Letter

Fractional edge reconstruction in integer quantum Hall phases

Udit Khanna ,[1,2] Moshe Goldstein ,[1] and Yuval Gefen[2]
[1]*Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University, Tel Aviv 6997801, Israel*
[2]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

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Protected edge modes are the cornerstone of topological states of matter. The simplest example is provided by the integer quantum Hall state at Landau level filling unity, which should feature a single chiral mode carrying electronic excitations. In the presence of a smooth confining potential it was hitherto believed that this picture may only be partially modified by the appearance of additional counterpropagating integer-charge modes. Here, we demonstrate the breakdown of this paradigm: The system favors the formation of edge modes supporting fractional excitations. This accounts for previous observations, and leads to additional predictions amenable to experimental tests.

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Introduction. Edge modes are responsible for many of the exciting properties of quantum Hall (QH) states [1]: While the bulk of a QH state is gapped, the edge supports one-dimensional gapless chiral modes [2]. Although several transport properties of these modes are universal and determined by the topological invariants characterizing the bulk state, their detailed structure depends on the interplay between the edge confining potential, electron-electron interaction, and disorder-induced backscattering. As the confining potential is made less steep, the chiral edges of integer [3–8] and fractional [9–20] QH phases and the helical edges of time-reversal-invariant topological insulators [21] may undergo a quantum phase transition (or "edge reconstruction"), while the bulk state remains untouched. Edge reconstruction may be driven by charging or exchange effects and leads to a change in the position, ordering, number, and/or nature of the edge modes.

Arguably the simplest example is provided by the edge of the $\nu = 1$ QH state. When confined by a sharp potential, this state supports a single gapless chiral integer mode with charge $e^* = 1$; the electronic density steeply falls from its bulk value to zero at the edge. Smoothening the confining potential and accounting for the incompressibility of QH states leads to the formation of an outer, finite density reconstructed strip. Employing a self-consistent Hartree-Fock (HF) scheme, Chamon and Wen [5] found that this additional strip can be described as a $\nu = 1$ QH state [Fig. 1(a)]. Such a state allows the local density to assume an integer value, leading to a smooth variation of the coarse-grained density from its bulk value to zero. Reconstruction introduces an additional pair of counterpropagating gapless chiral modes at the edge. The HF approximation is limited to Slater-determinant states, entailing these to be integer modes ($e^* = 1$). Exact diagonalization of the $\nu = 1$ phase [5] (and of fractional phases [15–18]) is consistent with the expected picture, but is limited to very small systems, rendering it hard to confirm the precise filling factor of the side strip or the nature of edge modes.

Recent transport experiments on the $\nu = 1$ state [22,23] have led to some surprising observations regarding the edge structure. Exciting the $\nu = 1$ edge at a quantum point contact (QPC), Ref. [22] observed a flow of energy but not charge upstream from the QPC, possibly indicating the presence of upstream neutral modes. Reference [23] has studied the interference of the edge modes in an electronic Mach-Zehnder interferometer. As the bulk filling factor is reduced from 2 to less than 1, reduction in the visibility of the interference pattern has been observed, with full suppression for $\nu \leqslant 1$. This is another indication of the presence of upstream neutral modes [24]. However, it is inconsistent with Chamon and Wen's picture of only integer-charge modes, which can lead to upstream charge propagation, but not to upstream neutral modes. Reference [23] also found a fractional conductance plateau with $g = 1/3 \times e^2/h$ by partially pinching off a QPC in the $\nu = 1$ bulk state. This too is incompatible with the edge structure of Fig. 1(a). To cap it all, the conductance plateau observed was accompanied by shot noise with a quantized Fano factor 1, which seems to suggest the edge modes do possess an integer charge. Fractional modes were also observed at the $\nu = 1$ edge through direct imaging of the local density [25,26] as well as in recent transport experiments [27].

