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Spin relaxation in a strongly correlated quantum Hall ferromagnet

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The spin relaxation of a strongly correlated electron system hosted in a narrow AlAs quantum well is studied experimentally in the regime of the quantum Hall ferromagnet, i.e., near odd fillings of the integer quantum Hall effect. The relaxation rate is evaluated by analyzing the linewidth of the electrically detected electron-spin resonance. The measured spin dynamics is shown to undergo a radical modification in tilted magnetic fields. Increasing of the tilt angle resulted not only in the decrease of the relaxation time at the exact odd fillings by almost an order of magnitude, but also in the essentially different dependence of the relaxation rate on the filling factor at highest tilt angles. In the case of a perpendicular magnetic field the relaxation rate reached its minimal value near the exact odd filling and slowly increased when the filling factor ν was altered. In contrast, at large tilt angles relaxation turned out to be the fastest at some ν in close vicinity of the exactly odd number. The observed effects may be ascribed to the renormalization of the spin excitation spectra by the strong e-e interaction.

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I. INTRODUCTION

The plentiful physics of the quantum Hall effect (QHE) becomes even richer in the presence of strong Coulomb interaction, as it brings about a vast amount of intriguing phenomena, including Wigner crystallization [1,2], ferromagnetic Stoner-like transitions around even fillings [3,4], the existence of spin-texture excitations [5–7], that appearance of gapped states at fractional filling factors [8,9], etc. Even though extensive research effort is devoted to studies of QHE in the strongly correlated limit, a complete understanding of this effect has yet to be achieved. In contrast to numerous publications where the ground state and the excitation spectra [3,4,10–20] of strongly interacting electrons are studied in the QHE regime, the present paper aims to study the dynamical properties of such a system, namely, the electron spin relaxation in a strongly correlated quantum Hall ferromagnet. Furthermore, spin relaxation of a two-dimensional (2D) electron at high magnetic fields is essentially a many-electron process and thus may be especially useful in probing the effects of strong e-e interactions.

It is commonly accepted that the limit of strong e-e correlations is achieved in the QHE when the characteristic Coulomb energy E_c exceeds the typical splitting of the single-particle levels, i.e., the cyclotron gap Δ_c is inversely proportional to the effective mass m^*. The large values of m^* diminish Δ_c and favor the dominance of E_c. Thus the weakly interacting regime is established in the semiconductor heterostructures with a small effective mass of electrons, for instance, in GaAs quantum wells. The spin dynamics of electrons is thoroughly studied in such structures, especially in the integral QHE regime around odd filling factors [21–30]. Note that such states posses almost full spin polarization and hence are traditionally referred to as a quantum Hall ferromagnet. Here we aim to study the spin relaxation around odd fillings in the system with effective mass large enough to reach the limit of strong e-e interaction and, as it will be demonstrated later in the manuscript, e-e interactions will greatly affect the dynamics of electron spins. There exist several types of semiconductor heterostructures hosting the two-dimensional electron system (2DES) with suitable m^* including AlAs-based quantum wells [31], ZnO/MgZnO heterostructures [32], hole channels in GaAs wells [33], etc. The experiments reported in the present manuscript were carried out on one of the most promising of them, namely, a narrow AlAs quantum well.

Let us evaluate the overall strength of the electron-electron interaction in such structures. It is conventionally determined by the ratio of the Coulomb energy to the Fermi energy at zero magnetic field, namely, by the parameter $r_s = \frac{1}{\sqrt{\pi n}} \frac{m^* e^2}{\epsilon \hbar^2}$ for the 2D systems. Here n stands for the electron density and ϵ for the dielectric constant of the material. For our samples this parameter varies from 7 at the lowest density studied to 4 in case of the largest n. In GaAs-based 2D electron systems with the same densities, r_s is much smaller and equals 1.4 and 0.8. In the QHE regime the characteristic Coulomb energy E_c may be estimated as $\frac{e^2}{\epsilon l_b}$, where ϵ is the dielectric constant of the material and $l_b = \sqrt{\hbar/eB}$ is the magnetic length. Then for the AlAs well at the typical magnetic field of 6 T the value of E_c is approximately 110 K, whereas single-particle cyclotron [34] and spin splittings [35] may be estimated as 30 and 8 K, respectively, and are dominated by E_c, proving that the strongly interacting regime of QHE is indeed established in the structures under study. For comparison, the GaAs-based 2D electron systems are characterized by the ratio $E_c/\hbar\omega_c$ equal to only 0.7 at the same magnetic field.

