Hydrodynamics of nonintegrable systems from a relaxation-time approximation

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We develop a general kinetic theory framework to describe the hydrodynamics of strongly interacting, nonequilibrium quantum systems in which integrability is weakly broken, leaving a few residual conserved quantities. This framework is based on a generalized relaxation-time approximation; it gives a simple, but surprisingly accurate, prescription for computing nonequilibrium transport even in strongly interacting systems. We validate the predictions of this approximation against matrix product operator calculations on chaotic quantum spin chains, finding surprisingly good agreement. We show that despite its simplicity, our framework can capture phenomena distinctive to strongly interacting systems, such as widely separated charge and energy diffusion constants.

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Hydrodynamics has experienced a revival in the past decade as an effective theory of strongly interacting quantum matter far from equilibrium [1–10]. A major factor in this revival has been the advent of new experimental platforms, from quark-gluon plasmas [11] to strongly interacting ultracold gases [12,13] and pristine solid-state systems that feature strong interactions and long mean free times [14–17]. Hydrodynamics is particularly rich for low-dimensional fluids, featuring transport anomalies such as long-time tails [18–23]; in *one* dimension, hydrodynamics is further enriched by the proximity of many realistic systems to integrability. In the integrable limit, conventional hydrodynamics breaks down, and a new framework, called "generalized hydrodynamics" (GHD), has been developed [24–50]. GHD incorporates the distinctive features of integrable dynamics: namely, the presence of infinitely many conservation laws and of stable ballistically propagating quasiparticles. This framework has led to quantitative explanations of many phenomena, including Drude weights [27,29,31,35], diffusion constants [39–41,51–53], and the presence of anomalous transport in strongly interacting spin chains [51,54–66].

Realistic systems, however, are only *approximately* integrable. On short timescales they obey GHD, but on the longest timescales they cross over to conventional hydrodynamics. A general theory of this crossover has remained elusive, despite recent progress [21,56,67–90]. In principle, one can write a collisional Boltzmann equation for weak integrability breaking [84,85]. However, in general the collision integral is intractable, as it depends on *all* the matrix elements of the integrability-breaking perturbation. In special cases, such as long-range interactions, slowly fluctuating noise, or weakly interacting systems, the integrability-breaking perturbation can itself be expressed in terms of GHD data [84,85]. More generally, however, integrability-breaking perturbations lie outside GHD: for example, umklapp scattering involves

large momentum transfer, and thus cannot be captured by a long-wavelength theory such as GHD. In the absence of the GHD framework, evaluating the collision integral is an intractable task.

This work addresses the question of integrability breaking from a fundamentally different perspective. Instead of microscopically deriving the collision integral, we adopt a simple but general approximation, which we call the "generalized relaxation time approximation" (GRTA), by analogy with the conventional relaxation time approximation (RTA) for weakly interacting electrons [91]. The GRTA assumes that there is a single dominant relaxation time that controls the onset of chaos. This assumption allows us to efficiently simulate dynamics away from the integrable limit. Although our approach resembles the conventional RTA in positing a unique relaxation time, its implementation and physical consequences are completely different. The RTA deals with nearly free particles, so their scattering kinematics is simple. By contrast, in an interacting integrable system, the momentum carried by each quasiparticle is a nonlinear functional of the full quasiparticle distribution function. Thus, when one describes a scattering process in an integrable system, not only the matrix elements but also the δ functions conserving momentum and energy are nontrivial to evaluate.

Instead, we implement the GRTA as follows. In GHD, one regards a system as locally being in a generalized Gibbs ensemble (GGE) [92–94], with chemical potentials for each conservation law [94]. The key step in our approach is to replace the local GGE with a *local* thermal Gibbs state, subject to the residual conservation laws, at some finite rate $1/\tau$ (where τ is the generalized relaxation time). The main assumption is that there is a unique *local* relaxation rate for the quasiparticle distribution function. This is justified under certain assumptions, and (as we discuss below) fails sometimes; however, we find that it is remarkably

accurate at reproducing numerical time evolution, even when the integrability-breaking perturbations are not especially small. For initial states far from equilibrium, the GRTA (unlike the RTA) gives rise to nontrivial relaxation dynamics, as the local equilibrium state is a nontrivial functional of the local quasiparticle distribution. Moreover, contrary to the simplest implementation of the RTA, the GRTA preserves conservation laws and is suitable to study hydrodynamics. Thus, we argue that the GRTA captures the "generic" crossover from generalized to conventional hydrodynamics.

