Metamagnetic phase transition in the ferromagnetic superconductor URhGe

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Ferromagnetic superconductor URhGe has orthorhombic structure and possesses spontaneous magnetization along the c axis. Magnetic field directed along the b axis suppresses ferromagnetism in the c direction and leads to a metamagnetic transition into polarized paramagnetic state in the b direction. The theory of these phenomena based on the specific magnetic anisotropy of this material in the (b, c) plane is given. Line of the first order metamagnetic transition ends at a critical point. The Van der Waals-type description of behavior of physical properties near this point is developed. The triplet superconducting state destroyed by orbital effect is recreated in the vicinity of the transition. It is shown that the reentrance of superconductivity is caused by the sharp increase of magnetic susceptibility in the b direction near the metamagnetic transition. The specific behavior of the upper critical field in the direction of spontaneous magnetization in UCoGe and in UGe_2 related to the field dependence of magnetic susceptibility is discussed.

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I. INTRODUCTION

Investigations of uranium superconducting ferromagnets UGe_2, URhGe, and UCoGe continue to attract attention mostly due to the quite unusual nature of its superconducting states created by the magnetic fluctuations (see the recent experimental [1] and theoretical [2] reviews and references therein). They have orthorhombic crystal structure and the anisotropic magnetic properties. The spontaneous magnetization is directed along the a axis in UGe_2 and along the c axis in URhGe and UCoGe. The ferromagnetic state in the two last materials is suppressed by the external magnetic field H_y directed along b crystallographic direction. In URhGe at field $H_y = H_{cr} \approx 12$ T the second order phase transition to ferromagnetic state is transformed to the transition of the first order [3]. The superconducting state suppressed [4] in much smaller fields $H_y \approx 2$ T reappears in the vicinity of the first order transition in field interval (9, 13) T. The phenomenological theory of this phenomenon has been developed in Ref. [5] (see also Ref. [2]). According to this theory the state arising in fields above the suppression of spontaneous magnetization in the c direction is the paramagnetic state.

There was established, however [3,6,7], that in fields above H_{cr} the magnetization along the b direction looks like it has field independent "spontaneous" component

$$M_y = M_{y0} + \chi_y H_y. \tag{1}$$

This state is called polarized paramagnetic state. The formation of this state is related to so-called metamagnetic transition observed in several heavy-fermion compounds (see the paper [8] and the more recent publication [9] and references therein). To take into account the formation of polarized paramagnetic state one must introduce definite modifications in the treatment performed in Ref. [5]. Here I present the corresponding derivation.

The paper is organized as follows. In Sec. II after the brief reminder of results of the paper [5] the description of the metamagnetic transition is presented. It is based on the specific phenomenon of magnetic anisotropy in URhGe obtained with local spin-density approximation calculations by Alexander Shick [10]. After the general consideration of the metamagnetic transition the modifications introduced by the uniaxial stress are considered. Then the Van der Waals-type theory of phenomena near the metamagnetic critical point is developed and some physical properties are discussed.

The phenomenon of the reentrant superconducting state is explained in Sec. III. It is shown that the recreation of superconductivity is caused by the sharp increase in the magnetic susceptibility [7] in the b direction near the metamagnetic transition. This section also contains the qualitative description of the specific behavior of the upper critical field in direction of spontaneous magnetization in UCoGe and in UGe_2 related to the field dependence of magnetic susceptibility. The Conclusion contains the summary of the results.

II. METAMAGNETIC TRANSITION IN URhGe

As in the previous publications (Refs. [2,5]) I shall use x, y, z as the coordinates pinned to the corresponding crystallographic directions a, b, c. The Landau free energy of an orthorhombic ferromagnet in magnetic field $\mathbf{H}(\mathbf{r}) = H_y\hat{y}$ is

$$F = \alpha_z M_z^2 + \beta_z M_z^4 + \delta_z M_z^6 + \alpha_y M_y^2 + \beta_y M_y^4 + \delta_y M_y^6 + \beta_{yz} M_z^2 M_y^2 - H_y M_y. \tag{2}$$

Here

$$\alpha_z = \alpha_{z0}\left(T - T_{c0}^c\right), \quad \alpha_y > 0, \tag{3}$$

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and I bear in mind the terms of the sixth order in powers of M_z, M_y and also the fact that in the absence of a field in the x direction the magnetization along the hard x direction $M_x = 0$.

