

## Cubic spin-orbit coupling and anomalous Josephson effect in planar junctions

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Spin-orbit coupling in two-dimensional systems is usually characterized by Rashba and Dresselhaus spin-orbit coupling (SOC) linear in the wave vector. However, there is a growing class of materials which instead support dominant SOC cubic in the wave vector (cSOC), while their superconducting properties remain unexplored. By focusing on Josephson junctions in Zeeman field with superconductors separated by a normal cSOC region, we reveal a strongly anharmonic current-phase relation and complex spin structure. An experimental cSOC tunability enables both tunable anomalous phase shift and supercurrent, which flows even at the zero-phase difference in the junction. A fingerprint of cSOC in Josephson junctions is the *f*-wave spin-triplet superconducting correlations, important for superconducting spintronics and supporting Majorana bound states.

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Spin-orbit coupling (SOC) and its symmetry breaking provide versatile opportunities for materials design, bringing relativistic phenomena to the fore of the condensed matter physics [1–6]. While for decades SOC was primarily studied to elucidate and manipulate normal-state properties, including applications in spintronics and quantum computing [7–15], there is a growing interest to examine its role on superconductivity [16–21].

Through the coexistence of SOC and Zeeman field, a conventional spin-singlet superconductivity can acquire spin-dependent long-range proximity effects [20,22–24] as well as support topological superconductivity and host Majorana bound states, a building block for fault-tolerant quantum computing [25–27]. In both cases, Josephson junctions (JJs) provide a desirable platform to acquire spin-triplet superconductivity through proximity effects [28–38]. In contrast, even seemingly well-established intrinsic spin-triplet superconductivity in Sr<sub>2</sub>RuO<sub>4</sub> [39] is now increasingly debated [40,41].

Extensive normal-state studies of SOC in zinc-blende heterostructures usually distinguish the resulting spin-orbit fields due to broken bulk inversion symmetry, Dresselhaus SOC, and surface inversion asymmetry, Rashba SOC, and focus on their dominant linear dependence in the wave vector,  $\mathbf{k}$  [10,15]. In this linear regime, with matching strengths of these SOC's it is possible to strongly suppress the spin relaxation [42] and realize a persistent spin helix (PSH) [43,44] with a controllable spin precession over long distances [45–47].

While typically  $\mathbf{k}$ -cubic SOC contributions (cSOC) in heterostructures are neglected or considered just detrimental perturbations (for example, limiting the stability of PSH [45–47]), a more complex picture is emerging. Materials advances suggest that such cSOC, shown in Fig. 1(a), not only has to be included, but may also dominate the normal-state properties [48–57]. However, the role of cSOC in superconducting heterostructures is unexplored. It is unclear if cSOC is detrimental or desirable to key phenomena such as Joseph-

son effect, spin-triplet superconductivity, or Majorana bound states.

To address this situation and motivate further cSOC studies of superconducting properties, we consider JJs depicted in Fig. 1(b), where *s*-wave superconductors (S) are separated by a normal region with cSOC which is consistent with the two-dimensional (2D) electron or hole gas, confined along the *z* axis [48,53]. While the commonly expected current-phase

