Letter

Emergence of spin-active channels at a quantum Hall interface

Amartya Saha [1], Suman Jyoti De [2], Sumathi Rao [2], Yuval Gefen [3], and Ganpathy Murthy [1]
[1]*Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506-0055, USA*
[2]*Harish-Chandra Research Institute, HBNI, Chhatnag Road, Jhunsi, Allahabad 211019, India*
[3]*Department of Condensed Matter Physics, Weizmann Institute, 76100 Rehovot, Israel*

(Received 5 October 2020; revised 9 December 2020; accepted 13 January 2021; published 1 February 2021)

We study the ground state of a system with an interface between $\nu = 4$ and $\nu = 3$ in the quantum Hall regime. Far from the interface, for a range of interaction strengths, the $\nu = 3$ region is fully polarized but $\nu = 4$ region is unpolarized. Upon varying the strength of the interactions and the width of the interface, the system chooses one of two distinct edge/interface phases. In phase A, stabilized for wide interfaces, spin is a good quantum number, and there are no gapless long-wavelength spin fluctuations. In phase B, stabilized for narrow interfaces, spin symmetry is spontaneously broken at the Hartree-Fock level. Going beyond Hartree-Fock, we argue that phase B is distinguished by the emergence of gapless long-wavelength spin excitations bound to the interface, which can be detected by a measurement of the relaxation time T_2 in nuclear magnetic resonance.

DOI: 10.1103/PhysRevB.103.L081401

Introduction. In the integer quantum Hall effect (IQHE) [1], a two-dimensional electron gas subjected to a strong perpendicular magnetic field displays a Hall conductivity quantized in integral units of $\frac{e^2}{h}$ at low temperatures. These systems are the simplest examples of topological insulators [2]. Their bulk is insulating, and the underlying band topology manifests itself in chiral current-carrying edge states which are protected against localization. The bulk band topology dictates the charge Hall and thermal Hall conductances. Further, because the kinetic energy is quantized into degenerate Landau levels (LLs), partially filled LLs host strong electron correlations, leading to quantum Hall ferromagnetism [3,4] and the fractional quantum Hall effects [5].

It has long been realized that within the constraints of bulk band topology, a variety of reconstructed edge phases are possible. Much theoretical work exists on edge reconstructions, with most reconstructions being driven by electrostatic considerations: the "desire" of the electron fluid to perfectly neutralize the positive background competing with the "desire" to form incompressible droplets. In the simplest reconstructions spin plays no role [6–11]. At the edges of quantum Hall ferromagnets, states with broken spin and/or edge translation symmetry are known to occur in the Hartree-Fock (HF) approximation [12–15].

It is clear from previous work that edge reconstructions can generate counterpropagating pairs of chiral charge modes. Going beyond this, one can ask whether exchange can lead to the emergence of a pair of chiral, neutral, spin-active edge modes. Since spin is involved, it is natural to look for interfaces where at least one of the two bulk states is a QH ferromagnet.

Motivated by these considerations, we investigate an interface between a $\nu = 4$ singlet region and a fully polarized $\nu = 3$ region in the HF approximation. In the following, we will use the words edge/interface interchangeably. Our tuning parameters are the width of the interface in units of magnetic length ($\tilde{w} = w/\ell$), where the background charge is assumed to vary smoothly between $\nu = 4$ and $\nu = 3$, and the strength of the Coulomb in units of the cyclotron energy ($\tilde{E}_c = \frac{e^2}{\ell\hbar\omega_c}$). We find two robust phases: For large $\tilde{w}$ we find phase A: HF single-particle levels are spin polarized and three of them cross the Fermi energy, as required by the total $S_z = 0$ in the $\nu = 4$ bulk and the total $S_z = 3/2$ in the $\nu = 3$. For smaller $\tilde{w}$ we find phase B, with spontaneously broken $U(1)$ spin-rotation symmetry, with a single HF level crossing the Fermi energy.

Phase A has a pair of counterpropagating, spin-resolved chiral charge modes in addition to the one chiral charge mode required by topology. Phase B, as we argue in the discussion, manifests a pair of chiral, counterpropagating spin-active neutral modes bound to the interface, in addition to the required charged chiral. Any probe sensitive to gapless long-wavelength spin excitations, such as nuclear magnetic resonance (NMR), will be able to distinguish the two phases.

