Strong tunable spin-spin interaction in a weakly coupled nitrogen vacancy spin-cavity electromechanical system

Wei Xiong,[1,2,3] Jiaojiao Chen,[4] Baolong Fang,[5] Mingfeng Wang,[1,*] Liu Ye,[3,†] and J. Q. You [2,‡]

[1]*Department of Physics, Wenzhou University, Zhejiang 325035, China*
[2]*Interdisciplinary Center of Quantum Information and Zhejiang Province Key Laboratory of Quantum Technology and Device,
Department of Physics and State Key Laboratory of Modern Optical Instrumentation, Zhejiang University, Hangzhou 310027, China*
[3]*School of Physics and Material Science, Anhui University, Hefei 230601, China*
[4]*Hefei Preschool Education College, Hefei 230013, China*
[5]*Institute of Advanced Manufacturing Engineering, Hefei University, Hefei 230022, China*

The long coherence time of a single nitrogen vacancy (NV) center spin in diamond is a crucial advantage for implementing quantum information processing. However, the realization of strong coupling between single NV spins is challenging. Here, we propose a method to greatly enhance the interaction between two single NV spins in diamond which are only weakly coupled to an electromechanical cavity. Owing to the presence of a critical point for the linearized electromechanical subsystem, the coupling between a single NV spin and the high-frequency polariton (formed by the mechanical and cavity modes) can be fully decoupled, but the coupling between the single NV spin and the low-frequency polariton is, however, greatly enhanced. Thus, AC Stark shift of the single NV spin can be measured. With the low-frequency polariton as a quantum bus, a strong coupling between two single NV centers is achievable. This effective strong coupling can ensure coherent quantum-information exchange between two spin qubits in the weakly coupled spin-cavity electromechanical system.

I. INTRODUCTION

A nitrogen vacancy (NV) center spin in diamond [1,2], with long coherence time [3–5] and high tunability [2], is a promising qubit candidate for quantum information processing [6,7]. However, spin-spin interaction is usually too weak to efficiently implement quantum-information exchange [8–10]. One popular method to overcome this drawback is the use of an ensemble containing many NV spins [11–14]. Thus, the coupling strength between NV spin ensembles can be efficiently enhanced [15–17]. However, it is difficult for the ensemble to implement direct single-qubit manipulation, and the coherence time is also greatly shortened due to the inhomogeneous broadening [18–20]. Another potential approach is to couple NV spins to the nanomechanical resonator [21–34], but the required strong magnetic gradient remains challenging experimentally [21,27], and the strain force is inherently tiny for the ground states of NV spins [32,33]. Also, interacting spins with squeezed photons to enhance the coupling are proposed [35–37], but external noises may be introduced to the considered system. In addition, there are important efforts to couple remote NV spins via an optical network link [38,39] or a superconducting bus [40].

Recently, optomechanical and electromechanical systems have attracted much interest in testing macroscopic quantum properties because of their appealing applications in quantum information science [41–50]. Based on remarkable progress in experiments [51–59], we can propose an experimentally accessible approach to realizing strong spin-spin coupling in a hybrid spin-cavity electromechanical system, where the single NV spin is only weakly coupled to the cavity mode. By applying a strong driving field to the cavity, two hybrid modes arising from the linearized strong coupling between the cavity and mechanical modes are generated, namely, the high-frequency and low-frequency polaritons. Tuning the linearized electromechanical coupling strength to a certain value (i.e., a critical point) by varying the driving field, we can have the electromechanical subsystem reach the *critical* regime. When operating the hybrid system around this critical point, the coupling between the single NV spin and the high-frequency polariton is totally suppressed if the cavity frequency detuning from the driving field is much larger than that of the mechanical resonator. However, the coupling strength between the NV spin and the low-frequency polariton is greatly *enhanced,* more than three orders of magnitude of the single spin-cavity coupling. This strong coupling allows one to measure the AC Stark shift of a single NV spin. Taking the low-frequency polariton as a quantum bus, strong spin-spin coupling can be then induced. The results indicate that, even for the weakly coupled spin-cavity electromechanical system, it is promising to probe the spin-qubit states and implement

