Letter

Microscopic calculation of spin torques in textured antiferromagnets

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A microscopic calculation is presented for the spin-transfer torques (STTs) and damping torques in metallic antiferromagnets (AFs). It is found that the sign of the STT is opposite to that in ferromagnets (FMs) because of the AF transport character, and the current-to-STT conversion factor is enhanced near the AF gap edge. The dissipative torque parameter β_n and the damping parameter α_n for the Néel vector arise from spin relaxation of electrons. Physical consequences are demonstrated for the AF domain wall motion using collective coordinates, and some similarities to the FM case are pointed out such as intrinsic pinning and the specialty of $\alpha_n = \beta_n$. A recent experiment on a ferrimagnetic GdFeCo near its angular-momentum compensation temperature is discussed.

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Manipulation of spin textures using electric current forms an intriguing subfield of spintronics. The effect of currents on ferromagnetic (FM) textures is well understood through the spin angular momentum transfer between the conduction electrons and magnetization [1–3]. However, a similar picture is not feasible in antiferromagnets (AFs) [4–6] since the magnetic order parameter and conduction electrons do not carry macroscopic spin angular momenta [7–13]. This makes microscopic studies indispensable for understanding spin torques in AFs.

In FMs, electrons moving in a spin texture with exchange coupling exhibit a spin polarization:

$$\langle \hat{\boldsymbol{\sigma}} \rangle \propto \mathbf{n} \times (\mathbf{v} \cdot \nabla)\mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is the magnetic order parameter (magnetization vector for FM), and $\mathbf{v}$ is a velocity that characterizes the electron flow (spin current for FM). The spin polarization arises as a reactive response [14] and exerts a reaction torque, known as the spin-transfer torque (STT), on the FM spins. In AFs, Xu *et al.* [7] and Swaving and Duine [8] numerically obtained the same form of spin polarization as Eq. (1) with $\mathbf{n}$ now representing the Néel vector. Analogous to FM, this spin polarization emerges through a reactive process and gives rise to a torque that conserves total angular momentum, which may thus be called the STT. However, in contrast to FM, the coefficient cannot be determined by a macroscopic argument based on the conservation law. Moreover, there is in general another type of torque, called the β torque, that arises as a dissipative response due to spin relaxation [15,16], the analytic expression of which is yet to be determined for AFs.

In this letter, we present a microscopic calculation of the STT, the β torque, and the damping torques in AF metals. A careful treatment is given to the effects of spin relaxation, which we model by magnetic impurities. We find a STT proportional to the electric current but with a coefficient different from that in FMs. The β torque is proportional to the spin-relaxation rate. Interestingly, both torques in AFs drive the texture in the opposite direction than those in FMs. Using collective coordinates, it is shown that only the β torque drives AF domain walls (DWs) [9,10] because the effect of STT is nullified by an effect like the intrinsic pinning in FMs. Finally, a recent experiment on the current-assisted DW motion in ferrimagnets at the angular-momentum compensation temperature [17] is discussed.

We consider the s-d model consisting of localized spins (H_S), conduction electrons ($H_{\rm el}$), and the s-d exchange interaction ($H_{\rm sd}$) between them:

$$H = H_S + H_{\rm el} + H_{\rm sd}. \tag{2}$$

The space dimensionality d can be arbitrary in the general formulation, but explicit calculations will be done for a two-dimensional square lattice $d = 2$.

We first sketch the derivation of the equations that describe long-wavelength, low-frequency dynamics of AF spins coupled to conduction electrons. We start with the lattice model:

$$H_S = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_i \left(S_i^z\right)^2, \tag{3}$$

$$H_{\rm sd} = -J_{\rm sd} \sum_i \mathbf{S}_i \cdot c_i^\dagger \boldsymbol{\sigma}\, c_i, \tag{4}$$

where $\mathbf{S}_i$ is a localized, classical spin at site i, $J > 0$ is the AF exchange coupling constant between nearest-neighbor (n.n.) sites $\langle i, j \rangle$, and $K > 0$ is the easy-axis magnetic anisotropy constant. In $H_{\rm sd}$, $c_i^\dagger = (c_{i\uparrow}^\dagger, c_{i\downarrow}^\dagger)$ is the electron creation operator at site i, $\boldsymbol{\sigma}$ is a vector of Pauli matrices, and $J_{\rm sd}$ is the s-d exchange coupling constant.