In the following, we will set up the problem, explain our computation briefly, and describe the two phases in HF. We address the important issue of fluctuations beyond HF in the discussion, before addressing potential experimental signatures. Details of the robustness of the two phases with respect to the Zeeman coupling ($\tilde{E}_Z = \frac{E_Z}{\hbar\omega_c}$), the screening of the interaction, the number of Landau levels kept in our calculation, and other details, are relegated to the Supplemental Material (SM) [16].

Edge between $\nu = 4$ and $\nu = 3$ quantum Hall states. The geometry of the interface between the $\nu = 4$ and $\nu = 3$ QH systems is shown in Fig. 1. In the noninteracting limit, the $\nu = 4$ bulk will have the Landau levels (LLs) $|0\uparrow, 0\downarrow, 1\uparrow, 1\downarrow\rangle$

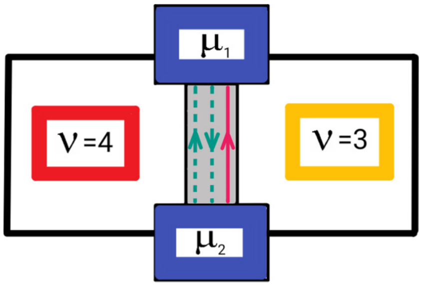

FIG. 1. A schematic diagram of our setup with an interface between bulk $\nu = 4$ and $\nu = 3$ IQHE states. The solid line (red online) is a downstream chiral charged mode required by topology. The pair of dashed lines (green online) depict either spin-resolved charged chiral modes (phase *A*) or gapless spin-active chiral modes (phase *B*).

occupied, while the $\nu = 3$ bulk has the LLs $|0\uparrow, 0\downarrow, 1\uparrow\rangle$ occupied. At the edge between the two, we expect the $1\downarrow$ LL to smoothly cross the chemical potential μ from below as one moves rightwards (from $\nu = 4$ to $\nu = 3$), leading to a single chiral charged edge mode with $\downarrow$ spin.

As interactions grow we expect a greater tendency towards spin polarization (QH ferromagnetism). However, the $\nu = 3$ and $\nu = 4$ states polarize at the different values of $\tilde{E}_c$. There is a range of $\tilde{E}_c$ where the $\nu = 4$ bulk remains unpolarized, while the $\nu = 3$ bulk is fully polarized. Now it is not obvious how many μ crossings, and hence chiral modes, there should be: The result will depend on the details of the interface. Our goal is to study the possible edge phases that can exist as our tuning parameters $\tilde{w}, \tilde{E}_c$ are varied.

Our Hamiltonian is

$$H = \hbar\omega_c \sum_{n,k,s} c^\dagger_{nks} c_{nks} + \frac{E_Z}{2} \sum_{n,k} (c^\dagger_{nk\downarrow} c_{nk\downarrow} - c^\dagger_{nk\uparrow} c_{nk\uparrow}) + \frac{1}{2L_x L_y} \sum_{\mathbf{q}} v(q) : (\rho_b(\mathbf{q}) - \rho_e(\mathbf{q}))(\rho_b(-\mathbf{q}) - \rho_e(-\mathbf{q})) : \tag{1}$$

Using n for the Landau level index and k for the guiding center index (defined below), the electron density operator is $\rho_e(x, y) = \sum_s \Psi^\dagger_s(x, y)\Psi_s(x, y)$, where the electron field operator is $\Psi_s(x, y) = \sum_{n,k} \Phi_{nk}(x, y) c_{nks}$, with c_{nks} being canonical fermion operators. $v(q)$ and $\rho_e(\mathbf{q})$ are the Fourier transforms of the long-ranged screened Coulomb potential and $\rho_e(x, y)$, respectively. We model the background charge density ρ_b as changing linearly from $4\rho_0 \equiv 4/2\pi\ell^2$ to $3\rho_0$ over a distance $\tilde{w}$ in the $\hat{y}$ direction (Fig. 2). Note that the background charge density preserves translation invariance in the x direction. As in real samples, the Zeeman coupling $\tilde{E}_Z > 0$ (but $\tilde{E}_Z \ll \tilde{E}_c, 1$), with the spin symmetry of the Hamiltonian being $U(1)$.

For the unscreened Coulomb interaction, at $\tilde{E}_Z = 0$ in HF, for $2.52 < \tilde{E}_c < 2.90$ the bulk $\nu = 4$ ground state ($0\uparrow, 0\downarrow, 1\uparrow, 1\downarrow$ occupied) is unpolarized and the bulk $\nu = 3$ ground state ($0\uparrow, 1\uparrow, 2\uparrow$ occupied) is fully polarized. As E_z increases the range of $\tilde{E}_c$ changes.