Boltzmann equation. GHD describes the dynamics of integrable systems in terms of their quasiparticles. We characterize quasiparticles with a given quantum number ("rapidity") λ by their density $\rho_\lambda(x, t)$. Note that λ is a shorthand for both continuous and discrete labels. The distribution of quasiparticles $\rho_\lambda(x, t)$ is in one-to-one correspondence with a local equilibrium macrostate [95]. In an integrable system with conserved charges $\{\hat{Q}_n\}$, local equilibrium can be equivalently characterized by a generalized Gibbs ensemble (GGE) density matrix $\hat{\rho}_{\text{GGE}} = Z^{-1} e^{-\sum_n \beta_n \hat{Q}_n}$. In integrable systems, quasiparticles scatter elastically with phase shifts leading to Wigner time delays [29,33]: the effective velocity $v_\lambda^{\text{eff}}[\rho]$ of a quasiparticle with rapidity λ depends on the density of all the other quasiparticles [24,25,46,96]. Transport properties can be inferred from the fact that quasiparticles carry some charge $h_i(\lambda)$, where i labels the conserved charges of the integrable system. The density of charge i reads $q_i(x, t) = \int d\lambda\, h_i(\lambda) \rho_\lambda(x, t)$, with the associated Euler current $j_i(x, t) = \int d\lambda\, h_i(\lambda) \rho_\lambda(x, t) v_\lambda^{\text{eff}}[\rho] + \cdots$, where "$\cdots$" represents higher-order (diffusive) corrections [39–41,52] that will be negligible for our purposes. The conservation laws $\partial_t q_i + \partial_x j_i = 0$ form the basis of GHD [24,25].

We now imagine perturbing such an integrable system with Hamiltonian $\hat{H}_0$ by a small, nonintegrable perturbation $\hat{V}$ of order g that destroys all but a few conservation laws. We assume that the expressions for charges and currents are unchanged—neglecting $O(g)$ corrections to these quantities, and force terms that are treated elsewhere [44]. The leading effect of the nonintegrable perturbation is to thermalize quasiparticle distributions at long times $t \gg O(g^{-2})$. Integrability breaking endows the GHD equation with a collision integral

$$\partial_t \rho_\lambda + \partial_x\big(v_\lambda^{\text{eff}}[\rho] \rho_\lambda\big) = \mathcal{I}_\lambda[\rho] \tag{1}$$

that mixes quasiparticle sectors. This collision integral $\mathcal{I}_\lambda$ can in principle be derived perturbatively using Fermi's golden rule (FGR), and is $O(g^2)$ [84,85,87]. It involves the matrix elements (form factors) of the integrability breaking perturbations, which can be expressed in terms of hydrodynamical data only for noninteracting systems, and for perturbations involving low momentum transfer such as slowly varying noisy potentials or long-range interactions [84]. Equation (1) was analyzed within linear response in Ref. [84], and it was shown to lead to diffusive hydrodynamics in general.

Generalized relaxation-time approximation. For most physical integrability-breaking perturbations, the matrix elements of the perturbation cannot be expressed in terms of hydrodynamic data. In the few cases in which the collision integrals can be written down explicitly, they are impractical to implement numerically, even for simple physical processes

like particle loss in a Bose gas [97]. For context, we remark that even for weakly interacting fermions, collision integrals are often approximated by using the relaxation-time approximation (RTA), which suffices to capture most of the relaxation physics and to describe experiments. Here, we introduce a generalized relaxation-time approximation (GRTA), which amounts to choosing a simple form for the collision integral:

$$\partial_t \rho_\lambda + \partial_x\big(v_\lambda^{\text{eff}}[\rho] \rho_\lambda\big) = -(\rho_\lambda - \rho_\lambda^{\text{Gibbs}}[\rho])/\tau. \tag{2}$$

This right-hand side enforces local thermalization on a typical relaxation timescale τ as follows: $\rho_\lambda^{\text{Gibbs}}[\rho]$ is a nonlinear functional of the state ρ_λ, defined as the distribution of quasiparticles of a Gibbs state with the same value of the conserved quantities q_α ($\alpha = 1, \ldots, N$ corresponding to the charges preserved by the integrability breaking perturbation) as the state ρ_λ. For example, consider a Bose gas where the integrability-breaking perturbation preserves energy E, particle number N, and momentum P. Then the distribution $\rho_\lambda^{\text{Gibbs}}[\rho]$ corresponds to the (boosted) Gibbs ensemble density matrix $\hat{\rho}_{\text{Gibbs}} = \frac{1}{Z} e^{-\beta(\hat{H} - \mu \hat{N} - \nu \hat{P})}$, where β, μ, and ν are chosen so that the average particle number, energy, and momentum are the same as in the state ρ_λ. By definition, we have $\int d\lambda (\rho_\lambda - \rho_\lambda^{\text{Gibbs}}) h_\alpha(\lambda) = 0$, ensuring the conservation of the charges $\hat{Q}_\alpha$.