We consider a two-sublattice unit cell m with localized spins, $\mathbf{S}_{A,m}$ and $\mathbf{S}_{B,m}$, on each sublattice, and define the Néel component $\mathbf{n}_m$ and the uniform component $\mathbf{l}_m$ by [18]

$$\mathbf{n}_m = \frac{\mathbf{S}_{A,m} - \mathbf{S}_{B,m}}{2S}, \quad \mathbf{l}_m = \frac{\mathbf{S}_{A,m} + \mathbf{S}_{B,m}}{2S}, \tag{5}$$

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where $S = |\mathbf{S}_i|$ is the (constant) magnitude of the localized spins. We assume the spatial variations are slow for $\mathbf{n}_m$ and $\mathbf{l}_m$ and adopt a continuum description $\mathbf{n}_m \to \mathbf{n}(\mathbf{r})$ and $\mathbf{l}_m \to \mathbf{l}(\mathbf{r})$. We also exploit the exchange approximation, $|\mathbf{l}| \ll 1$ and neglect higher order terms in $\mathbf{l}$ [19]. It then follows from $|\mathbf{l} \pm \mathbf{n}| = 1$ that $|\mathbf{n}| = 1$ and $\mathbf{n} \cdot \mathbf{l} = 0$.

Since the magnetization $\mathbf{l}$ in Eq. (5) contains a texture-induced unphysical component [20], it is convenient to work with the physical magnetization [21],

$$\tilde{\mathbf{l}} \equiv \mathbf{l} + \frac{a}{2}(\partial_x \mathbf{n}), \tag{6}$$

where a is the lattice constant, and the x axis is chosen along the bond connecting two sites in the unit cell. This preserves the constraints, $\tilde{\mathbf{l}} \cdot \mathbf{n} = 0$ and $|\mathbf{n}| = 1$ within the exchange approximation and simplifies the formalism. In terms of $\tilde{\mathbf{l}}$ and $\mathbf{n}$, the Lagrangian density is given by [21]

$$\mathcal{L}_S = s_n[\tilde{\mathbf{l}} \cdot (\mathbf{n} \times \dot{\mathbf{n}}) - \mathcal{H}_S - \mathcal{H}_{\rm sd}], \tag{7}$$

$$\mathcal{H}_S = \frac{zJS}{\hbar}\left[\tilde{\mathbf{l}}^2 + \frac{a^2}{4d}\sum_{i=1}^{d}(\partial_i \mathbf{n})^2 - \frac{K}{zJ}n_z^2\right], \tag{8}$$

$$\mathcal{H}_{\rm sd} = -\frac{M}{s_n}(\tilde{\mathbf{l}} \cdot \hat{\boldsymbol{\sigma}}_l + \mathbf{n} \cdot \hat{\boldsymbol{\sigma}}_n), \tag{9}$$

where $s_n = 2\hbar S/(2a^d)$ is the density of staggered angular momentum, z is the number of n.n. sites of a given site, $\hat{\boldsymbol{\sigma}}_l$ and $\hat{\boldsymbol{\sigma}}_n$ are the uniform and staggered components of the electron spin density, and $M = J_{\rm sd}S$. The equations of motion are obtained as

$$\begin{aligned}\dot{\mathbf{n}} &= \mathbf{H}_l \times \mathbf{n} + \mathbf{t}_n,\\ \dot{\tilde{\mathbf{l}}} &= \mathbf{H}_n \times \mathbf{n} + \mathbf{H}_l \times \tilde{\mathbf{l}} + \mathbf{t}_l,\end{aligned} \tag{10}$$