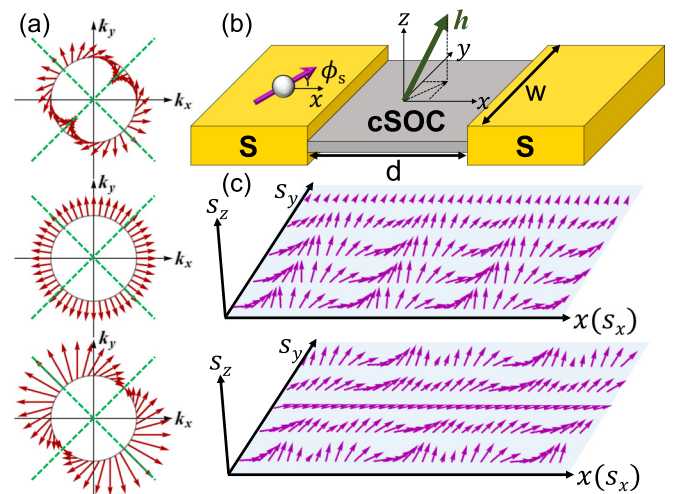


FIG. 1. (a) Spin-orbit fields in  $k$  space for Rashba cubic spin-orbit coupling (cSOC) ( $\alpha_c = -1$ ), Dresselhaus cSOC ( $\beta_c = -1$ , middle), and both ( $\alpha_c = \beta_c = -1$ , bottom). (b) Schematic of the Josephson junction. The middle region hosts cSOC and an effective Zeeman field,  $\mathbf{h}$ , between the two *s*-wave superconductors (S). (c) Spin textures in the cSOC region resulting from the normal-incident electrons with in-plane spin orientations [see Fig. 1(b)] when S is at normal state; the upper (lower) panel  $\alpha_c = 1, \beta_c = 0$  ( $\alpha_c = \beta_c = 1$ ). The in-plane spin orientations of the incident electrons  $\phi_s$  are from 0 (bottom row) to  $\pi/2$  (top row).

relation (CPR) is  $I(\varphi) = I_c \sin(\varphi + \varphi_0)$  [19,58], where  $I_c$  is the JJ critical current and  $\varphi_0$  the anomalous phase ( $\varphi_0 \neq 0, \pi$ ), we reveal that CPR can be strongly anharmonic and host Majorana bound states. Instead of the  $p$ -wave superconducting correlations for linear SOC, their  $f$ -wave symmetry is the fingerprint of cSOC.

To study cSOC, we consider an effective Hamiltonian

$$H = \frac{1}{2} \int d\mathbf{p} \hat{\psi}^\dagger(\mathbf{p}) H(\mathbf{p}) \hat{\psi}(\mathbf{p}), \quad (1)$$

where  $H(\mathbf{p}) = \mathbf{p}^2/2m^* + \boldsymbol{\sigma} \cdot \mathbf{h} + H_{\text{cSOC}}(\mathbf{p})$ , with momentum,  $\mathbf{p} = (p_x, p_y, 0)$  [see Fig. 1(b)]; effective mass,  $m^*$ ; Pauli matrices,  $\boldsymbol{\sigma}$ ; effective Zeeman field,  $\mathbf{h}$ , realized from an externally applied magnetic field or through magnetic proximity effect [6,59]; and the cSOC term [48,49,53,54]

$$H_{\text{cSOC}}(\mathbf{p}) = \frac{i\alpha_c}{2\hbar^3} (p_-^3 \sigma_+ - p_+^3 \sigma_-) - \frac{\beta_c}{2\hbar^3} (p_-^2 p_+ \sigma_+ + p_+^2 p_- \sigma_-), \quad (2)$$

expressed using cSOC strengths  $\alpha_c$  and  $\beta_c$ , for Rashba and Dresselhaus terms, where  $p_\pm = p_x \pm ip_y$  and  $\sigma_\pm = \sigma_x \pm i\sigma_y$ . The field operator in spin space is given by  $\hat{\psi}(\mathbf{p}) = [\psi_\uparrow(\mathbf{p}), \psi_\downarrow(\mathbf{p})]^T$ , with  $\uparrow, \downarrow$  spin projections.