Here, we propose another picture of the reconstructed edge of the $\nu = 1$ phase, and show that it accounts for all these seemingly contradictory observations. We establish that reconstruction may introduce a different type of counterpropagating modes, namely *fractionally charged* ($e^* = 1/3$) modes. This is the case when the strip of electrons separated at the edge forms a $\nu = 1/3$ Laughlin state [Fig. 1(b)] instead of the commonly assumed $\nu = 1$ state (such an edge structure was first suggested in Ref. [23]). To go beyond the constraints of the HF approximation [which imply an integer (0 or 1) occupation of each single-particle state], we follow the approach by Meir [12] and treat the two edge configurations depicted in Fig. 1 as variational states, and compare their respective energies for different strip

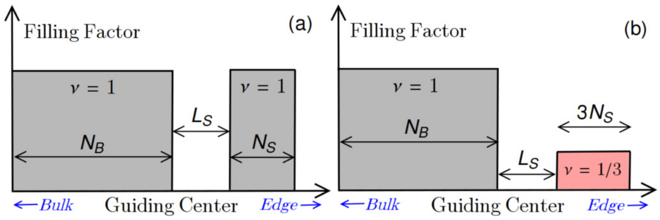

FIG. 1. Schematic representation of two possible configurations at the reconstructed edge of the $\nu = 1$ state. Letting the confining potential become smoother, N_S electrons may separate from the bulk by L_S guiding centers, forming a strip of (a) a $\nu = 1$ state [5] or (b) a $\nu = \frac{1}{3}$ Laughlin state.

size (N_S) and separation (L_S) as a function of the slope of the confining potential. We find that for smooth slopes the fractionally reconstructed edge [Fig. 1(b)] is energetically favorable. Our analysis then demonstrates that *fractional* edge reconstruction may be much more robust than integer reconstruction.

The intricate edge structure involving a downstream $e^* = 1$ mode along with a pair of counterpropagating $e^* = 1/3$ modes has several experimental consequences. First, with such an edge structure the two-terminal (electrical) conductance would vary from $g_{2T} = e^2/h$ in a long sample (with full edge equilibration) to $g_{2T} = 5/3 \times e^2/h$ in a short sample (with no equilibration) [28,29]. This would be a *smoking gun* signature of the edge structure proposed here. Second, in the presence of disorder-induced tunneling and intermode interactions, the counterpropagating modes $e^* = 1$ and $1/3$ are renormalized to two effective modes of charge $e^*_\uparrow$ and $e^*_\downarrow$ [13,14,28] (here, $\uparrow/\downarrow$ denote the upstream/downstream modes). When biased, the upstream mode can carry a heat flow, which, in the particularly interesting case of $e^*_\uparrow = 0$ and $e^*_\downarrow = 2/3$, may appear without an accompanying upstream charge flow. Such neutral modes have been observed in hole-conjugate QH states [22,30–34]. Bias of the neutral modes can cause stochastic noise in the charge modes through the generation of quasihole-quasiparticle pairs [31,35–37]. Below we show that this could account for the aforementioned Fano factor 1 [23]. Moreover, neutral modes may also lead to suppression of interference in Mach-Zehnder interferometers [24], in line with existing experiments.

Basic setup. We consider a $\nu = 1$ state on a disk. In the symmetric gauge, $e\vec{A}/\hbar = (-y/2\ell^2, x/2\ell^2)$, the wave function of single-particle states in the lowest Landau level are $\phi_m(\vec{r}\,) = (r/\ell)^m e^{-im\theta_r} e^{-(\frac{r}{2\ell})^2}/\sqrt{2^{m+1}\pi m!\ell^2}$, where (r, θ_r) are the polar components of $\vec{r}$ in the x-y plane; ϕ_m is an angular momentum eigenfunction with eigenvalue $\hbar m$, centered at $r = \sqrt{2m}\ell$ where ℓ is the magnetic length. Assuming spin-polarized electrons and neglecting higher Landau levels, the Hamiltonian is $H = H_{ee} + H_c$, where H_{ee} is the interaction part while H_c is a circularly symmetric one-body confining potential. Denoting $E_c = e^2/\epsilon_0\ell$, $H_{ee} = (E_c/2)\sum_{m_1,m_2,n} V^{ee}_{m_1 m_2;n} c^\dagger_{m_1+n} c^\dagger_{m_2} c_{m_2+n} c_{m_1}$ and $H_c = E_c \sum_m V^c_m c^\dagger_m c_m$, where V^{ee} is the two-body Coulomb matrix element and V^c is the matrix element of the confining potential. The total angular momentum L is a good quantum number. The edge confining potential reads [12]

$$V_c(r) = \begin{cases} 0, & r < r_0 - \frac{w\ell}{2}, \\ \frac{s}{w\ell}\left(r - r_0 + \frac{w\ell}{2}\right), & r_0 - \frac{w\ell}{2} < r < r_0 + \frac{w\ell}{2}, \\ s, & r > r_0 + \frac{w\ell}{2}, \end{cases} \tag{1}$$

where r_0 is the radius of a compact $\nu = 1$ state. The dimensionless parameter s sets the overall height of the potential, which we henceforth fix to $s = 7$. The steepness of the potential is controlled by the dimensionless width w.

