Interaction of vector Bose gases with fermionic superfluids

Fei-Jie Huang,[1] Qi-Hui Chen,[2] Yong-Ping Fu,[3] Wei Guo,[1] Yifen Zhao,[1] and Hai Yang[1]
[1]*Department of Physics, Kunming University, Kunming 650214, China*
[2]*School of Physical Science and Technology, Southwest Jiaotong University, Chengdu 610031, China*
[3]*Department of Physics, West Yunnan University, Lincang 677000, China*

(Received 15 September 2020; revised 7 May 2021; accepted 25 May 2021; published 8 June 2021)

We study the effects of vector Bose gases, which are described by massive vector Bose fields, on fermionic superfluids in the low-energy region. It is demonstrated that the vector Bose gases can give rise to the Meissner effect, the flux quantization, and the Josephson effect, which are similar to those of the electromagnetic field in fermionic superfluids. However, unlike the electromagnetic field where the Meissner effect is unrelated to mass and the quantized flux is a constant, the Meissner effect of the vector Bose gases can be related to mass via a classical kinetic energy, and the value of the quantized flux is tunable. Our analyses also show that, although the origin model violates gauge invariance, the vector Bose gases in fact are gauge-invariant Maxwell-Chern-Simons systems under a Gauss constraint. It is also proposed that these effects can be observed by using the spin-1 cold atomic Bose gases in experiments.

DOI: 10.1103/PhysRevB.103.224512

I. INTRODUCTION

Vector Bose gases are bosonic systems which are described by massive vector Bose fields. In high-energy physics, the massive vector Bose fields describe mesons [1,2] and may be related to dark matter and dark energy [3–5]. In condensed matter systems, all spin-1 bosonic systems can be described by massive vector Bose fields, such as the alkali atomic Bose gases [6–18], the triplet pairing systems of p-wave superconductors [19–26] and superfluids [27–30], and spin-1 Haldane phase systems [31–36], etc. Without considering the interactions between bosons, these kinds of vector Bose fields are referred to as vector Bose gases.

The vector Bose gases are described by the Proca equation [37,38] in relativity, which is similar to the motion equation of the electromagnetic field. However, due to the vector Bose gases carry mass, there are two significant differences between the vector Bose gases and the electromagnetic field. One is that the vector Bose gases violate gauge invariance, which is an essential characteristic of the electromagnetic field. The other is that the vector Bose gases can have a nonrelativistic approximation, while the nonrelativistic approximation is absent in the electromagnetic field due to it being massless. This last characteristic enables us to investigate the properties of the vector Bose gases in real experiments.

In this paper, the effects of vector Bose gases on fermionic superfluids are investigated. Since the relativistic vector Bose gases are still difficult to realize in real experiments, the vector Bose gases are investigated in their nonrelativistic case, i.e., in their low-energy region. This investigation starts from the relativistic Lagrangian of the vector Bose gases, then performs a mass transformation to map the vector Bose gases to the nonrelativistic state. Due to the vector fields used to describe the vector Bose gases, the interactions between the vector Bose gases and superfluids can be studied by analogy with that of the electromagnetic field.

In the nonrelativistic state, one of the major kinds of the vector Bose gases is the spin-1 Bose gases which are named to highlight their spin dynamics [6–18,39–48]. The spin-1 Bose gases are usually researched in the spinor wave functions [39–48]. However, they can also be described naturally in terms of the vector fields, with correspondences between the vector fields and the spinor wave functions (see Appendix A). When viewing from the vector fields, a better understanding of the spin-1 Bose gases from the relativistic perspective can be developed, and the familiar methods of studying the electromagnetic field can be used to study the phenomena of spin-1 Bose gases. Moreover, not only the spin-1 Bose gases but also the investigation of the vector Bose gases can help further develop the understanding of the phenomena of condensed matter systems interacting with the massive vector Bose fields, which include triplet pairing superconducting and superfluid systems which initially are fermionic systems [19–30].

The paper is organized as follows. In Sec. II, the nonrelativistic vector Bose gases model from the relativistic Lagrangian are set up. In Sec. III A, we focus on the Meissner effect of the vector Bose gases interacting with the fermionic superfluids. The flux quantization and the Josephson effect of the vector Bose gases are presented in Sec. III B. In Sec. III C, we explore the conditions of forming a Maxwell-Chern-Simons model in the vector Bose gases. In Sec. III D, we briefly discuss the experimental proposals. Last, we draw conclusions in Sec. IV.

II. FORMULATION

The relativistic Lagrangian of the vector Bose gases is given by [38]

$$\mathcal{L}_B = \tfrac{1}{2}\mathcal{G}^*_{\mu\nu}\mathcal{G}_{\mu\nu} - \tfrac{1}{2}\mathcal{G}^*_{\mu\nu}(\partial_\mu\phi_\nu - \partial_\nu\phi_\mu) - \tfrac{1}{2}(\partial_\mu\phi^*_\nu - \partial_\nu\phi^*_\mu)\mathcal{G}_{\mu\nu} - m_B^2\phi^*_\mu\phi_\mu. \tag{1}$$

The first term of Eq. (1) is a kinetic energy term constructed using the antisymmetric tensor fields $\mathcal{G}_{\mu\nu}(\boldsymbol{x}, t)$, where μ and ν are four-dimensional direction indices, $\boldsymbol{x} = (x, y, z)$ is the space parameter, and t is the time parameter. For general considerations, the vector Bose fields in Eq. (1) are regarded as carrying charges, thus Eq. (1) includes the conjugate fields $\mathcal{G}^*_{\mu\nu}(\boldsymbol{x}, t)$. It should be noted that our results also hold in the neutral vector Bose gases. The fourth term of Eq. (1) is a mass term representing the rest energy described by the four-dimensional vector fields $\phi_\mu(\boldsymbol{x}, t)$, where m_B is the mass of the vector Bose gases, and $\phi^*_\mu(\boldsymbol{x}, t)$ are the conjugate fields. The second and third terms are the coupling between the tensor fields $\mathcal{G}_{\mu\nu}(\boldsymbol{x}, t)$ and the vector fields $\phi_\mu(\boldsymbol{x}, t)$, from which the constraint $\mathcal{G}_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$ is obtained through the Lagrange equation. Equation (1) is similar to the Lagrangian of the electromagnetic field where the difference between the two is that the former has an additional mass term, which induces the vector Bose gases to lose the gauge invariance.