To describe S regions in Fig. 1(b), we use an  $s$ -wave BCS model with a two-electron amplitude in spin-Nambu space  $\Delta(\psi_\uparrow^\dagger \psi_\downarrow^\dagger) + \text{H.c.}$ , given by the effective Hamiltonian in particle-hole space

$$\mathcal{H}(\mathbf{p}) = \begin{pmatrix} H(\mathbf{p}) - \mu \hat{1} & \hat{\Delta} \\ \hat{\Delta}^\dagger & -H^\dagger(-\mathbf{p}) + \mu \hat{1} \end{pmatrix}, \quad (3)$$

where  $\mu$  is the chemical potential and  $\hat{\Delta}$  is a  $2 \times 2$  gap matrix in spin space. The field operators in the rotated particle-hole and spin basis are  $\hat{\psi} = (\psi_\uparrow, \psi_\downarrow, \psi_\uparrow^\dagger, -\psi_\downarrow^\dagger)^T$ .

To calculate the charge current, we use its quantum definition where no charge sink or source is present. Therefore, the time variation of charge density vanishes,  $\partial_t \rho_c \equiv 0 = \lim_{\mathbf{r} \rightarrow \mathbf{r}'} \sum_{\sigma\tau\sigma'\tau'} [\psi_{\sigma\tau}^\dagger(\mathbf{r}') \mathcal{H}_{\sigma\tau\sigma'\tau'}(\mathbf{r}) \psi_{\sigma'\tau'}(\mathbf{r}) - \psi_{\sigma\tau}^\dagger(\mathbf{r}') \mathcal{H}_{\sigma\tau\sigma'\tau'}^\dagger(\mathbf{r}') \psi_{\sigma'\tau'}(\mathbf{r})]$ .  $\mathcal{H}_{\sigma\tau\sigma'\tau'}$  is the component form of  $\mathcal{H}$ , with spin (particle-hole) label  $\sigma$  ( $\tau$ ), and  $\mathbf{r} \equiv (x, y, 0)$ . From the current conservation, the charge current density is  $\mathbf{J} = \int d\mathbf{r} \{ \hat{\psi}^\dagger(\mathbf{r}) \vec{\mathcal{H}}(\mathbf{r}) \hat{\psi}(\mathbf{r}) - \hat{\psi}^\dagger(\mathbf{r}) \vec{\mathcal{H}}(\mathbf{r}) \hat{\psi}(\mathbf{r}) \}$ , where  $\mathcal{H}(\mathbf{r})$  is obtained by substituting  $\mathbf{p} \equiv -i\hbar(\partial_x, \partial_y, 0)$ . The arrow directions indicate the specific wave functions that the  $\mathcal{H}$  operates on. By an exact diagonalization of  $\mathcal{H}$ , we obtain spinor wave functions  $\hat{\psi}^{l,r,m}(\mathbf{p})$  within the left ( $x < 0$ ) and right ( $x > d$ ) S region and the middle normal region ( $0 < x < d$ ) in Fig. 1(b). The wave functions and generalized velocity operators  $v_x^{l,r,m}$  are continuous at the junctions, i.e.,  $\hat{\psi}^l = \hat{\psi}^m|_{x=0}$ ,  $\hat{\psi}^m = \hat{\psi}^r|_{x=d}$ ,  $v_x^l \hat{\psi}^l = v_x^m \hat{\psi}^r|_{x=0}$ , and  $v_x^m \hat{\psi}^m = v_x^r \hat{\psi}^r|_{x=d}$ . The spinor wave functions are given in the Supplemental Material [60].

The complexity of  $\mathcal{H}$  precludes simple solutions and we evaluate the wave functions and supercurrent numerically. To reduce the edge effects, we consider Fig. 1(b) geometry with  $W/d \gg 1$  [61]. This approach has been successfully used to study supercurrent in junctions with PSH, Weyl semimetals, phosphorene, and twisted bilayer graphene [62–68]. The calculated supercurrent is normalized by  $I_0 = 2|e\Delta|/\hbar$ , where  $e$

