Letter

Dynamical torque from Shiba states in *s*-wave superconductors

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Magnetic impurities inserted in an *s*-wave superconductor give rise to spin-polarized in-gap states called Shiba states. We study the back-action of these induced states on the dynamics of the classical moments. We show that the Shiba state pertains to both reactive and dissipative torques acting on the precessing classical spin that can be detected through ferromagnetic resonance measurements. Moreover, we highlight the influence of the bulk states as well as the effect of the finite linewidth of the Shiba state on the magnetization dynamics. Finally, we demonstrate that the torques are a direct measure of the even and odd frequency triplet pairings generated by the dynamics of the magnetic impurity. Our approach offers noninvasive alternative to the scanning tunneling microscopy techniques used to probe the Shiba states.

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Introduction. The quest for Majorana fermions is driven by their promise as a building block for a fault tolerant topological quantum computer [1]. Magnetic impurities in superconductors have been a prime area of research for realizing topological superconductors that can host such exotic quasiparticles [2–22]. The elementary unit for such a chain is a single magnetic impurity inserted in an *s*-wave superconductor, which can bind spin polarized subgap energy electrons in the so-called Shiba states [23].

The Shiba impurity states have been well studied theoretically [23–25] and experimentally revealed by the scanning tunneling microscopy (STM) technique. Unfortunately, such systems are hard to tune once in the superconductor, which drastically reduces the ability to explore various topological regimes. Driving the impurities, however, can result in the ability to achieve such a feat dynamically where the precessing frequency acts as the knob to control the topological transition [21]. Spin pumping and spin transfer torques in ferromagnets are just a few phenomena that pertain to magnetization dynamics [26,27] and which are staple dynamical methods for manipulating, transporting, and detecting spins in magnetic systems. Recently, it has been shown theoretically that such dynamics result in controllable shifts in the Shiba energies that show up as features in the differential conductance in transport measurements [28]. Thus, dynamical magnetic impurities are a promising platform for engineering topological superconductor [21]. In this paper, we take a step forward and investigate a single time-dependent magnetic impurity of size S in an *s*-wave superconductor (SC), in particular the back-action effects of the "stirred" electrons in the SC on the spin dynamics. In the adiabatic limit, we find a universal reactive torque pertaining to the Shiba state that is geometrical in nature:

$$\boldsymbol{\tau}_R(t) = (n_S - 1/2)\, F_s[\boldsymbol{n}(t)]\, \dot{\boldsymbol{n}}(t)\,, \tag{1}$$

where n_S is the occupation of the Shiba state, $F_s[\boldsymbol{n}(t)] = S\boldsymbol{B}\cdot\boldsymbol{n(t)} = 1/2$ is the radial Berry curvature of the Shiba state, $\boldsymbol{B}$ is the magnetic field, and $\boldsymbol{n}(t) = \boldsymbol{S}(t)/S$ is the precessing classical spin direction. Changing n_S is equivalent to effectively changing the classical spin length as $S \to S - (n_S - 1/2)$. Berry-phase induced torques and their effects on classical spins in normal metals have been investigated previously in several important works [29–33]. However, the origin of the torque in all those instances is different from that described here pertaining to precessing spins in superconductors. In

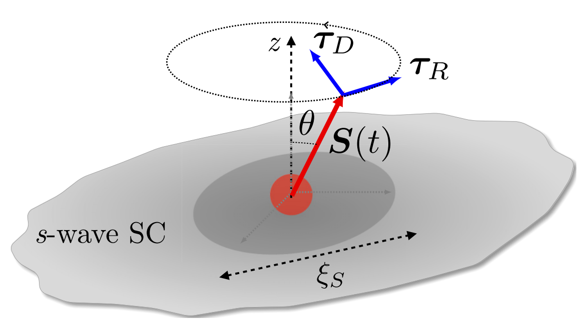

FIG. 1. Sketch of the combined system. A classical precessing spin (red) at angle θ with respect to the z axis is coupled via exchange interaction to an *s*-wave superconductor (grey). A localized Shiba state of size ξ_S is formed underneath which is spin polarized and can act back on the classical spin precession. Both reactive ($\boldsymbol{\tau}_R$) and dissipative ($\boldsymbol{\tau}_D$) torques are present which affect the dynamics of the classical spin.