In practice, the relativistic vector Bose gases are hard to obtain, so the focus is shifted to the vector Bose gases in their low-energy region; in other words, we focus on the nonrelativistic state of the gases. To this end, we perform a mass transformation to Eq. (1) to cancel the mass part of the vector Bose gases so as to stand out the low-energy region.

The mass transformation can be described by $\phi_\mu(\boldsymbol{x}, t) = \varphi_\mu(\boldsymbol{x}, t)e^{-im_Bt}$, where $\varphi_\mu(\boldsymbol{x}, t) = (\boldsymbol{\varphi}, i\varphi_0)$ denote the low-energy vector fields of the vector Bose gases, $\boldsymbol{\varphi}(\boldsymbol{x}, t)$ is the space component, and $\varphi_0(\boldsymbol{x}, t)$ is the time component. Performing the mass transformation, Eq. (1) becomes

$$\mathcal{L}_B = -\tfrac{1}{2}G^*_{\mu\nu}G_{\mu\nu} - m_BG^*_{\mu4}\varphi_\mu + m_B\varphi^*_\mu G_{\mu4} + m_B^2\varphi^*_0\varphi_0, \quad (2)$$

where $G_{\mu\nu}$ represent the low-energy antisymmetric tensor fields that have removed the influence of mass. After the mass transformation, the space component mass term $m_B^2\phi^*_i\phi_i$ ($i = x, y, z$) in Eq. (1) has been eliminated, leaving only the time component mass term $m_B^2\varphi^*_0\varphi_0$. Equation (2) is described by the low-energy fields and can be viewed as a series expansion expanding by the mass m_B. In Eq. (2), the relativistic invariance has lost and the relativistic vector Bose gases have been mapped into a nonrelativistic expansion.

By analogy with the electromagnetic field, the corresponding electric and magnetic fields of the vector Bose gases can be defined. The electric field is defined as $\boldsymbol{E} = -\nabla\varphi_0 - \partial_t\boldsymbol{\varphi}$, and the magnetic field as $\boldsymbol{B} = \nabla \times \boldsymbol{\varphi}$; then Eq. (2) reads

$$\mathcal{L}_B = \boldsymbol{E}^* \cdot \boldsymbol{E} - \boldsymbol{B}^* \cdot \boldsymbol{B} + im_B\boldsymbol{E}^* \cdot \boldsymbol{\varphi} - im_B\boldsymbol{\varphi}^* \cdot \boldsymbol{E} + m_B^2\varphi^*_0\varphi_0. \quad (3)$$

Equation (3) is expressed by fields similar to the electromagnetic field, which enables us to extract the characteristics similar to that of the electromagnetic field to study the vector Bose gases. Note that here the terms of the electric field and magnetic field are borrowed to dub the wave functions of the vector Bose gases. These terms have no relation with the real electromagnetic field.

III. RESULT AND DISCUSSION

A. The Meissner effect

Subsequently, the effects of the nonrelativistic vector Bose gases on fermionic superfluids are investigated. The Lagrangian of charged vector Bose gases interacting with s-wave fermionic superfluids is given by

$$\mathcal{L} = \psi^\dagger i\hbar\frac{\partial}{\partial t}\psi - \psi^\dagger\left\{\frac{1}{2m_s}\left[\boldsymbol{p} - \frac{g}{2}(\boldsymbol{\varphi} + \boldsymbol{\varphi}^*)\right]^2 + \frac{g}{2}(\varphi_0 + \varphi^*_0) - \mu\right\}\psi + \Delta(\psi^\dagger\psi^\dagger + \text{H.c.}) + \mathcal{L}_B, \quad (4)$$

where $\psi(\psi^\dagger)$ is the wave function of superfluids; m_s and $\boldsymbol{p}$ are the mass and momentum of superfluid particles, respectively; μ is the chemical potential; and Δ is the energy gap. The interacting terms in Eq. (4) take the form $\psi^\dagger g(\varphi_\mu + \varphi^*_\mu)\psi/2$, in which g is the coupling strength between the vector Bose gases and superfluids, and the conjugate fields φ^*_μ are included to maintain the Hermitian.

According to the Lagrange equations, the motion equations of the vector Bose gases can be expressed as

$$\nabla \cdot \boldsymbol{E} = \frac{1}{2}g\rho - m_B^2\varphi_0 - im_B\nabla \cdot \boldsymbol{\varphi}, \quad \nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B},$$
$$\nabla \cdot \boldsymbol{B} = 0, \quad \nabla \times \boldsymbol{B} = \frac{1}{2}\boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{E} - im_B\boldsymbol{E} + im_B\frac{\partial}{\partial t}\boldsymbol{\varphi}. \quad (5)$$

Here $\rho = \psi^\dagger\psi$ is the particle density of superfluids and $\boldsymbol{J} = g\psi^\dagger\frac{1}{m_s}[\boldsymbol{p} - \frac{g}{2}(\boldsymbol{\varphi} + \boldsymbol{\varphi}^*)]\psi$ is the current density. Comparing to the Maxwell equations, for the vector Bose gases carrying mass, Eq. (5) have the terms reflecting the mass effect.

At first sight, the expressions of Eq. (5) are similar to the motion equations of the electromagnetic field and seem no relation with the Schrödinger equation of nonrelativistic spin-1 Bose gases. However, when we change the low-energy vector fields to the spinor wave functions, Eq. (5) turn into the Schrödinger equation, which manifests that indeed Eq. (5) are the nonrelativistic motion equations of the vector Bose gases, see Appendix A. Notice that the motion equations of the electromagnetic field described by vector fields cannot has a nonrelativistic approximation, because the electromagnetic field is massless, this is a crucial difference in motion equations between the vector Bose gases and the electromagnetic field.