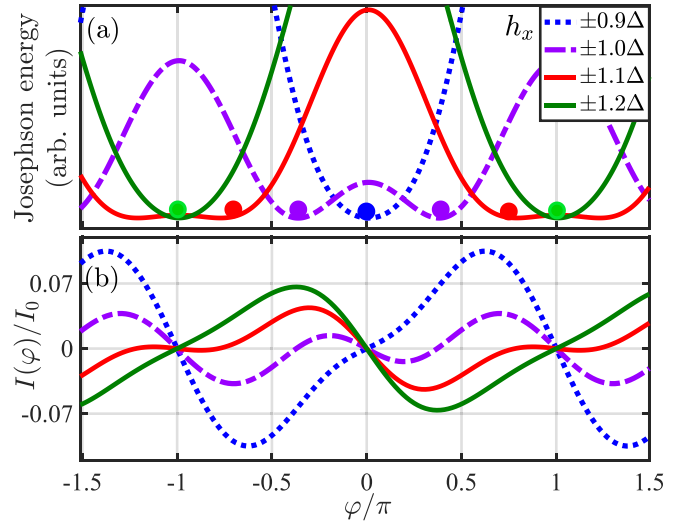


FIG. 2. (a) Josephson energy and (b) associated supercurrent evolution with the superconducting phase difference  $\varphi$ . Zeeman field values,  $h_x$ , are chosen near a  $0$ - $\pi$  transition. The other parameters are  $\alpha_c = \pm 0.1$  and  $\beta_c = 0$ ,  $\mu = \Delta$ ,  $h_y = 0$ .

is the electron charge, and  $\Delta$  the energy gap in S. The energies are normalized by  $\Delta$ , lengths by  $\xi_S = \hbar/\sqrt{2m^*\Delta}$ , and cSOC strengths by  $\Delta\xi_S^3$ . The junction length is set at  $d = 0.3\xi_S$ .

To investigate the role of cSOC on the ground-state Josephson energy,  $E_{\text{GS}}$ , and the CPR obtained from the supercurrent  $I(\varphi) \propto \partial E_{\text{GS}}/\partial\varphi$ , we first consider a simple situation with only Rashba cSOC ( $\alpha_c \neq 0$ ,  $\beta_c = 0$ ) and effective Zeeman field  $h_x$  ( $h_y = h_z = 0$ ). The evolution of  $E_{\text{GS}}$  with  $|h_x|$ , where its minima are denoted by dots in Fig. 2(a), shows a continuous transition from  $\varphi = 0$  to  $\pi$  state (blue to green dot). For  $\varphi_0 \neq 0$ ,  $E_{\text{GS}}$  minima come in pairs at  $\pm\varphi_0$  [69]. The corresponding CPR reveals in Fig. 2(b) a competition between the standard,  $\sin\varphi$ , and the next harmonic,  $\sin 2\varphi$ , resulting in  $I(-\varphi) = -I(\varphi)$ . There is no spontaneous current expected in a Josephson junction with SOC,  $I(\varphi = 0) = 0$ , but only  $I_c$  reversal with  $h_x$ . Such a scenario of a continuous and symmetric  $0$ - $\pi$  transition is well studied without SOC in S/ferromagnet/S JJs due to the changes in the effective magnetization or a thickness of the magnetic region [70–77].

While our previous results suggest no direct cSOC influence on CPR, a simple in-plane rotation of  $\mathbf{h}$ ,  $h_x = 0$ ,  $h_y \neq 0$ , drastically changes this behavior. This is shown in Fig. 3(b) where, at fixed  $|h_y| = 2.4\Delta$ , we see a peculiar influence of a finite Rashba cSOC which is responsible for the anomalous Josephson effect with spontaneous current,  $I(\varphi = 0) \neq 0$ , and strong anharmonic CPR that cannot be described by  $I(\varphi) = I_c \sin(\varphi + \varphi_0)$ . Unlike in Fig. 3(a), a relative sign between  $\alpha_c$  and  $h$  alters the CPR and Josephson energy, where the ground states  $\varphi_0$  appear at single points [green, red dots in Fig. 3(a)], consistent with  $\varphi_0 \propto \alpha_c h_y$ .

