ and (b) with $E_e = 2E_{e/2}$. Solid and dashed lines correspond to states with opposite fermion parity. The Kramers degeneracy at $2\pi n$ is lifted due to TRS breaking, while the rest of the crossings survive, being protected by fermion parity. As a result, the ground state is either given by the blue or green branch.

scales E_e and $E_{e/2}$. Then, for a fixed phase the energy of the ground state, formed by a pair of $\mathbb{Z}_4$ PFs, is given by (green branch in Fig. 4)

$$E_g = -E_e \cos(\phi/2) - \sqrt{\delta^2 + E_{e/2}^2 \cos^2 \frac{\phi}{4}}$$
$$\approx -E_e \cos(\phi/2) - E_{e/2} \max_m \cos \frac{\phi - 4\pi m}{4}. \quad (17)$$

We note that only Kramers degeneracies at $2\pi n$ are lifted due to broken TRS, and all the other crossings remain, as they are protected by fermion parity conservation (finite-size effects being neglected) [18,40]. If LZT can be neglected (condition given below), the phase potential of the JJ is $U = -E_J \cos\phi + E_g$, which allows us to calculate the instanton action for tunneling between the lowest minima:

$$S_{4\pi} = 2S_0\left[1 + \frac{1}{8}\left(1 + \ln\frac{16E_J}{E_e}\right)\frac{E_e}{E_J} + \frac{1}{16}\left(1 + \ln\frac{8E_J}{E_{2/3}}\right)\frac{E_{e/2}}{E_J}\right]. \quad (18)$$

From this we can determine the critical current for the insulating regime,

$$I_m^{4\pi} = e64\sqrt{2\pi}E_c\left(\frac{2E_J}{E_c}\right)^{3/4} e^{-S_{4\pi}}\frac{R_Q}{R}. \quad (19)$$

As long as $\delta \gg \delta_{4\pi} = \sqrt{2v_{4\pi}E_{e/2}}$ (negligible LZT), the above assumption is valid, while in the limit $\delta \ll \delta_{8\pi} = \sqrt{4v_{8\pi}E_{e/2}}$ the LZT probability is almost unity, which allows us to treat the phase potential as effectively 8π periodic [and reproduce the results derived above in the absence of degeneracy lifting; see Eq. (16)]. The splitting δ is controlled by the local magnetic field, and in this case the dependence is monotonic—the larger the field, the larger is δ.

Discussion and conclusions. The above analysis provides a promising method to establish the presence of PFs in systems that are expected to support these exotic topological bound states. The method consists of measuring the I-V characteristics of the current-biased junction in an underdamped regime at different values of splitting δ at avoided crossings. As shown in Ref. [49], the splitting due to the finite-size effect is oscillating around zero as a function of magnetic field (similar to junctions supporting MBSs [52,53]). As a result, if the magnetic field is varied, the system oscillates between the regimes of low and high LZT probabilities with significantly different values of the peak $V_m = RI_m$ (due to different effective periodicities of the phase potential). Moreover, for systems with TRS (without a magnetic field) one can switch to a state with reduced periodicity applying local magnetic fields (δ is monotonically increasing with the field). An underdamped regime of a Josephson junction with $E_J \gg E_c$, which is crucial to observe effects caused by quantum phase fluctuations, is technically challenging, but nevertheless possible with the proper choice of an environment with high impedance [58–60]. The results obtained here may be generalized to systems hosting $\mathbb{Z}_n$ PFs with any integer n. The voltage peak will be at

$$I_m = e32\sqrt{\pi n^3/l^3}E_c\left(\frac{2E_J}{E_c}\right)^{3/4} e^{-S_{2\pi n/l}}\frac{R_Q}{R}, \quad (20)$$

where $l < n$ is the reduced periodicity factor arising from finite-size effects or TRS breaking. The generalized formula is valid as long as the Cooper-pair tunneling is dominating over any fractional quasiparticle tunneling. The tunneling action is given by $S_{2\pi n/l} = nS_0/l + \cdots$, where the correction is determined by the terms corresponding to fractional quasiparticle tunneling. Thus, I_m changes significantly if l goes from $l = 1$ (negligible splitting) to $l > 1$. This nonmonotonic behavior of I_m is specific only for topological junctions, which provides a straightforward way to distinguish a junction hosting Majorana fermions [45] or parafermions. The latter may have additional symmetries (such as TRS), which allow more complicated mechanisms for reducing phase periodicity and, therefore, richer phenomena, which can be observed experimentally.

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