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Fig. 1 we show a sketch of the combined dynamical spin-SC system, and highlight the reactive ($\boldsymbol{\tau}_R$) and dissipative ($\boldsymbol{\tau}_D$) torques that act back on the classical spin. The rest of the paper is dedicated to derive Eq. (1) microscopically, to take into account a finite Shiba linewidth, which in turn leads to a dissipative torque ($\boldsymbol{\tau}_D$), as well as for the effects of the bulk (nonlocalized states in the continuum) states in the superconductor. Further, we show how these torques are a measure of various even- and odd-frequency triplet SC pairings induced by the dynamics.

Model Hamiltonian. The model Hamiltonian for the dynamical system in Fig. 1, which describes both two-dimensional (2D) and three dimensional (3D) setups, can be written as $H_{\rm tot}(t) = (1/2)\int d\boldsymbol{r}\Psi^\dagger(\boldsymbol{r})H_{\rm BdG}(t)\Psi(\boldsymbol{r})$ where the Bogolioubov–de Gennes Hamiltonian reads

$$\begin{aligned} H_{\rm BdG}(t) &= H_0 + V_i(t)\,, \\ H_0 &= \epsilon_p\tau_z + \Delta\tau_x\,, \\ V_i(t) &= -J\boldsymbol{S}(t)\cdot\boldsymbol{\sigma}\,\delta(\boldsymbol{r})\,, \end{aligned} \tag{2}$$

with H_0 and $V_i(t)$ being the bare Bogolioubov–de Gennes Hamiltonian for the s-wave superconductor and its coupling to the classical magnetic impurity, respectively, written in the Nambu basis $\Psi(\boldsymbol{r}) = [c_\uparrow(\boldsymbol{r}), c_\downarrow(\boldsymbol{r}), c_\downarrow^\dagger(\boldsymbol{r}), -c_\uparrow^\dagger(\boldsymbol{r})]^T$. Also, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ and $\boldsymbol{\tau} = (\tau_x, \tau_y, \tau_z)$ are the Pauli spin matrices in the spin and particle hole subspace, respectively. The spectrum of free electrons is given as $\epsilon_p = p^2/2m - \mu$, where m, p, and μ are the electron mass, momentum, and chemical potential, respectively. Δ is the superconducting order parameter and J defines the coupling between the classical spin $\boldsymbol{S}(t) = S(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and the electrons in the superconductor. In the following, we assume circular precession, i.e., $\phi = \Omega t$, with Ω and θ representing the precession frequency and angle the classical spin makes with z axis, respectively. The magnetic impurity generates also a scalar potential but, for simplicity, we neglect such a contribution in this work. The dynamics of the classical spin is described by the Landau-Lifshitz-Gilbert (LLG) equation:

$$\dot{\boldsymbol{S}}(t) = -\boldsymbol{S}(t)\times(\gamma\boldsymbol{B}(t) - \langle\boldsymbol{\sigma}(t)\rangle + \beta\dot{\boldsymbol{S}}(t))\,, \tag{3}$$

where γ and β are the gyromagnetic coupling and the Gilbert damping, respectively, and $\boldsymbol{\tau}_{\rm R}(t) = J\boldsymbol{S}(t)\times\langle\boldsymbol{\sigma}(t)\rangle$ is the total torque acting on the classical spin by the superconductor, with $\langle\boldsymbol{\sigma}(t)\rangle$ being the spin expectation value in the superconductor at the position of the impurity in the steady state. This term can change both the resonance frequency and the Gilbert damping. Note that in typical setups $\boldsymbol{B}(t) = B_0\boldsymbol{z} + \boldsymbol{B}_\perp(t)$, with $|\boldsymbol{B}_\perp(t)| \ll B_0$ (small angle precession) and $\gamma B_0 \equiv \Omega_0$ defines the resonance frequency in the absence of the Shiba states.

Rotating frame description. It is convenient to analyze the dynamics by using the rotating wave description approach. Due to the circular precession of the magnetic impurity, the symmetry of the system allows us to perform a unitary transformation $U(t)$ that renders the problem fully static. Hence, we can write $\Psi(\boldsymbol{r}, t) = U(t)\Phi(\boldsymbol{r})e^{-iEt}$ such that the time independent Schrodinger equation can be written as $H_{\rm rot}\Phi(\boldsymbol{r}) = E\Phi(\boldsymbol{r})$ with $H_{\rm rot} = U^\dagger(t)H_{\rm tot}(t)U(t) + i\dot{U}^\dagger(t)U(t)$ or

$$H_{\rm rot} = H_{\rm tot}(0) - b\sigma_z\,, \tag{4}$$

where $U(t) = \exp(-ib\sigma_z t)$ and $b = \Omega/2$ is the fictitious magnetic field perpendicular to the plane of the superconductor.