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*mfwang@wzu.edu.cn
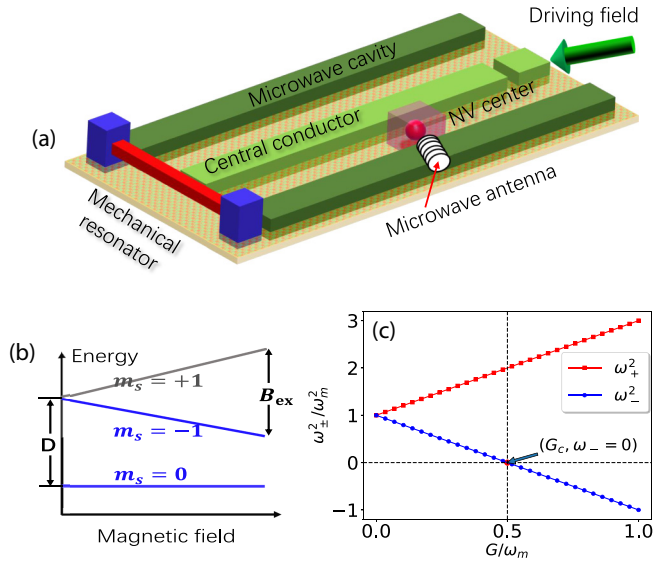†yeliu@ahu.edu.cn
‡jqyou@zju.edu.cn

FIG. 1. (a) Schematic diagram of the proposed hybrid system. The single nitrogen vacancy (NV) center spin, located d distance away from the central conductor, is weakly coupled to an electromechanical cavity and driven by a microwave field via a microwave antenna. This on-chip cavity is also driven by a strong microwave field. (b) The level structure of the triplet ground state $S = 1$ in an NV center, where D is the zero-field splitting, and B_{ex} is an external magnetic field to lift the degenerate states $m_s = \pm 1$. The lowest two sublevels $m_s = 0, -1$ act as a two-level quantum system (i.e., qubit). (c) Frequencies of the two hybrid modes, generated by the strong coupling between the cavity and mechanical modes, vs the electromechanical coupling strength in units of the mechanical frequency.

polariton-mediated quantum information processing with single spin qubits.

II. MODEL AND HAMILTONIAN

We consider a hybrid quantum system consisting of a *single* NV spin weakly coupled to an electromechanical cavity [Fig. 1(a)], where the NV spin in diamond with a spin $S = 1$ triplet ground state [Fig. 1(b)] is located d distance away from the central conductor of the coplanar-waveguide resonator. Hereafter, we term this waveguide resonator as a (on-chip) cavity to distinguish it from the mechanical resonator. The Hamiltonian of the hybrid system can be written as (setting $\hbar = 1$) $H_{tot} = H_{NV} + H_{EM} + H_I + H_D$, where $H_{NV} = \frac{1}{2}\omega_{NV}\sigma_z$, with the transition frequency $\omega_{NV} = D - g_e\mu_B B_{ex}$ between the lowest two levels of the triplet ground state of the NV. Here, $D \approx 2.87$ GHz is the zero-field splitting, $g_e = 2$ is the Landé factor, μ_B is the Bohr magneton, and B_{ex} is the external magnetic field to lift the near-degenerate states $|m_s = \pm 1\rangle$. The second term $H_{EM} = \omega_a a^\dagger a + \omega_m b^\dagger b - ga^\dagger a(b + b^\dagger)$ is the Hamiltonian of the cavity electromechanical subsystem [60], where ω_a is the frequency of the cavity mode when the mechanical resonator is at its equilibrium position, ω_m is the frequency of the mechanical mode, and g is the single-photon coupling strength between the cavity and mechanical modes. The third term H_I describes the magnetic coupling between the single NV spin and the cavity mode. Under the

rotating-wave approximation, $H_I = \lambda(a^\dagger\sigma_- + a\sigma_+)$, with $\lambda = 2g_e\mu_B B_{0,rms}(d)$ [40], where $B_{0,rms}(d) = \mu_0 I_{rms}/2\pi d$, with μ_0 being the permeability of vacuum and $I_{rms} = \sqrt{\hbar\omega_a/2L_a}$. To estimate λ, $\omega_a \sim 2\pi \times 2$ GHz and $L_a \sim 2$ nH are chosen [61]. For $d \sim 50$ μm, $\lambda \sim 2\pi \times 70$ Hz, and $d \sim 50$ nm gives $\lambda \sim 2\pi \times 7$ KHz. Obviously, the estimated spin-cavity coupling strength is smaller than the typical decay rate of the cavity with the gigahertz frequency and quality factor $Q \sim 3 \times 10^4$ [61,62], i.e., $\lambda < \kappa = \omega_a/Q \sim 1$ MHz. This indicates that the spin-cavity coupling is in the *weak-coupling* regime. The last term $H_D = \Omega_d a^\dagger \exp(-i\omega_d t) + \Omega_d^* a \exp(i\omega_d t) - \Omega_{NV}\sigma_+ \exp(-i\omega_d t) - \Omega_{NV}^*\sigma_- \exp(i\omega_d t)$ involves the driving fields acting on the cavity mode and the NV spin. Due to the large Ω_d, the electromechanical coupling strength can be amplified to $G \equiv g\sqrt{N} > \kappa$, where $N = |\langle a\rangle|^2$ is the intracavity mean photon number. This amplification is achieved by linearizing the Hamiltonian with $a = \langle a\rangle + \delta a$ and $b = \langle b\rangle + \delta b$, where $\langle a\rangle = -\Omega_d/(\Delta_a - i\kappa)$ and $\langle b\rangle = g|\langle a\rangle|^2/(\omega_m - i\gamma_m)$ are the mean values of the operators a and b in the steady state, respectively [63], with $\Delta_a = \omega_a - \omega_d - g(\langle b\rangle + \langle b\rangle^*)$ and γ_m being the decay rate of the mechanical mode. As the driving field on the cavity will indirectly cause the NV spin to flip, an additional microwave field with frequency ω_d and amplitude Ω_{NV} is imposed to the NV spin via a microwave antenna [see Fig. 1(a)], so as to cancel this flip. Then the linearized Hamiltonian of the hybrid spin-cavity electromechanical system is given by [63]