Physically, the GRTA assumes that local relaxation is controlled by a single relaxation rate. Of course, realistic FGR collision integrals have a lot more structure, involving a hierarchy of relaxation rates. However, we expect this approximation to capture the key physics of integrability-breaking. One can formalize this intuition as follows. The relaxation of charges in the presence of weak integrability-breaking is captured by the equation $\partial_t Q_i = -\sum_j \Gamma_{ij} Q_j$, where $\mathbf{\Gamma}$ is a matrix that is itself a functional of the equilibrium state [84,85]. The spectrum of the matrix $\mathbf{\Gamma}$ contains zero modes corresponding to the residual conserved charges, as well as other eigenmodes that capture the characteristic decay rates. If there is a gap between the zero modes and the decaying modes, one can identify this gap with $1/\tau$, and replace the matrix $\mathbf{\Gamma}$ with a projector onto modes that decay at rate $\sim 1/\tau$, which is justified at long enough times where $e^{-t/\tau}$ will dominate exponentials decaying with faster rates. The GRTA corresponds to replacing $\mathbf{\Gamma}^{-1} \approx \tau$ for all decaying charges, which approximately coincides with the projection approach, provided that all residual conserved currents have approximately similar overlaps with the slowest-decaying modes of $\mathbf{\Gamma}$. (This construction indicates that the GRTA will fail whenever there are arbitrarily slowly relaxing modes, as we expect on physical grounds, and also when the currents of residual charges have very different overlaps with the slowest-relaxing modes of $\mathbf{\Gamma}$.)

We evaluate the right-hand side of Eq. (2) as follows. We compute the (density of) conserved charges q_α (say particle number, momentum, and energy) in the state $\rho_\lambda(x, t)$, and we invert the equation of states of the model—known from the equilibrium thermodynamic Bethe ansatz (TBA) [98]—to find the Lagrange multipliers (in our example, β, μ, and ν) of the Gibbs state corresponding to those values. Using the TBA, we then compute the density of quasiparticles $\rho_\lambda^{\text{Gibbs}}[\rho]$ corresponding to those Lagrange multipliers and thus I_λ [99].

Note that we use the equation of states of the *unperturbed* (integrable) model. This is justified perturbatively by the fact that the integrability-breaking perturbation smoothly modifies thermodynamic quantities and the equation of states (with small changes if the perturbation is weak), while it dramatically affects the dynamics at long times. We take τ to be an unknown constant, a single phenomenological parameter to be determined by comparing the solution of Eq. (2) to numerics or experiments.

Numerical solution. To implement this GRTA scheme numerically, we develop a general numerical scheme to solve (1), which can be used both near and far from equilibrium. Following the numerical methods of Refs. [28,44,49] in the integrable case, we find it convenient to work with the "normal modes" of GHD, which are given by the occupation ratios (Fermi factors) $n_\lambda = \rho_\lambda / \rho_\lambda^{\mathrm{tot}}$, where $\rho_\lambda^{\mathrm{tot}} = \rho_\lambda + \rho_\lambda^h$ is the total density of states at rapidity λ, and ρ_λ^h is the density of holes. There is a one-to-one correspondence between the density of quasiparticles ρ_λ and the occupation ratios n_λ, provided by the Bethe equations. In terms of n_λ, the Boltzmann equation (1) takes the *advection* form

$$\partial_t n_\lambda + v_\lambda^{\mathrm{eff}}[n]\partial_x n_\lambda = I_\lambda[n], \qquad (3)$$

where I_λ is simply related to $\mathcal{I}_\lambda[\rho]$ [99]. We then solve this equation by finite elements, discretizing space, time, and rapidity. We use a backward first-order scheme $n_\lambda(x,t) = n_\lambda(x - v_\lambda^{\mathrm{eff}}[n(x,t)]\Delta t, t - \Delta t) + \Delta t I_\lambda[n(x,t)]$, where crucially, the velocity and collision integrals on the right-hand side are evaluated at time t to improve stability. We solve this equation by iteration, and we check convergence with respect to the small parameters Δt, Δx, and $\Delta\lambda$.

