η-pairing ground states in the non-Hermitian Hubbard model

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The introduction of non-Hermiticity has greatly enriched the research field of traditional condensed matter physics and eventually led to a series of discoveries of exotic phenomena. We investigate the effect of non-Hermitian imaginary hoppings on the Hubbard model. The exact bound-pair solution shows that the electron-electron correlation suppresses the non-Hermiticity, resulting in off-diagonal long-range order (ODLRO) ground state in the attractive Hubbard model. In a large U limit, such non-Hermiticity contributes an extra minus sign in the virtual exchange of the particles. As a consequence, the energy of the effective spin model describing the behavior of the ground state and low energy excitations will be reversed. The corresponding ground state experiences a transition from antiferromagnetism to ferromagnetism characterized by the appearance of a non-decaying correlation function. The numerical result indicates that the η-pairing ground state exits in 1D and 2D systems and is insensitive to the disorder. We further propose a protocol to adiabatically generate the η-pairing state. Our results provide a promising approach for the non-Hermitian strongly correlated system.

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I. INTRODUCTION

Non-Hermitian Hamiltonian is not only an extension of standard quantum mechanics, but it can also describe dissipative systems in a minimalistic fashion. In the last decade, non-Hermitian systems have gained a great deal of attention, especially due to rapid progresses in experimental implementations of non-Hermiticity. Such experiments include examples from photonics [1–8], acoustics [9], vacancy centers in solids [10], and cold atoms [11], where non-Hermiticity was introduced through judiciously incorporating gain and loss [12–16]. It has been revealed that non-Hermiticity drastically alters the properties of a number of well-known quantum phenomena that have been established in the Hermitian physics, ranging from quantum phase transitions [17–19], quantum critical behavior [20,21], topological phases [22–30], to magnetism [31]. However, since most of these previous studies relied on single-particle or mean-field descriptions, investigation of many-body physics in non-Hermitian systems is still in its infancy.

Superconductivity is one of the most striking quantum many-body phenomena, which has been a subject of intensive investigation in condensed matter physics. Recent advances in quantum simulations of the Hubbard model with ultracold atoms have offered a multifunctional platform to unveil such properties of the strongly correlated system [32–44]. Of particular interest is the generation of the η-pairing states, which exhibit off-diagonal long-range order (ODLRO), and thus are superconducting. This stimulates a plethora of protocols to generate transient nonequilibrium superconductivity in Hubbard models [45–50]. Inspired by the pioneer work [45], a plethora of protocols to generate transient nonequilibrium superconductivity in repulsive Hubbard models are proposed. More recently, a Floquet protocol to engineer η-pairing superfluid in attractive Hubbard model is also studied [51]. On the other hand, most previously studied non-Hermitian Hubbard system does possess the inelastic Hubbard interaction [52–55], in which the bound states possess the complex energy. From the dynamical perspective, such states, especially the η-pairing state, are not stable. Then a question arises: under the non-Hermitian framework, is there a scheme to prepare bound states with real energy and make the energy of η-pairing state as low as possible? If yes, what is the magnetism of such non-Hermitian system? In this work, we attempt to answer the question by focusing on the Hubbard model with non-Hermitian imaginary hopping rather than complex-valued interaction.

A number of findings are in order. (i) The non-Hermitian imaginary hopping can indeed induce a robust η-pairing ground state for a wide range of parameters U (particle-particle interaction) and t (hopping strength), by considering the bipartite non-Hermitian Hubbard system. An exact solution of the bound pair is employed to elucidate the underlying paring mechanism and pave the way to extend the results to dilute gas. (ii) In the large U limit, the antiferromagnetic and ferromagnetic states of both physical and η spins always coexist, but the energy of the two is different. In the Hermitian Hubbard model, the energy of the antiferromagnetic state is lower than that of the ferromagnetic state. The presence of the imaginary hopping leads to a minus sign in the virtual exchange of the particles and hence reverses the energy of two such states. The ground state undergoes a transition from an antiferromagnetic to a ferromagnetic state. Correspondingly, the correlation function of doublon hopping remains a constant number rather than decaying in terms of the power-law

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form as in the antiferromagnetic ground state. Specifically, in the repulsive Hubbard model ($U > 0$), the presence of imaginary hopping leads to a change from antiferromagnetic to ferromagnetic correlations of physical spins, whereas in the attractive Hubbard model ($U < 0$), the η-pairing state with ferromagnetic correlation becomes the ground state. In this sense, the η-pairing state is set to the ground state through a non-Hermitian setting. These results were absent in the previously studied non-Hermitian correlated system. (iii) Numerical results of 1D and 2D systems with corrugation patterns indicate that the presence of the η-pairing ground state is insensitive to the disorder and the strength of the interaction even though the on-site interaction breaks the $SO(4)$ symmetry, which suggests a promising scheme in a real experiment. We further demonstrate that such insensitive property can be served as the building block to generate a η-pairing superconducting state. It is hoped that these results can stimulate further studies of both fundamental aspects and potential applications of the non-Hermitian correlated system.

The rest paper is organized as follows. Section II discusses the non-Hermitian Hubbard model, wherein the exact two-particle solution and effective spin Hamiltonian in large U limit are investigated. The underlying mechanism of the formation of the η-pairing ground state is discussed. Section III shows the numerical results and the analytical understanding of superconductive η-pairing ground state. Section IV demonstrates the transition from a conventional paring to η-pairing superconductive ground state. Section V concludes this paper. Some details of our calculations are placed in Appendixes.