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II. SAMPLES

The sample studied was the 4.5-nm-wide AlAs/AlGaAs quantum well grown by molecular beam epitaxy in the [001] direction. As it was demonstrated earlier [34], in such structures the electrons tend to occupy the single valley located at the X-point of the Brillouin zone along the growth direction and are characterized by an isotropic in the plane of the structure mass of $m^* = 0.26\ m_0$ [34], where m_0 stands for the free electron mass. Note that in the structures with similar single-particle mass, pronounced interaction-induced effects have already been detected, including effective mass and spin susceptibility renormalization [19,36], ferromagnetic behavior of the nominally unpolarized even fillings [3,4], softening of spin-flip modes [15], etc. The low-temperature electron density could be varied in the range from 1.5×10^{11} cm^{-2} to 4.5×10^{11} cm^{-2} by means of persistent photoconductivity. The mobility was around 4×10^4 cm^2/Vs at a temperature of 4.2 K.

The standard Hall bar was formed on the samples with the aid of photolithography, and Ohmic contacts were created by soldering and annealing indium in the forming gas atmosphere. The samples were mounted inside of the He-3 pot of a cryostat with a superconducting magnet so that the experiments were performed in magnetic fields of up to 15 T and at temperatures as low as 500 mK. The tilt angle θ between the magnetic field and the normal to the plane of the sample could be varied. Typical magnetoresistance of the 2D electron channel hosted in the structure under study is presented in Ref. [35].

III. EXPERIMENTAL METHOD

A number of various experimental techniques have been developed thus far to probe the spin dynamics of 2D electron systems, and most of them are optical [37,38]. Yet in the AlAs-based 2DES the optical measurements are substantially complicated by the inderect gap of this semiconductor, making the spin resonance method especially useful. This manuscript represents a study of the spin relaxation in the QHE regime in the strongly correlated regime. The spin-relaxation rate is evaluated from the linewidth of the conduction electron spin resonance [39]. The conventional method of electron spin resonance (ESR) detection is based on the high sensitivity of the longitudinal 2D channel resistance to the absorption of the microwave radiation and is explained in detail in our previous publications [40,41].

The detection of ESR was based on the sensitivity of the longitudinal resistance of the 2D channel to the microwave radiation absorption [42]. In order to increase the signal-to-noise ratio, the standard double lock-in technique was implemented, so that the first lock-in amplifier measured the resistance of electron channel, while the second one was tuned to the frequency of the amplitude modulation of the microwave radiation and measured the variation of the 2DES resistance due to the radiation absorption. Spin resonance was then observed as a peak in this variation δR_{xx} when the magnetic field was slowly swept and the radiation frequency was kept constant. The hyperfine interaction of electron and nuclear spins is known to cause dynamic polarization of nuclear spins in the GaAs/AlGaAs heterostructure [43] and complicate the precise measurements of the ESR width by distorting the ESR line shape. However, under typical experimental conditions, no signs of this effect were registered in the samples under study. The linewidth of the ESR peaks was equal in the case of up and down magnetic field sweeps and did not depend on the sweep rates. The radiation was delivered to the sample through an oversized waveguide and was injected into the waveguide quasioptically with the aid of two collimator lenses. Note that the radiation frequencies are large enough for a quasioptical approach to perform well. The power of the radiation was small enough to produce almost no heating of the sample. The ESR amplitude was linear with respect to the incident power, and the width of the ESR peak did not depend on it in a wide range of radiation power. These facts are illustrated in Figs. 1(d) and 1(e).