In the absence of precession, a magnetic impurity in an s-wave superconductor gives rise to a Shiba state within the superconducting gap at energy, $E_S = \Delta(1-\alpha^2)/(1+\alpha^2)$ and $\alpha = \pi\nu_0 JS$ is the dimensionless impurity strength in terms of the normal phase density of states ν_0. For finite precession of the impurity, the coherence peaks split due to the fictitious magnetic field, therefore it becomes easier to break the Cooper pair and thus lower the energy of the excitation.

When the impurity spin precesses, a general solution to the eigenvalue problem has a complicated form [see the Supplemental Material (SM) [34–40] for details], but in the deep Shiba limit, $\alpha \approx 1$ and in the adiabatic regime, $b/\Delta \ll 1$, the effective Shiba energy acquires the simple expression $E_S' \approx E_S - b\cos\theta$ in leading order in b/Δ. The corresponding wave function is $|\Phi_S\rangle \approx |\Phi_S^0\rangle + (X/2)\sin\theta|\Phi_S^1\rangle$, where $|\Phi_S^0\rangle = [\cos(\theta/2), \sin(\theta/2), \cos(\theta/2), \sin(\theta/2)]^T$ and $|\Phi_S^1\rangle = [\sin(\theta/2), -\cos(\theta/2), \sin(\theta/2), -\cos(\theta/2)]^T$ scaled up to a normalization factor $1/\sqrt{N}$ where $N = (1+\alpha^2)^2/(2\pi\nu_0\alpha\Delta)$ [34], and $X = \frac{b}{\Delta}\frac{(1+\alpha^2)^2}{4\alpha^2} \ll 1$. The dynamics induces a coupling between the static Shiba state to its spin partner in the continuum. Considering either the electron or hole component, the result can be interpreted in the context of an adiabatically driven spin 1/2 particle in an effective magnetic field $B_{\rm eff} = b/X$. The average spin at the site of the impurity pertaining to the Shiba state can be calculated from the above renormalized wave functions $|\Phi_S\rangle$ in leading order in b/Δ, and accounting for their occupation [34]:

$$\langle\boldsymbol{\sigma}_S(t)\rangle \approx \frac{2n_S - 1}{N}[(1 - X\cos\theta)\boldsymbol{n}(t) + X\boldsymbol{z}]\,, \tag{5}$$

where $n_S \equiv n_S(b,\theta)$ is the occupation number for the Shiba state which itself can depend on the driving. The misalignment of the Shiba state spin and the classical moment is due to the competition between the local exchange field and the fictitious global magnetic field acting along the $\boldsymbol{z}$ direction. Using Eq. (5), the universal torque can be evaluated as presented in Eq. (1), with $F_s[\boldsymbol{n}(t)] = 1/2$ corresponding to the Berry curvature of the effective spin 1/2. However, this contribution is due to the purely isolated Shiba state in the absence of any relaxation channels. To account for the full out-of-equilibrium properties, including the bulk states and to account for the various dissipation effects, in the following we analyze the dynamical problem by employing the Green's function (GF) technique.

GF approach. The bare retarded GF of the superconductor in the rotating frame is

$$\tilde{G}_0(\omega) = -\frac{\pi\nu_0}{2}\sum_{\sigma=\pm1}\frac{\omega + \sigma b + \Delta\tau_x}{\sqrt{\Delta^2 - (\omega+\sigma b)^2}}(1 + \sigma\sigma_z)\,, \tag{6}$$

where $\omega = \omega + i0^+$ and $\sigma = +1(-1)$ for $\uparrow$ ($\downarrow$). The coupling to the impurity spin in the rotating frame can be accounted for via the Dyson's equation that relates the full GF to the bare one, or $[\tilde{G}^R(\omega)]^{-1} = \tilde{G}_0^{-1}(\omega) - V_i(0) + i\Gamma$. Here, Γ is a phenomenological Dynes broadening added to the self-energy that accounts for the relaxation processes in the superconductor. That, in turn, allows us to write

$$\tilde{G}^R(\omega) = \frac{\pi \nu_0}{D(\omega, \alpha, b, \theta)} M(\omega, \alpha, b, \theta), \tag{7}$$