We now calculate the Meissner effect associated with the vector Bose gases. The London equation is written as $\boldsymbol{J} = -\frac{1}{\lambda^2}(\boldsymbol{\varphi} + \boldsymbol{\varphi}^*)$, where $\lambda = \lambda(\Delta)$ is the penetration length of the superfluids containing the energy gap [49]. Considering the curl of $\nabla \times \boldsymbol{B}$ in Eq. (5), we obtain

$$\frac{\partial^2}{\partial t^2}\text{Re}(\boldsymbol{B}) + 2m_B\frac{\partial}{\partial t}\text{Im}(\boldsymbol{B}) - \nabla^2\text{Re}(\boldsymbol{B}) + \lambda^{-2}\text{Re}(\boldsymbol{B}) = 0, \quad (6)$$

where $\text{Re}(\boldsymbol{B})$ and $\text{Im}(\boldsymbol{B})$ are the real and imaginary parts of $\boldsymbol{B}$. Equation (6) is the Meissner effect equation for the vector Bose gases in superfluids. The significant feature of Eq. (6) is the appearance of the time derivative containing the mass of the vector Bose gases. Without this term, Eq. (6) is reduced to the Meissner effect equation of an electromagnetic field in superconductors.

Selecting the ansatz of the magnetic field as $\boldsymbol{B} = \boldsymbol{B}_0 \exp[-\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t]$, where $\boldsymbol{B}_0$ is the amplitude, $\boldsymbol{k}$ is the wave vector, and ω is the real frequency, and then substituting the ansatz into Eq. (6):

$$k = (\lambda^{-2} - \omega^2 - 2m_B\omega)^{\frac{1}{2}}. \tag{7}$$

In Eq. (7), $k = 0$ is the critical point at which the vector Bose gases destroy superfluids. If the frequency ω increases any further, then the wave vector $\boldsymbol{k}$ will change from real to imaginary, and the superfluidity is destroyed.

Considering $k = 0$, the frequency in Eq. (7) has the expression $\omega = \omega_\pm = -m_B \pm \sqrt{m_B^2 + \lambda^{-2}}$, where ω_+ and ω_- are the critical points corresponding to oppositely charged vector Bose gases. These critical points are related to the mass of the vector Bose gases, and this fact is different from that of the electromagnetic field for its critical frequency point being independent of mass. The term $-m_B$ in the critical frequency indicates that the mass has been removed, and the critical frequency only represents the kinetic energy of the vector Bose gases. If $m_B \gg \lambda^{-1}$, which means that the kinetic energy is much smaller than the mass energy, then ω_+ is approximately equal to

$$\omega_+ = \frac{\lambda^{-2}}{2m_B}. \tag{8}$$

This is a well-known classical kinetic energy expression, which demonstrates that the vector Bose gases destroy superfluids in a classical kinetic energy once the momentum matches the characteristic length λ of superfluids.

The expression of Eq. (8) also can be obtained from the Schrödinger formalism of Eq. (6), see Appendix B, which demonstrates that the neutral vector Bose gases also can generate the Meissner effect when interacting with superfluids. Meanwhile, the Schrödinger formalism also shows that the spin-1 Bose gases can display the Meissner effect in the spinor wave function representation.

B. The flux quantization and Josephson effect

Next, let us discuss the flux quantization and the Josephson effect of the vector Bose gases. The superfluid wave function is written as $\psi(\boldsymbol{x}, t) = \sqrt{\rho}e^{i\phi}$, where ρ is the superfluid density and $\phi = \phi(\boldsymbol{x}, t)$ is the superfluid phase. Then, assuming that ρ is constant, and integrating the current density $\boldsymbol{J}$ in Eq. (5) over a closed path, we obtain

$$\oint \boldsymbol{J}\cdot d\boldsymbol{l} + g^2\frac{\rho}{m_s}\oint \mathrm{Re}(\boldsymbol{\varphi})\cdot d\boldsymbol{l} = g\frac{\hbar\rho}{m_s}\oint \nabla\phi\cdot d\boldsymbol{l}. \tag{9}$$

Here $\mathrm{Re}(\boldsymbol{\varphi})$ is the real part of $\boldsymbol{\varphi}$ and $d\boldsymbol{l}$ is the path integral factor. For Eq. (9), laying the closed path in a region without superfluids, the current density integration on the left is eliminated. Using the relation $\boldsymbol{B} = \nabla\times\boldsymbol{\varphi}$, the second term in Eq. (9) becomes a flux term $\oint \mathrm{Re}(\boldsymbol{\varphi})\cdot d\boldsymbol{l} = \int \mathrm{Re}(\boldsymbol{B})\cdot d\boldsymbol{S}$, where S is the surface surrounded by the closed path. Because ϕ is a phase, the right-hand side of Eq. (9) turns out to be $\oint \nabla\phi\cdot d\boldsymbol{l} = 2\pi n$, where n is an integer. Therefore, Eq. (9) can be written as

$$\int \mathrm{Re}(\boldsymbol{B})\cdot d\boldsymbol{S} = 2\pi\hbar n/g. \tag{10}$$

Equation (10) describes the flux quantization of the vector Bose gases in superfluids. The flux in Eq. (10) is a result of the antisymmetric tensor fields $G_{\mu\nu}$, and the quantization is attributed to the coherent phase of superfluids. The expression of the flux quantization of the vector Bose gases is similar to that of the electromagnetic field. However, unlike the value of the flux quantization of the electromagnetic field is a constant, the value of the vector Bose gases can be tuned, for the coupling constant g in the Eq. (10) can be adjusted. Therefore, what is obtained is a tunable flux quantization when considering the vector Bose gases interacting with the superfluids.