where $\mathbf{H}_n = \partial\mathcal{H}_S/\partial\mathbf{n}$ and $\mathbf{H}_l = \partial\mathcal{H}_S/\partial\tilde{\mathbf{l}}$ are the effective fields coming from the spin part ($\mathcal{H}_S$), and

$$\mathbf{t}_n = \frac{M}{s_n}\mathbf{n} \times \langle\hat{\boldsymbol{\sigma}}_l\rangle, \tag{11}$$

$$\mathbf{t}_l = \frac{M}{s_n}(\mathbf{n} \times \langle\hat{\boldsymbol{\sigma}}_n\rangle + \tilde{\mathbf{l}} \times \langle\hat{\boldsymbol{\sigma}}_l\rangle), \tag{12}$$

are the spin torques from electrons ($\mathcal{H}_{\rm sd}$). We calculate $\langle\hat{\boldsymbol{\sigma}}_l\rangle$ and $\langle\hat{\boldsymbol{\sigma}}_n\rangle$ in response to an applied electric field $\mathbf{E}$ or to the time-dependent $\mathbf{n}$ and $\tilde{\mathbf{l}}$ using the Kubo formula [22,23].

To be explicit, we consider tight-binding electrons on a two-dimensional square lattice described by

$$H_{\rm el} = -t\sum_{\langle i,j\rangle}(c_i^\dagger c_j + \text{H.c.}) + V_{\rm imp}, \tag{13}$$

where the first term expresses n.n. hopping, and

$$V_{\rm imp} = u_{\rm i}\sum_l c_l^\dagger c_l + u_{\rm s}\sum_{l'}\mathbf{S}_{l'}^{\rm imp}\cdot c_{l'}^\dagger\boldsymbol{\sigma}\, c_{l'}, \tag{14}$$

defines coupling to nonmagnetic and magnetic impurities. Combined with $H_{\rm sd}$, the hopping term gives upper and lower (spin-degenerate) bands $\pm E_{\mathbf{k}} = \pm\sqrt{\varepsilon_{\mathbf{k}}^2 + M^2}$ in a uniform AF state, where $\varepsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y)$. We take a directional average of $\mathbf{S}_j^{\rm imp}$ with second moment $\overline{S_z^2}$ ($\overline{S_\perp^2}$) for the component parallel (perpendicular) to $\mathbf{n}$. In the Born approximation, they appear through $\gamma_{\rm n} = \pi n_{\rm i}u_{\rm i}^2\nu$, $\gamma_\perp = \pi n_{\rm s}u_{\rm s}^2\,\overline{S_\perp^2}\,\nu$, and $\gamma_z = \pi n_{\rm s}u_{\rm s}^2\,\overline{S_z^2}\,\nu$, where $n_{\rm i}$ and $n_{\rm s}$ are the respective impurity concentrations, and $\nu = \frac{1}{N}\sum_{\mathbf{k}}\delta(|\mu| - E_{\mathbf{k}})$ is the density of states per spin (N is the total number of sites) with the chemical potential μ measured from the AF gap center.

Vertex correction is necessary for a proper account of spin conservation or its weak violation. Here, it is evaluated in the ladder approximation:

$$\Pi_{\sigma\bar{\sigma}} = \frac{2}{\pi\nu\tau^2}\frac{\mu^2}{\mu^2 - M^2}\frac{1}{Dq^2 - i\omega + \tau_\varphi^{-1} + \tau_{\rm s}^{-1}}. \tag{15}$$