If instead of  $\mu = \Delta$ , we consider a regime  $\mu \gg \Delta$ , the evolution of Josephson energy from Fig. 2(a) changes. While  $0$ - $\pi$  transitions with  $|h_x|$  remain, there are no longer global minima with  $\varphi \neq 0, \pi$  and the CPR reveals a stronger anharmonicity. In contrast, for  $\mu \gg \Delta$ , the anomalous Josephson effect from Fig. 3 remains robust and similar  $\varphi_0$  states are accessible [60].

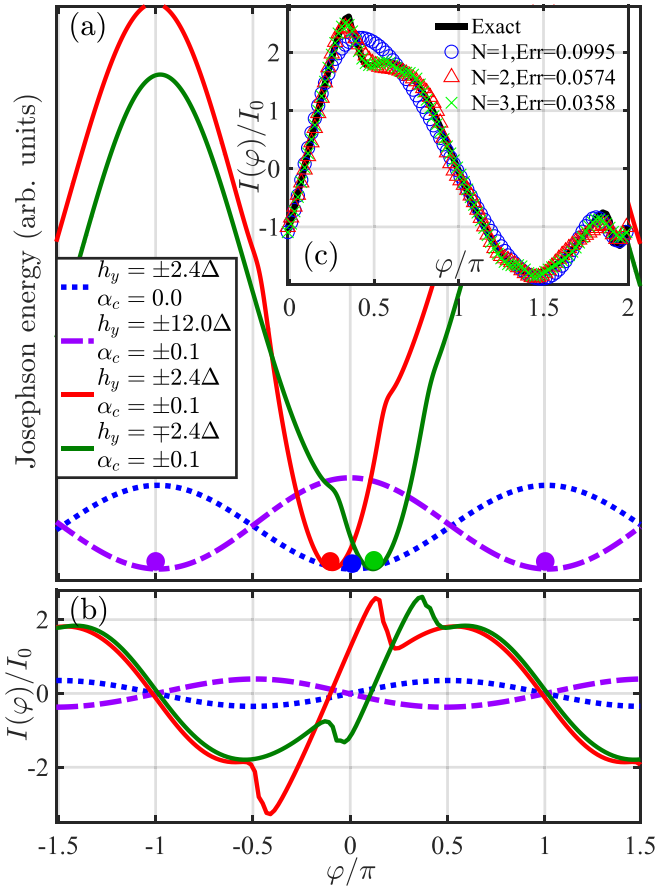


FIG. 3. (a) Josephson energy and (b) related supercurrent evolution with the superconducting phase difference  $\varphi$  Zeeman field,  $h_y$ , at a fixed magnitude and varying Rashba cSOC strength  $\alpha_c$  are considered. The other parameters are  $\beta_c = 0$ ,  $\mu = \Delta$ ,  $h_x = 0$ . (c) Three fits to the green curve in (b) using the generalized CPR from Eq. (4) with  $N = 1, 2, 3$  harmonics.

Simple harmonics used to describe anharmonic CPR in high-temperature superconductors [78,79] here are not very suitable. By generalizing a short-junction limit for CPR [77,78,80], we identify a much more compact form where only a small number of terms gives an accurate description. To recognize the importance of SOC and two nondegenerate spin channels,  $\sigma$ , we write

$$I(\varphi) \approx \sum_{n=1}^N \sum_{\sigma=\pm} \frac{I_n^\sigma \sin(n\varphi + \varphi_{0n}^\sigma)}{\sqrt{1 - \tau_n^\sigma \sin^2(n\varphi/2 + \varphi_{0n}^\sigma/2)}}, \quad (4)$$

where  $\tau_n^\sigma$  is the normal region transparency for spin channel  $\sigma$ . With only few lowest terms in this expansion ( $N = 1, 2, 3$ ), shown in Fig. 3(c) with the corresponding errors, it is possible to very accurately describe strong CPR anharmonicities for anomalous Josephson effect. To achieve the relative error from  $N = 3$  expansion in Eq. (4), in a standard  $\{\sin, \cos\}$  expansion, with the corresponding phase shifts as extra fitting parameters, requires  $N > 20$  [60].