IV. RESULTS AND DISCUSSION

Extensive studies of the electron *g*-factor that determines the magnetic field position of ESR at a given radiation frequency for the narrow AlAs wells may be found in one of our previous works [35]. The electron *g*-factor turned out to be around 1.88 at large magnetic fields and increased as the magnetic field was decreased due to the spin-orbit (SO) coupling of the spin degree of freedom and the orbital motion of an electron in the QHE regime. In the present manuscript the linewidth of the spin resonance was investigated around small odd filling factors. The resultant dependence of the deduced relaxation rate $1/\tau$ on the filling factor ν around $\nu = 1$ is presented in Fig. 1(a). The electron density was equal to 1.5×10^{11} cm^{-2}. In case the magnetic field was perpendicular to the plane of the 2DES (black open squares), the value $1/\tau$ achieved its minimal value at exactly $\nu = 1$ and slowly increased if ν was varied. Such behavior is similar to the experimental data reported for GaAs-based 2D electrons [26] and may be ascribed to the creation of extensive delocalized electrons as ν moves away from unity. These additional electrons may serve as effective scatterers of nonequilibrium spins and are generated by thermal fluctuations. Their quantity is governed by the ratio of the spin splitting energy gap enhanced by exchange interaction to the temperature of the system and grows quickly away from unity filling, as this gap is maximal at the exactly $\nu = 1$ [44,45].

If the magnetic field was tilted by the nonzero angle θ as illustrated in the inset of Fig. 1(a), the spin dynamics became radically different. The correspondent experimental data is denoted by blue solid circles for $\theta = 30^\circ$ and by red open circles for $\theta = 40^\circ$ in Fig. 1(a). The profound acceleration of the spin relaxation in tilted fields is illustrated in Fig. 1(b), where the spin resonance peaks measured at $\nu = 1$ are demonstrated for $\theta = 0^\circ$ and 40°. The linewidth of spin resonances is clearly different, and this difference is well beyond the experimental uncertainty. This observation becomes even more surprising if one recalls that the larger the value of θ, the greater the magnetic field of the unity filling and the correspondent spin splitting gap Δ_s. Thus one would expect the retardation of the spin dynamics instead of acceleration. Furthermore, the longitudinal resistance of the 2D channel around unity filling