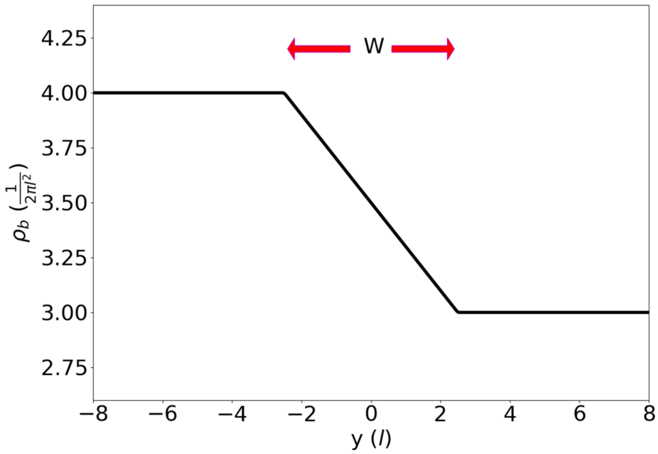

FIG. 2. Dependence of the background charge density on $\hat{y}$. The charge density is uniform in the $\hat{x}$ direction.

We work in the Landau gauge $\vec{A} = -B_0 y\hat{x}$ with the magnetic field pointing in the positive $\hat{z}$ direction. The magnetic length $\ell = \sqrt{\frac{\hbar}{eB_0}}$. The Hamiltonian has translation invariance along x. The one-body wave functions are

$$\Phi_{n,k}(x, y) = \frac{e^{ikx} e^{-\frac{(y-k\ell^2)^2}{2\ell^2}}}{\sqrt{L_x n! 2^n \sqrt{\pi}\,\ell}} H_n\left(\frac{y - k\ell^2}{\ell}\right). \tag{2}$$

The x coordinate (along the edge) has periodic boundary conditions with period L_x to discretize k, which defines the guiding center position $Y(k) = k\ell^2$. The interface is centered at $y = 0$ with $\nu = 4$ as the bulk ground state for $y \ll -\ell$ and $\nu = 3$ as the bulk ground state for $y \gg \ell$. We use spin-unrestricted HF, looking for solutions that preserve the translation invariance in x. Thus, k is a good single-particle quantum number in HF. Since we allow for Landau level and spin mixing, we work with a total of eight basis states for each value of k (four Landau levels, each with $\uparrow$ and $\downarrow$ spin). The HF states are specified by the matrix $\Delta_{ns,n's'}(k) = \langle c^\dagger_{n's'k} c_{nsk}\rangle$, which is obtained self-consistently by diagonalizing the effective one-body HF Hamiltonian. The chemical potential μ is chosen to maintain overall charge neutrality. We use a screened Coulomb potential of the form $v(q) = \frac{2\pi E_c}{q+q_0}$, where q_0, the screening parameter, is chosen to be $q_0\ell = 0.01$. Using this method we obtain the phase diagram in the parameters $\tilde{w}$, $\tilde{E}_c$. The SM [16] contains the details.

Phase diagram in the HF approximation. There are two edge phases, as shown in Fig. 3, separated by a first-order phase transition. In phase *A* there are three μ crossings of single-particle levels, each spin resolved. In phase *B* there is only a single self-consistent energy level that crosses μ. In addition, the HF state of phase *B* shows a spontaneous breaking of the $U(1)$ spin symmetry.

The main features of the phase diagram result from the competition between (i) the interface potential, controlled by the width $\tilde{w}$ of the interface region, (ii) the electrostatic repulsion, and (iii) the spin stiffness. All three are controlled by the Coulomb interaction. For large values of $\tilde{w}$, it is energetically favorable for the system to approximately neutralize the background potential by creating an extra pair of