We consider two classes of variational states (shown in Fig. 1), corresponding to an integer [Chamon-Wen [5], Fig. 1(a)] and a fractional [Fig. 1(b)] reconstructed edge. Both are controlled by two parameters: the total occupancy N_S of the reconstructed edge strip, and the number L_S of empty orbitals separating it from the bulk. The latter contains N_B electrons, such that the total number of electrons $N_S + N_B$ is fixed (to be 100). The Chamon-Wen family of states includes the compact edge configuration ($N_S = 0 = L_S$) which is the ground state for sharp confining potentials. For smoother confining potentials, the lowest energy state is expected to be at nonzero N_S and L_S. In this case, a comparison of the energies of the states in the two classes determines whether fractionally charged modes could appear at the edge of the $\nu = 1$ phase.

Variational ansatz: Integer edges. Figure 1(a) represents a Slater-determinant state of $N_S + N_B$ electrons. It can be written as $|N_B, 0\rangle \otimes |N_S, N_B + L_S\rangle$, where

$$|N, L\rangle = c^\dagger_{L+N-1}\, c^\dagger_{L+N-2} \cdots c^\dagger_{L+1}\, c^\dagger_L |0\rangle. \tag{2}$$

The energy and angular momentum of each state in the integer class of reconstructions can be found easily once the Coulomb matrix elements are known [38].

Variational ansatz: Fractional edges. Figure 1(b) represents the product state of a Slater determinant ($|N_B, 0\rangle$) with an annulus of the $\nu = 1/3$ Laughlin state, containing N_S electrons starting at the guiding center $m = N_B + L_S$. The (unnormalized) wave function corresponding to the annulus is

$$\prod_{i=1}^{N_S}\left[z_i^{N_B+L_S}\right]\left[\prod_{i<j}(z_i - z_j)^3\right] e^{-\frac{1}{4}\sum_i |z_i|^2}, \tag{3}$$

where $z_n = x_n - iy_n$ is the coordinate of the nth particle. The energy and angular momentum of states in this class involve the Coulomb energy and average occupations of the Laughlin state [Eq. (3)]. We evaluate these using standard classical Monte Carlo techniques [38].

Results. Figure 2 shows the total energies and the ground state densities for the two class of variational states at different confining potentials. In Figs. 2(a) and 2(b) the blue dots correspond to integer edges while the red dots correspond to the fractional edge states. For a sharp confining potential [$w < 10$, Fig. 2(a)] the lowest energy state is the one with the minimal angular momentum (in this case $4950\hbar$). This corresponds to the unreconstructed $\nu = 1$ state with a single chiral edge mode. Figure 2(c) shows the electronic density in this case, which drops monotonically from $1/2\pi\ell^2$ to 0.

For smoother potentials [$w > 10$, Fig. 2(b)] the lowest energy state has a much larger angular momentum ($5256\hbar$ for $w = 10.2$ with $N_S = 18$ and $L_S = 0$) than the compact state. Correspondingly, Fig. 2(d) shows that the density varies