where

$M(\omega, \alpha, b, \theta) = M_0 + \boldsymbol{M}_1 \cdot \boldsymbol{\sigma} + \tau_x \otimes (M_2 + \boldsymbol{M}_3 \cdot \boldsymbol{\sigma})$ is a 4×4 matrix and $D(\omega, \alpha, b, \theta) = \omega_1\omega_2(1 + \alpha^4 - 2\alpha^2 \cos^2\theta) - 2\alpha \cos\theta(1 - \alpha^2)[(b + \omega)\omega_2 + (b - \omega)\omega_1]$, where $\omega_1 = \sqrt{\Delta^2 - (\omega + b)^2}$ and $\omega_2 = \sqrt{\Delta^2 - (\omega - b)^2}$. The full Shiba state energy is found from the solutions of $D(\omega, \alpha, b, \theta) = 0$ [34]. In the absence of precession, the lesser GF, $\tilde{G}^<(\omega) = n_F(\omega)[\tilde{G}^A(\omega) - \tilde{G}^R(\omega)]$ where $n_F(\omega) = [\exp(\beta\omega) + 1]^{-1}$ is the Fermi distribution function with $\beta = 1/k_B T$, k_B being the Boltzmann constant and T is the temperature. The advanced GF instead satisfies $\tilde{G}^A(\omega) = [\tilde{G}^R(\omega)]^\dagger$. In this work, we are not considering the microscopic mechanisms behind Γ, but rather focus on its manifestations on the ferromagnetic resonance (FMR) signal. For finite precession and in the rotating frame the fictitious magnetic field b leads to a spin dependent shift in the Fermi distribution function and is no longer an identity operator:

$$\tilde{n}_F \equiv \tilde{n}_F(\omega, b) = f_0(\omega, b) + f_s(\omega, b)\sigma_z, \tag{8}$$

where $f_{0,s}(\omega, b) = [n_F(\omega + b) \pm n_F(\omega - b)]/2$. Note that $\tilde{n}_F(\omega, b)$ does not commute with $\tilde{G}^R(\omega)$, and the lesser GF is found as Ref. [35] (also Ref. [34]):

$$\tilde{G}^<(\omega) = \tilde{n}_F \tilde{G}^A_S - \tilde{G}^R_S \tilde{n}_F + \tilde{G}^R_S (V_i \tilde{n}_F - \tilde{n}_F V_i)\tilde{G}^A_S. \tag{9}$$

The instantaneous spin expectation value at the position of the impurity in the rotating frame is

$$\langle \tilde{\boldsymbol{\sigma}}(\mathbf{0}) \rangle = \frac{-i}{2\pi} \int_{-\infty}^{\infty} d\omega \, \mathrm{Tr}\left[\left(\boldsymbol{\sigma} \otimes \frac{1 + \tau_z}{2}\right) \tilde{G}^<(\omega)\right], \tag{10}$$

which contains both the in-gap (Shiba) and the bulk (continuum of states) contributions, respectively. Here, the $(1 + \tau_z)/2$ term is introduced in order to account for only the electron components. For a static impurity spin, the imaginary part of the integrand in Eq. (10) is a Lorentzian function located at the Shiba energies and the expectation value of the spins is nonzero for a finite Shiba linewidth. The contribution to $\langle \tilde{\boldsymbol{\sigma}}(\mathbf{0}) \rangle$ is only due to the Shiba states and it points along the classical spin direction resulting in zero net torque in the absence of precession, as expected. For the dynamic case, the spin expectation value has contribution from both the Shiba and the bulk states, as discussed below.

Effect of the Shiba states. The in-gap Shiba contribution stems from the range of integration $\omega \in [-\Delta + b, \Delta - b]$ in Eq. (10). While later on we will evaluate this term fully numerically, let us next consider the deep Shiba limit [21,36,41] and $b/\Delta \ll 1$ so that these states are well separated from the bulk. As mentioned before, the average spin of the Shiba state for precessing case is no longer along the classical spin direction $\boldsymbol{n}$ [34] and hence, a finite torque acts on the classical spin due to this deviation which in turn will affect its dynamics. After lengthy but straightforward calculations, we find compact analytical expressions for the spin expectations values that pertain to the reactive and dissipative torques,