Next, we discuss the influence of the flux in Josephson junctions. The current density $\boldsymbol{J}$ can be written as

$$\boldsymbol{J} = g\rho\frac{\hbar}{m_s}\nabla\Phi, \tag{11}$$

where $\boldsymbol{p} = \hbar\nabla\Phi$ is the superfluid momentum and $\Phi = \phi - \phi_B$ represents the effective superfluid phase where $\phi_B = \frac{g}{\hbar}\int \mathrm{Re}(\boldsymbol{B})\cdot d\boldsymbol{S}$ is the flux phase provided by the vector Bose gases. The expression $-\hbar\nabla\phi_B$ in the superfluid momentum indicates that the flux of the vector Bose gases jointly drives the supercurrent.

Assuming the boundary condition at the surface of two superfluid systems is expressed as $\nabla\psi_1 = \boldsymbol{\kappa}\psi_2$, where $\boldsymbol{\kappa}$ is the tunneling strength between the two systems near the surface, and $\psi_1(\psi_2)$ represents the boundary (bulk) wave function of the first (second) system [49]. Here $\boldsymbol{\kappa}$ contains not only the tunneling of Cooper pairing but also the tunneling of the supercurrent driving by the flux of the vector Bose gases. Combining the boundary condition $\nabla\psi_1 = \boldsymbol{\kappa}\psi_2$ and the momentum eigenequation $-i\hbar\nabla\psi_1 = \boldsymbol{p}\psi_1$, a relation can be expressed as $\hbar(\nabla\Phi_1)\psi_1 = -i\hbar\boldsymbol{\kappa}\psi_2$, where $\psi_1 = \sqrt{\rho_1}e^{i\Phi_1}$, and $\psi_2 = \sqrt{\rho_2}e^{i\phi_2}$, in which $\rho_1(\rho_2)$, $\Phi_1(\phi_2)$ are the superfluid density and the effective phase (the bulk phase) of the first (second) system, respectively. Here the effective phase $\Phi_1 = \phi_1 - \phi_B$ includes the influence of the flux, where ϕ_1 is the bulk phase of the first system. Substituting ψ_1 and ψ_2 into Eq. (11), the tunneling current density has the expression

$$\boldsymbol{J} = \boldsymbol{J}_0 \sin\left[\phi_2 - \phi_1 + \frac{g}{\hbar}\int \mathrm{Re}(\boldsymbol{B})\cdot d\boldsymbol{S}\right], \tag{12}$$

where $\boldsymbol{J}_0 = g\rho_1\frac{\hbar\boldsymbol{\kappa}}{m_s}(\frac{\rho_2}{\rho_1})^{1/2}$ is the tunneling amplitude. Equation (12) shows that the Josephson effect is influenced by the flux of the vector Bose gases in superfluids. The reason for this result is also due to the kinetic energy of the vector Bose gases being described by the antisymmetric tensor fields. It should be noted that the value of the flux in Eq. (12) can be tuned, since the coupling constant g is tunable.

IV. THE MAXWELL-CHERN-SIMONS MODEL OF THE VECTOR BOSE GASES

In this section, we discuss the Maxwell-Chern-Simons model of the vector Bose gases. In the canonical quantization, the independent dynamic parameter of the vector Bose gases is the three-dimensional space vector field $\boldsymbol{\varphi}$, and the corresponding conjugate parameter is the electric field $\boldsymbol{E}$. The scale field φ_0 is not dynamic and depends on the dynamic parameters from the first equation of Eq. (5): $\varphi_0 = \frac{1}{2}m_B^{-2}g\rho -$

$im_B^{-1}\nabla\cdot\boldsymbol{\varphi} - m_B^{-2}\nabla\cdot\boldsymbol{E}$. If $|\varphi_0|^2$ can be neglected, or m_B^2 can be neglected in the case of the much lighter of the mass comparing to the kinetic energy, then Eq. (3) changes to an effective Lagrangian $\mathcal{L}_{B,\mathrm{eff}} = \boldsymbol{E}^*\cdot\boldsymbol{E} - \boldsymbol{B}^*\cdot\boldsymbol{B} + im_B\boldsymbol{E}^*\cdot\boldsymbol{\varphi} - im_B\boldsymbol{\varphi}^*\cdot\boldsymbol{E}$. We further take the Gauss constraint $\nabla\cdot\boldsymbol{E} = 0$, then the effective Lagrangian $\mathcal{L}_{B,\mathrm{eff}}$ is gauge invariance under the gauge transformation $\varphi_0 \to \varphi_0 - \partial_t\alpha$, $\boldsymbol{\varphi} \to \boldsymbol{\varphi} + \nabla\alpha$ [$\alpha(\boldsymbol{x}, t)$ is an analytic function]. This shows that after performing the mass transformation, we have isolated the mass term that violates gauge invariance and highlighted the gauge invariance terms in the vector Bose gases.

Let $\varphi_\mu \to \varphi_\mu/\sqrt{2}$, the effective Lagrangian becomes $\mathcal{L}_{B,\mathrm{eff}} = \frac{1}{2}(\boldsymbol{E}^*\cdot\boldsymbol{E} - \boldsymbol{B}^*\cdot\boldsymbol{B}) + m_B\mathrm{Im}(\boldsymbol{\varphi}^*\cdot\boldsymbol{E})$, where $\mathrm{Im}(\boldsymbol{\varphi}^*\cdot\boldsymbol{E})$ is the imaginary part of $\boldsymbol{\varphi}^*\cdot\boldsymbol{E}$. Defining $E_\mu = \frac{1}{2}\varepsilon_{\mu\nu\lambda}F_{\nu\lambda}$, $(\mu, \nu, \lambda = x, y, z)$, where $F_{\nu\lambda}$ is the duality field of the electric field, we obtain

$$\mathcal{L}_{B,\mathrm{eff}} = \frac{1}{2}(\boldsymbol{E}^*\cdot\boldsymbol{E} - \boldsymbol{B}^*\cdot\boldsymbol{B}) + \frac{1}{2}m_B\varepsilon_{\mu\nu\lambda}\mathrm{Im}(\varphi_\mu^* F_{\nu\lambda}). \quad (13)$$

Equation (13) is a Maxwell-Chern-Simons Lagrangian [50–52]. Therefore, under the Gauss constraint $\nabla\cdot\boldsymbol{E} = 0$, the effective model of the vector Bose gases is related to a gauge-invariant Maxwell-Chern-Simons system.