II. MODEL

We consider a non-Hermitian Hubbard model on a bipartite lattice

$$H = i\sum_{j,l}\sum_{\sigma=\uparrow,\downarrow} t_{jl}(c_{j,\sigma}^{\dagger}c_{l,\sigma} + c_{l,\sigma}^{\dagger}c_{j,\sigma}) + U\sum_{j} n_{j,\uparrow}n_{j,\downarrow}, \quad (1)$$

with the following notation: the operator $c_{j,\sigma}$ ($c_{j,\sigma}^{\dagger}$) is the usual annihilation (creation) operator of a fermion with spin $\sigma \in \{\uparrow, \downarrow\}$ at site j, and $n_{j,\sigma} = c_{j,\sigma}^{\dagger}c_{j,\sigma}$ is the number operator for a particle of spin σ on site j; the symbol $i = \sqrt{-1}$ represents an imaginary number; U and t_{jl} are required to be real and play the role of interaction and kinetic energy scales, respectively; the system can be divided into two sublattices A and B such that $t_{jl} = 0$ whenever $j \in \{A\}$ and $l \in \{A\}$ or $j \in \{B\}$ and $l \in \{B\}$. For convenience and clarity, the number of the sites and the particles are denoted by N and M, respectively. The non-Hermiticity of H stems from the imaginary hopping it_{jl} that can be realized by the judicious design of the loss and the magnetic flux [56,57] which are within the reach of cold atom experiments [31,58]. Notice that such non-Hermiticity is distinct from the complex particle-particle interaction adopted to describe the inelastic collision of two particles [53,55], wherein the second-order process mediates a complex spin-exchange interaction. Evidently, the imaginary hopping inevitably competes with the interaction leading to the unique properties of the considered system. Specifically, the effective spin Hamiltonian of Eq. (1) is Hermitian in large U limit. It can be expected that the introduction of such non-Hermiticity will significantly alter the magnetic correlation of the parent Hermitian system, which will be demonstrated in the following section.

In this paper, we focus on whether the system can favor the ground state with η-pairing superconductivity in this non-Hermitian setting. To gain physical insight into this system, we first investigate the symmetry of the considered model. It has two sets of commuting SU(2) symmetries. The first is the spin symmetry characterized by the generators

$$s^{+} = (s^{-})^{\dagger} = \sum_{j} s_{j}^{+}, \quad (2)$$

$$s^{z} = \sum_{j} s_{j}^{z}, \quad (3)$$

where the local operators $s_{j}^{+} = c_{j,\uparrow}^{\dagger}c_{j,\downarrow}$ and $s_{j}^{z} = (n_{j,\uparrow} - n_{j,\downarrow})/2$ obey the Lie algebra, i.e., $[s_{j}^{+},\ s_{j}^{-}] = 2s_{j}^{z}$, and $[s_{j}^{z},\ s_{j}^{\pm}] = \pm s_{j}^{\pm}$. The spin quantum number s_{c} is related to the eigenvalues of the operator $s^{2} = (s^{z})^{2} + (s^{+}s^{-} + s^{-}s^{+})/2$, i.e., $s_{\mathrm{c}}(s_{\mathrm{c}} + 1)$. Large values of s_{c} corresponds to ferromagnetism. The second often referred to as η-symmetry has the generators

$$\eta^{+} = (\eta^{-})^{\dagger} = \sum_{j} \eta_{j}^{+}, \quad (4)$$

$$\eta^{z} = \sum_{j} \eta_{j}^{z}, \quad (5)$$

with $\eta_{j}^{+} = \lambda c_{j,\uparrow}^{\dagger}c_{j,\downarrow}^{\dagger}$ and $\eta_{j}^{z} = (n_{j,\uparrow} + n_{j,\downarrow} - 1)/2$ satisfying commutation relation, i.e., $[\eta_{j}^{+},\ \eta_{j}^{-}] = 2\eta_{j}^{z}$, and $[\eta_{j}^{z},\ \eta_{j}^{\pm}] = \pm\eta_{j}^{\pm}$. Again, the quantum number η_{c} is related to the operator $\eta^{2} = (\eta^{z})^{2} + (\eta^{+}\eta - +\eta^{-}\eta+)/2$, with eigenvalues $\eta_{\mathrm{c}}(\eta_{\mathrm{c}} + 1)$. Here we assume a bipartite lattice and $\lambda = 1$ for $j \in \{A\}$ and -1 for $j \in \{B\}$. Notice that under a particle-hole transformation, $c_{j,\downarrow} \to \lambda c_{j,\downarrow}^{\dagger}$, which maps the attractive Hubbard model to a repulsive one in the parent Hermitian Hamiltonian (1), the role of the two sets of SU(2) generators is interchanged. Straightforward algebra shows that

$$[H, \eta^{\pm}] = \pm U\eta^{\pm}, \quad (6)$$

$$[H, \eta^{z}] = 0, \quad (7)$$

which indicates that one can construct many exact eigenstates

$$H|\psi_{c}(m)\rangle = mU|\psi_{c}(m)\rangle\ (m = M/2 = 0, 1, \dots), \quad (8)$$

where $|\psi_{c}(m)\rangle = \Omega^{-1}(\eta^{+})^{m}|\mathrm{Vac}\rangle$, with $|\mathrm{Vac}\rangle$ being the vacuum state of fermion $c_{j,\sigma}$ and renormalization coefficient $\Omega = \sqrt{C_{N}^{m}}$. What makes the state is special is the fact that it has been shown to have ODLRO in the form of doublon-doublon correlations, $\langle\psi_{c}(m)|\eta_{i}^{+}\eta_{j}^{-}|\psi_{c}(m)\rangle =$ const, $(i \neq j)$. This relation provides a possible definition of superconductivity, as a finite value of this quantity can be shown to imply both the Meissner effect and flux quantization [59]. Correspondingly, the large values of the η_{c} quantum number are related to a staggered ODLRO and superconductivity [60,61]. It is worth pointing out that the imaginary hopping does not change the η-pairing state, only the variation of U will alter its energy.

A. η-pairing state in two-particle subspace

Based on the symmetry of the system, we first elucidate the paring mechanism through the exact solution within the two-particle subspace. Supposing that the Hamiltonian (1) describes a 1D homogeneous ring system in which $it_{jl} = it\delta_{l,j+1}$. Owing to the translation symmetry, the basis of such invariant subspace can be constructed as follow:

$$|\phi_0^-(K)\rangle = \frac{1}{\sqrt{N}}\sum_j e^{iKj}c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger|\mathrm{Vac}\rangle, \quad (9)$$

$$|\phi_r^\pm(K)\rangle = \frac{1}{\sqrt{2N}}e^{iKr/2}\sum_j e^{iKj}(c_{j,\uparrow}^\dagger c_{j+r,\downarrow}^\dagger \quad (10)$$

$$\pm c_{j,\downarrow}^\dagger c_{j+r,\uparrow}^\dagger)|\mathrm{Vac}\rangle, \quad (11)$$

and

$$\frac{s^\pm}{\sqrt{2}}|\phi_r^+(K)\rangle = \frac{1}{\sqrt{N}}e^{iKr/2}\sum_j e^{iKj}c_{j,\pm\uparrow}^\dagger c_{j+r,\pm\uparrow}^\dagger|\mathrm{Vac}\rangle, \quad (12)$$