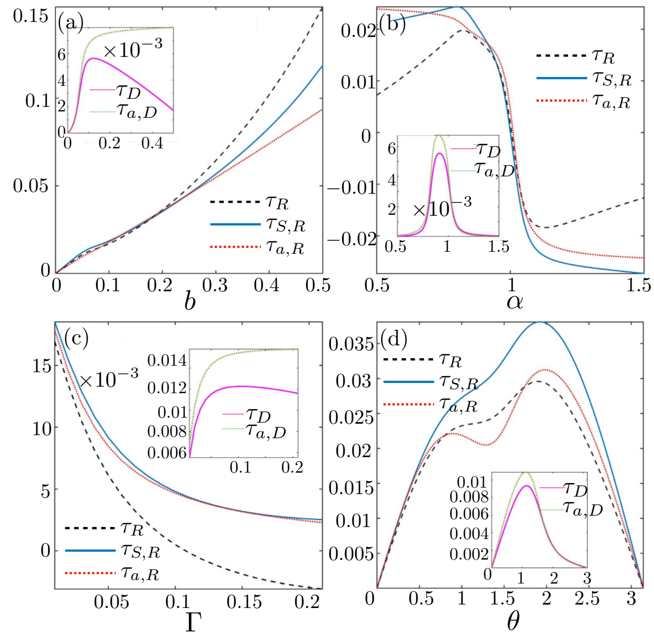

FIG. 2. Variation of the reactive and dissipative torques. Main: full Shiba $\tau_{S,R}$) (blue solid line), approximate Shiba $\tau_{a,R}$ (red dotted line) and total τ_R (black dashed line) reactive torques, respectively, as a function of b, α, Γ, and θ [shown in (a), (b), (c), and (d), respectively]. We assumed $\alpha = 0.9$ [(a), (c), and (d)], $\theta = \pi/6$ [(a), (b), and (c)], $\Gamma = 0.01$ [(a), (b), and (d)], and $b = 0.1$ [(b), (c), and (d)]. The insets in the plots show the dependence of the full τ_D (full magenta) and approximate $\tau_{a,D}$ (dotted green) dissipative torques, respectively, on the corresponding parameters, with the other values being the same as for the main plots.

respectively, and at $T \to 0$:

$$\langle \tilde{\sigma}_{a,R} \rangle \approx -b(n_+ + n_- \cos\theta); \quad \langle \tilde{\sigma}_{a,D} \rangle \approx \Gamma_S n_-, \tag{11}$$

$$n_\pm = \frac{1}{2\pi} \sum_{s=\pm 1} s^p \arctan\left(\frac{E'_S + sb}{\Gamma_S}\right), \tag{12}$$

where $\Gamma_S = (2/N)\Gamma$ is the effective Shiba linewidth and $p = 0(1)$ for $n_{+(-)}$. Equations (11) supplemented with the plots in Fig. 2 are the main findings of this work. In the rotating frame, we can write the torque stemming from the Shiba state only as

$$\boldsymbol{\tau}_S = \langle \tilde{\sigma}_{a,R} \rangle \boldsymbol{n} \times \boldsymbol{z} + \langle \tilde{\sigma}_{a,D} \rangle \boldsymbol{n} \times (\boldsymbol{z} \times \boldsymbol{n}), \tag{13}$$

where the first (second) term corresponds to the reactive (dissipative) torque $\boldsymbol{\tau}_{S,R}$ ($\boldsymbol{\tau}_{S,D}$). We see that both the reactive and dissipative torques vanish for either $b = 0$ or $\theta = 0, \pi$, as expected. Moreover, the reactive torque can be casted in the form shown in Eq. (5), by identifying $n_S \equiv 1/2 - (n_+ + n_- \cos\theta)$ as the occupation number of the Shiba state (see the SM).

A few comments are in order. In order to extract the dissipative torque, we accounted for the linewidth Γ not only in the denominator (that reflects the Shiba state lifetime through Γ_S), but also in the numerator M_0 and $\boldsymbol{M}$. Our expansion goes beyond the effective Shiba approximations discussed in, for example, Ref. [36], where they neglect contributions of Γ in the numerator, and which would naively lead to a vanishing dissipative torque. Our theory shows, we believe, one of the first instances where such an effective approach is not sufficient for the case of Shiba states in superconductors.