$$H_{lin} = \tfrac{1}{2}\Delta_{NV}\sigma_z + H_{LEM} + \lambda(\delta a^\dagger\sigma_- + \delta a\sigma_+), \quad (1)$$

where $\Delta_{NV} = \omega_{NV} - \omega_d$, and $H_{LEM} = \Delta_a\delta a^\dagger\delta a + \omega_m\delta b^\dagger\delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger)$ is the linearized Hamiltonian of the cavity electromechanical subsystem. In deriving Eq. (1), we assume $\lambda\langle a\rangle = \Omega_{NV}$ [63], where $\langle a\rangle$ is controlled by Ω_d and can be determined via the input-output theory [64] by probing the output field of the cavity. There were proposals for measuring a weak coupling strength λ [65–67], and they were demonstrated experimentally [68,69]. With a given λ, the condition $\lambda\langle a\rangle = \Omega_{NV}$ can be readily satisfied because both Ω_d and Ω_{NV} are precisely tunable. Here, the effects of the driving fields are included in Δ_a and G, while quantum behaviors of the system are kept in the linearized Hamiltonian in Eq. (1) via the fluctuation operators δa and δb. In the linearized Hamiltonian H_{LEM}, there are counterrotating terms $G\delta a\delta b$ and $G\delta a^\dagger\delta b^\dagger$, which are related to the squeezing effect in the system. As shown below, when G approaches its critical value, these counterrotating terms play an increasingly important role [70], and polaritons of novel properties can be formed.

III. STRONG COUPLING BETWEEN A SINGLE NV SPIN AND THE LOW-FREQUENCY POLARITON

Assisted by a strong driving field, the mechanical mode can strongly couple to the cavity mode via the linearized electromechanical coupling strength G, yielding two hybrid modes (i.e., polaritons) with eigenfrequencies

$$\omega_\pm^2 = \tfrac{1}{2}\left[\Delta_a^2 + \omega_m^2 \pm \sqrt{(\Delta_a^2 - \omega_m^2)^2 + 16G^2\Delta_a\omega_m}\right], \quad (2)$$
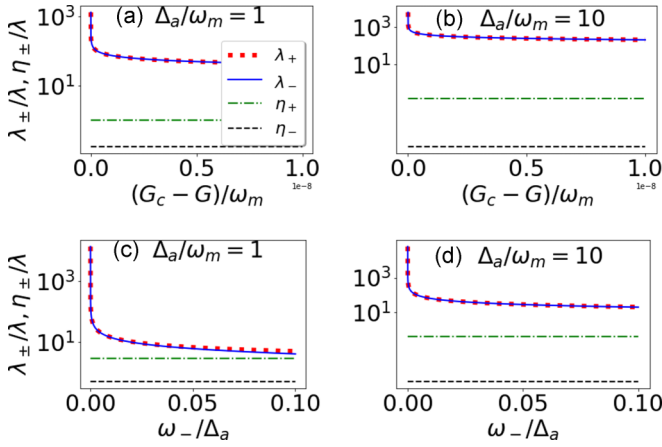
FIG. 2. Coupling strength between the nitrogen vacancy (NV) spin and the high- (low-) frequency polaritons vs the dimensionless parameters $(G_c - G)/\omega_m$ and ω_-/Δ_a. Here, $\Delta_a/\omega_m = 1$ in both (a) and (c), and $\Delta_a/\omega_m = 10$ in both (b) and (d).