Energy transport in spin chains. The GRTA approach has the advantage of being very general, and it can be applied to chaotic spin chains near integrability. To illustrate this, we consider the spin-$\frac{1}{2}$ XXZ spin chain with integrability-breaking perturbations

$$\hat{H} = \sum_i \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z \right) + \hat{V}, \qquad (4)$$

with anisotropy $\Delta = \frac{1}{2}$, and $\hat{V} = h_x \sum_i (-1)^i \hat{S}_i^x$ or $\hat{V} = J_x' \sum_i (-1)^i \hat{S}_i^x \hat{S}_{i+1}^x$. When $\hat{V} = 0$, this model is integrable, and energy transport is purely ballistic as the total energy current is a conserved quantity. As higher-order corrections vanish exactly, energy transport can be captured extremely well by GHD [28]. The staggered perturbation $\hat{V}$ breaks integrability and the $U(1)$ symmetry of the XXZ model.

We consider energy transport in the Hamiltonian (4) by preparing a local region with temperature $T = 10$ embedded in a uniform equilibrium background with temperature $T = 2$ [100]. We simulate the dynamics of this system up to time $t = 20$ by evolving the density matrix using time-evolving block decimation (TEBD) [101–103] and compare with the GRTA (2) for various values of τ. We compare the local temperature profiles $T(x,t)$ between the two approaches, using the equilibrium equation of state of Eq. (4) to convert energy density to temperature. (This accounts for the shift in the equilibrium energy density due to the perturbation $\hat{V}$, which can readily be captured using perturbation theory.) We find a best fit for the
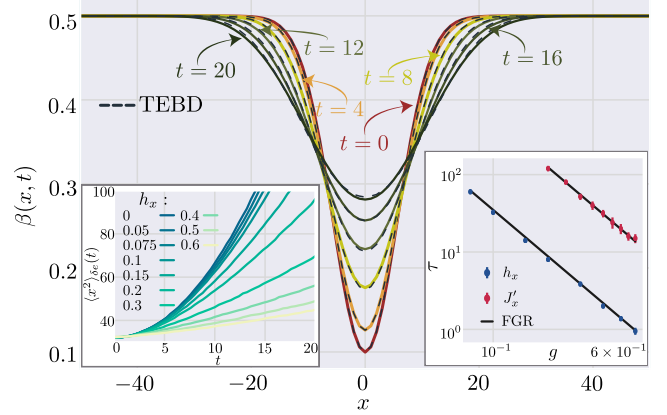

FIG. 1. Energy transport in nonintegrable spin chains: inverse temperature profiles $\beta(x,t) = 1/T(x,t)$ in an XXZ spin chain with a staggered transverse field h_x breaking integrability. The TEBD data for $h_x = 0.2$ are described very well by Eq. (3) and GRTA with $\tau \simeq 8$. Left inset: Variances of the energy profiles vs time from TEBD, for various values of h_x, showing a crossover between ballistic and diffusive transport. Right inset: The fitted values of τ agree with the FGR scaling (5) for both $g = h_x$ (staggered x-fields) and $g = J_x'$ (staggered xx-couplings).

single parameter τ by matching the full temperature profiles from the TEBD simulations and the GRTA.

We find that the GRTA is able to describe the nonintegrable dynamics of (4) remarkably well with a single parameter τ for each $\hat{V}$, for various values of h_x or J_x' ranging from 0.05 to 0.6, corresponding to almost two decades in τ. Moreover, the fitted values of τ all agree very well with the simple FGR scaling

$$\tau \simeq Cg^{-2}, \qquad (5)$$

with $C \approx 0.32(5)$ for $g = h_x$ (staggered x-fields), and $C \approx 4.95(5)$ for $g = J_x'$ (staggered xx-couplings). This is remarkable, as in general we expect that relaxation times should depend on temperature, and the initial state considered has a wide range of temperatures. Allowing for limited dependence of τ on the state ρ—such as through the local temperature— might be necessary to capture strongly nonequilibrium setups with even wider temperature ranges. While the variance of the profiles of the local perturbation in energy grows quadratically (indicating ballistic transport) in the integrable case, it crosses over to linear (diffusive) growth for times $t \gg \tau$.

This scaling implies that the whole time evolution for all values of g we consider can be described quite accurately using a single free parameter C. While we obviously expect corrections to this GRTA approach, combined with the expected FGR scaling (5), it clearly captures most of the physics of integrability breaking. Surprisingly, the GRTA is able to describe energy transport even for strongly chaotic chains for which the relaxation time τ is $O(1)$.