where N is an even number and $K = 2n\pi/N$ is the momentum vector indexing the subspace. r represents the relative distance between the two particles. These bases are eigenvectors of the operators s^2 and s^z, which satisfies

$$s^2|\phi_r^-(K)\rangle = 0, \quad (13)$$

$$s^z|\phi_r^-(K)\rangle = 0, \quad (14)$$

$$s^2|\phi_r^+(K)\rangle = 2|\phi_r^+(K)\rangle, \quad (15)$$

$$s^z|\phi_r^+(K)\rangle = |\phi_r^+(K)\rangle. \quad (16)$$

Evidently, each subspace labeled by K can be further decomposed into four subspaces with $(s,\ s^z) = (0,\ 0)$, $(1,\ 0)$ and $(1,\ \pm1)$ in term of spin symmetry. Aiding by the detailed calculation in Appendix, the bound pair emerges in the $(0,\ 0)$ subspace with eigenenergy being $\epsilon_K = \mathrm{sgn}(U)\sqrt{U^2 + 4\lambda_K^2}$ in which $\lambda_K = 2it\cos(K/2)$. The bound pair state is $|\varphi_K^{\mathrm{b}}\rangle = \sum_r f_K^-(r)|\phi_r^-(K)\rangle$ with

$$f_K^-(j) = \begin{cases} 1/\sqrt{2},\ j = 0 \\ e^{-\beta j},\ j \neq 0 \end{cases}, \quad (17)$$

where $\beta = \ln[(-U \pm \sqrt{U^2 + 4\lambda_K^2})/2\lambda_K]$. Here $\pm$ denotes negative and positive U, respectively. In the absence of on-site interaction U, only the scattering eigenstate with imaginary eigenenergy presents and the system does not accommodate the bound pair state. The nonzero interaction U leads to the emergence of the bound pair. When $|U| > |4t|$, the system possesses the full real bound pair spectrum. However, a small U results in the appearance of the imaginary bound pair energy. The corresponding eigenstate is in the form of an oscillation damping wave rather than a monotonic damping wave of the Hermitian parent system. Notice that if $|U| \leqslant |4t|$, then an exceptional point (EP) $|U| = |2\lambda_{K_c}|$ presents, at which the coalescent eigenstate approaches to a unidirectional plane wave with $\beta = 0$ or π corresponding to $K = 0$ or 2π. In this sense, the non-Hermiticity of the system is suppressed through the pairing mechanism. The emergence of real energy is the consequence of the competition between the on-site interaction and imaginary hopping. Furthermore, when $U < 0$, the lowest real eigenenergy appears in the $K = \pi$ subspace no matter whether the system possesses the full real spectrum. The corresponding ground state is η-pairing state with the form

$$|\phi_0^-(K)\rangle = (\eta^+)/\sqrt{N}|\mathrm{Vac}\rangle. \quad (18)$$

Here the *ground state* refers to the steady eigenstate with the lowest eigenenergy, where the so-called steady eigenstate means that its Dirac probability does not change with time. From the dynamical perspective, all the eigenstates with real eigenenrgy belong to this class except for the coalescent state, since the Dirac probability of such state increases over time in power law according to the order of coalescence [44]. In the current non-Hermitian setting, the energies of η-pairing states are real rather than complex that appears in the non-Hermitian Hubbard system with complex interaction. Under the long-term time evolution, the η-pairing state exhibits distinct dynamic behaviors in two such non-Hermitian systems. Note in passing that the behavior of the ground state is in stark difference from the Hermitian system, i.e., $it \to t$. In that case, the ground state of the two-particle system locates on the $K = 0$ rather than the $K = \pi$ subspace such that the η-pairing state has the highest eigenenergy than the other bound pair state, which can be clearly seen by comparing Figs. 1(c1)–1(c2). It also demonstrates that the imaginary hopping flips the bound pair spectrum of the parent Hermitian spectrum

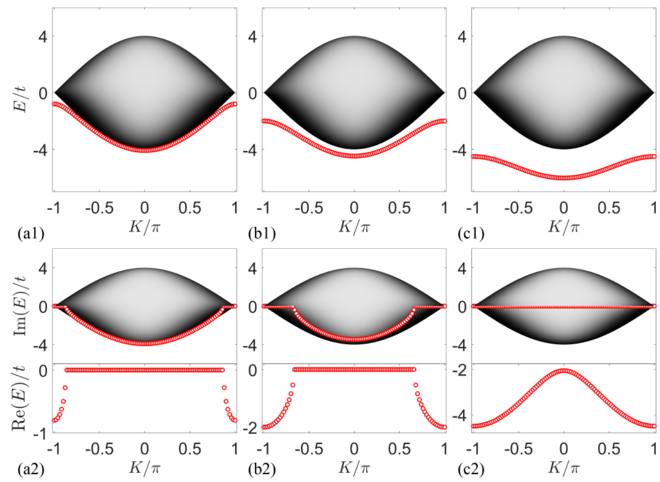

FIG. 1. Comparison of the two-particle spectrum within the subspace (0,0) between the non-Hermitian setting and its parent Hermitian system for (a) $U = -0.8t$, (b) $-2t$, and (c) $-4.5t$, respectively. The upper and lower panels present the spectrum of the Hermitian and non-Hermitian system, respectively. The red circle and gray shading denote the bound pair and scattering state. The parent Hermitian system can be obtained by assuming $it \to t$. For the Hermitian system, the bound pair with the lowest energy lies in the $K = 0$ subspace while the ground state of two-particle non-Hermitian setting locates on the subspace indexed by $K = \pi$. It is shown that the presence of the imaginary hopping not only makes all scattering energy bands imaginary but also reverses the whole bound band. In the condition of small U, there can exist an EP characterized by the divergence of $\partial\epsilon_K/\partial K$. Such non-Hermiticity alters significantly the paring mechanism and hence favors superconductivity.

so that the steady ground state experiences a transition from conventional paring ($K = 0$) to η-paring ($K = \pi$) state.