A. Transition ferro-para

Let us remind first the treatment developed in Ref. [5] undertaken in the assumption $\beta_y > 0$. Then in the constant magnetic field $\mathbf{H} = H_y\hat{y}$ the equilibrium magnetization projection along the y direction

$$M_y \approx \frac{H_y}{2\left(\alpha_y + \beta_{yz}M_z^2\right)} \tag{4}$$

is obtained by minimization of free energy (2) in respect to M_y neglecting the higher order terms. Substituting this expression back to (2) we obtain

$$F = \alpha_z M_z^2 + \beta_z M_z^4 + \delta_z M_z^6 - \frac{1}{4}\frac{H_y^2}{\alpha_y + \beta_{yz}M_z^2}, \tag{5}$$

that gives after expansion of the denominator in the last term,

$$F = -\frac{H_y^2}{4\alpha_y} + \tilde{\alpha}_z M_z^2 + \tilde{\beta}_z M_z^4 + \tilde{\delta}_z M_z^6 + \ldots, \tag{6}$$

where

$$\tilde{\alpha}_z = \alpha_{z0}(T - T_{c0}) + \frac{\beta_{yz}H_y^2}{4\alpha_y^2}, \tag{7}$$

$$\tilde{\beta}_z = \beta_z - \frac{\beta_{yz}}{\alpha_y}\frac{\beta_{yz}H_y^2}{4\alpha_y^2}, \tag{8}$$

$$\tilde{\delta}_z = \delta_z + \frac{\beta_{yz}^2}{\alpha_y^2}\frac{\beta_{yz}H_y^2}{4\alpha_y^2}. \tag{9}$$

Thus, in a magnetic field perpendicular to the direction of spontaneous magnetization the Curie temperature decreases as

$$T_c = T_c(H_y) = T_{c0} - \frac{\beta_{yz}H_y^2}{4\alpha_y^2\alpha_{z0}}. \tag{10}$$

The coefficient $\tilde{\beta}_z$ also decreases with H_y and reaches zero at

$$H_y = H^\star = \frac{2\alpha_y^{3/2}\beta_z^{1/2}}{\beta_{yz}}. \tag{11}$$

At this field under fulfillment the condition,

$$\frac{\alpha_{z0}\beta_{yz}T_{c0}}{\alpha_y\beta_z} > 1 \tag{12}$$

the Curie temperature (10) is still positive and the phase transition from the ferromagnetic to the paramagnetic state becomes the transition of the first order [Fig. 1(a)]. The point $(H^\star, T_c(H^\star))$ on the line paramagnet-ferromagnet phase transition is a tricritical point. The qualitative field dependences of the normalized Curie temperature $t_c(H_y) = \frac{T_c(H_y)}{T_{c0}}$ and $b(H_y) = \frac{\tilde{\beta}_z}{\beta_z}$ are plotted in Fig. 1(a).

On the line of the first order phase transition from the ferromagnet to the paramagnet state the M_z component of magnetization drops from $M_z^\star$ to zero [2]. The M_y component jumps from $M_y \approx \frac{H^\star}{2(\alpha_y+\beta_{yz}M_z^{\star 2})}$ to $M_y \approx \frac{H^\star}{2\alpha_y}$. Then at

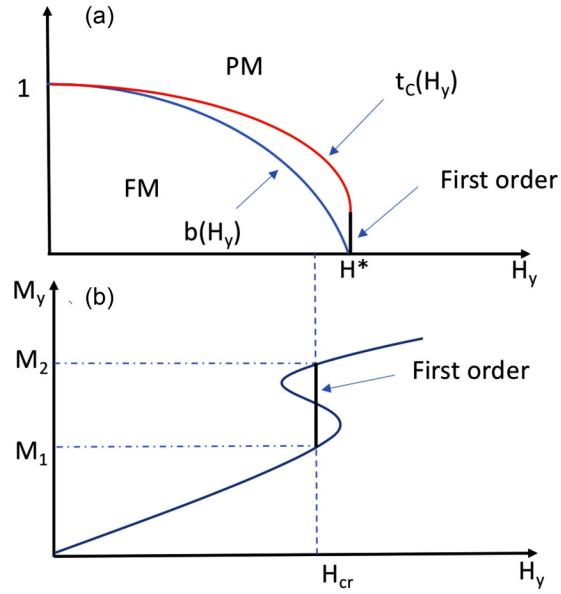

FIG. 1. (a) Schematic behavior of the normalized Curie temperature $t_c(H_y) = \frac{T_c(H_y)}{T_{c0}}$ and coefficient $b(H_y) = \frac{\tilde{\beta}_z}{\beta_z}$. FM and PM stand for ferromagnetic and paramagnetic phases. (b) Schematic dependence $M_y(H_y)$ at $T < T_{cr}$ and $H_{cr} < H^\star$.

fields $H_y > H^\star$

$$M_y \approx \frac{H_y}{2\alpha_y} \tag{13}$$

proportional to the external field. This contradicts experimental observations [3,6,7] which demonstrate the presence of a "spontaneous" part of magnetization in the field above the transition in accordance with Eq. (1).