Key insights into the CPR and an explicit functional dependence for the  $\varphi_0$  state are obtained by a systematic  $I(\varphi)$  symmetry analysis with respect to the cSOC ( $\alpha_c, \beta_c$ ) and Zeeman field or, equivalently, magnetization ( $h_{x,y,z}$ ) parameters

[60]. We find that  $h_z$  plays no role in inducing the  $\varphi_0$  state; it only produces  $I(\varphi)$  reversals, explaining our focus on  $h_z = 0$  (Figs. 2 and 3).

These properties are expressed as an effective phase shift to a sinusoidal CPR,  $\sin(\varphi + \varphi_0)$ , extracted from Eq. (4). We again distinguish small- and large- $\mu$  regime ( $\mu = \Delta$  vs  $\mu = 10\Delta$ ). In the first case, for the JJ geometry from Fig. 1, we obtain

$$\varphi_0 \propto \Gamma_y(\alpha_c^2 + \Gamma_1\beta_c^2)h_x\beta_c + \Gamma_x(\alpha_c^2 - \Gamma_2\beta_c^2)h_y\alpha_c, \quad (5)$$

where the parameters  $\Gamma_{1,2,x,y}$  are introduced through their relations,  $\Gamma_2 > \Gamma_1$ ,  $\Gamma_1 < 1$ ,  $\Gamma_2 > 1$ ,  $\Gamma_y(h_y = 0) = \Gamma_x(h_x = 0) = 1$ ,  $\Gamma_y(h_y \neq 0) < 1$ ,  $\Gamma_x(h_x \neq 0) < 1$ . These relations are modified as  $\mu$  and  $\mathbf{h}$  change. For  $\mu \gg \Delta$ , the functional dependence for the  $\varphi_0$  state is simplified:

$$\varphi_0 \propto (\alpha_c^2 - \Gamma_1\beta_c^2)h_x\beta_c + (\alpha_c^2 - \Gamma_2\beta_c^2)h_y\alpha_c, \quad (6)$$

where  $\Gamma_2 > \Gamma_1$  and  $\Gamma_{1,2} > 1$ . Therefore,  $\varphi_0$  state occurs when  $\mathbf{h}$  shifts  $\mathbf{p} \perp$  to  $\mathbf{I}(\varphi)$  and thus alters the SOC [60].

Taken together, these results reveal that cSOC in JJ supports a large tunability of the Josephson energy, anharmonic CPR, and the anomalous phase, key to many applications, from post-CMOS logic, superconducting spintronics, quiet qubits, and topological quantum computing. Realizing  $\pi$  states in JJs is desirable for improving rapid single flux quantum (RSFQ) logic, with operation  $> 100$  GHz [81,82] and enhancing coherence by decoupling superconducting qubits from the environment [83]. However, common approaches for  $\pi$  states using JJs combining  $s$ - and  $d$ -wave superconductors or JJs with ferromagnetic regions [78,79] pose various limitations. Instead, extensively studied gate-tunable SOC [10,38,45,53,54,84] could allow not only a fast transformation between 0 and  $\pi$  states in JJs with cSOC, but also an arbitrary  $\varphi_0$  state to tailor desirable CPR.