B. η-pairing state in the large U limit

Now we turn to investigate the situation with arbitrary filling but in the large U limit system. Following the standard step of quantum mechanics, the system can be divided into the kinetic part H' and interaction part H_0, where

$$H' = i\sum_{j,l}\sum_{\sigma=\uparrow,\downarrow} t_{jl}(c^\dagger_{j,\sigma}c_{l,\sigma} + c^\dagger_{l,\sigma}c_{j,\sigma}), \tag{19}$$

$$H_0 = U\sum_j n_{j,\uparrow}n_{j,\downarrow}. \tag{20}$$

Here the imaginary hopping is assumed to be homogeneous $it_{jl} = it\delta_{l,j+1}$. In the strongly correlated regime $|U| \gg t$, the kinetic term H' can be treated as a perturbation and one can derive an effective Hamiltonian for the degenerate space. The second-order perturbation theory gives the effective η-spin Hamiltonian regarding the doublon-hole creation and recombination process as

$$H_{\mathrm{eff}} = mU + \frac{4t^2}{U}\sum_j\left(\boldsymbol{\eta}_j\cdot\boldsymbol{\eta}_{j+1} - \frac{1}{4}\right), \tag{21}$$

where $\boldsymbol{\eta}_j = (\eta^x_j,\ \eta^y_j,\ \eta^z_j)$. In Appendix, a simple two-site case is provided to elucidate this mechanism. This indicates that the non-Hermitian virtual exchange mediates an interaction between the pseudo spins. The eigenstate of the lowest eigenenergy within each doublon subspace is the η-pairing state with different pair numbers when $U < 0$. Similarly, for the case of repulsive Hubbard model at half filling, the effective Heisenberg Hamiltonian describing the behavior of the ground state and low energy excitations can be given as

$$H^1_{\mathrm{eff}} = -\frac{4t^2}{U}\sum_j\left(\boldsymbol{s}_j\cdot\boldsymbol{s}_{j+1} - \frac{1}{4}\right). \tag{22}$$

Evidently, the non-Hermitian imaginary hopping contributes an extra minus sign in the second order perturbation theory. For $U < 0$, this leads to a change from antiferromagnetic to ferromagnetic correlations of η spins. For $U > 0$, this leads to a change from antiferromagnetic to ferromagnetic correlations of real spins s.

III. η-PAIRING STATE IN SYSTEM WITH MIXED HOPPINGS

Reference [62] states that the ground state is unique with $s = 0$ (M is even) if the attractive Hubbard model ($U < 0$) is considered, and the η-pairing state is not the ground state [59]. Hence, many efforts have been done to seek the strongly correlated electronic models with superconducting η-pairing ground state [63–66]. In the aforementioned sections, we have demonstrated that the η -pairing state can be either the ground state of the system under the large U limit, or the ground state of the two-particle subspace with nonzero U. Then a natural question arises: (i) for any nonzero U, is the η-pairing state still the ground state of the system in the subspace of other particle numbers? (ii) If yes, can the existing 1D results be extended to 2D or higher dimensional system? (iii) If the disorder is introduced, does the property of ground state be changed?

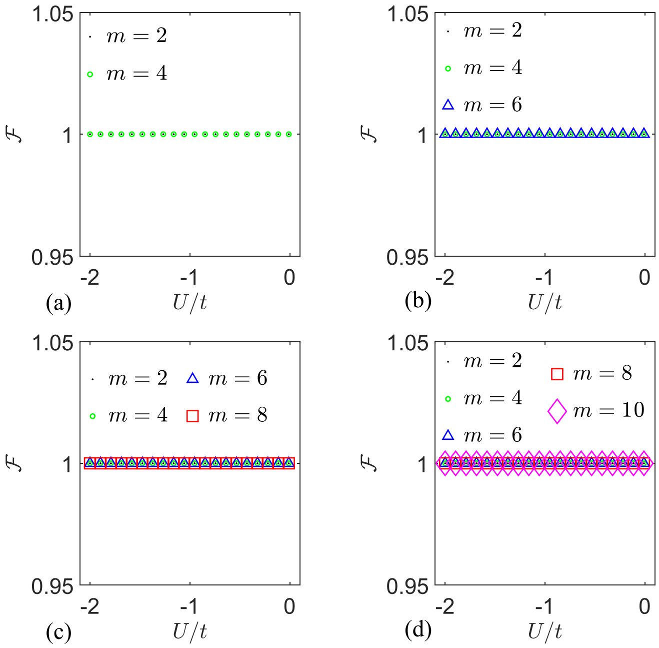

FIG. 2. Plot of the fidelity $\mathcal{F}$ as a function of the interaction strength U for the homogeneous 1D systems: (a) four-site system with filled particle pairs $m = 2$, and 4; (b) six-site system with filled particle pairs $m = 2, 4$, and 6; (c) eight-site system with filled particle pairs $m = 2$, 4, 6, and 8; and (d) ten-site system with filled particle pairs $m = 2, 4, 6, 8$, and 10. It is shown that the system possesses the η-pairing ground state as long as U is switched on.

To answer these questions, we first investigate the 1D non-Hermitian homogeneous system by setting $it_{jl} = it\delta_{l,j+1}$. The fidelity $\mathcal{F}$ defined as

$$\mathcal{F} = |\langle\psi_g(m)|\psi_c(m)\rangle|, \tag{23}$$

is introduced to measure the similarity between the ground state in the subspace with m particle pairs and the η-pairing state. Although the formal eigen solution of the 1D Hubbard model can be obtained through the Bethe ansatz method, it is difficult to identify the property of the ground state in the non-Hermitian setting theoretically. Hence, we perform the numerical simulation to check such quantity. Figure 2 shows that the ground state is the η-pairing state in the subspaces with different particle numbers even though a small U presents. Combining with the conclusions of two-particle and large U cases, we can infer that the η-pairing state has the lowest energy comparing with other bound pairs for any even M.