To illustrate the predictive power of the GRTA, we study energy transport for a more complicated inhomogeneous initial state, for various values of the staggered field h_x, comparing the GRTA to TEBD (Fig. 2). Note that there is no free parameter here, as the values of the relaxation time $\tau(h_x)$ are fixed from the analysis of Gaussian initial states in Fig. 1, and they follow approximately Eq. (5). The agreement is
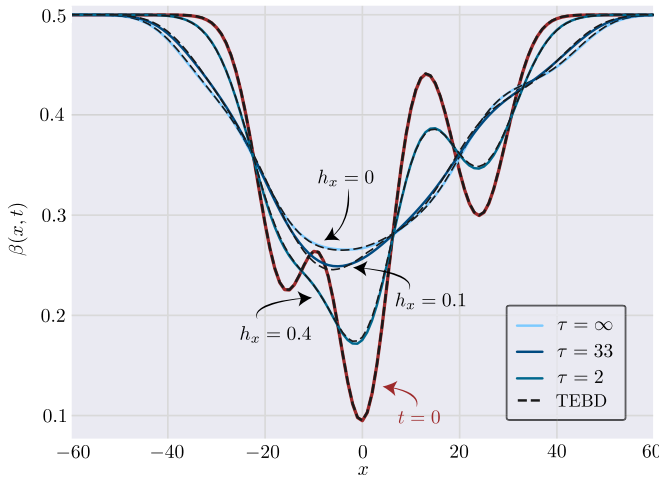
FIG. 2. Generic energy transport in chaotic spin chains: inverse temperature profiles $\beta(x,t) = 1/T(x,t)$ at time $t = 20$ in an XXZ spin chain with a staggered transverse field h_x breaking integrability, comparing TEBD and GRTA starting from a nontrivial inhomogeneous initial state. The values of τ in GRTA for each h_x were determined from Fig. 1.

remarkable, and it illustrates that the GRTA captures energy transport in this generic nonintegrable spin chain not only qualitatively, but also to a large extent quantitatively (the error between GRTA and TEBD is at most 2%) (see Fig. 2).

Hydrodynamics of nonintegrable Bose gases. We also used the GRTA to capture the crossover from generalized to conventional hydrodynamics in one-dimensional Bose gases, described by the Lieb-Liniger model [99]. We considered integrability-breaking perturbations that either relax or preserve momentum, and we implemented both far-from-equilibrium free expansions into vacuum of a cloud of atoms which models experiments on ultracold Bose gases [43,104–114], and linear-response setups where the initial state is a small local perturbation on top of an equilibrium Gibbs state. For Bose gases, one can consider integrability-breaking perturbations that conserve momentum, as well as energy and particle number. We briefly summarize some key findings [99]. (i) For perturbations that conserve energy, particle number, and momentum, we recover the hydrodynamics of a conventional fluid, with separate heat and sound peaks. (ii) For perturbations that conserve only energy and particle number, we find well-separated diffusion constants for these

two conserved quantities. This is a natural consequence of GHD, since the energy and particle-number Drude weights are different, but it illustrates the strongly interacting nature of the dynamics we are able to capture. (In noninteracting systems, these quantities would be linked by the Wiedemann-Franz law.) (iii) Finally, although the GRTA assumes a uniform relaxation time τ, starting from nonequilibrium states we find that different charges can approach their equilibrium values at different rates. This is due to the highly nonlinear nature of the GRTA, discussed above.

Discussion. In this work, we have introduced the GRTA as a numerically efficient approximation to study the nonequilibrium dynamics of systems with weak integrability-breaking. The GRTA treats integrability-breaking in a rather drastic approximation, where all but the residual conserved charges decay on a single timescale τ. Nevertheless, this approximation works surprisingly well to capture the hydrodynamics of physically relevant integrability-breaking perturbations $\hat{V}$ (such as a staggered transverse field in the XXZ model) at the cost of introducing $\tau(g) = Cg^{-2}$ with a single fit parameter C. Many natural extensions of this method suggest themselves. For instance, in cases in which some charges relax much slower than others, we can treat the dynamics of the fast charges within the GRTA (treating the slow modes as conserved) and then relax the slow charges separately. This could be relevant, for example, in ultracold atomic experiments, where integrability-breaking due to collisions can be much faster than atom loss or momentum relaxation due to the trap. Another natural extension would be to add noise to the GRTA equations (of strength given by the fluctuation-dissipation theorem). Finally, our implementation of the integrable dynamics itself has been restricted to Euler scale hydrodynamics. An important open question is how to develop an efficient scheme for numerically solving the GHD equations beyond the Euler scale [39]; incorporating the GRTA into this scheme would allow us to answer currently open questions about the fate of anomalous diffusion in nonintegrable spin chains [86].

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