V. EXPERIMENTAL PROPOSAL

In this work, the nonrelativistic vector Bose gases interacting with fermionic superfluids was studied. In the nonrelativistic case, one of the major kinds of the vector Bose gases is the spin-1 Bose gases. The cold atomic spin-1 Bose gases such as ^{23}Na or ^{87}Rb [6–18] have been realized in real experiments, and, experimentally, these gases would be the appropriate candidates in the case of the superfluids also in a proper selection to obtain the striking experimental signatures. With respect to studying the Maxwell-Chern-Simons model of the vector Bose gases, one choice is that the lighter mass atomic gases, such as ^{7}Li, can be selected to cancel the influence of the m_B^2 term in Eq. (3). However, since it is a requirement that neglecting the scalar potential, or the constrain that the kinetic energy is much larger than the mass is tough to obtain, it may still be difficult to realize the Maxwell-Chern-Simons model in real experiments.

VI. CONCLUSION

In summary, we have investigated the effects of the vector Bose gases on fermionic superfluids. Our results show that, unlike the electromagnetic field, the destruction of the Meissner effect of vector Bose gases is related to mass. In the low-energy region, the vector Bose gas can destroy the superfluid states as long as it obtains a classical kinetic energy in which the momentum matches the penetration length of the superfluids. Similarly to those of the electromagnetic field, the flux quantization and the Josephson effect of the vector Bose gases also appear in superfluids. However, instead of being just a constant as in the electromagnetic field, the value of the quantized flux of the vector Bose gases can be tunable. The study also shows that under a Gauss constraint, the vector Bose gases are related to the gauge-invariant Maxwell-Chern-Simons system in certain suitable situations. We also propose that the spin-1 cold atomic Bose gases can be used to observe these effects. The investigations of this work may also be beneficial for the study of related systems in high-energy physics.

ACKNOWLEDGMENTS

This work was supported by the Yunnan Local Colleges Applied Basic Research Projects (No. 2017FH001-001, No. 2017FH001-112, and No. 2019FH001(-077)), the General Projects of Yunnan Education Department (No. 2018JS393), the National Natural Science Foundation of China (No. 12047576), the National Science Foundation of China (No. 11805029), and the Yunnan Applied Basic Research Projects (No. 2017FD250 and No. 2021FB145).

APPENDIX A: CORRESPONDENCE BETWEEN THE MOTION EQUATIONS OF THE VECTOR BOSE GASES AND THE NONRELATIVISTIC SPIN-1 BOSE GASES

The motion equations of the vector Bose gases are described by Eq. (5) in the main text,

$$\nabla\cdot\boldsymbol{E} = -m_B^2\varphi_0 - im_B\nabla\cdot\boldsymbol{\varphi}, \quad (A1a)$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}, \quad (A1b)$$

$$\nabla\cdot\boldsymbol{B} = 0, \quad (A1c)$$

$$\nabla\times\boldsymbol{B} = \frac{\partial}{\partial t}\boldsymbol{E} - im_B\boldsymbol{E} + im_B\frac{\partial}{\partial t}\boldsymbol{\varphi}. \quad (A1d)$$

Here $(\boldsymbol{\varphi}, \varphi_0)$ denote the low-energy vector fields of the vector Bose gases, m_B is the mass of the vector Bose gases, $\boldsymbol{E} = -\nabla\varphi_0 - \partial_t\boldsymbol{\varphi}$ and $\boldsymbol{B} = \nabla\times\boldsymbol{\varphi}$ are the corresponding electric and magnetic fields, respectively; t is the time parameter. In Eq. (S1), for explicit correspondence, the particle density ρ and the current density $\boldsymbol{J}$ of Eq. (5) have been dropped.

Equation (A1d) can be rewritten in the low-energy vector fields as

$$\left(\nabla^2 - \frac{m_B^2c^2}{\hbar^2}\right)\varphi_0 + \nabla\cdot\left(\frac{\partial}{c\partial t}\boldsymbol{\varphi} - i\frac{m_Bc}{\hbar}\boldsymbol{\varphi}\right) = 0, \quad (A2a)$$

$$\nabla\times\nabla\varphi_0 = 0, \quad (A2b)$$

$$\nabla\cdot(\nabla\times\boldsymbol{\varphi}) = 0, \quad (A2c)$$

$$\frac{\partial^2}{c^2\partial t^2}\boldsymbol{\varphi} - i\frac{2m_B}{\hbar}\frac{\partial}{\partial t}\boldsymbol{\varphi} - \nabla^2\boldsymbol{\varphi} + \nabla\left(\frac{\partial}{c\partial t}\varphi_0 - i\frac{m_Bc}{\hbar}\varphi_0 + \nabla\cdot\boldsymbol{\varphi}\right) = 0. \quad (A2d)$$

In Eq. (A1d), the Planck constant $\hbar$ and the light velocity c have been restored. Since Eq. (A2b) and Eq. (A2d) are just the Bianchi identities, only Eq. (A2a) and Eq. (A2c) need to be analyzed.