Now we turn to examine the second and the third questions by introducing the disordered imaginary hopping and interaction, which always presents in real experiments. The corresponding disordered Hamiltonian can be obtained by taking two sets of random numbers $\{t_{j,l}\}$ and $\{U_{j,l}\}$ around t and U in Eq. (1). The random number parameter can be taken as

$$t_{j,l} = t + \mathrm{rand}(-a, a),\ U_{j,l} = U + \mathrm{rand}(-b, b), \tag{24}$$

where $\mathrm{rand}(-a, a)$ denotes a uniform random number within $(-a, a)$. Again, we employ $\overline{\mathcal{F}}$ to identify the similarity

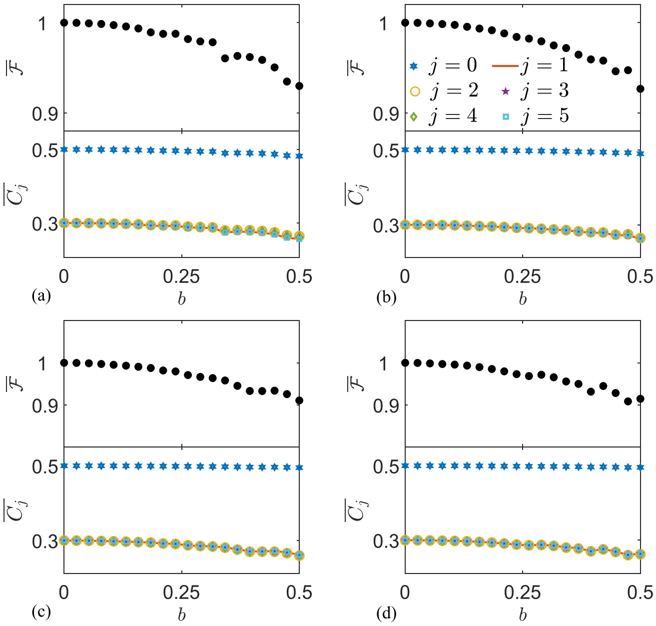

FIG. 3. Plots of the overlap $\overline{\mathcal{F}}$ and correlator $\overline{C_j}$ as a function of the strength of interaction disorder b for (a) $t=1$, $a=0.1t$, $U=-0.5t$ (b) $t=1$, $a=0.3t$, $U=-1.5t$ (c) $t=1$, $a=0$, $U=-4t$, and (d) $t=1$, $a=0.2t$, $U=-4t$. The numerical simulation is performed for the 6 site 1D Hubbard model at half filling and $s^z=0$. Here the strength of the hopping disorder a is set to be constant for each subfigure and the correlator $\overline{C_j}$ is averaged over all sites separated by a distance j. When $b=0$, no matter what value a takes, as long as U is nonzero, one can always get a perfect η-pairing ground state. The variation of $\overline{C_j}$ indicates that the increase of b will not result in the significant change of the η-pairing ground state; the coexistence of hopping and interaction disorders does not cause the ground state to deviate from the perfect η-pairing state, which can be seen from (c) and (d). Therefore the performance of the correlator is the consequence of the interplay between two such disorders, which provides a scheme to prepare η-pairing ground state in the experiment.

between the ground state and η-pairing state, where the overline indicates the disorder average. Figure 3 shows that if U is homogeneous ($b=0$) then the ground state is the η-pairing state in the subspace with particle numbers $M=2$, 4, 6, and 6 for each subfigure. The corresponding energies are U, $2U$, $3U$, and $3U$ respectively. It indicates that the formation of the η-pairing ground state is immune to the hopping disorder. However, the introduction of disordered U will cause the ground state of the system to deviate from the η-pairing state. The underlying mechanism is clear, that is, the system fulfills the η symmetry even in the presence of the hopping disorder, but if one introduces disorder into the interaction, this symmetry will be destroyed leading to such deviation. One may think that the disorder of interaction scrambles the background spin configuration and disturb the spin correlation. Then a question arises: to what extent does the ground state maintain the superconductivity? To capture superconductivity, the doublon-doublon correlator

$$C_j=\sum_i\langle\eta_i^+\eta_{i+j}^-\rangle/N \tag{25}$$

is introduced. It is averaged over all sites separated by a distance j. For the target state $|\psi_c(m)\rangle$, the expectation value can be given as

$$\langle\psi_c(m)|\eta_i^+\eta_{i+j}^-|\psi_c(m)\rangle=\begin{cases}\frac{m(N-m)}{N(N-1)}, \text{ for } j\neq 0\\ \frac{m}{N}, \text{ for } j=0\end{cases}. \tag{26}$$

Notice that it is irrelative to the distance j and hence the correlator C_j obeys the same law such that $C_j=m(N-m)/[N(N-1)]$ for $j\neq 0$ or $C_j=m/N$ for $j=0$. In Fig. 3, the correlator of the ground state $\overline{C_j}$ and the fidelity $\overline{\mathcal{F}}$ are calculated and averaged over 100 disorder configurations. It demonstrates that $\overline{\mathcal{F}}$ is around 0.9 and the correlator $\overline{C_j}$ stays at a nonzero value ensuring the ground state of the system possesses the superconductivity even though the strong inhomogeneity of interaction presents.

Now we switch gears to the cases of the 2D system. In Fig. 4, the disordered 2D system is sketched. For simplicity, we fix the strength of the hopping disorder a and examine two quantities $\overline{\mathcal{F}}$ and $\overline{C_2}$. It is shown that the system still possesses the η-pairing state even though the small homogeneous U and the disordered imaginary hopping present, which is similar to that of the 1D system. Although the disorder U affects the correlation of ground state, the correlator $\overline{C_2}$ has a small fluctuation around the value of uniform case supporting the η-pairing superconducting ground state. Therefore one can conclude that all the results of 1D can be extended to 2D lattice system. It can be expected that this conclusion is still valid for the higher dimensional bipartite system.

IV. DYNAMICAL TRANSITION FROM NORMAL TO η-PAIRING GROUND STATES

In this section, we focus on how does the η-paring state can be generated from a normal paring state. To observe such a transition, we consider a 1D Hubbard system. The corresponding Hamiltonian can be given as

$$\begin{aligned}H=&-\sum_{j,\sigma=\uparrow,\downarrow}it(c_{j,\sigma}^{\dagger}c_{j+1,\sigma}+\text{H.c.})\\&+U\sum_j n_{j,\uparrow}n_{j,\downarrow}+\sum_{j,\sigma=\uparrow,\downarrow}\mu_{j,\sigma}n_{j,\sigma},\end{aligned} \tag{27}$$

where the term $\mu_{j,\sigma}$ represents spin-dependent local potentials. Here the hopping and local potential are chosen to have the forms $t(\tau)=\lambda_t\tau$ and $\mu_{j,\sigma}(\tau)=\mu_{j,\sigma}(0)e^{-\lambda_\mu\tau}$, which are varied with time τ so as to generate η-paring state. We prepare the initial state $|\varphi(0)\rangle=c_{1,\uparrow}^{\dagger}c_{1,\downarrow}^{\dagger}c_{2,\uparrow}^{\dagger}c_{2,\downarrow}^{\dagger}|\text{Vac}\rangle$ as the ground state of an noninteracting Hamiltonian $H(0)$ by setting $t(0)=0$, and $\mu_{1,\sigma}(0)=\mu_{2,\sigma}(0)=-2$ otherwise $\mu_j=0$. The introduction of the inhomogeneous $\mu_{j,\sigma}$ is to break the degeneracy of the ground state when the imaginary potential is switched off at the very beginning. And the exponential form of $\mu_{1(2),\sigma}(\tau)=\mu_{1(2),\sigma}(0)e^{-\lambda_\mu\tau}$ ensures that the final Hamiltonian $H(\tau_f)$ possesses a η-paring state whose energy is the minimum value of the real part of the energy spectrum. Figures 5(a) and 5(b) shows the variation of the instantaneous eigen spectrum of $H(\tau)$ when the system parameters vary. Evidently, the lowest eigenenergy has neither experienced the level crossing nor level coalescent so that it is always a