where ω_+ and ω_- correspond to the high- and low-frequency polaritons, respectively. These eigenfrequencies are directly obtained by diagonalizing the linearized Hamiltonian H_{LEM} in Eq. (1) (see Ref. [63] for details). With the eigenvectors of the hybrid modes, Eq. (1) becomes

$$\mathcal{H} = \tfrac{1}{2}\Delta_{\mathrm{NV}}\sigma_z + \omega_+ a_+^\dagger a_+ + \omega_- a_-^\dagger a_-$$
$$+ \lambda_+(a_-^\dagger\sigma_- + a_-\sigma_+) + \lambda_-(a_-^\dagger\sigma_+ + a_-\sigma_-)$$
$$+ \eta_+(a_+^\dagger\sigma_- + a_+\sigma_+) + \eta_-(a_+^\dagger\sigma_+ + a_+\sigma_-), \quad (3)$$

where $\lambda_\pm = \lambda\cos\theta(\Delta_a \pm \omega_-)/2\sqrt{\Delta_a\omega_-}$ denotes the effective coupling strength between the NV spin and the low-frequency polariton, and $\eta_\pm = \lambda\sin\theta(\Delta_a \pm \omega_+)/2\sqrt{\Delta_a\omega_+}$ is the effective coupling strength between the NV spin and the high-frequency polariton. The parameter θ is defined by $\tan(2\theta) = 4G\sqrt{\Delta_a\omega_m}/(\Delta_a^2 - \omega_m^2)$. Both $\lambda_\pm$ and $\eta_\pm$ can be *tuned* by the driving field on the cavity. From Eq. (2), one can see that ω_+^2 increases with the linearized optomechanical coupling G, but ω_-^2 decreases [see Fig. 1(c)]. When G reaches a certain value,

$$G = G_c \equiv \tfrac{1}{2}\sqrt{\Delta_a\omega_m}, \quad (4)$$

critical phenomenon occurs [71], where the frequency of the low-frequency polariton vanishes (i.e., $\omega_- = 0$) [see the red point in Fig. 1(c)]. Here, we consider the case with both $G \to G_c$ (i.e., $\omega_- \to 0$) and $\Delta_a/\omega_m \gg 1$, where $\lambda_+ \approx \lambda_- \to \frac{\lambda}{2}\sqrt{\Delta_a/\omega_-} \gg \lambda$, $\eta_+ \to \lambda\omega_m/\Delta_a \ll \lambda$, and $\eta_- \to 0$. This indicates that the coupling between the NV spin and the high-frequency polariton can be ignored in Eq. (3). Correspondingly, the coupling between the *single* NV spin and the low-frequency polariton is greatly *enhanced* due to the extremely small value of ω_- and tunable parameter Δ_a. Thus, raising the photon occupation number in the cavity to have $G \equiv g\sqrt{N}$ reach G_c, we can use the critical behavior of the effective spin-polariton system to enhance the coupling between the spin and the low-frequency polariton.

In Fig. 2, we plot the coupling strength between the NV spin and the high- (low-) frequency polariton vs the dimensionless parameters $(G_c - G)/\omega_m$ and ω_-/Δ_a with $\Delta_a/\omega_m =$
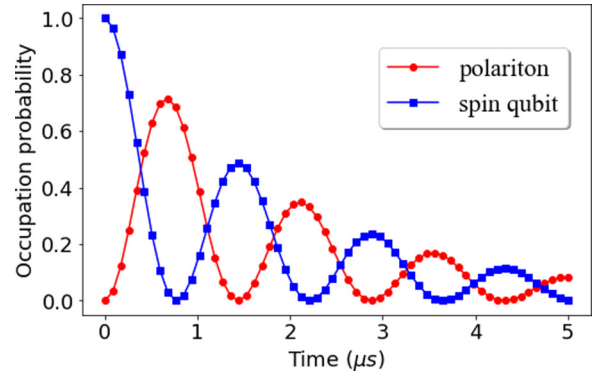

FIG. 3. The mean occupation number of the low-frequency polariton and the occupation probability of the nitrogen vacancy (NV) spin vs the evolution time, where the polariton is initially prepared in the ground state, and the spin qubit is in the excited state. We choose $\lambda_+ = 2\pi \times 3.5$ MHz, $\gamma_\perp = 1$ KHz, and $\kappa = 1$ MHz.