This describes diffusion, dephasing, and relaxation of transverse spin density, generalizing the result of Ref. [25] to include the effects of magnetic impurities. Here, $\tau^{-1} = 2[\gamma_+ + (M/\mu)^2\gamma_-]$, with $\gamma_\pm = \gamma_{\rm n} + \gamma_z \pm 2\gamma_\perp$, is the electron scattering rate, and

$$\frac{1}{\tau_\varphi} = \frac{4M^2}{\mu^2}\left[\frac{\mu^2 + M^2}{\mu^2 - M^2}\gamma_{\rm n} + 3\gamma_\perp + \frac{2(2\mu^2 + M^2)}{\mu^2 - M^2}\gamma_z\right], \tag{16}$$

$$\frac{1}{\tau_{\rm s}} = 4(\gamma_\perp + \gamma_z), \tag{17}$$

are, respectively, the spin-dephasing rate [24,25] and the (transverse) spin-relaxation rate. In Eq. (15), q^{-1} (ω^{-1}) is the typical length (time) scale of the AF spin texture (dynamics), and D is the diffusion constant. We assume $q\ell_\varphi \ll 1$ and $\omega\tau_\varphi \ll 1$, where $\ell_\varphi = \sqrt{D\tau_\varphi}$ is the spin-dephasing length, and let $q, \omega \to 0$ in the results. The constant terms in the denominator $\tau_\varphi^{-1} + \tau_{\rm s}^{-1}$ reflect spin nonconservation in the electron system. The spin dephasing (τ_φ^{-1}), characteristic of AFs and absent in FMs, is dominated by nonmagnetic impurities and vanishes at $M = 0$ [24], whereas $\tau_{\rm s}^{-1}$ comes solely from magnetic impurities and is essentially the same as that in FMs [22]. It is convenient to decompose the former as $\tau_\varphi^{-1} = \tau_{\varphi 0}^{-1} + \tau_{\varphi 1}^{-1}$, where $\tau_{\varphi 0}^{-1}$ ($\propto \gamma_{\rm n}$) is the contribution from nonmagneic impurities and $\tau_{\varphi 1}^{-1}$ is from magnetic impurities. The "dissipated" spin angular momentum via $\tau_{\varphi 0}^{-1}$ is actually transferred to the AF spin system.

We calculate electron spin density induced by an external electric field $\mathbf{E}$ in the presence of spin texture (for current-induced torques) or induced by time-dependent spins $\dot{\mathbf{n}}$ and $\dot{\tilde{\mathbf{l}}}$ (for damping torques). We assume weak spin relaxation, $\gamma_z, \gamma_\perp \ll \gamma_{\rm n}$ and retain terms of lowest nontrivial order. The calculations are straightforward along the lines of Refs. [22,23,25]; see [26] for details.

Results.—We obtained

$$\langle\hat{\boldsymbol{\sigma}}_n\rangle = -\frac{s_n}{M}[\beta_n(\mathbf{v}_n\cdot\nabla)\mathbf{n} + \alpha_n\dot{\mathbf{n}}], \tag{18}$$

$$\langle\hat{\boldsymbol{\sigma}}_l\rangle = \frac{s_n}{M}[\mathbf{n} \times (\mathbf{v}_n\cdot\nabla)\mathbf{n} - \alpha_l\dot{\tilde{\mathbf{l}}}], \tag{19}$$

which are consistent with previous studies [7–12] and lead to the torques

$$\mathbf{t}_n = -(\mathbf{v}_n\cdot\nabla)\mathbf{n} - \alpha_l\,\mathbf{n} \times \dot{\tilde{\mathbf{l}}}, \tag{20}$$

$$\mathbf{t}_l = -\beta_n\,\mathbf{n} \times (\mathbf{v}_n\cdot\nabla)\mathbf{n} - \alpha_n\,\mathbf{n} \times \dot{\mathbf{n}}. \tag{21}$$

We retained dominant contributions, which come from $\langle\hat{\boldsymbol{\sigma}}_l\rangle$ for $\mathbf{t}_n$, and $\langle\hat{\boldsymbol{\sigma}}_n\rangle$ for $\mathbf{t}_l$.