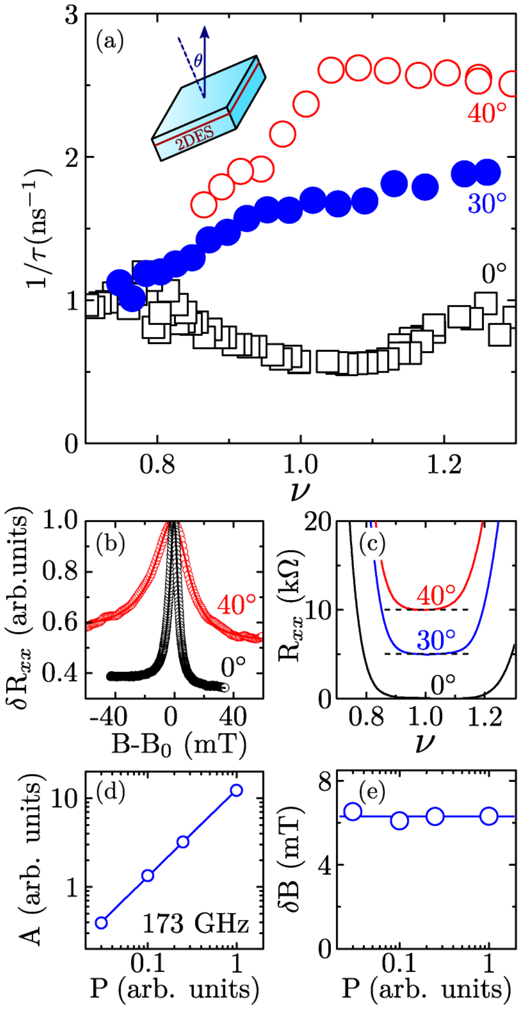

FIG. 1. (a) The filling factor dependence of the relaxation rate $1/\tau$ measured around unity filling at 0.5 K and at different tilt angles θ. Open black squares indicate values of $1/\tau$ in the case of perpendicular magnetic field, while solid blue and open red circles denote data for $\theta = 30^\circ$ and $\theta = 40^\circ$, respectively. The electron density was around 1.5×10^{11} cm^{-2}. (b) Typical line shapes of electron-spin resonance measured at $\nu = 1$, at $\theta = 0^\circ$ (black circles), and $\theta = 40^\circ$ (red circles). The ESR signal is normalized for clarity. The radiation frequency was equal to 165.8 GHz and 221.7 GHz, respectively. The resonant magnetic fields B_o were measured to be 6.318 and 8.24 T, respectively. (c) The longitudinal resistance of the 2D channel measured at $\theta = 0^\circ$, 30°, and 40° vs the filling factor. The data is offset for clarity. (d) The double logarithmic plot of the ESR amplitude measured around unity filling vs radiation power. The radiation frequency equals 173 GHz. The solid line represents the linear fit with slope of 0.98. (e) The power dependence of the ESR linewidth δB for the same radiation frequency of 173 GHz.

does not experience any drastic modification for all the angles θ studied, as is illustrated in Fig. 1(c).

We would like to emphasize that not only the value of $1/\tau$ at the exact $\nu = 1$ became almost an order of magnitude larger for the largest θ studied than for the case of perpendicular magnetic field, but the shape of the spin-relaxation rate dependence on the filling factor turned out to be fundamentally different. For example, for $\theta = 30^\circ$ the value of $1/\tau$ decreased almost monotonically with decreasing ν without any peculiarity around unity filling. If θ was further increased, a maximum in the filling factor dependence of spin relaxation is formed at some ν close to $\nu = 1$. We would like to highlight that this observation seems to be counterintuitive, as the spin gap and hence the relaxation time should be maximal at the exact $\nu = 1$ [44,45]. These findings contradict what was observed in conventional GaAs-based 2DES, as discussed later, and challenge the straightforward understanding of spin-relaxation process in the QHE regime, highlighting the need for further theoretical and experimental effort. Note that the existent spin-relaxation theories (see, for example, [25]) are based on the approximations valid in case the cyclotron gap is larger than the Coulomb energy. This condition is strongly violated in the system under study, rendering such theories inapplicable.

Let us compare the behavior of the spin-relaxation rate deduced from the ESR linewidth in GaAs-based 2DES reported in Ref. [26] and in AlAs wells presented here. In the case of the perpendicular magnetic field, the filling factor dependence of $1/\tau$ is at least qualitatively the same, as discussed earlier in the manuscript. Increasing tilt angles led to the acceleration of spin relaxation in both materials, yet in GaAs quantum wells the filling factor dependence of $1/\tau$ remained essentially the same, namely, at the exact odd ν $1/\tau$ reached its minimal value, in contrast to the experimental observations reported here. This contradiction thus seems to be induced by the strong e-e interaction, as the 2DES in GaAs represents a rather weakly interacting system.

A similar behavior of the spin dynamics in tilted fields was observed around the next odd filling factor $\nu = 3$. Note that the 2D electron density was increased from the value of 1.5 to around 4.5×10^{11} cm^{-2} so that the magnetic field corresponding to $\nu = 3$ was almost the same as that of $\nu = 1$ in a previous set of experiments. The measured filling factor dependencies of the spin relaxation for the same three θ are demonstrated in Fig. 2(a). Black open squares correspond to the experimental data in the case of a perpendicular magnetic field, whereas the solid blue and open red circles denote the $1/\tau$ dependence for $\theta = 30^\circ$ and 40°, respectively. Typical R_{xx} of the sample around a filling factor of 3 is depicted in Fig. 2(b) for the same θ. The key difference in spin dynamics around $\nu = 1$ and $\nu = 3$ is that tilting the magnetic field resulted in a less pronounced increase of $1/\tau$ for the filling factor of 3 than for $\nu = 1$, although the tilt angles studied were the same. This fact once again indicates the critical role of e-e interactions in the observed modification of spin relaxation, as increased electron density typically results in less pronounced effects of e-e correlations [19,36].

The reported radical modification of spin dynamics in tilted fields and its correlation with the strength of e-e interactions is clear from an experimental point of view, yet the theoretical description of a quantum Hall ferromagnet in a strongly interacting regime is still lacking to-date, highlighting the need for further research effort and limiting us to only a qualitative discussion of the possible reasons for the observed effect. We argue that such pronounced effects are caused by the appearance of new spin modes or softening of existing ones in tilted fields and in the presence of strong e-e interaction.