First, Eq. (A2a) is discussed. Using the reverse mass transformation $(\varphi_0, \boldsymbol{\varphi}) = (\phi_0, \boldsymbol{\phi})e^{im_Bt}$, where $(\phi_0, \boldsymbol{\phi})$ are four-dimensional relativistic vector fields of Eq. (1) in the main text, Eq. (A2a) changes to

$$\left(\nabla^2 - \frac{m_B^2c^2}{\hbar^2}\right)\phi_0 + \nabla\cdot\frac{\partial}{c\partial t}\boldsymbol{\phi} = 0. \quad (A3)$$

It should be noted that the Proca equation reads as $(\nabla^2 - \frac{\partial^2}{c^2\partial t^2})\phi_0 - \frac{m_B^2c^2}{\hbar^2}\phi_0 = 0$, which leads to Eq. (A2c) becoming

$$\frac{\partial}{c\partial t}\left(\frac{\partial}{c\partial t}\phi_0 + \nabla\cdot\boldsymbol{\phi}\right) = 0. \tag{A4}$$

The expression in the bracket of the above equation is just the expression of the Lorentz gauge of the Proca equation: $\frac{\partial}{c\partial t}\phi_0 + \nabla\cdot\boldsymbol{\phi} = 0$. Therefore, Eq. (A2a) has no dynamic meaning, it is just the Lorentz gauge which automatically appears in the spin-1 fields.

Next, in Eq. (A2c), since the first term of Eq. (A2c) is in c^{-2} order and it is the smallest term comparing to others, this term can be neglected in nonrelativistic approximation. It should also be noted that the Lorentz gauge of the Proca equation can be expressed as $\frac{\partial}{c\partial t}\varphi_0 - i\frac{m_Bc}{\hbar}\varphi_0 + \nabla\cdot\boldsymbol{\varphi} = 0$, leading to Eq. (A2c) becoming

$$i\hbar\frac{\partial}{\partial t}\boldsymbol{\varphi} = -\frac{\hbar^2}{2m_B}\nabla^2\boldsymbol{\varphi}, \tag{A5}$$

where the above equation is a Schrödinger equation of the vector fields $\boldsymbol{\varphi}$.

To correspond Eq. (A2a) to the Schrödinger equation of spin-1 Bose gases described by the spinor wave functions, Eq. (A2a) should be rewritten in the S_z representation of spin-1, where $S_z = \mathrm{diag}(1, 0, -1)$ is the third spin component. The spin operators of the vector fields read as $S_k = \int d^3x\varepsilon_{ijk}\mathcal{G}_{i4}^*\phi_j$, where $\mathcal{G}_{i4}^*$ are four-dimensional relativistic tensor fields of Eq. (1) in the main text, i, j, k are space indices. Therefore, in the nonrelativistic approximation, the third spin component is written as

$$\begin{aligned} S_z &= im_Bc\int d^3x(\varphi_2^*\varphi_1 - \varphi_1^*\varphi_2) \\ &= \int d^3x(\psi_+^*, \psi_0^*, \psi_-^*)\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}\begin{pmatrix}\psi_+\\ \psi_0\\ \psi_-\end{pmatrix}, \end{aligned} \tag{A6}$$

in which $\psi_\pm = \sqrt{\frac{m_Bc}{2}}(\varphi_1 \mp i\varphi_2)$, $\psi_0 = \sqrt{m_Bc}\varphi_3$.

In Eq. (A6), the wave function $\Psi = (\psi_+, \psi_0, \psi_-)^T$ is just the spinor wave function describing spin-1 Bose gases, using this wave function, Eq. (A5) can be written to

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m_B}\nabla^2\Psi. \tag{A7}$$

This is just the Schrödinger equation of spin-1 Bose gases described by spinor wave functions. Therefore, the motion equations of the vector Bose gases correspond to the motion equations of the nonrelativistic spin-1 Bose gases.

APPENDIX B: THE MEISSNER EFFECT IN THE SCHRÖDINGER FORMALISM

The Meissner effect of the vector Bose gases is described by Eq. (6) in the main text

$$\frac{\partial^2}{\partial t^2}\mathrm{Re}(\boldsymbol{B}) + 2m_B\frac{\partial}{\partial t}\mathrm{Im}(\boldsymbol{B}) - \nabla^2\mathrm{Re}(\boldsymbol{B}) + \lambda^{-2}\mathrm{Re}(\boldsymbol{B}) = 0, \tag{B1}$$

where λ is the penetration length of the superfluids and $\mathrm{Re}(\boldsymbol{B})[\mathrm{Im}(\boldsymbol{B})]$ is the real (imaginary) part of $\boldsymbol{B}$. If $\boldsymbol{B} = \nabla\times\boldsymbol{\varphi}$ is substituted to the above equation, then the Planck constant $\hbar$ and the light velocity c are restored, we have

$$\frac{\partial^2}{c^2\partial t^2}\boldsymbol{\varphi} - i\frac{2m_B}{\hbar}\frac{\partial}{\partial t}\boldsymbol{\varphi} - \nabla^2\boldsymbol{\varphi} + \lambda^{-2}\boldsymbol{\varphi} = 0. \tag{B2}$$

Neglecting the first term of the above equation for its c^{-2} order, Eq. (B2) changes to

$$i\hbar\frac{\partial}{\partial t}\boldsymbol{\varphi} = -\frac{\hbar^2}{2m_B}\nabla^2\boldsymbol{\varphi} + \frac{\hbar^2\lambda^2}{2m_B}\boldsymbol{\varphi}. \tag{B3}$$

Equation (B3) is the Schrödinger formalism of the Meissner effect, which also demonstrates that the neutral vector Bose gases can generate the Meissner effect when interacting with superfluids.

Taking the ansatz of the vector fields as $\boldsymbol{\varphi} = \boldsymbol{\varphi}_0\exp(-\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t)$, where $\boldsymbol{\varphi}_0$ is the amplitude, $\boldsymbol{k}$ is the wave vector, and ω is the real frequency, then substituting the ansatz into Eq. (B3), at the critical point $k = 0$, we obtain

$$\omega = \frac{\lambda^{-2}}{2m_B}. \tag{B4}$$

This is the destroying energy expressed by Eq. (8) in the main text. The form of Eq. (B4) is a well-known classical kinetic energy expression, which results from the nonrelativistic approximation that the kinetic energy is much smaller than the mass energy. This approximation is the primary assumption that allows the square term of the time derivative in Eq. (A2c) and Eq. (B2) to be neglected.