Current-induced torques.—The first terms in $\mathbf{t}_l$ and $\mathbf{t}_n$ are current-induced torques, which are proportional to the charge current density $\mathbf{j} = \sigma_{xx}\mathbf{E} = 2e^2 D\nu\mathbf{E}$, via

$$\mathbf{v}_n = -\frac{\hbar}{2es_n}\mathcal{P}_n\mathbf{j}. \tag{22}$$

The velocity $\mathbf{v}_n$ quantifies the STT, and we identify

$$\mathcal{P}_n = \frac{\mu M}{\mu^2 - M^2}, \tag{23}$$

to be the conversion factor from a charge current to STT. Note that $|\mathcal{P}_n|$ can be significantly larger than unity near the AF gap edge ($|\mu| \gtrsim |M|$). This contrasts to the case of FMs, in which the corresponding factor $|P|$ is less than or equal to unity. The current-induced torque in $\mathbf{t}_l$ is characterized by a dimensionless parameter:

$$\beta_n = \frac{2(\gamma_\perp + \gamma_z)}{M} = \frac{\hbar}{2M\tau_s}, \tag{24}$$

which originates from magnetic impurities, i.e., from spin relaxation, and is therefore a dissipative torque. The spin dephasing due to nonmagnetic impurities ($\tau_{\varphi 0}^{-1}$) is microscopically a reactive process and does not contribute to β_n, whereas that from $\tau_{\varphi 1}^{-1}$, combined with the self-energy terms, results in a contribution proportional to τ_s^{-1}. Along with the contribution originating from τ_s^{-1} in Eq. (15), it leads to Eq. (24). The obtained two current-induced torques are related via $\beta_n\mathbf{n}\times$, which is reminiscent of the relation between the reactive and dissipative torques in FMs; we call the former $[-(\mathbf{v}_n\cdot\nabla)\mathbf{n}]$ the STT in AFs and the latter $[-\beta_n\mathbf{n}\times(\mathbf{v}_n\cdot\nabla)\mathbf{n}]$ the β_n torque. The expression of β_n in terms of τ_s and $M = J_{sd}S$ is also shared by FM [15,22].

The above two current-induced torques change their signs across the AF gap [see Eq. (23)], reflecting the fact that electrons in the upper and lower AF bands have mutually opposite spin directions. This feature of the STT was suggested in Ref. [8]. Interestingly, the driving direction is opposite to the naïve expectation based on the two-FM picture of AFs. Namely, for $\mu < 0$, the spin polarization on the Fermi surface is positive (dominated by majority spin carriers), but the driving direction is opposite to the direction of electron flow. This is due to the intersublattice hopping in AFs, namely, the electron spins exert torques on oppositely pointing neighboring spins, so the sign of the torques is reversed from that of FMs [27]. The same is true for $\mu > 0$.

Damping torques.—The second terms in Eqs. (20) and (21) describe damping. The damping parameters are calculated as

$$\alpha_n = \left[\gamma_\perp + \gamma_z + \frac{M^2}{\mu^2}(\gamma_\perp - \gamma_z)\right]\frac{2\hbar\nu}{s_n}, \tag{25}$$

$$\alpha_l = \frac{(\mu^2 - M^2)(\mu^2 + M^2)}{\mu^2}\frac{\nu}{s_n}\tau. \tag{26}$$

While α_n arises from spin relaxation (magnetic impurities), α_l does not necessitate it. Rather, α_l is proportional to τ, like conductivity, hence can be very large in good metals. These features were pointed out in Refs. [29,30] based on first-principles calculations.

It is interesting to compare α_n with the Gilbert damping in FMs:

$$\alpha_F = \sum_\sigma (\gamma_{z,\sigma}\nu_{\bar\sigma} + \gamma_{\perp,\sigma}\nu_\sigma)\frac{\hbar}{s_0}, \tag{27}$$

obtained based on the same spin-relaxation model (magnetic impurities) [22]. Here, $\gamma_{\alpha,\sigma} = \pi n_s u_s^2 \overline{S_\alpha^2}\,\nu_\sigma$ ($\alpha = \perp, z$), ν_σ ($\sigma = \uparrow, \downarrow$) is the density of states of electrons with spin σ, and $s_0 = \hbar S/a^d$ is the angular-momentum density. We see that, in the limit of spin-degenerate bands ($\nu_\uparrow = \nu_\downarrow$) and isotropic magnetic impurities ($\gamma_\perp = \gamma_z$), the above expressions of α_n (for AFs) and α_F (for FMs) coincide. Therefore, in the current model of AFs, the ratio β_n/α_n is of order unity, similar to FMs [14].