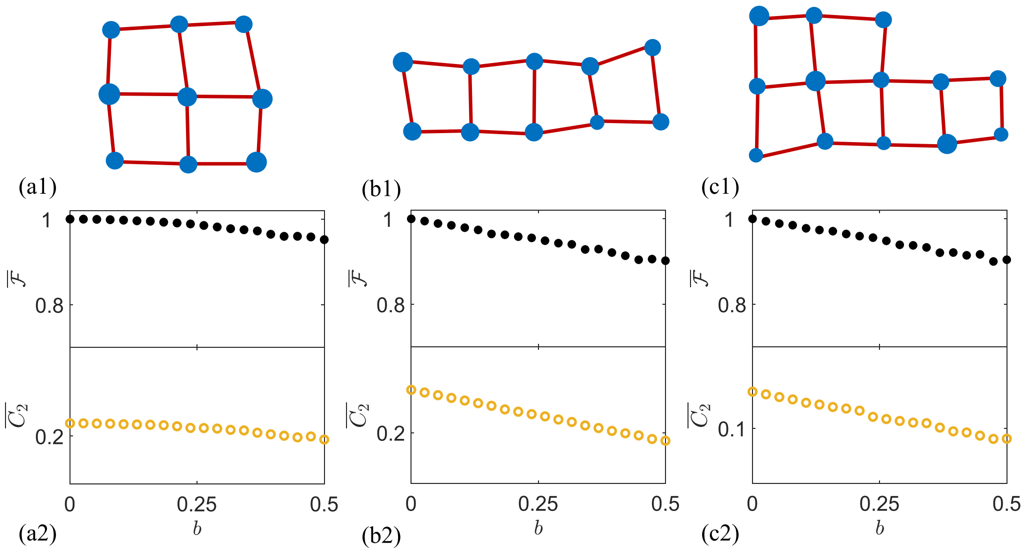

FIG. 4. Numerical simulation of 2D corrugation pattern for (a1) eight-site Hubbard model with four filled particles, (b1) nine-site Hubbard model with 6 filled particles, and (c1) 13 site Hubbard model with four filled particles. The different sizes of solid circles and different lengths of red edges denote the disorder of the interaction strength U_j and hopping strength t_j of Eq. (24). (a2)–(c2) Plots of $\overline{\mathcal{F}}$ and $\overline{C_2}$ as function of disorder strength b. The system parameters are (a) $t = 1$, $a = 0$, $U = -0.8t$ (b) $t = 1$, $a = 0.2t$, $U = -0.8t$ and (c) $t = 1$, $a = 0.2t$, $U = -0.5t$. Similar with 1D system, the induced fluctuation of $\overline{C_2}$ is minor in comparison with disorder free case ($b = 0$) indicating that the existing result of 1D can be extended to 2D or higher dimensional system.

real number. As such, one can envisage that the system can drive adiabatically the initial state $|\varphi(0)\rangle$ towards the η-paring state. To verify and demonstrate the above analysis, numerical simulations are performed to investigate the dynamics of the adiabatic evolution. We compute the time evolution of an eigenstate by using a uniform mesh in the time discretization for the time-dependent Hamiltonian $H(\tau)$. The parameters λ_t and λ_μ are chosen to be a small number such that the evolved state remains in the eigenstate denoted by the red line in Figs. 5(a) and 5(b). The fidelity $\mathcal{F}_g$ defined as

$$\mathcal{F}_g = |\langle\psi_c(m)|\mathcal{T}\exp\left[-i\int_0^\tau H(\tau')d\tau'\right]|\varphi(0)\rangle|, \tag{28}$$

is employed to witnesses the formation of the η-pairing state. The plot in Fig. 5 shows that the final evolved state approaches to the η-pairing state, which agrees with our prediction.

Before ending this section, we want to point out that the imaginary hopping plays the key to achieve the η-paring ground state, however, it does not mean that the system must have the η-pairing ground state as long as the imaginary hopping is applied. The transition of the conventional pairing (zero momentum) to η pairing (finite momentum $K = \pi$) always requires a process. Although the parent Hermitian system ($it \to t$) favors the superconductivity when large $-U$ is assumed, the presence of the imaginary hopping can lead to a unique η -pairing state with lower energy due to the additional minus sign of the virtual exchange of particles. The conclusion still holds even though a small $-U$ applies. On the other hand, the proposed dynamic scheme does not apply to the non-Hermitian Hubbard model with inelastic Hubbard interaction, that is $U \neq U^*$. Although the spectrum of inelastic Hubbard model can be obtained by changing $U \to -U$ and swapping real/imaginary components of the eigenvalues of H, the dynamic behavior of the two Hamiltonians is completely different. In the latter case, the η -pairing state possesses the complex eigenenergy such that one cannot design an adiabatic process to generate such state and make it lie at the lowest energy of the bound states. In other words, the mechanism for preparing η-pairing state is different for the two non-Hermitian settings. These findings pave the way to understand the η-spin ferromagnetic state of the non-Hermitian strongly correlated system.