[1] J. D. Walecka, A theory of highly condensed matter, Ann. Phys. **83**, 491 (1974).

[2] P. G. Reinhard, M. Rufa, J. Maruhn, W. Greiner, and J. Friedrich, Nuclear ground-state properties in a relativistic meson-field theory, Z. Phys. A **323**, 13 (1986).

[3] T. Hambye, Hidden vector dark matter, J. High Energy Phys. 01 (2009) 028.

[4] J. Arakawa, A. Rajaraman, and T. M. P. Tait, Emergent Solution to the Strong CP Problem, Phys. Rev. Lett. **123**, 161602 (2019).

[5] C. G. Böhmer and T. Harko, Dark energy as a massive vector field, Eur. Phys. J. C **50**, 423 (2007).

[6] D. M. Stamper-Kurn, M. R. Andrews, A. P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, Optical Confinement of a Bose-Einstein Condensate, Phys. Rev. Lett. **80**, 2027 (1998).

[7] J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur, and W. Ketterle, Spin domains in ground-state Bose-Einstein condensates, Nature **396**, 345 (1998).

[8] H.-J. Miesner, D. M. Stamper-Kurn, J. Stenger, S. Inouye, A. P. Chikkatur, and W. Ketterle, Observation of Metastable States in Spinor Bose-Einstein Condensates, Phys. Rev. Lett. **82**, 2228 (1999).

[9] M.-S. Chang, C. D. Hamley, M. D. Barrett, J. A. Sauer, K. M. Fortier, W. Zhang, L. You, and M. S. Chapman, Observation of Spinor Dynamics in Optically Trapped ^{87}Rb Bose-Einstein Condensates, Phys. Rev. Lett. **92**, 140403 (2004).
[10] J. M. Higbie, L. E. Sadler, S. Inouye, A. P. Chikkatur, S. R. Leslie, K. L. Moore, V. Savalli, and D. M. Stamper-Kurn, Direct Nondestructive Imaging of Magnetization in a Spin-1 Bose-Einstein Gas, Phys. Rev. Lett. **95**, 050401 (2005).
[11] L. E. Sadler, J. M. Higbie, S. R. Leslie, M. Vengalattore, and D. M. Stamper-Kurn, Spontaneous symmetry breaking in a quenched ferromagnetic spinor Bose-Einstein condensate, Nature **443**, 312 (2006).
[12] M. Vengalattore, J. Guzman, S. R. Leslie, F. Serwane, and D. M. Stamper-Kurn, Periodic spin textures in a degenerate $F = 1^{87}$Rb spinor Bose gas, Phys. Rev. A **81**, 053612 (2010).
[13] J. Guzman, G.-B. Jo, A. N. Wenz, K. W. Murch, C. K. Thomas, and D. M. Stamper-Kurn, Long-time-scale dynamics of spin textures in a degenerate $F = 1^{87}$Rb spinor Bose gas, Phys. Rev. A **84**, 063625 (2011).
[14] H. K. Pechkis, J. P. Wrubel, A. Schwettmann, P. F. Griffin, R. Barnett, E. Tiesinga, and P. D. Lett, Spinor Dynamics in an Antiferromagnetic Spin-1 Thermal Bose Gas, Phys. Rev. Lett. **111**, 025301 (2013).
[15] D. M. Stamper-Kurn and M. Ueda, Spinor Bose gases: Symmetries, magnetism, and quantum dynamics, Rev. Mod. Phys. **85**, 1191 (2013).
[16] R. P. Anderson, M. J. Kewming, and L. D. Turner, Continuously observing a dynamically decoupled spin-1 quantum gas, Phys. Rev. A **97**, 013408 (2018).
[17] Y. Eto, H. Shibayama, H. Saito, and T. Hirano, Spinor dynamics in a mixture of spin-1 and spin-2 Bose-Einstein condensates, Phys. Rev. A **97**, 021602(R) (2018).
[18] X. Chai, D. Lao, K. Fujimoto, R. Hamazaki, M. Ueda, and C. Raman, Magnetic Solitons in a Spin-1 Bose-Einstein Condensate, Phys. Rev. Lett. **125**, 030402 (2020).
[19] A. P. Mackenzie and Y. Maeno, The superconductivity of Sr_2RuO_4 and the physics of spin-triplet pairing, Rev. Mod. Phys. **75**, 657 (2003).
[20] C. Kallin and A. J. Berlinsky, Is Sr_2RuO_4 a chiral p-wave superconductor?, J. Phys. Condens. Matter **21**, 164210 (2009).
[21] J. D. Strand, D. J. Bahr, D. J. Van Harlingen, J. P. Davis, W. J. Gannon, and W. P. Halperin, The transition between real and complex superconducting order parameter phases in UPt_3, Science **328**, 1368 (2010).
[22] Y. Maeno, S. Kittaka, T. Nomura, S. Yonezawa, and K. Ishida, Evaluation of spin-triplet superconductivity in Sr_2RuO_4, J. Phys. Soc. Jpn. **81**, 011009 (2012).
[23] E. R. Schemm, W. J. Gannon, C. M. Wishne, W. P. Halperin, and A. Kapitulnik, Observation of broken time-reversal symmetry in the heavy-fermion superconductor UPt_3, Science **345**, 190 (2014).
[24] H. Kim, K. Wang, Y. Nakajima, R. Hu, S. Ziemak, P. Syers, L. Wang, H. Hodovanets, J. D. Denlinger, P. M. R. Brydon, D. F. Agterberg, M. A. Tanatar, R. Prozorov, and J. Paglione, Beyond triplet: Unconventional superconductivity in a spin-3/2 topological semimetal, Sci. Adv. **4**, 4513 (2018).
[25] S. Ran, C. Eckberg, Q.-P. Ding, Y. Furukawa, T. Metz, S. R. Saha, I. Liu, M. Zic, H. Kim, J. Paglione, and N. P. Butch, Nearly ferromagnetic spin-triplet superconductivity, Science **365**, 684 (2019).
[26] P. Coleman, Y. Komijani, and E. J. König, Triplet Resonating Valence Bond State and Superconductivity in Hund's Metals, Phys. Rev. Lett. **125**, 077001 (2020).
[27] A. J. Leggett, A theoretical description of the new phases of liquid ^{3}He, Rev. Mod. Phys. **47**, 331 (1975).
[28] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor & Francis, London, 1990).
[29] C. Wu and J. E. Hirsch, Mixed triplet and singlet pairing in ultracold multicomponent fermion systems with dipolar interactions, Phys. Rev. B **81**, 020508(R) (2010).
[30] T. Shi, J.-N. Zhang, C.-P. Sun, and S. Yi, Singlet and triplet Bardeen-Cooper-Schrieffer pairs in a gas of two-species fermionic polar molecules, Phys. Rev. A **82**, 033623 (2010).
[31] F. D. M. Haldane, Continuum dynamics of the 1-D Heisenberg antiferromagnet: Identification with the $O(3)$ nonlinear sigma model. Phys. Lett. A **93**, 464 (1983); Nonlinear Field Theory of Large-Spin Heisenberg Antiferromagnets: Semiclassically Quantized Solitons of the One-Dimensional Easy-Axis Nel State, Phys. Rev. Lett. **50**, 1153 (1983).
[32] W. Lu, J. Tuchendler, M. von Ortenberg, and J. P. Renard, Direct observation of the Haldane gap in high magnetic fields, Phys. Rev. Lett. **67**, 3716 (1991).
[33] A. V. Sologubenko, T. Lorenz, J. A. Mydosh, A. Rosch, K. C. Shortsleeves, and M. M. Turnbull, Field-Dependent Thermal Transport in the Haldane Chain Compound NENP, Phys. Rev. Lett. **100**, 137202 (2008).
[34] S. Ward, M. Mena, P. Bouillot, C. Kollath, T. Giamarchi, K. P. Schmidt, B. Normand, K. W. Krämer, D. Biner, R. Bewley, T. Guidi, M. Boehm, D. F. McMorrow, and C. Rüegg, Bound States and Field-Polarized Haldane Modes in a Quantum Spin Ladder, Phys. Rev. Lett. **118**, 177202 (2017).
[35] M. Fujihala, T. Sugimoto, T. Tohyama, S. Mitsuda, R. A. Mole, D. H. Yu, S. Yano, Y. Inagaki, H. Morodomi, T. Kawae, H. Sagayama, R. Kumai, Y. Murakami, K. Tomiyasu, A. Matsuo, and K. Kindo, Cluster-Based Haldane State in an Edge-Shared Tetrahedral Spin-Cluster Chain: Fedotovite $K_2Cu_3O(SO_4)_3$, Phys. Rev. Lett. **120**, 077201 (2018).
[36] A. N. Poddubny and M. M. Glazov, Topological Spin Phases of Trapped Rydberg Excitons in Cu_2O, Phys. Rev. Lett. **123**, 126801 (2019).
[37] A. Proca, Sur la théorie ondulatoire des électrons positifs et négatifs, J. Phys. Radium **7**, 347 (1936).
[38] W. Greiner, *Relativistic Quantum Mechanics*, 3rd ed. (Springer-Verlag, Berlin, 2000).
[39] T. Ohmi and K. Machida, Bose-Einstein condensation with internal degrees of freedom in alkali atom gases, J. Phys. Soc. Jpn. **67**, 1822 (1998).
[40] T.-L. Ho, Spinor Bose Condensates in Optical Traps, Phys. Rev. Lett. **81**, 742 (1998).
[41] C. K. Law, H. Pu, and N. P. Bigelow, Quantum Spins Mixing in Spinor Bose-Einstein Condensates, Phys. Rev. Lett. **81**, 5257 (1998).
[42] T.-L. Ho and S. K. Yip, Fragmented and Single Condensate Ground States of Spin-1 Bose Gas, Phys. Rev. Lett. **84**, 4031 (2000).
[43] H. Saito, Y. Kawaguchi, and M. Ueda, Breaking of Chiral Symmetry and Spontaneous Rotation in a Spinor Bose-Einstein Condensate, Phys. Rev. Lett. **96**, 065302 (2006).