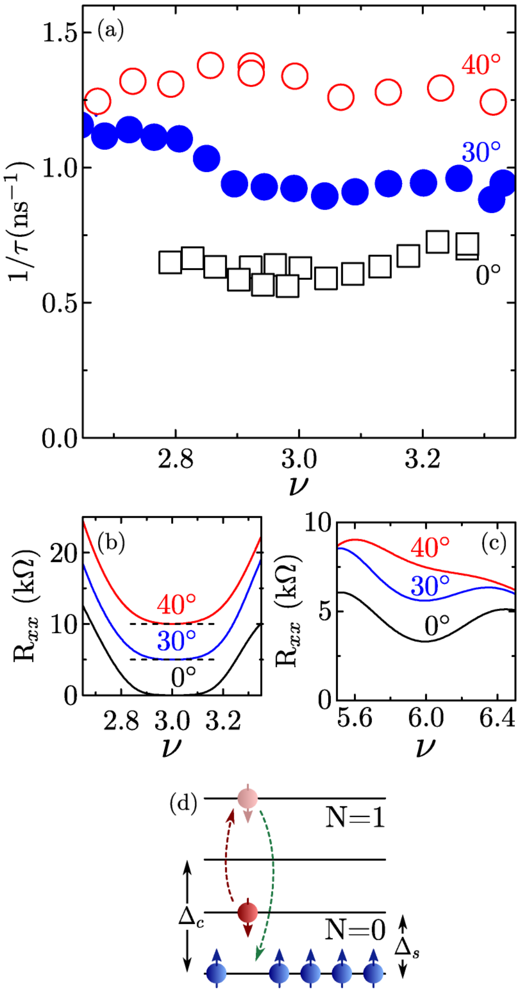

FIG. 2. (a) The filling factor dependence of the relaxation rate $1/\tau$ measured around $\nu = 3$ at 0.5 K and at different tilt angles θ. Open black squares indicate values of $1/\tau$ in the case of perpendicular magnetic field, while solid blue and open red circles denote data for $\theta = 30^\circ$ and $\theta = 40^\circ$, respectively. The electron density was around 4.5×10^{11} cm^{-2}. (b) The longitudinal resistance of the 2D channel measured at $\theta = 0^\circ$, 30°, and 40° vs the filling factor. The data is offset for clarity. (c) The longitudinal resistance of the 2D channel measured at $\theta = 0^\circ$, 30°, and 40° vs the filling factor at around $\nu = 6$. (d) Schematic representation of a single-particle energy spectrum at unity filling.

The relaxation of a spin in a quantum Hall ferromagnet is typically slow, as both the energy and the spin should be dissipated. While energy relaxation is relatively fast, the rate of spin dissipation is small, as there exist only a limited number of ways the spin may be rotated. The first option to dissipate spin that comes to mind is to convert the spin degree of freedom in the orbital motion of the electron [25] by the spin-orbit interaction. In other words, SO coupling induces the transition of an electron between the Landau levels accompanied by the change of its spin orientation. Taking into account the positive sign of the electron g-factor and the Dresselhaus-like SO interaction in AlAs [35], the actual process of spin relaxation is as schematically depicted in Fig. 2(d). The electron with the nonequilibrium spin orientation is first transferred to another Landau level without changing its spin and is then transferred to the lowest Landau level by SO coupling. However, increasing the tilt angle leads to the slowing of the latter process, contradicting the experimental observations, because for the given filling factor the energy splitting between these two levels $\Delta_c + \Delta_s$ increases as $\Delta_s \sim 1/\cos\theta$.

Another way to relax spin is to interact with some agent that is capable of accommodating excessive spins. For instance, the nonequilibrium polarization of electrons may be dissipated by simultaneous rotation of nuclear and electron spins coupled by the hyperfine interaction. Yet in AlAs the hyperfine interaction is damped compared to GaAs [46], and even in GaAs-based quantum wells this relaxation channel is weak under typical conditions [25].