B. Transition ferro - polarized para

The part of free energy depending on M_y,

$$F_y = \alpha_y M_y^2 + \beta_y M_y^4 + \delta_y M_y^6 + \beta_{yz}M_z^2M_y^2 - H_yM_y, \tag{14}$$

can be used also far from the transition to the ferromagnetic state in the temperature region where M_z is not small. The important fact obtained with the local spin-density approximation calculations [10] is that the coefficient $\beta_y < 0$. In the frame of the isotropic Fermi liquid model the negativeness of the fourth order term in the expansion of the free energy in power of magnetic moment is usually ascribed to the peculiar behavior of the electron density of states (see the review [11] and references therein). In the orthorhombic URhGe this specific magnetocrystalline anisotropy reveals itself in the system of magnetic moments localized on the uranium atoms [12].

The M_y component of magnetization is determined by the equation

$$2\tilde{\alpha}_y M_y + 4\beta_y M_y^3 + 6\delta_y M_y^5 = H_y, \tag{15}$$

where

$$\tilde{\alpha}_y = \alpha_y + \beta_{yz}M_z^2. \tag{16}$$

Taking into account the third order term we obtain

$$M_y \approx \frac{H_y}{2\tilde{\alpha}_y} - \frac{\beta_y H_y^3}{2\tilde{\alpha}_y^4}. \tag{17}$$

The coefficient $\beta_y < 0$ and we see that the increase of magnetization occurs faster than it was according to Eq. (4).

The shape of $M_y(H_y)$ depends on the temperature and pressure dependence of coefficients $\alpha_y, \beta_y, \delta_y$. In particular, the coefficient $\tilde{\alpha}_y(T)$ is decreasing function of temperature and at temperature decrease the field dependence of M_y transfers from the monotonous growth taking place at $\beta_y^2 < \frac{5}{3}\tilde{\alpha}_y\delta_y$ to the S-shape dependence realizing at $\beta_y^2 > \frac{5}{3}\tilde{\alpha}_y\delta_y$. This transformation occurs at some temperature $T_{\rm cr}$ such that in the dependence $H_y(M_y)$ appears an inflection point. It is determined by the equations

$$\frac{\partial H_y}{\partial M_y} = 0, \quad \frac{\partial^2 H_y}{\partial M_y^2} = 0 \tag{18}$$

having common solution

$$M_{\rm cr}^2 = -\frac{\beta_y}{5\delta_y}, \tag{19}$$

at $\beta_y^2 = \frac{5}{3}\tilde{\alpha}_y\delta_y$. The corresponding critical field is

$$H_{\rm cr} = H_y(M_{\rm cr}) = \frac{16}{5\sqrt{3}}\frac{\tilde{\alpha}_y^{3/2}}{|\beta_y|^{1/2}}. \tag{20}$$

At $T < T_{\rm cr}$ the inequality

$$\beta_y^2 > \tfrac{5}{3}\tilde{\alpha}_y\delta_y \tag{21}$$

is realized and the equation $\frac{\partial H_y}{\partial M_y} = 0$ acquires two real solutions, hence, the field dependence of M_y acquires the S shape plotted at Fig. 1(b). Equilibrium transition from the lower to the upper part of the curve $M_y(H_y)$ corresponds to a vertical line connecting the points M_1 and M_2 defined by the Maxwell rule $\int_1^2 M(H)dH = 0$. The integration is performed along the curve $M_y(H_y)$. The M_y component of magnetization jumps from M_1 to M_2 [see Fig. 1(b)].

At temperatures above $T_{\rm cr}$ the jump transforms into the crossover which is the temperature-field region characterized by the fast growth M_y. The lower boundary of this region roughly coincides with the Curie temperature (see Fig. 2). The Curie temperature decreasing with growth of magnetization M_y

$$T_c(H_y) = T_{c0} - \frac{\beta_{yz}M_y^2}{\alpha_{z0}} \tag{22}$$

falls down to zero or even to negative value at sharp increase of M_y in the vicinity of the critical field $H_{\rm cr}$ and the ferromagnetic order along the z direction disappears. Thus, at $T < T_{\rm cr}$ and $H_y = H_{\rm cr}$ we have the phase transition of the first order from the ferromagnetic state with spontaneous magnetization along the z direction to the polarized paramagnetic state with induced magnetization along the y direction (Fig. 2).