Equations of AF spin dynamics.—With the obtained torques and $\mathcal{H}_S$ [Eq. (8)], the equations of motion are explicitly written as

$$\dot{\mathbf{n}} = \tilde{J}\tilde{\mathbf{l}}\times\mathbf{n} - (\mathbf{v}_n\cdot\nabla)\mathbf{n}, \tag{28}$$

$$\begin{aligned}\dot{\tilde{\mathbf{l}}} = &-(c^2\tilde{J}^{-1}\nabla^2\mathbf{n} + \tilde{K}n^z\hat{z})\times\mathbf{n}\\ &+[\alpha_n\dot{\mathbf{n}} + \beta_n(\mathbf{v}_n\cdot\nabla)\mathbf{n}]\times\mathbf{n}\\ &+\mathbf{n}[\tilde{\mathbf{l}}\cdot(\mathbf{v}_n\cdot\nabla)\mathbf{n}],\end{aligned} \tag{29}$$

with $c = (zJSa)/(\hbar\sqrt{d})$, $\tilde{J} = 2zJS/\hbar$, and $\tilde{K} = 2SK/\hbar$. Damping terms in the first equation are dropped, as they are higher order in $\tilde{\mathbf{l}}$. Solving Eq. (28) for $\tilde{\mathbf{l}}$ as $\tilde{\mathbf{l}} = \tilde{J}^{-1}\mathbf{n}\times[\dot{\mathbf{n}} + (\mathbf{v}_n\cdot\nabla)\mathbf{n}]$ and substituting it in Eq. (29), one can obtain a closed equation for $\mathbf{n}$:

$$\begin{aligned}\ddot{\mathbf{n}}\times\mathbf{n} = &(c^2\nabla^2\mathbf{n} + \tilde{J}\tilde{K}n_z\hat{z})\times\mathbf{n}\\ &-\tilde{J}[\alpha_n\dot{\mathbf{n}} + \beta_n(\mathbf{v}_n\cdot\nabla)\mathbf{n}]\times\mathbf{n}\\ &-[(\mathbf{v}_n\cdot\nabla)\dot{\mathbf{n}}]\times\mathbf{n}.\end{aligned} \tag{30}$$

This differs slightly from Ref. [8] due to the difference in H_{sd} (i.e., $\mathbf{l}$ vs $\tilde{\mathbf{l}}$) and leads to the magnon dispersion

$$\begin{aligned}\omega = &\sqrt{c^2\mathbf{q}^2 + \tilde{J}\tilde{K} + \frac{(\mathbf{v}_n\cdot\mathbf{q} - i\tilde{J}\alpha_n)^2}{4} + i\tilde{J}\beta_n\mathbf{v}_n\cdot\mathbf{q}}\\ &\pm\frac{\mathbf{v}_n\cdot\mathbf{q} - i\tilde{J}\alpha_n}{2},\end{aligned} \tag{31}$$

where damping enters only through α_n and β_n.