V. SUMMARY

In summary, we have systematically studied the effect of the non-Hermitian imaginary hopping on the low-lying energy spectrum of the Hubbard model. The analytical solution within the two-particle subspace shows that the introduction of the imaginary hopping results in a full imaginary scattering spectrum and a flip of the bound pair spectrum comparing to its Hermitian parent model. It indicates that the particle-particle correlation suppresses the non-Hermiticity making the ground state to be η-pairing state with ODLRO. In the large U limit, the magnetism of the Hubbard model is altered fundamentally due to the interplay between the particle-particle interaction and non-Hermitian imaginary hopping. In particular, a change from antiferromagnetic to ferromagnetic correlations of η spins occurs. When the η-spin ferromagnetic ground state presents, the total spin of any two points is $\langle(\boldsymbol{\eta}_i + \boldsymbol{\eta}_j)^2\rangle = 1$ ($i \neq j$), and is irrelevant to the distance between two η spins. As a consequence, the correlation functions $\langle\eta_i^\dagger\eta_j^-\rangle$ and $\langle\eta_i^z\eta_j^z\rangle$ are connected with each other because $\langle\boldsymbol{\eta}_i\cdot\boldsymbol{\eta}_j\rangle = 1/4$ ($\langle\eta_i^\dagger\eta_j^-\rangle = 1/4 - \langle\eta_i^z\eta_j^z\rangle$ for any $i \neq j$). In the thermodynamic limit, such constant does not approach 0 suggesting that η-spin ferromagnetic state has ODLRO and thus favors the superconductivity. What needs to be emphasized

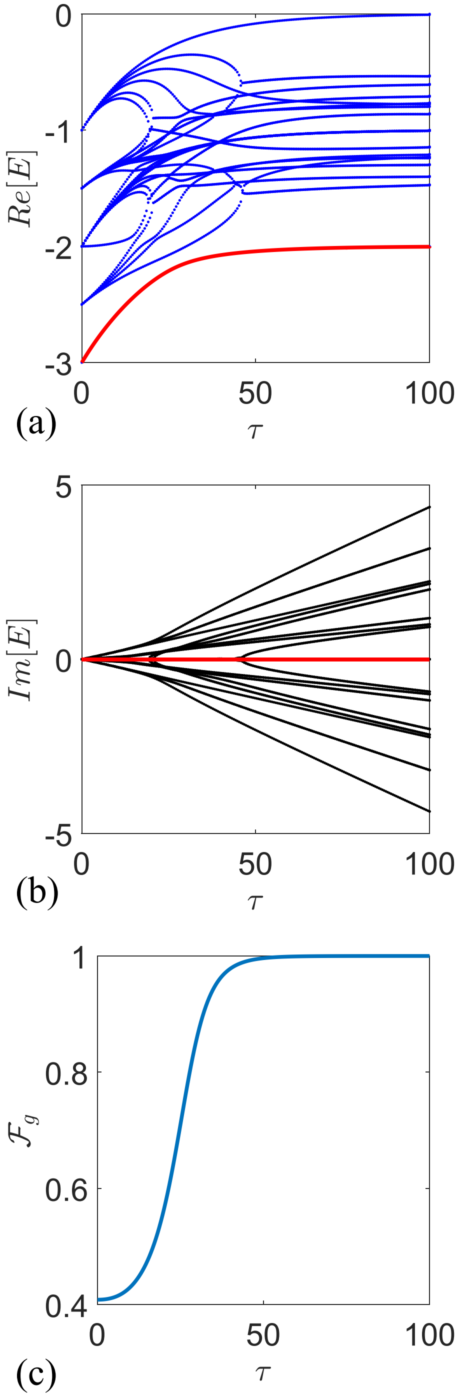

FIG. 5. Dynamical generation of the η-pairing superconducting ground state. [(a) and (b)] depict the variation of the instantaneous energy spectrum with respect to time τ, and (c) presents the fidelity $\mathcal{F}_g$. The numerical simulation is performed for the six-site Hubbard model with four filled particles. The other system parameters are $\lambda_t = 0.01$, $\lambda_\mu = 0.05$, and $\mu_{1(2),\sigma}(0) = 0.5$. The time τ is in units of t^{-1}, where t is the energy scale of the Hamiltonian. The adiabatic path denoted by red line is chosen to avoid the level crossing and coalescing such that the evolved state stay at the ground state. The variation of $\mathcal{F}_g$ indicates that the ground state undergoes a transition from conventional paring to η paring.

here is that although the considered system only has the nearest-neighbor interaction, the expectation values of both $\langle \eta_i^z \eta_j^z \rangle$ and $\langle \eta_i^\dagger \eta_j^- \rangle$ in the ferromagnetic state exhibit long-range correlation. However, for the η-spin antiferromagnetic ground state, the correlation function will decay with the increase of the relative distance between the two η spins in terms of the power-law form since the low-lying spectrum is gapless in the thermodynamic limit. The transition of the ground state holds for any pair filled, that is, the ground state in each invariant subspace is $(\eta^+)^m|\text{Vac}\rangle$ with m being the pairs of particles. Through numerical simulation of 1D and 2D non-Hermitian Hubbard system, we demonstrate that the η-pairing ground state can still survival albeit a small negative U presents. This evidence is robust against disorder even if the system does not fulfill the $SO(4)$ symmetry. Our results open a new avenue toward populating a η-pairing ground state and suppressing antiferromagnetic correlation of η spins in the attractive Hubbard model.

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APPENDIX A: TWO-PARTICLE SOLUTIONS

In this section, we show the detailed calculation for the two-particle solution in each invariant subspace. For the simplicity, we only focus on the solutions in subspaces (0, 0) and (1, 0), since the solution in subspace (1, ± 1) can be obtained directly from that in subspace (1, 0) by operator $s^\pm$. A two-particle state can be given as

$$|\varphi_K^\pm\rangle = \sum_r f_{K,k}^\pm(r)|\phi_r^\pm(K)\rangle, (f_K^+(0) = f_{K,k}^-(-1) = 0), \quad \text{(A1)}$$

where r denotes the relative distance between the two particles and the wave function $f_{K,k}^\pm(r)$ obeys the Schrödinger equations

$$Q_r^K f_{K,k}^+(r+1) + Q_{r-1}^K f_{K,k}^+(r-1) + \big[(-1)^n Q_r^K \delta_{r,N_0} - \varepsilon_K\big] f_{K,k}^+(r) = 0 \quad \text{(A2)}$$

and

$$Q_r^K f_{K,k}^-(r+1) + Q_{r-1}^K f_{K,k}^-(r-1) + \big[U\delta_{r,0} + (-1)^n Q_r^K \delta_{r,N_0} - \varepsilon_K\big] f_{K,k}^-(r) = 0, \quad \text{(A3)}$$