1, 10. When $G \to G_c$ or $\omega_- \to 0$, the coupling between the NV spin and the low-frequency polariton can be extremely strong [see Figs. 2(a) and 2(c)]. With increasing Δ_a, $\lambda_\pm$ can be further enhanced [see Figs. 2(b) and 2(d)]. In principle, ω_- can be very close to zero, and the detuning Δ_a can be sufficiently large. These can yield $\lambda_\pm$ in the *strong-coupling* regime (i.e., $\lambda_\pm \geqslant \kappa$) to allow coherent quantum-information exchange between the single NV spin and the low-frequency polariton. In this regime, rotating-wave approximation is still valid, and the counterrotating term related to λ_- in Eq. (3) can be safely ignored. Thus, around the critical point $G = G_c$ and when $\Delta_a \gg \omega_m$, the Hamiltonian in Eq. (1) reduces to the Janes-Cumming model,

$$\mathcal{H}_{\mathrm{JC}} = \tfrac{1}{2}\Delta_{\mathrm{NV}}\sigma_z + \omega_- a_-^\dagger a_- + \lambda_+(a_-^\dagger\sigma_- + a_-\sigma_+), \quad (5)$$

when considering the interaction between the single NV spin and the low-frequency polariton. To estimate λ_+, we choose $\Delta_a = 10^6\omega_-$, then $\lambda_+/\lambda = 0.5 \times 10^3$, which indicates that λ_+ can be approximately enhanced three orders of magnitude of λ in our hybrid system. Specifically, with $\lambda = 2\pi \times 7$ KHz, as estimated above for $d = 50$ nm, $\lambda_+ \sim 2\pi \times 3.5$ MHz, larger than the decay rate of the gigahertz cavity with quality factor $\sim 10^4$. This implies that the coupling between the single NV spin and the low-frequency polariton is in the strong-coupling regime. Theoretically, the value of λ_+ can be even larger due to the extremely small ω_-.

When decoherence is considered, the dynamics of the above low-frequency polariton-spin system can be governed by a master equation

$$\frac{d\rho}{dt} = -i[\mathcal{H}_{\mathrm{JC}}, \rho] + \kappa\mathcal{D}[a_-]\rho + \gamma_\perp\mathcal{D}[\sigma_-]\rho + \gamma_\parallel\mathcal{D}[\sigma_z]\rho, \quad (6)$$

where $\mathcal{D}[o]\rho = o\rho o^\dagger - \tfrac{1}{2}(o^\dagger o\rho + \rho o^\dagger o)$ for a given operator o, and $\gamma_\perp$ ($\gamma_\parallel$) is the transversal (longitudinal) relaxation rate of the NV spin. Experimentally, $\gamma_\perp \gg \gamma_\parallel$ [72], so the longitudinal relaxation rate can be ignored. To solve Eq. (6), we choose $\gamma_\perp = 1$ KHz and $\kappa = 1$ MHz, for example. By individually tuning the driving fields acting on the cavity and the NV spin, we can have $\Delta_{\mathrm{NV}} = \omega_-$ in Eq. (5). In Fig. 3, we show the time evolutions of the mean occupation number $\langle a_-^\dagger a_- \rangle$

of the low-frequency polariton and the occupation probability of the NV spin qubit. Initially, the low-frequency polariton is prepared in the ground state, and the NV spin is in the excited state $|m_s = -1\rangle \equiv |1\rangle$. It can be seen that Rabi oscillations between the low-frequency polariton and the NV spin occur owing to the strong coupling, despite the decoherence in the hybrid system.

IV. EFFECTIVE STRONG COUPLING BETWEEN TWO NV SPINS AND THE AC STARK SHIFT

We consider two separated NV center spins coupled to the low-frequency polariton mode with coupling strengths $\lambda_+^{(1)}$ and $\lambda_+^{(2)}$. We can use this low-frequency polariton as a quantum interface to achieve an effective strong spin-spin coupling in the dispersive regime, i.e., $\zeta_i = \lambda_+^{(i)}/|\delta_i| \ll 1$, where $\delta_i = \Delta_{\mathrm{NV}}^{(i)} - \omega_-$, $i = 1, 2$, are the frequency detunings of the two NV spins from ω_-. With the Fröhlich-Nakajima transformation [73,74], the effective spin-spin Hamiltonian can be obtained as [63]

$$\mathcal{H}_{\mathrm{eff}} = \sum_{i=1}^{2} \frac{1}{2} \Delta_{\mathrm{eff}}^{(i)} \sigma_z^{(i)} + g_{\mathrm{eff}} \left[\sigma_+^{(1)} \sigma_-^{(2)} + \sigma_-^{(1)} \sigma_+^{(2)} \right], \quad (7)$$