[44] M.-C. Chung and S. Yip, Phase diagrams for spin-1 bosons in an optical lattice, Phys. Rev. A **80**, 053615 (2009).
[45] S. S. Natu and E. J. Mueller, Spin waves in a spin-1 normal Bose gas, Phys. Rev. A **81**, 053617 (2010).
[46] Y. Kawaguchi, N. T. Phuc, and P. B. Blakie, Finite-temperature phase diagram of a spin-1 Bose gas, Phys. Rev. A **85**, 053611 (2012).
[47] G. Lang and E. Witkowska, Thermodynamics of a spin-1 Bose gas with fixed magnetization, Phys. Rev. A **90**, 043609 (2014).
[48] H. M. Hurst, J. H. Wilson, J. H. Pixley, I. B. Spielman, and S. S. Natu, Real-space mean-field theory of a spin-1 Bose gas in synthetic dimensions, Phys. Rev. A **94**, 063613 (2016).
[49] K. Fossheim and A. Sudbø, *Superconductivity Physics and Applications* (John Wiley & Sons, Chichester, 2004).
[50] J. Schonfeld, A mass term for three-dimensional gauge fields, Nucl. Phys. B **185**, 157 (1981).
[51] S. Deser, R. Jackiw, and S. Templeton, Three-Dimensional Massive Gauge Theories, Phys. Rev. Lett. **48**, 975 (1982); Topologically massive gauge theories, Ann. Phys. **140**, 372 (1982).
[52] C. R. Hagen, A new gauge theory without an elementary photon, Ann. Phys. **157**, 342 (1984).