Bulk effects. The above torques account only for the in-gap contributions stemming from the Shiba poles, while the full average spin value at the impurity can be evaluated only numerically. Interestingly, in the general case, the integrand in Eq. (10) is nonzero even when $\Gamma = 0$ for $|\omega| > \Delta - b$. Hence, the precession of the impurity can result in a finite contribution of the bulk states to the dynamical torques, absent in the static case. The total spin expectation value can be written as $\langle\tilde{\boldsymbol{\sigma}}\rangle = \langle\tilde{\boldsymbol{\sigma}}_B\rangle + \langle\tilde{\boldsymbol{\sigma}}_S\rangle$ (and similarly for the torques), denoting a sum of the bulk and Shiba contributions, respectively. In Fig. 2 we show the total reactive torque (τ_R) and the Shiba parts of the reactive torque ($\tau_{S,R}$) as a function of various parameters, such as b, α, Γ, and θ, calculated numerically. The difference between τ_R and $\tau_{S,R}$ accounts for the bulk contribution to the reactive torque. The plot of the dissipative torque (τ_D), which originates only from the Shiba state, is shown in the insets of Fig. 2. To compare the full numerics with the analytical results describing the Shiba contribution, we also plot the reactive ($\tau_{a,R}$) and dissipative torques ($\tau_{a,D}$) calculated from Eq. (11) and shown by the dotted line in Fig. 2. We see a very good agreement between the analytic expressions and the Shiba contribution obtained from the numerics for small b [Fig. 2(a)] and in the deep Shiba limit [Fig. 2(b)]. Furthermore, in this limit, τ_R and $\tau_{S,R}$ coincide, proving that the Shiba state is responsible for the reactive torque, while the bulk contribution tends to zero. Moving away from the deep Shiba limit, there is a finite bulk contribution to the reactive torque [see Fig. 2(b)]. Figures 2(c) and 2(d) show the behavior of the dynamical torques as a function of the Dynes broadening factor Γ and θ, respectively. By increasing Γ the difference between τ_R from $\tau_{S,R}$ increases also. Note that since $\boldsymbol{\tau}_D \propto \boldsymbol{n} \times \dot{\boldsymbol{n}}$ is always positive, as per the LLG equation given in Eq. (3) it corresponds to a damping like/dissipation torque. From Fig. 2(b), τ_D peaks at α corresponding to $E_S' = 0$ for the given parameters as can be seen from the analytic expression Eq. (11). Experimentally, that would result in a strong enhancement of the FMR linewidth.

Unconventional pairing. The occurrence of odd frequency superconductivity has been recently discussed for a static impurity [42,43]. Here we show that spin precession leads to generation of unconventional pairing that is directly related to the experimentally accessible dynamical torques. Similar to Ref. [43], we consider the adiabatic deep Shiba limit and expansion of the numerator and denominator of $\tilde{G}^R_\pm(\omega)$ in zeroth and first order in Γ, respectively, we can write

$$\tilde{G}^R_\pm(\omega) \approx \frac{\pi\nu_0(\tau_0 \pm \tau_x)}{\omega \mp E_S' + i\Gamma_S}(M_0 \pm \boldsymbol{M}\cdot\boldsymbol{\sigma}), \tag{14}$$

where M_0 [$\boldsymbol{M} = (M_x, 0, M_z)$] is a scalar (vector) that depends on the precession frequency b, angle θ and impurity strength α. Equation (14) is the second main result of our work. In this limit, the anomalous part of the retarded GF in the rotating frame, $\tilde{F}(\omega)$, corresponds to the term $\propto \tau_x$ in Eq. (14) (see the SM). The reactive torque for a given frequency ω originating from the Shiba state (which when integrated gives the total torque) can then be written in terms of the anomalous pairing as follows:

$$\tau_{S,R}(\omega) = \mathrm{Im}\big[\tilde{F}_o(\omega) f_s(\omega)\sin\theta - \tilde{F}_e(\omega) f_0^o(\omega)\big], \tag{15}$$

where the even and odd frequency triplet pairing components above are defined as $F_o(\omega) = \sum_\sigma F^o_{\sigma\bar{\sigma}}(\omega)$, $F_e(\omega) = \sum_\sigma F^e_{\sigma\sigma}(\omega)$ with $\tilde{F}^{e/o}_{\sigma\sigma'}(\omega) = \tilde{F}_{\sigma\sigma'}(\omega) \pm \tilde{F}^*_{\sigma\sigma'}(-\omega)$. Morerover, $f_0^o(\omega) = f_0(\omega) - 1/2$ while $f_s(\omega)$ is even under $\omega \to -\omega$. As seen from Eq. (15), the reactive torque is generated by two types of pairing: (*i*) a b dependent induced triplet pairing and (*ii*) an odd frequency pairing term independent of b (in leading order in b). Measuring the reactive torque experimentally through FMR can act as a probe to such unconventional pairing. Furthermore, when the Shiba state is completely filled or empty ($f_s = 0$), a finite reactive torque establishes the presence of precession induced triplet superconducting pairing. The total reactive torque can be obtained by integrating over ω. Such dynamical generation of triplet pairing and its connection to the torques can be utilized to both manipulate and detect the topological phase diagram of a chain of Shiba impurities and eventually of the emergent Majorana fermions by standard spintronic techniques [44]. Nevertheless, such a study is beyond the scope of this paper and it is left for future work.