DW motion.—Here, we study the AF DW motion using collective coordinates. Since $\mathcal{L}_S$ [Eq. (7)] is written with $\mathbf{n}$ and $\tilde{\mathbf{l}}$, we consider collective coordinates for both $\mathbf{n}$ and $\tilde{\mathbf{l}}$ [21]. Assuming for $\mathbf{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ a DW form, $\cos\theta(x,t) = \pm\tanh[\frac{x - X(t)}{\lambda}]$ and $\phi(x,t) = \phi_0(t)$, where $\lambda = a\sqrt{zJ/4Kd}$ is the DW width, we treat the DW position $X(t)$ and the angle $\phi_0(t)$ as dynamical variables [31]. As for $\tilde{\mathbf{l}}$, we expand it as [21]

$$\tilde{\mathbf{l}}(x,t) = [\,l_\theta(t)\,\mathbf{e}_\theta + l_\phi(t)\,\mathbf{e}_\phi\,]\,\varphi_0(x) + \cdots, \tag{32}$$

where $\mathbf{e}_\theta \equiv \partial_\theta\mathbf{n}$ and $\mathbf{e}_\phi \equiv \mathbf{n}\times\mathbf{e}_\theta$ are orthonormal vectors normal to $\mathbf{n}$. The function $\varphi_0(x) = [\cosh\frac{x - X}{\lambda}]^{-1}$ reflects the spatial profile of $\mathbf{n}\times\dot{\mathbf{n}}$ and naturally extracts l_θ and l_ϕ in the first term of $\mathcal{L}_S$ [Eq. (7)]. The obtained Lagrangian $L_{DW} = 2s_n(\pm\dot{X}l_\phi - \lambda\dot{\phi}_0 l_\theta) - H_S$ shows that l_ϕ and l_θ are canonical conjugate to X and ϕ_0, respectively. The equations of motion

are given by

$$\pm\lambda\,\dot{l}_\phi = \beta_n v_n - \alpha_n \dot{X}, \tag{33}$$

$$\pm\dot{X} = \pm v_n + v_J\, l_\phi + \alpha_l \lambda \dot{l}_\phi, \tag{34}$$

$$\dot{l}_\theta = \alpha_n\, \dot{\phi}_0, \tag{35}$$

$$\lambda\, \dot{\phi}_0 = -v_J l_\theta - \alpha_l \lambda \dot{l}_\theta, \tag{36}$$

where $v_J = 4JS\lambda/\hbar$, and $\pm$ is the topological charge of the AF DW. The first two equations describe the translational motion, and the remaining two describe the rotational motion of the DW plane (defined by the Néel vector). Unlike in FMs [32], these two motions are decoupled in AFs. The term $\pm v_n$ in Eq. (34) describes the spin-transfer effect, and $\beta_n v_n$ in Eq. (33) describes the momentum-transfer effect (a force on the DW). The terms with α_l are negligible in effect but retained here for the sake of comparison with FMs (see below).

Let us overview the translational motion of AF DWs under a stationary v_n [9]. When $\alpha_n = \beta_n = 0$, l_ϕ is a constant of motion. With an initial condition $l_\phi = 0$ (no canting), the DW moves at a constant velocity $\dot{X} = v_n$ by the spin-transfer effect [8]. If the DW is initially canted $l_\phi = l_\phi^0$, the constant velocity is modified to $\dot{X} = v_n \pm v_J\, l_\phi^0$. For finite α_n, l_ϕ is no longer conserved and approaches a terminal value $l_\phi \to \mp(1 - \beta_n/\alpha_n)(v_n/v_J)$. Then from Eq. (34), the DW velocity approaches

$$\dot{X} \;\to\; v_n - \left(1 - \frac{\beta_n}{\alpha_n}\right) v_n = \frac{\beta_n}{\alpha_n}\, v_n, \tag{37}$$

which is solely determined by the β_n torque. If $\beta_n = 0$, the spin-transfer effect is completely nullified by the canting $l_\phi = \pm v_n/v_J$, and the aforementioned steady movement eventually ceases [9]. This is quite similar to the intrinsic pinning in FMs. For finite β_n, canting l_ϕ is reduced, and the cancellation of the spin-transfer effect is incomplete. Finally, the case $\beta_n = \alpha_n$ is special in that there is no canting, and the spin-transfer effect is undisturbed.