An insight to the phase evolution and circuit operation of JJs with cSOC is provided by generalizing the classical model of resistively and capacitively shunted junction (RSCJ) [85]. The total current,  $i$ , is the sum of the displacement current across the capacitance,  $C$ , normal current characterized by the resistance,  $R$ , and  $I(\varphi)$ ,

$$\frac{\varphi_0}{2\pi} C \frac{d^2\varphi}{dt^2} + \frac{\varphi_0}{2\pi R} \frac{d\varphi}{dt} + I(\varphi) = i, \quad (7)$$

where  $\varphi_0$  is the magnetic flux quantum and  $I(\varphi)$  yields a generally anharmonic CPR, as shown from Eq. (4), which can support 0,  $\pi$ , and turnable  $\varphi_0$  states. As we have seen from Figs. 2 and 3, this CPR tunability is accompanied by the changes in Josephson energy, which in turn is responsible for the changes in effective values of  $C$ ,  $R$ , and the nonlinear Josephson inductance. This JJ tunability complements using voltage or flux control [86,87].

In JJs with ferromagnetic regions,  $I_c$  is the tunable  $I_c$  by changing the underlying magnetic state [32,88,89]. In JJs with cSOC, tuning  $I_c$  could be realized through gate control by changing the relative strengths of  $\alpha_c$  and  $\beta_c$ , even at zero Zeeman field. This is shown in Fig. 4 by calculating  $\text{Max}[I(\varphi)]$  with  $\varphi \in [0, 2\pi]$ . In the low- $\mu$  regime, the maximum  $I_c$  occurs at the slightly curved region near the symmetry lines  $|\alpha_c| = |\beta_c|$ . For the high- $\mu$  regime, the region of maximum  $I_c$  evolves into inclined symmetry lines,  $|\alpha_c| = \mathcal{A}|\beta_c|$ ,  $\mathcal{A} < 1$ .



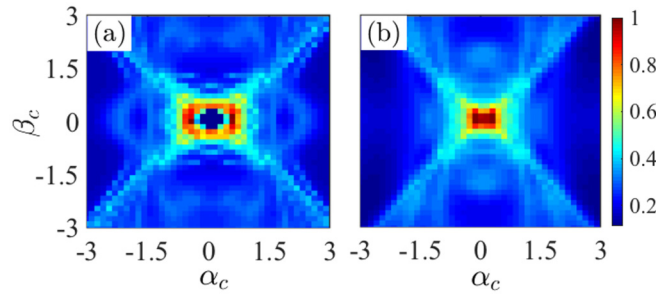


FIG. 4. Normalized critical supercurrent as a function of cSOC strength  $\alpha_c$  and  $\beta_c$  for (a)  $\mu = \Delta$  and (b)  $\mu = 10\Delta$ . The Zeeman field is set to zero.

Similar to linear SOC, in the diffusive regime for cSOC, one expects that the minimum in  $I_c$  occurs near these symmetry lines because of the presence of long-range spin-triplet supercurrent [63,90,91].

We expect that a hallmark of JJs with cSOC goes beyond CPR and will also influence the spin structure and symmetry properties of superconducting proximity effects. Linear SOC is responsible for mixed singlet-triplet superconducting pairing [16], while with Zeeman or exchange field it is possible to favor spin-triplet proximity effects which can become long range [20,33] or host Majorana bound states [25,26]. To explore the proximity effects in the cSOC region, we calculate superconducting pair correlations using the Matsubara representation for the anomalous Green's function,  $F(\tau; \mathbf{r}, \mathbf{r}')$  [92],

$$F_{ss'}(\tau; \mathbf{r}, \mathbf{r}') = +\langle T_\tau \psi_s(\tau, \mathbf{r}) \psi_{s_1}(0, \mathbf{r}') \rangle (-i\sigma_{s_1 s'}^y), \quad (8)$$

where  $s, s', s_1$  are spin indices, the summation is implied over  $s_1$ ,  $\tau$  is the imaginary time,  $\psi_s$  is the field operator, and  $T_\tau$  denotes time ordering of operators [60].