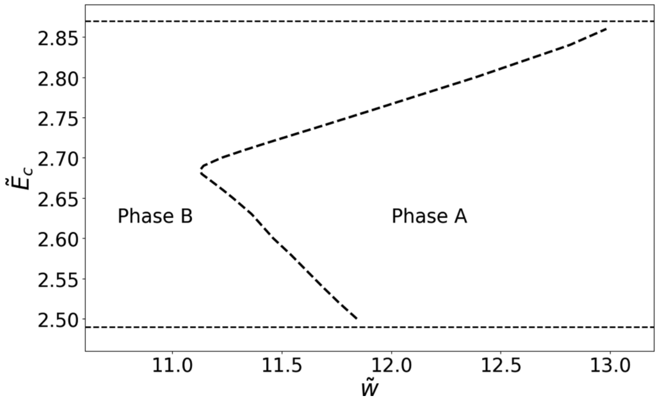

FIG. 3. Phase diagram in the parameter space $\tilde{E}_c$ and $\tilde{w}$ at $\tilde{E}_Z = 0.03$, $q_0\ell = 0.01$. At this value of $\tilde{E}_Z$ the $\nu = 4$ bulk state is a singlet and $\nu = 3$ fully polarized for $2.49 < \tilde{E}_c < 2.87$. For values of $\tilde{E}_c < 2.7$ Landau level mixing is weak, and the spin stiffness increases with $\tilde{E}_c$. This raises the cost of phase B over phase A, leading to the phase boundary moving towards smaller $\tilde{w}$. For $\tilde{E}_c > 2.7$ Landau level mixing decreases the spin stiffness, thereby favoring phase B. The transition is first order in HF.

counterpropagating edge modes, spreading the electron density over a larger region. In this phase, the spins of the chiral modes (assuming one associated with each single-particle μ crossing) remain well defined. For smaller values of $\tilde{w}$, it becomes energetically favorable to have a single HF level crossing μ. The requirement that the spin polarization at each k change by $\frac{3\hbar}{2}$ in going from $\nu = 4$ to $\nu = 3$ necessitates a spin rotation (and Landau level index rotation) of all single-particle HF levels.

Let us further examine the HF solution. For this paragraph only, we will make the naive assumption that each single-particle crossing of μ represents a chiral mode. The single particle energy levels and the spin components of the levels are plotted in Figs. 4 and 5. From the energy dispersions in Fig. 4 we see that two of the modes are downstream and one is upstream. The spin of these modes can also be identified:

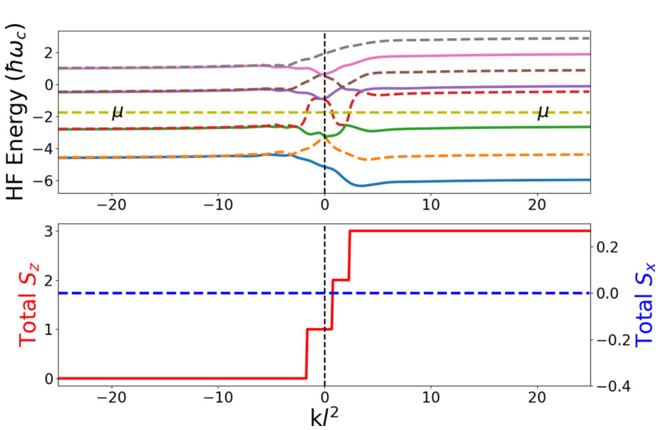

FIG. 4. Phase A in the HF approximation: The upper panel shows the single-particle energy dispersion and the lower panel shows the total S_z and S_x values (in units of $\frac{\hbar}{2}$) as a function of the guiding center position. The parameter values are $\tilde{E}_c = 2.52$, $\tilde{w} = 13.0$ and $\tilde{E}_Z = 0.03$, $q_0\ell = 0.01$.

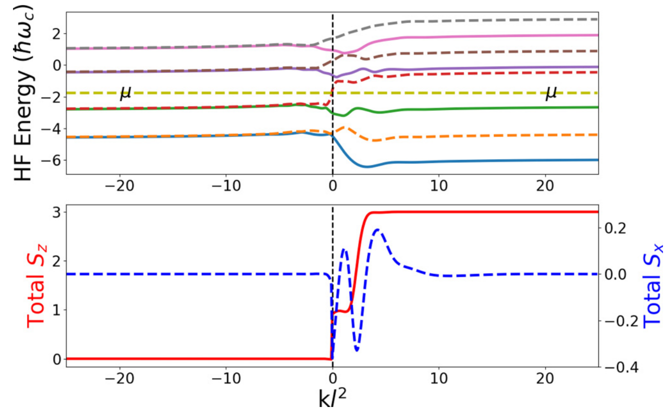

FIG. 5. Phase B in the HF approximation: The upper panel shows the single-particle energy levels and the lower panel shows the total S_z and S_x values (in units of $\frac{\hbar}{2}$) as a function of the guiding center position. The parameter values are $\tilde{E}_c = 2.52$, $\tilde{w} = 10.0$ and $\tilde{E}_Z = 0.03$, $q_0\ell = 0.01$.