It is instructive to make a more detailed comparison with FMs. In FMs, the current-driven DW motion is described by

$$\pm\lambda\,\dot{\phi}_0 = \beta v_s - \alpha \dot{X}, \tag{38}$$

$$\pm\dot{X} = \pm v_s + v_K \sin 2\phi_0 + \alpha\lambda\dot{\phi}_0, \tag{39}$$

where, now, X and ϕ_0 are coupled. (Here, ϕ_0 is defined by the uniform magnetization, and $\pm$ is the topological charge of the FM DW.) A close similarity to Eqs. (33) and (34) is evident, and here, ϕ_0 plays the role of l_ϕ. The effect of current appears in $v_s = -(\hbar/2es_0)Pj$, where P is the current polarization factor, and the velocity $v_K = K_\perp S\lambda/2\hbar$ is defined with the hard-axis anisotropy constant $K_\perp$. At low current, $v_s < v_K$, and with $\beta = 0$, the DW plane tilts by $\phi_0 = \frac{1}{2}\sin^{-1}(v_s/v_K)$ and the DW ceases to move $\dot{X} = 0$. This is the intrinsic pinning in FMs [32,33]. If v_s exceeds v_K, v_K can not nullify the spin-transfer effect v_s, and the DW is released from intrinsic pinning. The corresponding term in Eq. (34) has the linearized form $v_J\, l_\phi$, which is justified since v_J of AFs is much larger than v_K of FMs (by 2–3 orders of magnitude), and the intrinsic pinning is robust in AFs. It is interesting to note the contrasting origins of intrinsic pinning; in FMs, it is the explicit breaking of spin rotation symmetry $K_\perp$, whereas in AFs, it is the AF order itself, i.e., spontaneous breaking. Finally, the case $\alpha = \beta$ provides a special solution $\phi_0 = 0$ and $\dot{X} = v_s$, like the case $\alpha_n = \beta_n$ for AFs.

Recently, current-assisted field-driven DW motion was experimentally studied in a *ferrimagnetic* GdFeCo near its angular-momentum compensation temperature [17]. The authors analyzed the data by the Landau-Lifshitz-Gilbert equation for the *uniform moment* **m** (parallel to **n**) and obtained a very large, negative value of $\beta/\alpha \simeq -100$. They assumed $\sim\beta P\mathbf{j}$ for the β-torque coefficient (that acts on **m**), with a small factor P ($\simeq$0.1) included. If, however, the main driving is the β_n torque that acts on the *Néel vector* **n**, as studied in this letter, we would conclude $\mathcal{P}_n\beta_n/\alpha_n \simeq -10$ (see Ref. [26]). While $\beta_n/\alpha_n \simeq 1$ as in FMs (for positive J_{sd} [34,35]), $|\mathcal{P}_n|$ can be significantly larger than unity near the AF gap edge. Therefore, the large value of $|\mathcal{P}_n| \sim 10$ may lie within the scope of the present results. The negative sign can be explained likewise from $\mathcal{P}_n$ with a negative μ, which reflects intersublattice hopping in AFs. Such "AF transport" picture in GdFeCo is supported by a recent magnetoresistance measurement [36].

In conclusion, we have presented a microscopic model calculation of current-induced torques and damping torques in AF metals, paying attention to the effects of spin relaxation (and spin dephasing). A formulation in terms of the Néel vector and physical magnetization is given to study the AF spin dynamics in metallic AFs with s-d exchange interaction. The current-induced torques are found to be opposite in direction to those of FMs, reflecting the AF transport character, and the current-to-STT conversion factor can be significantly larger than that in FMs. These results seem to be relevant to the recent experiment on GdFeCo.

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[27] We have confirmed this by introducing the next n.n. hopping t'. As t' starts dominating, the sign of the STT changes from negative to positive in the $\mu < 0$ region, and vice versa for $\mu > 0$, in conformity with the change of transport character from AF like to FM like [28]. Note that the torques in FMs also change their signs in a tight-binding model as μ is increased from the band bottom to the top since the majority-minority spins are switched and the sign of P is reversed.

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