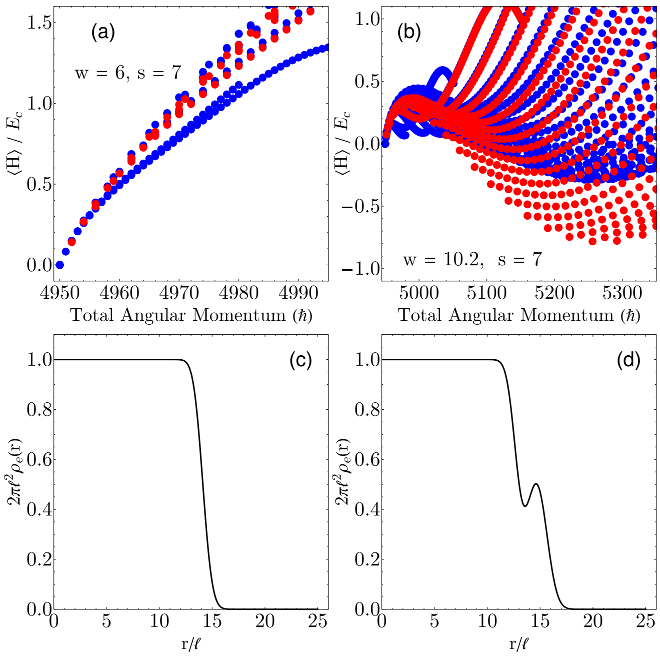

FIG. 2. Variational analysis for $N_S + N_B = 100$ and $s = 7$. (a), (b) The energy of the two variational states as a function of the total angular momentum at (a) $w = 6.0$ and (b) $w = 10.2$. The energy of the unreconstructed state has been subtracted to make comparison easier. The blue (red) dots correspond to states with $\nu = 1$ ($\nu = \frac{1}{3}$) reconstruction at the edge. For sharp edges ($w < 10$) the ground state is the one with minimum angular momentum, implying that $L_S = 0$, hence no edge reconstruction. In this case, we expect a single downstream edge mode supporting $e^* = 1$ quasiparticles. For smooth edges ($w > 10$) the ground state shifts to a higher angular momentum sector implying that the electronic disk expands and the edge undergoes reconstruction. (b) shows that a fractional reconstruction is energetically favorable to an integer reconstruction. This is true for all $w > 10$. Thus the reconstructed edge supports counterpropagating modes with fractional charges. (c) and (d) depict the electronic densities of the ground state at (c) $w = 6.0$ and (d) $w = 10.2$. The nonmonotonic variation of density at the edge is another signature of the presence of additional emergent modes.

nonmonotonically at the edge [44]. The states with a fractional edge are found to have a lower energy than the states with an integer edge *whenever reconstruction is favored* [45]. This is the main result of this work. We have verified that it does not depend on the precise form of the confining potential [38]. We now turn to discuss the experimental consequences of such a reconstruction and compare them to the observations reported in literature so far.

Two-terminal conductance. Let us consider the setup shown in Fig. 3, where the edge structure is based on our analysis of a disk geometry. The chiral modes emanating from the source (S) are biased with respect to those emerging from the drain (D). Due to disorder-induced intermode tunneling, the counterpropagating chirals at each edge will equilibrate over a typical length $\ell_{\rm eq}$. For a fully equilibrated edge ($L \gg \ell_{\rm eq}$), the two-terminal conductance is e^2/h, as expected for the $\nu = 1$ QH state. Note that this would be the case for both sharp and smooth edges and for both integer and fractional reconstructions.

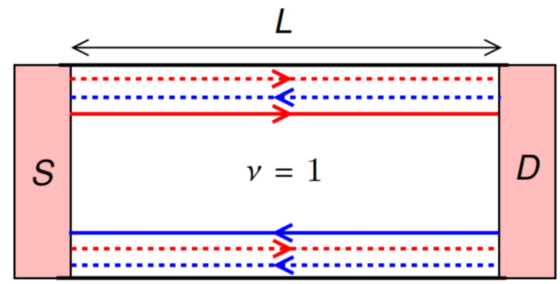

FIG. 3. Two-terminal transport experiment at $\nu = 1$, with an edge structure as calculated for a disk geometry (cf. text). The solid (dashed) lines indicate the integer (fractional) chirals at the two edges of the sample. The red (blue) chirals are biased (unbiased) due to the source S (drain D). For $L \ll \ell_{\rm eq}$ ($\ell_{\rm eq}$ is the intermode equilibriation length) the conductance is $g = 5/3 \times e^2/h$ ($3 \times e^2/h$) for fractional (integer) edge reconstruction [cf. Figs. 1(b) and 1(a)]. For a fully equilibrated edge ($L \gg \ell_{\rm eq}$), the conductance reduces to $g = e^2/h$ in both cases, as expected for the unreconstructed $\nu = 1$ state.

For $L \ll \ell_{\rm eq}$, the detailed structure of the edge underlies the conductance. For a sharp edge transport takes place through a single integer chiral, hence the electric conductance would retain the values e^2/h. This is different for smooth edges. The electric conductance is sensitive to the number as well as the nature of the modes; with a pair of counterpropagating fractional edges, the electric conductance becomes $5/3 \times e^2/h$ [28,29]. Such an observation would uniquely identify the edge structure proposed here [Fig. 1(b)]—a smoking gun signature of fractional edge reconstruction [46].