For a translationally invariant SOC region, spin-triplet correlations in Fig. 5, obtained from Eq. (8), provide a striking difference between linear and cubic SOC. Unlike the  $p$ -wave symmetry for linear Rashba SOC [Figs. 5(a) and 5(b)], we see that the  $f$ -wave symmetry is the fingerprint for cSOC, retained with only  $\alpha_c \neq 0$  [Figs. 5(c) and 5(d)] or both  $\alpha_c, \beta_c \neq 0$  [Figs. 5(e) and 5(f)]. Remarkably, unlike the commonly sought  $p$ -wave symmetry, we confirm that with a suitable orientation of the Zeeman field cSOC also supports Majorana flatbands [60].

While we are not aware of any Josephson effect experiments in 2D systems dominated by cSOC, our studied parameters are within the range of already reported measurements. Choosing  $m^*$  of an electron mass, and  $\Delta = 0.2$  meV, which is similar for both AI and proximity-induced superconductivity [38,93], the characteristic length becomes  $\xi_S \approx 14$  nm. The resulting cSOC strength from Fig. 3(b) with  $\alpha_c \Delta \xi_S^3 \approx 50$  eV  $\text{\AA}^3$  is compatible with the values in 2D

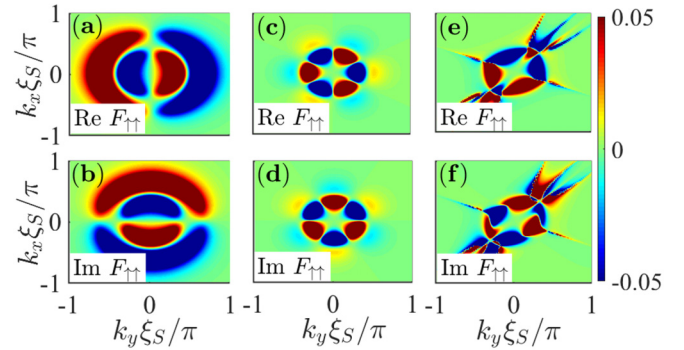


FIG. 5. Real and imaginary parts of equal-spin superconducting correlations in  $k$  space;  $\xi_S = \hbar/\sqrt{2m^*}\Delta$  is the characteristic length. (a), (b) Linear Rashba,  $\alpha = 1$ . (c), (d) cSOC,  $\alpha_c = 1, \beta_c = 0$ . (e), (f) cSOC,  $\alpha_c = \beta_c = 1$ . The other parameters are the same for all panels.

electron and hole gases [55,56]. The Zeeman splitting  $2.4 \times 0.2$  meV is available by applying magnetic field in large  $g$ -factor materials [10], or from magnetic proximity effects, measured in 2D systems to reach up to  $\sim 20$  meV [6]. Even though we have mostly focused on the tunable Rashba SOC, the Dresselhaus SOC can also be gate tunable [45,94], offering a further control of the anomalous Josephson effect.

Our results reveal that the cSOC in JJs provides versatile opportunities to design a superconducting response and test its unexplored manifestations. The anomalous Josephson effect could serve as a sensitive probe to quantify cSOC. While identifying the relevant form of SOC is a challenge even in the normal state [10,12], in the superconducting state already a modest SOC can give a strong anisotropy in the transport properties [22,24,95,96] and enable extracting the resulting SOC. Identifying SOC, either intrinsic, or generated through magnetic textures, remains important for understanding which systems could host Majorana bound states [37,97–110]. While in semiconductor heterostructures the normal-state cubic SOC was mostly studied in a zinc-blende phase, interesting opportunities also exist in the wurzite phase [111–113].

With the advances in gate-tunable structures and novel materials systems [38,53–56,93,114], the functional dependence of the anomalous phase  $\varphi_0$  and the  $f$ -wave superconducting correlations could also enable decoupling of the linear and cubic SOC contributions [60]. For the feasibility of such decoupling, it would be useful to consider methods employed in the studies of the nonlinear Meissner effect [115–124]. Even small corrections to the supercurrent from the magnetic anisotropy of the nonlinear Meissner response offer a sensitive probe to distinguish different pairing-state symmetries.

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