Finally, let us give some estimates for the possible FMR frequency shift $\delta\Omega_r \sim \Omega_0/(4S+1)$ of a spin S impurity (details in the SM), where Ω_0 is the bare Larmor frequency. For example, considering the experimental system of Moire patterns of adsorbates on a conventional SC (Pb) with SC gap $\Delta \approx 3$ meV and $S = 1$, described in Ref. [45], $\delta\Omega_r = 20$ GHz for $\Omega_0 = 100$ GHz $< \Delta$. Similarly, for an impurity spin $S = 5/2$ corresponding to the transition metals [46], $\delta\Omega_r \approx 9$ GHz, which is within the current experimental resolution [47].

Conclusion and outlook. In conclusion, we have investigated the dynamical torques acting on a classical spin S precessing in an s-wave superconductor. We found that the torques originate both from the Shiba and the bulk states, with the former contribution having a geometrical (Berry-phase) origin that shifts the FMR frequency. Using various theoretical methods, we showed that a finite linewidth of the Shiba state results in an extra damping of the precession of the classical spin. Finally, we showed that classical spin precession generates unconventional superconducting pairings which is directly reflected into the dynamical torques. Our results offer a noninvasive alternative to the usual STM techniques to address and manipulate the Shiba states, similarly to atomic spins manipulation on metallic substrates [48,49]. Moreover, we expect our findings to be relevant for arrays of coupled dynamical magnetic impurities in superconductors that can harbor exotic Majorana fermionic states.