Neutral modes. Consider the fractional reconstruction of Fig. 1(b). Labeling the outermost channel as 1 and the innermost edge as 3 [cf. Fig. 4(a)], the low energy dynamics of the three modes is described by three chiral bosonic fields ϕ_j ($j = 1, 2, 3$) satisfying the Kac-Moody algebra, $[\phi_{j_1}(x), \phi_{j_2}(x')] = i\pi [K^{-1}]_{j_1,j_2} \,\mathrm{sgn}(x - x')$, where the K matrix is diagonal with $K_{1,1} = 3$, $K_{2,2} = -3$, $K_{3,3} = 1$. The inner two modes are counterpropagating charge modes of $\nu = 1$ and $\nu = 1/3$ type. This is precisely the edge structure of the hole-conjugate $\nu = 2/3$ FQH state. Since L_S is typically small [44], in the presence of disorder-induced backscattering and interactions the two charge modes can hybridize [Fig. 4(a)], resulting in a downstream charged mode ϕ_c and an upstream neutral mode ϕ_n, which are effectively decoupled at low energies [13]. This K matrix is diagonal with $K_{1,1} = 3$, $K_{c,c} = 1$, $K_{n,n} = -1$. We note that here the outermost mode (ϕ_1) is kept untouched (cf. Fig. 4).

The experimental consequences of this emergent neutral mode are similar to the neutral modes in hole-conjugate states. For instance, it can lead to an upstream thermal current, which was reported in Ref. [22], accompanied by an upstream shot noise (see below) [47,48]. The presence of the neutral mode can also hinder observation of interference effects in Mach-Zehnder setups [24] as reported in Ref. [23].

Fractional conductance plateau and noise. The presence of fractionally charged chiral modes at the edge has clear experimental consequences for transport measurements. Consider for example the single QPC setup of Fig. 4(b). Here, the bulk filling factor is $\nu = 1$ and the current is transmitted from the source (S1) to the drain (D1). When the QPC is fully open

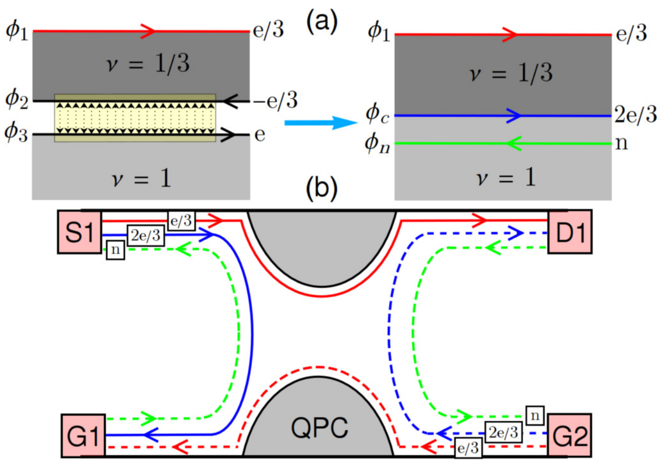

FIG. 4. (a) Renormalization of the inner two edge modes due to interactions and disorder-induced backscattering into a downstream charge (ϕ_c) and upstream neutral (ϕ_n) mode. Only the inner two modes are assumed to couple, since within the variational calculation, the width of the $\nu = 1/3$ strip increases as the edge potential is made smoother but the separation between the $\nu = 1$ and $\nu = 1/3$ regions remains constant. Thus the outermost edge mode (ϕ_1) can be assumed to be physically separated from the inner two modes ($\phi_{2,3}$) [20]. (b) A single QPC tuned to the transmission plateau $t = 1/3$. The bulk on both sides of the QPC is in the $\nu = 1$ state with a reconstructed and renormalized edge. Solid (dashed) lines correspond to biased (unbiased) modes.

then the conductance would be e^2/h, as expected from the bulk topological index. However, due to the edge structure discussed above, it is also possible to pinch off the QPC, so that only the outermost mode (ϕ_1) is transmitted while the inner two modes are completely reflected. In this case there would be a fractional conductance plateau at $1/3 \times e^2/h$ while the bulk filling factor remains 1. Such a plateau was reported in Ref. [23].

Interestingly, although the conductance is quantized, the system could exhibit shot noise on the conductance plateau. Under the assumption of coherent propagation of the neutral mode, and provided certain symmetry conditions are satisfied [35,49], the Fano factor is quantized. Such a quantized noise at the 1/3 conductance plateau has been reported in Ref. [23]. Below we sketch the underlying physics relying on our fractionally reconstructed edge picture.