Moving rightwards from large negative y, first the $1\downarrow$ level from the $\nu = 4$ region smoothly crosses μ from below, implying a downstream chiral mode with $\downarrow$ spin. Next the $2\uparrow$ level crosses μ from above, producing an upstream chiral mode with $\uparrow$ spin. Finally the $0\downarrow$ level crosses μ from below, producing another downstream chiral mode with $\downarrow$ spin. The spin at the interface changes by three units (each unit is $\frac{\hbar}{2}$) as mandated by the spin polarizations of the bulk states. The average value of S_x remains zero confirming that the chiral levels have well-defined spins. Phase B has only one chiral downstream mode. Here again, the S_z at the interface does change by three units as is required, but the average value of S_x is nonzero here and there is spin rotation at the interface.

Fluctuations beyond HF. HF is known to overpredict order, because it neglects quantum fluctuations. HF can be taken as reliable for single-particle spectra but should be supplemented by reasoning based on effective field theory (EFT) when questions of spontaneously broken symmetry and collective modes arise. We proceed by (i) identifying the correct EFT, (ii) matching the HF phases to those of the EFT, and (iii) looking at quantum fluctuations beyond mean field in the EFT, and the implied consequences for physical observables in our system.

The $SU(2)$ spin symmetry of our electronic Hamiltonian is broken down to $U(1)$ by the Zeeman coupling, and the edge is a quasi-1D system. Thus, the relevant EFT in the spin sector is the XXZ model in a Zeeman field in 1D [17–20]. $H_{xxz} = -J\sum S_x(n)S_x(n+1) + S_y(n)S_y(n+1) + \Delta S_z(n)S_z(n+1) - E_z\sum S_z(n)$. Quantum Hall ferromagnetism constrains $J, \Delta > 0$ to have the ferromagnetic sign. Since the low-energy modes involve collective rotations of a whole region near the edge the value of the spin at each "site" of the XXZ model is large [16] $S \gg 1$.

The XXZ parameters J, Δ are functions of $\tilde{w}, \tilde{E}_c$. To match the phases in HF with those of H_{xxz}, we take the classical limit of H_{xxz}. For $\Delta < 1$ and $E_z < 4J(1-\Delta)$, the XXZ model spontaneously breaks the $U(1)$ symmetry classically, while for $\Delta > 1$, it does not. We conclude that $\Delta < 1$ in phase B of HF, while $\Delta > 1$ in phase A.

Coming to quantum fluctuations, the Mermin-Wagner theorem [21] ensures that a continuous symmetry cannot be spontaneously broken in 1D, even at zero temperature. Hence the spontaneous breaking of the $U(1)$ spin-rotation symmetry seen in HF (and the classical limit of H_{xxz}) will not survive quantum fluctuations. However, as shown by a combination of exact solution [19] and bosonization [20], H_{xxz} *still has two distinct phases for* $\Delta > 0$. The distinction between the phases lies in the presence of gapless long-wavelength spin excitations for $\Delta < \Delta_c$, while they are absent for $\Delta > \Delta_c$. The details are in the SM [16].

The physical consequences for our system are striking. Phase B will have, in addition to the charged chiral edge mode predicted in HF, a pair of gapless, chiral, counterpropagating spin modes bound to the interface. Phase A has three charged spin-resolved chiral modes (two downstream and one upstream) with no gapless long wavelength spin-flip excitations.

The classical analysis of H_{xxz} shows that the system can undergo the $B \rightarrow A$ transition even for $\Delta < 1$ upon increasing E_z. It should thus be possible to drive the $B \rightarrow A$ transition in a given sample by applying an in-plane field.

Coming to experiment, any probe that couples to low-energy long-wavelength spin fluctuations can be used to tell phases A and B apart. One such probe is NMR. The nuclear spin moments couple to the external field via their own Zeeman term and to the electronic spins via the hyperfine interaction. The total electronic spin polarization is measured by the Knight shift [22] of the frequencies of NMR resonance lines and has been used to measure the total spin polarization of QH ferromagnets [23–25]. The macroscopic nuclear spin moment relaxes via the inhomogeneous distribution of local effective magnetic fields (with relaxation time T_1) and via true energy relaxation by emitting and absorbing low-energy electronic spin degrees of freedom (the relaxation time T_2). Clearly, T_2 is the relevant quantity to detect the presence of absence of gapless electronic spin excitations. A transition from A to B will lead to a dramatic increase of the energy relaxation rate of nuclear spins and thus a decrease of T_2. Our system requires a local measurement of T_2, which may be on the verge of feasibility [26,27].