where $\Delta_{\mathrm{eff}}^{(i)} = \Delta_{\mathrm{NV}}^{(i)} + \lambda_+^{(i)} \zeta_i (1 + 2N_{\mathrm{pl}})$ are the effective transition frequencies of the two NV spins, which depend on the mean occupation number $N_{\mathrm{pl}} = \langle a_-^\dagger a_- \rangle$ of the low-frequency polariton, and $g_{\mathrm{eff}} = \frac{1}{2}[\lambda_+^{(1)} \zeta_2 + \lambda_+^{(2)} \zeta_1]$ is the effective spin-spin coupling strength induced by the low-frequency polariton. This coupling can allow coherent quantum-information exchange between the two separated NV spins when they are in the strong-coupling regime. For convenience and without loss of generality, we assume $\lambda_+^{(1)} = \lambda_+^{(2)} = \lambda_+ = 2\pi \times$ 3.5 MHz for $d = 50$ nm and $\delta_1 = \delta_2 = 2\pi \times 35$ MHz, which leads to $g_{\mathrm{eff}} = 2\pi \times 350$ KHz ~ 2.2 MHz. This value is comparable with the decay rate of the gigahertz microwave cavity with quality factor $Q = 10^4$. In fact, a microwave cavity with higher quality factor (e.g., $Q = 10^5$) can be fabricated experimentally [75]. Therefore, it is reasonable to conclude that the effective spin-spin coupling can reach the strong-coupling regime. To obtain this strong coupling, we assume $d = 50$ nm. With current nanofabrication technology, it has been achieved by placing an NV spin within ~ 30 nm inside an on-chip cavity of length ~ 1 cm [76]. Therefore, our proposal is experimentally feasible. Owing to the larger size of the cavity compared with a single NV spin, it allows one to place multiple spins in a cavity for a scalable network.

Furthermore, we show that the weakly coupled spin-cavity electromechanical system allows one to probe the AC Stark shift of the spin energy level induced by a single excitation of the low-frequency polariton, i.e., $N_{\mathrm{pl}} = \langle a_-^\dagger a_- \rangle = 1$. To obtain this, we consider that the spin is *dispersively* coupled to the low-frequency polariton, where $\zeta = \lambda_+/|\delta| \ll 1$, with $\delta = \Delta_{\mathrm{NV}} - \omega_-$. Under this condition, the Fröhlich-Nakajima transformation can be used to diagonalize the Hamiltonian in Eq. (5) to

$$\mathcal{H}_{\mathrm{AC}} = \frac{1}{2}(\Delta_{\mathrm{NV}} + \lambda_+ \zeta + 2\lambda_+ \zeta a_-^\dagger a_-)\sigma_z + \omega_- a_-^\dagger a_-, \quad (8)$$

where $\zeta = \lambda_+/|\delta|$ is the zero-point energy of the low-frequency polariton. Obviously, the frequency of the NV spin is shifted by both the zero-point energy $\lambda_+ \zeta$ and the low-frequency polariton-dependent Stark shift $2\lambda_+ \zeta N_{\mathrm{pl}}$. At the single polariton excitation $N_{\mathrm{pl}} = 1$, the Stark shift is $2\lambda_+ \zeta$. To estimate this Stark shift, we choose $\lambda_+ = 2\pi \times 3.5$ MHz, $\delta = 2\pi \times 35$ MHz, and $\kappa = 1$ MHz. These parameters ensure that the dispersive condition is valid and gives rise to $2\lambda_+ \zeta \sim 2\pi \times 0.7$ MHz. As shown above, the value of λ_+ can be much larger with increasing Δ_a/ω_-, so the Stark shift induced by a single polariton excitation is observable. Comparing with strong-coupling cases [77,78], our proposal greatly reduces experimental difficulties. It provides a promising way to realize coherent quantum-information exchange between two separated NV spins and can be used to probe the AC Stark shift of the single NV spin in the weakly coupled spin-cavity electromechanical systems.

With the effective Hamiltonian in Eq. (7), the two-qubit iSWAP gate can be realized at $\Delta_{\mathrm{eff}}^{(1)} \approx \Delta_{\mathrm{eff}}^{(2)}$. In the interaction picture, the Hamiltonian in Eq. (7) becomes

$$\mathcal{H}_{\mathrm{eff}}^I = g_{\mathrm{eff}} \left[\sigma_-^{(1)} \sigma_+^{(2)} + \sigma_+^{(1)} \sigma_-^{(2)} \right]. \quad (9)$$

The corresponding time evolution operator reads

$$\mathcal{U}(t) = \exp\left(-i\mathcal{H}_{\mathrm{eff}}^I t \right). \quad (10)$$