Consider the setup shown in Fig. 4(b). The source S1 on the upper left-hand side of the QPC biases both charge modes emanating from it with the same voltage (say V). The current in the two modes is $I_1 = V/3 \times e^2/h$, $I_c = 2V/3 \times e^2/h$, and the total current is thus $I = I_1 + I_c = V \times e^2/h$. The current ($I_i$, $i = 1, c$) in a given mode is related to the corresponding quasiparticle density (n_i) through $I_1 = e/3 \times v_1 n_1$ and $I_c = 2e/3 \times v_c n_c$, where v_i are the corresponding velocities, implying $v_1 n_1 = v_c n_c$. Therefore if N quasiparticles of charge $\frac{1}{3}$ emanate from the S1 in time τ, then N quasiparticles of charge $\frac{2}{3}$ also emanate in the same time interval. The total current (I) is $I = e/3 \times N/\tau + 2e/3 \times N/\tau = eN/\tau$.

Now, on the upper right-hand side of the QPC, the outermost $e/3$ mode is biased while the inner $2e/3$ mode is grounded, and therefore the two modes will equilibrate through tunneling processes, which would also create excitations in the neutral mode. If there were N quasiparticles in ϕ_1, then after equilibration with ϕ_c there would be $N/3$ quasiparticles left in both charged modes and $2N/3$ neutral excitations in the upstream neutral mode. These neutral excitations would move to the lower right-hand side of the QPC and decay into quasiparticle-quasihole pairs in the charge modes. This generates stochastic noise in the charged modes because each decay process can randomly generate either a quasiparticle (quasihole) in the outermost (inner) mode or vice versa. This decay process would lead to a stochastic tunneling of $N/3$ electronic excitations into ϕ_c, which eventually reach the drain D1. Similarly, on the lower left-hand side of the QPC, a biased $2e/3$ mode flows in parallel to an unbiased $e/3$ mode. Their mutual equilibration would again generate $2N/3$ neutral excitations. These decay on the upper left-hand side of the QPC and generate $2N/3$ excitations in the ϕ_1 mode entering the drain D1.

As a result of the above, the charge entering the drain in time τ is $Q = e/3 \times N/3 + 2e/3 \times N/3 + e/3 \times \sum_{i=1}^{2N/3} a_i + 2e/3 \times \sum_{i=1}^{N/3} b_i$, where a_i and b_i are random variables which take values ± 1 with equal probability, and describe the noise generated in the modes due to the neutral excitation decay described above. This implies that the average current arriving at the drain is $I_D = \langle Q \rangle/\tau = eN/3 = I/3$ (consistent with a transmission of 1/3). The variance of the charge is $\delta Q^2 = \langle Q^2 \rangle - \langle Q \rangle^2 = e^2/9 \times \sum_{i=1}^{2N/3} a_i^2 + 4e^2/9 \times \sum_{i=1}^{N/3} b_i^2 = 2Ne^2/9 = 2e/9 \times I\tau$. The effective Fano factor is $F_{\text{eff}} = \delta Q^2/I\tau \times 1/et(1-t)$. Using $t = 1/3$ we obtain $F_{\text{eff}} = 1$, which coincides with the observation of Ref. [23].

Conclusions. We have studied edge reconstruction at the boundary of $\nu = 1$ integer quantum Hall state. Previously reported Hartree-Fock calculations show that upon smoothening the confining potential a new strip of $\nu = 1$ QH state is formed at the edge, introducing counterpropagating *integer* modes [5]. Going beyond the mean-field approximation, we have performed a variational calculation, where we have compared the above ansatz to a new one, in which the electronic strip forms a $\nu = 1/3$ Laughlin state. We have found that such fractional reconstruction is always energetically favorable, implying that *fractional modes can appear at the boundary of integer QH states*. We have discussed the experimental consequences of such a fractionally reconstructed edge, which nicely square with previous measurements, and provide predictions for future experiments. Our finding sets the stage for a future detailed investigation of coherent as well as incoherent transport in designed geometries, implementing the idea of fractionally reconstructed edges.

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[45] We also performed a variational calculation for an edge structure where the filling factors follow the profile (from bulk to edge) $1 \to 2/3 \to 1 \to 0$, such that the edge has a similar structure to the $\nu = 2/3$ state. However, that configuration turned out to be energetically unfavorable compared with the structure in Fig. 1(b).

[46] The thermal Hall conductance is $3 \times \pi^2 k_B^2 T/3h$ for an unequilibrated edge for both integer and fractional reconstructions since it only depends on the number of chirals participating in transport. For an equilibrated edge it reduces to $\pi^2 k_B^2 T/3h$, as expected for the $\nu = 1$ QH state.

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