We leave several important questions for future analysis. (i) Are there phases besides A and B in a physically realistic model? It seems theoretically possible that in phase B, quantum fluctuations could gap out the spin excitations while leaving the chiral charged edge mode as the sole survivor. This phase, B^*, would be distinct from A because upstream modes (measurable in two-terminal interface charge/thermal conductance) are present in A but absent in B^*. (ii) What is the order of the $A \rightarrow B$ transition? The XXZ suggests a second order transition while HF implies first order. (iii) Can the gapless chiral spin modes in phase B carry charge? It seems possible that they can, based on the spin-charge relation in QH ferromagnets [3,4].

Acknowledgments. We would like to thank Udit Khanna for many illuminating discussions. We would also like to thank the International Center for Theoretical Sciences, Bangalore, for its hospitality and support during the workshop “Novel Phases of Quantum Matter” (Code: ICTS/Topmatter2019/12). A.S. and G.M. would like to thank the US-Israel Binational Science Foundation for its support via Grant No. 2016130. S.R. and G.M. would like to thank the VAJRA scheme of SERB, India for its support. Y.G. was also supported by CRC 183 (project C01) of the DFG, the Minerva Foundation, DFG Grant No. RO 2247/8-1, DFG Grant No. MI 658/10-1, and the GIF Grant No. I-1505-303.10/2019. We would also like to thank the University of Kentucky Center for Computational Sciences and Information Technology Services Research Computing for their support and use of the Lipscomb Compute Cluster and associated research computing resources.

[1] K. V. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).
[2] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[3] S. M. Girvin and A. H. MacDonald, in *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (John Wiley and sons, New York, 1997).
[4] K. Yang, K. Moon, L. Zheng, A. H. MacDonald, S. M. Girvin, D. Yoshioka, and S.-C. Zhang, Phys. Rev. Lett. **72**, 732 (1994).
[5] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[6] D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, Phys. Rev. B **46**, 4026 (1992).
[7] C. de C. Chamon and X. G. Wen, Phys. Rev. B **49**, 8227 (1994).
[8] J. Dempsey, B. Y. Gelfand, and B. I. Halperin, Phys. Rev. Lett. **70**, 3639 (1993).
[9] Y. Meir, Phys. Rev. Lett. **72**, 2624 (1994).
[10] J. Wang, Y. Meir, and Y. Gefen, Phys. Rev. Lett. **111**, 246803 (2013).
[11] A. H. MacDonald, Phys. Rev. Lett. **64**, 220 (1990).
[12] S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16419 (1993).
[13] M. Franco and L. Brey, Phys. Rev. B **56**, 10383 (1997).
[14] J. H. Oaknin, L. Martin-Moreno, and C. Tejedor, Phys. Rev. B **54**, 16850 (1996).
[15] U. Khanna, G. Murthy, S. Rao, and Y. Gefen, Phys. Rev. Lett. **119**, 186804 (2017).
[16] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.103.L081401 for technical details on the Hartree-Fock procedure, the stability of the phase diagram to various perturbations, etc.
[17] S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge University Press, Cambridge, UK, 2011).
[18] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, UK, 2004).
[19] F. D. M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980).
[20] H. J. Schulz, Phys. Rev. B **34**, 6372 (1986).
[21] N. D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966).
[22] C. Slichter, *Principles of Magnetic Resonance*, Springer Series in Solid-State Sciences (Springer, Berlin, Heidelberg, 1996).
[23] S. E. Barrett, G. Dabbagh, L. N. Pfeiffer, K. W. West, and R. Tycko, Phys. Rev. Lett. **74**, 5112 (1995).

[24] P. Khandelwal, A. E. Dementyev, N. N. Kuzma, S. E. Barrett, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **86**, 5353 (2001).

[25] K. R. Wald, L. P. Kouwenhoven, P. L. McEuen, N. C. van der Vaart, and C. T. Foxon, Phys. Rev. Lett. **73**, 1011 (1994).

[26] K. F. Yang, M. M. Uddin, K. Nagase, T. D. Mishima, M. B. Santos, Y. Hirayama, Z. N. Yang, and H. W. Liu, New J. Phys. **21**, 083004 (2019).

[27] K. Hashimoto, T. Tomimatsu, K. Sato, and Y. Hirayama, Nat. Commun. **9**, 2215 (2018).