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[1] S. D. Sarma, M. Freedman, and C. Nayak, npj Quantum Inf. **1**, 15001 (2015).
[2] T. P. Choy, J. M. Edge, A. R. Akhmerov, and C. W. J. Beenakker, Phys. Rev. B **84**, 195442 (2011).
[3] S. Nakosai, Y. Tanaka, and N. Nagaosa, Phys. Rev. B **88**, 180503(R) (2013).
[4] S. Nadj-Perge, I. K. Drozdov, B. A. Bernevig, and A. Yazdani, Phys. Rev. B **88**, 020407(R) (2013).
[5] B. Braunecker and P. Simon, Phys. Rev. Lett. **111**, 147202 (2013).
[6] J. Klinovaja, P. Stano, A. Yazdani, and D. Loss, Phys. Rev. Lett. **111**, 186805 (2013).
[7] M. M. Vazifeh and M. Franz, Phys. Rev. Lett. **111**, 206802 (2013).
[8] F. Pientka, L. I. Glazman, and F. von Oppen, Phys. Rev. B **88**, 155420 (2013).
[9] F. Pientka, L. I. Glazman, and F. von Oppen, Phys. Rev. B **89**, 180505(R) (2014).
[10] K. Pöyhönen, A. Westström, J. Röntynen, and T. Ojanen, Phys. Rev. B **89**, 115109 (2014).
[11] A. Heimes, P. Kotetes, and G. Schön, Phys. Rev. B **90**, 060507(R) (2014).
[12] I. Reis, D. J. J. Marchand, and M. Franz, Phys. Rev. B **90**, 085124 (2014).
[13] A. Westström, K. Pöyhönen, and T. Ojanen, Phys. Rev. B **91**, 064502 (2015).
[14] Y. Peng, F. Pientka, L. I. Glazman, and F. von Oppen, Phys. Rev. Lett. **114**, 106801 (2015).
[15] J. Röntynen and T. Ojanen, Phys. Rev. Lett. **114**, 236803 (2015).
[16] B. Braunecker and P. Simon, Phys. Rev. B **92**, 241410(R) (2015).
[17] J. Zhang, Y. Kim, E. Rossi, and R. M. Lutchyn, Phys. Rev. B **93**, 024507 (2016).
[18] S. Hoffman, J. Klinovaja, and D. Loss, Phys. Rev. B **93**, 165418 (2016).
[19] T. Neupert, A. Yazdani, and B. A. Bernevig, Phys. Rev. B **93**, 094508 (2016).
[20] L. Kimme and T. Hyart, Phys. Rev. B **93**, 035134 (2016).
[21] V. Kaladzhyan, P. Simon, and M. Trif, Phys. Rev. B **96**, 020507(R) (2017).
[22] G. M. Andolina and P. Simon, Phys. Rev. B **96**, 235411 (2017).
[23] H. Shiba, Prog. Theor. Phys. **40**, 435 (1968).
[24] L. Yu, Acta Phys. Sin. **21**, 75 (1965).
[25] A. I. Rusinov, JETP Lett. **9**, 85 (1969).
[26] M. D. Stiles and J. Miltat, in *Spin Dynamics in Confined Magnetic Structures III* (Springer, New York, 2006), pp. 225–308.
[27] J. C. Sankey, P. M. Braganca, A. G. F. Garcia, I. N. Krivorotov, R. A. Buhrman, and D. C. Ralph, Phys. Rev. Lett. **96**, 227601 (2006).
[28] V. Kaladzhyan, S. Hoffman, and M. Trif, Phys. Rev. B **95**, 195403 (2017).
[29] A. J. Millis, P. B. Littlewood, and B. I. Shraiman, Phys. Rev. Lett. **74**, 5144 (1995).
[30] C. Stahl, and M. Potthoff, Phys. Rev. Lett. **119**, 227203 (2017).
[31] M. Elbracht, S. Michel, and M. Potthoff, Phys. Rev. Lett. **124**, 197202 (2020).
[32] A. Suresh, U. Bajpai, and B. K. Nikolić, Phys. Rev. B **101**, 214412 (2020).
[33] U. Bajpai, and B. K. Nikolić, Phys. Rev. Lett. **125**, 187202 (2020).
[34] A. Mishra, S. Takei, P. Simon, and M. TrifSee Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.103.L121401 for the details on wave function approach and Green's function approach to evaluate the dynamic torques.
[35] S. Teber, C. Holmqvist, and M. Fogelström, Phys. Rev. B **81**, 174503 (2010).
[36] M. Ruby, F. Pientka, Y. Peng, F. von Oppen, B. W. Heinrich, and K. J. Franke, Phys. Rev. Lett. **115**, 197204 (2015).
[37] A. V. Balatsky, I. Vekhter, and J.-X. Zhu, Rev. Mod. Phys. **78**, 373 (2006).
[38] M. Kolodrubetz, D. Sels, P. Mehta, and A. Polkovnikov, Phys. Rep. **697**, 1 (2017).
[39] H. H. Haug and A.-P. Jauho, in *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer-Verlag, Berlin, 1998).
[40] H. Huang, C. Padurariu, J. Senkpiel, R. Drost, A. L. Yeyati, J. C. Cuevas, B. Kubala, J. Ankerhold, K. Kern, and C. R. Ast, Nat. Phys. **16**, 1227 (2020).
[41] V. Kaladzhyan, C. Bena, and P. Simon, J. Phys.: Condens. Matter **28**, 485701 (2016).
[42] D. Kuzmanovski, R. S. Souto, and A. V. Balatsky, Phys. Rev. B **101**, 094505 (2020).
[43] V. Perrin, F. L.N. Santos, G. C. Ménard, C. Brun, T. Cren, M. Civelli, and P. Simon, Phys. Rev. Lett. **125**, 117003 (2020).
[44] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Rev. Mod. Phys. **87**, 1213 (2015).
[45] N. Hatter, B. W. Heinrich, D. Rolf, and K. J. Franke, Nat. Commun. **8**, 2016 (2017).
[46] R. Žitko, Phys. B **536**, 230 (2018).
[47] P. Willke, W. Paul, F. D. Natterer, K. Yang, Y. Bae, T. Choi, J. Fernández-Rossier, A. J. Heinrich, and C. P. Lutz, Sci. Adv. **4**, eaaq1543 (2018).
[48] F. D. Natterer, K. Yang, W. Paul, P. Willke, T. Choi, T. Greber, A. J. Heinrich, and C. P. Lutz, Nature (London) **543**, 226 (2017).
[49] K. Yang, W. Paul, F. D. Natterer, J. L. Lado, Y. Bae, P. Willke, T. Choi, A. Ferrón, J. Fernández-Rossier, A. J. Heinrich *et al.*, Phys. Rev. Lett. **122**, 227203 (2019).