with $N_0 = (N-1)/2$ and the eigen energy ε_K in the invariant subspace indexed by K. Here factor $Q_r^K = -2\sqrt{2}it\cos(K/2)$ for $r = 0$ and $-2it\cos(K/2)$ for $r \neq 0$, respectively. U appears in the (0,0) subspace and therefore admits the bound pair solution. In the large N limit, we can neglect the effect of on-site potential $(-1)^{n+1}2it\cos(K/2)$ at N_0th site. The solution of (A3) is equivalent to that of the single-particle semi-infinite tight-binding chain system with nearest-neighbor (NN) hopping amplitude Q_j^K, and on-site potentials U at 0th site, respectively. Moreover the solution of (A2) corresponds to the same chain with infinite U. In this scenario, the bound state solution $|\varphi_K^\text{b}\rangle = \sum_r f_K^-(r)|\phi_r^-(K)\rangle$ can be determined by substituting the ansatz

$$f_K^-(j)=\begin{cases}1/\sqrt{2}, & j=0\\ e^{-\beta j}, & j\neq 0\end{cases} \tag{A4}$$

into the following equivalent Hamiltonian:

$$H_{\mathrm{eq}}^K=U|0\rangle\langle 0|+\sum_{i=0}^{\infty}\left(Q_i^K|i\rangle\langle i+1|+\mathrm{H.c.}\right). \tag{A5}$$

Straightforward algebra shows that $\beta=\ln[(-U\pm\sqrt{U^2+4\lambda_K^2})/2\lambda_K]$ where $\lambda_K=2it\cos(K/2)$ and $\pm$ denotes negative and positive U, respectively. Correspondingly, the energy of the bound pair is

$$\epsilon_K=\mathrm{sgn}(U)\sqrt{U^2-16t^2\cos^2(K/2)}. \tag{A6}$$

For the case of negative U, the lowest energy of bound pair is $\epsilon_\pi=-U$ locating on the subspace with $K=\pi$. As such the corresponding eigenstate is $|\phi_0^-(K)\rangle$ that represents a η-pairing state in the coordinate space with the form of $(\eta^+)/\sqrt{N}|\mathrm{Vac}\rangle$.

APPENDIX B: SIMPLE EXAMPLE OF A TWO-SITE CASE FOR THE EFFECTIVE HAMILTONIAN H_{eff}

To second order in perturbation theory, the effective Hamiltonian is given by

$$H_{\mathrm{eff}}^2=P_0H_0P_0+P_0H'P_1\frac{1}{E_0-H_0}P_1H'P_0+O\left(\frac{t^3}{U^2}\right), \tag{B1}$$

where P_0 is a projector onto the Hilbert subspace in which there are m lattice sites occupied by two particles with opposite spin orientation, and $P_1=1-P_0$ is the complementary projection. Here the energy E_0 of the unperturbed state is set to $E_0=mU$, where m denotes the number of doublons. Since H' acting on states in P_0 annihilates only one double occupied site, all states in $P_1H'P_0$ have exactly $m-1$ doubly occupied sites. Now we provide a detailed calculation of the two-site case for the effective Hamiltonian H_{eff}^2 which may shed light to obtain the effective Hamiltonian (21). In the simplest two-site case, $P_0=\sum_{\alpha\in\mathrm{d.o.}}|\alpha\rangle\langle\alpha|$ is the projection operator to the doublon subspace spanned by the configuration $\{|\mathrm{x}0\rangle,|0\mathrm{x}\rangle\}$, and $P_1=1-P_0=\sum_{a\notin\mathrm{d.o.}}|a\rangle\langle a|$ is the complementary projection. Here the abbreviation d.o. means the doubly occupied subspace and $|\mathrm{x}0\rangle=c_{1,\uparrow}^\dagger c_{1,\downarrow}^\dagger|\mathrm{Vac}\rangle$, $|0\mathrm{x}\rangle=c_{2,\uparrow}^\dagger c_{2,\downarrow}^\dagger|\mathrm{Vac}\rangle$. The first term of Eq. (B1) clear gives $P_0H_0P_0=U$. The second term can be simplified by noting: (i) the unperturbed energy E_0 is U; (ii) $P_1H'P_0$ annihilates the doubly occupied site. Then H_{eff} can be written as

$$\begin{aligned}H_{\mathrm{eff}}^2&=U+\sum_{\alpha,\beta\in\mathrm{d.o.}}\sum_{a,b\notin\mathrm{d.o.}}|\alpha\rangle\langle\alpha|H'|a\rangle\langle a|\\&\quad\times\frac{1}{U-H_0}|b\rangle\langle b|H'|\beta\rangle\langle\beta|\\&=U+\frac{1}{U}\sum_{\alpha,\beta\in\mathrm{d.o.}}\langle\alpha|(H')^2|\beta\rangle|\alpha\rangle\langle\beta|.\end{aligned} \tag{B2}$$

The second term describes the virtual exchange of the fermions. The non-Hermitian imaginary hopping brings about an additional sign to this process yielding that

$$H_{\mathrm{eff}}=U-\frac{2t^2}{U}(|\mathrm{x}0\rangle\langle 0\mathrm{x}|+|0\mathrm{x}\rangle\langle\mathrm{x}0|+|\mathrm{x}0\rangle\langle\mathrm{x}0|+|0\mathrm{x}\rangle\langle 0\mathrm{x}|). \tag{B3}$$

Combining the cases in the subspaces of $|\mathrm{xx}\rangle$ and $|\mathrm{Vac}\rangle$, the pseudo spin Hamiltonian can be given by the non-Hermitian Heisenberg-like model

$$H_{\mathrm{eff}}=U+\frac{4t^2}{U}\left(\boldsymbol{\eta}_1\cdot\boldsymbol{\eta}_2-\frac{1}{4}\right), \tag{B4}$$

where $\boldsymbol{\eta}_j=(\eta_j^x,\ \eta_j^y,\ \eta_j^z)$, and m can be 0, 1, and 2 denoting the number of pairs of the doublon subspace. Evidently, the ground state of H_{eff} is the η -spin ferromagnetic state with the form of $(\eta^+)^2|\mathrm{Vac}\rangle$. One can extend the result to the system with N sites, the corresponding effective Hamiltonian is given as

$$H_{\mathrm{eff}}=mU+\frac{4t^2}{U}\sum_j\left(\boldsymbol{\eta}_j\cdot\boldsymbol{\eta}_{j+1}-\frac{1}{4}\right). \tag{B5}$$

Hence, the ferromagnetic state of η spins aligned on the x-y plane is the η-pairing superconducting state.

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