The final state of the two-spin system described by Eq. (9) is

$$|\Psi_t\rangle = \mathcal{U}(t)|\Psi_0\rangle, \quad (11)$$

where $|\Psi_0\rangle$ is the initial state of the two-spin system. Specifically,

$$\begin{aligned}
|\Psi_0\rangle &= |g_1\rangle|g_2\rangle \to |\Psi_t\rangle = |g_1\rangle|g_2\rangle, \\
|\Psi_0\rangle &= |g_1\rangle|e_2\rangle \to |\Psi_t\rangle = \cos(g_{\mathrm{eff}}t)|g_1\rangle|e_2\rangle \\
&\quad - i\sin(g_{\mathrm{eff}}t)|e_1\rangle|g_2\rangle, \\
|\Psi_0\rangle &= |e_1\rangle|g_2\rangle \to |\Psi_t\rangle = \cos(g_{\mathrm{eff}}t)|e_1\rangle|g_2\rangle \\
&\quad - i\sin(g_{\mathrm{eff}}t)|g_1\rangle|e_2\rangle, \\
|\Psi_0\rangle &= |e_1\rangle|e_2\rangle \to |\Psi_t\rangle = |e_1\rangle|e_2\rangle.
\end{aligned} \quad (12)$$

When $g_{\mathrm{eff}}t = \pi/2$, Eq. (12) reduces to

$$\begin{aligned}
|g_1\rangle|g_2\rangle &\to |g_1\rangle|g_2\rangle, \\
|g_1\rangle|e_2\rangle &\to -i|e_1\rangle|g_2\rangle, \\
|e_1\rangle|g_2\rangle &\to -i|g_1\rangle|e_2\rangle, \\
|e_1\rangle|e_2\rangle &\to |e_1\rangle|e_2\rangle,
\end{aligned} \quad (13)$$

which is just the iSWAP gate.

For a realistic gate, the dissipations in the on-chip cavity, mechanical resonator, and two spins should be included. Near the critical point (i.e., $G \to G_c$), $\omega_- \to 0$. Hence, the annihilation operator of the low-frequency polariton [63] reduces to

$$a_- = \frac{1}{2}\left[\cos\theta \sqrt{\frac{\Delta_a}{\omega_-}}(\delta a - \delta a^\dagger) - \sin\theta \sqrt{\frac{\omega_m}{\omega_-}}(\delta b - \delta b^\dagger) \right]. \quad (14)$$
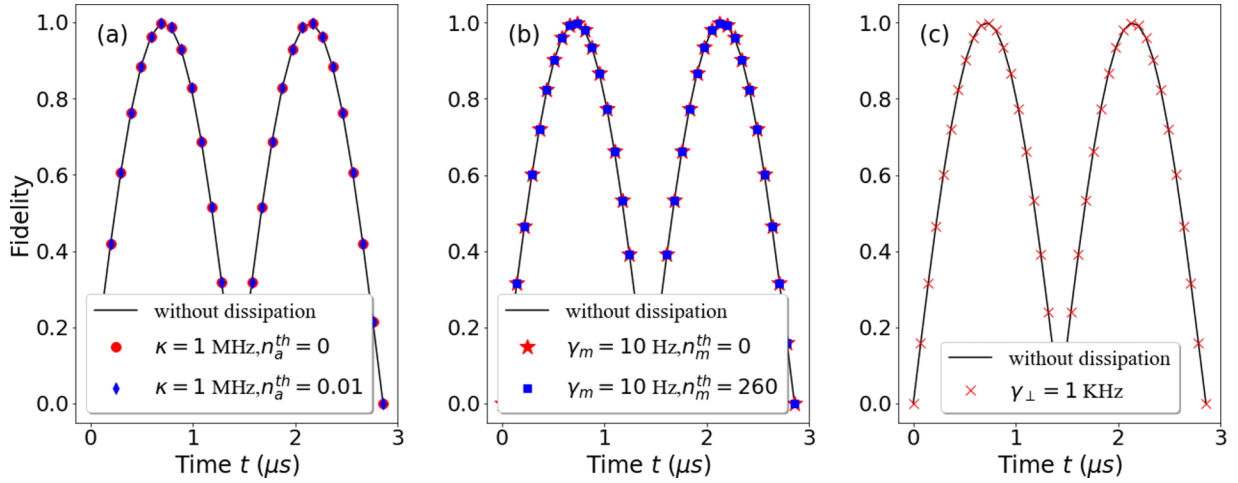
FIG. 4. The fidelity of the iSWAP gate vs the evolution time without and with (a) the decay rate of the cavity at zero ($n_c^{th} = 0$) and nonzero ($n_c^{th} = 0.01$, i.e., $T \sim 20$ mK) temperatures, (b) the decay rate of the mechanical resonator at zero ($n_m^{th} = 0$) and nonzero ($n_m^{th} = 260$, i.e., $T \sim 20$ mK) temperatures, and (c) the transversal relaxation rate. Here, we choose the parameters $\omega_- = 100$ Hz, $\Delta_a = 10^6 \omega_-$, $\omega_m = 10^5 \omega_-$, $\lambda = 2\pi \times 7$ KHz, $\lambda_+^{(1)} = \lambda_+^{(2)} = \frac{1}{2}\lambda\sqrt{\Delta_a/\omega_-}$, $\Delta_{NV}^{(1)} = \Delta_{NV}^{(2)} = \delta = 10\lambda_+^{(1)}$, and $g_{eff} = [\lambda_+^{(1)}]^2/\delta$.