The only other possible agent is then the spin excitations in the electron system itself with the large scale spin-texture modes (skyrmions) being one of the brightest examples [5,6]. To gain further insight into this possibility, let us examine the evolution of the electron energy spectrum in the quantum Hall ferromagnet in tilted magnetic fields. Strictly speaking, this energy spectrum consists of a number of excited states with nontrivial dispersions that are hard to calculate, especially in the limit of strong interactions. Thus we will restrict ourselves to a very simplified model where the properties of the system are analyzed in terms of renormalized Δ_c and Δ_s, i.e., in terms of renormalized electron g-factor and effective mass. While the limitations of this model are obvious, it is extensively used to explain transport and optical properties of the 2DES at tilted magnetic field in the QHE regime [3,4,19]. The renormalized values of both g-factor and mass deduced from the modification of the 2DES longitudinal resistance were reported in Ref. [19] for the very similar structure to the sample under study.

For simplicity, let us consider the case of $\nu = 1$. Then for a given filling factor the energy splitting between the lowest $0\uparrow$ and any other level either remains constant or grows if θ is increased. In contrast, the gap between the levels $0\downarrow$ and $1\uparrow$ equals $\delta E = \Delta_c - \Delta_s$, becomes smaller with increasing the tilt angle, and may even reach zero. The similar process around even fillings leads to the closing of the QHE gap [19] and even to the ferromagnetic phase transition [3,4]. The changes in δE are not small in our case, and this fact, for example, is illustrated by the substantial suppression of the resistance minima at large even fillings [see Fig. 2(c)], where the longitudinal resistance of the 2D channel around $\nu = 6$ for the tilted angles $\theta = 0^\circ, 30^\circ$ and 40° is demonstrated. Let us try to evaluate the evolution in δE with the angle θ. Following Ref. [19], we may write $\delta E = \hbar\omega_c(1 - \frac{g^*m^*}{2\cos\theta})$, and the combination of g^*m^* may be taken from Fig. 4 of the same reference. In the case of $n = 1.5 \times 10^{11}$ cm^{-2}, the value of δE changes by an order of magnitude from 8 K at $\theta = 0^\circ$ to approximately 1 K at the highest θ studied. For the higher density $n = 4.5 \times 10^{11}$ cm^{-2}, the renormalization of the g^*m^* value is less pronounced and the evolution of δE is less prominent as δE reaches the value of 5 K at the highest $\theta = 40^\circ$ and $\nu = 3$. Note that in the electron system with the strong e-e interaction the upper nominally empty Landau levels are inevitably populated due to the Landau-level mixing [4]. Furthermore, because of the screening effects the mixing is most effective for the closest in energy Landau levels. Thus diminishing of δE brings closer the two partially populated levels with opposite spin orientation and increases the number of electrons in the $1\uparrow$ state. Such electrons may serve as effective scatterers for the nonequilibrium spin population induced by the microwave absorption at ESR, and increase in their number may accelerate the spin-relaxation rate. Apart from that, the small value of δE compared to the characteristic exchange energy E_c may favor the formation of large-scale spin-texture excitations, effectively depolarizing spins [24,47]. Note that the maximal value of renormalized Δ_s and, as a result, the minimal δE, are achieved at the exact $\nu = 1$. This implies that the fastest relaxation of spin should be observed exactly around unity filling, in qualitative agreement with the experiment. The same discussion holds true for the $\nu = 3$ state. However, the changes in δE are smaller than at $\nu = 1$, and as a result, the effect should be less pronounced, in full accordance with the experimentally observed data. We would like to highlight that while close to zero values of δE induce the ferromagnetic phase transition around even fillings reported in numerous publications, diminishing of δE greatly impacts the spin dynamics around odd ones.

V. CONCLUSION

In conclusion, the spin dynamics was studied around odd fillings of the integer quantum Hall effect in the strongly correlated two-dimensional electron system hosted in a narrow AlAs quantum well. The relaxation of the electron spins was demonstrated to undergo a radical modification in tilted magnetic fields, as not only the relaxation rate $1/\tau$ itself became almost an order of magnitude higher at the largest tilt angles studied, but the filling factor dependence of $1/\tau$ became essentially different. In the case of zero tilt angle this dependence was rather weak, with a minimum at the exact $\nu = 1$; in contrast, at large angles the relaxation rate was maximal at some ν close to unity. The observed effects may be ascribed to the influence of the e-e interaction on the spin excitation spectra.

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