By substituting Eq. (14) into Eq. (7), the quantum dynamics of the considered system can be given by

$$
\begin{aligned}
\frac{d\rho}{dt} = &-i[\mathcal{H}_{eff}, \rho] + \kappa(n_a^{th} + 1)\mathcal{D}[\delta a]\rho \\
&+ \kappa n_a^{th}\mathcal{D}[\delta a^\dagger]\rho + \gamma_m(n_m^{th} + 1)\mathcal{D}[\delta b]\rho \\
&+ \gamma_m n_m^{th}\mathcal{D}[\delta b^\dagger]\rho + \gamma_\perp \mathcal{D}[\sigma_-]\rho,
\end{aligned} \tag{15}
$$

where $\mathcal{H}_{eff}$ is given by Eq. (7) and $n_{a(m)}^{th} = [\exp(\hbar\omega_{a(m)}/K_B T) - 1]^{-1}$ is the thermal photon (phonon) occupation in the cavity (mechanical) mode.

Figure 4 shows the numerical simulation for the fidelity of the iSWAP gate obtained using the effective Hamiltonian in Eq. (7) without and with dissipations in the cavity, mechanical resonators, and NV spins. Here, we choose $\omega_- = 100$ Hz, $\Delta_a = 10^6 \omega_-$, $\omega_m = 10^5 \omega_-$, $\lambda = 2\pi \times 7$ KHz, $\lambda_+^{(1)} = \lambda_+^{(2)} = \frac{1}{2}\lambda\sqrt{\Delta_a/\omega_-}$, $\Delta_{NV}^{(1)} = \Delta_{NV}^{(2)} = \delta = 10\lambda_+^{(1)}$, and $g_{eff} = [\lambda_+^{(1)}]^2/\delta$. These choices of the parameters ensure the harnessed approximation is valid. As the initial state, we prepare one spin qubit in its ground state, another spin qubit in the excited state, and both the cavity and the mechanical resonator are in the thermal state. In Fig. 4(a), we plot the iSWAP-gate fidelity vs the evolution time for different cavity decay rates (i.e., $\kappa = 0$, 1 MHz) at zero ($n_a^{th} = 0$) and nonzero ($n_a^{th} = 0.01$, i.e., $T \sim 20$ mK) temperatures, respectively. Compared with the results at zero temperature, it can be seen that the fidelity of the iSWAP gate is hardly affected by the decay rate of the cavity at $T \sim 20$ mK. This temperature is close to the temperatures often used for experiments on the on-chip cavity (e.g., the coplanar waveguide resonator). In Fig. 4(b), we plot the iSWAP-gate fidelity vs the evolution time for different decay rates of the mechanical resonator (i.e., $\gamma_m = 0$, 10 Hz) at zero ($n_m^{th} = 0$) and nonzero ($n_m^{th} = 260$, i.e., $T \sim 20$ mK) temperatures. It is shown that the iSWAP-gate fidelity is strongly robust against the dissipation in the mechanical resonator. Also, we plot the iSWAP-gate fidelity vs the evolution time by considering the transversal relaxation of the NV spins.

For simplicity, we assume the two NV spins have the same transversal relaxation, i.e., $\gamma_\perp = 1$ KHz. Since $\gamma_\perp \ll g_{eff}$, the gate fidelity is almost not affected by it. In short, Fig. 4 shows that the iSWAP-gate fidelity is robust against the dissipations in the cavity, mechanical resonator and spins. This is due to the fact that the polariton formed by the cavity and mechanical modes is dispersively coupled to the spins, and a strong coupling between the two separate spins can be induced in comparison with the decay rates of the cavity and spins.

V. CONCLUSIONS

We have proposed a method to realize a strong tunable spin-spin coupling in a hybrid quantum system consisting of NV spins weakly coupled to an elecromechanical cavity. By taking advantage of the critical behavior of the linearized electromechanical system, the high-frequency polariton can be *decoupled* from the NV spin, but the coupling between the low-frequency polariton and the NV spin can be greatly *enhanced*. With experimentally accessible parameters, this coupling can reach the strong-coupling regime. Thus, the AC Stark shift of the single NV spin induced by the single excitation of the low-frequency polariton can be resolved, and the polariton-mediated quantum-information exchange between spins can be realized. Our proposal can provide a feasible way to probe spin-qubit states and implement polariton-mediated quantum information processing with single spin qubits in the weakly coupled spin-cavity electromechanical systems.

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