The described jumplike transition is realized in the cylindrical specimen in the magnetic field parallel to the cylinder axis. In specimens of arbitrary shape with demagnetization factor n the transition occurs in some field interval where the specimen is filled by the domains with different magnetization.

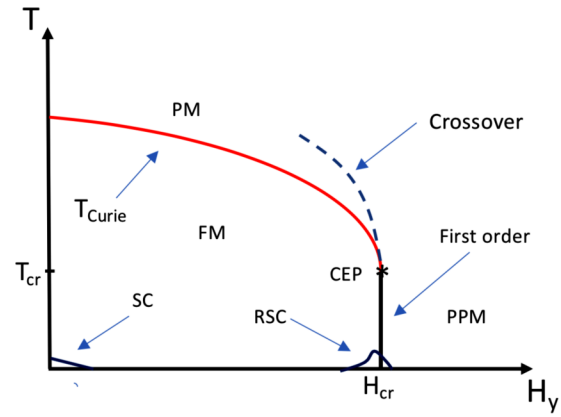

FIG. 2. Phase diagram UCoGe in magnetic field parallel to the *b*-crystallographic direction. PM, FM, and PPM denote paramagnetic, ferromagnetic, and polarized paramagnetic phases. CEP is the critical end point. SC and RSC are the superconducting and reentrant superconducting states.

When the critical field $H_{\rm cr}$ is smaller than the critical field of transition ferro-para $H^\star$, the ferro-para transition discussed in the previous section does not occur. At $T < T_{\rm cr}$ in fields H_y exceeding $H_{\rm cr}$, the field dependence of M_y component of magnetization behaves in accordance with Eq. (1) corresponding to the experimental observations.

C. Uniaxial stress effects

It is known that a hydrostatic pressure applied to URhGe crystals stimulates ferromagnetism and at the same time suppresses the superconducting state [13] and the reentrant superconducting state [14] as well. The latter is also shifted to a bit higher field interval. On the contrary, the uniaxial stress along the b direction suppresses the ferromagnetism decreasing the Curie temperature and stimulates the superconducting state so strongly that it leads to the coalescence of the superconducting and reentrant superconducting regions in the (H_y, T) phase diagram [15]. The phenomenological description of these phenomena was undertaken in the paper [16]. There it was shown that both coefficients α_z and α_y in the Landau free energy Eq. (2) acquire the linear uniaxial pressure dependence

$$\alpha_z(P_y) = \alpha_{z0}(T - T_{c0}) + A_z P_y, \tag{23}$$

$$\alpha_y(P_y) = \alpha_y - |A_y|P_y \tag{24}$$

corresponding to the moderate uniaxial pressure suppression of the Curie temperature

$$T_c(P_y) = T_{c0} - \frac{A_z P_y}{\alpha_{z0}}, \tag{25}$$

reported in Ref. [15] in the absence of an external field. However, under the external field along the y direction the drop of the Curie temperature Eq. (10) is accelerated

$$T_c(H_y, P_y)) \approx T_{c0} - \frac{A_z P_y}{\alpha_{z0}} - \frac{\beta_{yz}H_y^2}{4(\alpha_y(P_y))^2\alpha_{z0}} \tag{26}$$

in correspondence with the observed behavior. Moreover, the uniaxial stress causes strong decrease of the critical field Eq. (20)

$$H_{\rm cr} = H_y(M_{\rm cr}) = \frac{16}{5\sqrt{3}} \frac{(\tilde{\alpha}_y(P_y))^{3/2}}{|\beta_y|^{1/2}}. \tag{27}$$

D. Van der Waals-type theory near the critical point

The critical end point temperature for the first order transition in URhGe is $T_{\rm cr} = 4$ K and the critical field is $H_{\rm cr} = 12T$. Let us expand the function $H_y(M_y)$ at temperature slightly deviating from critical temperature $T = T_{\rm cr} + t$ and the magnetization near its critical value $M_y = M_{\rm cr} + m$. We have

$$h = H_y - H_{\rm cr} = bt + \left[\frac{\partial H_y}{\partial M_y}\bigg|_{t=0} + 2at\right]m + \frac{1}{2}\frac{\partial^2 H_y}{\partial M_y^2}\bigg|_{t=0} m^2 + \frac{1}{6}\frac{\partial^3 H_y}{\partial M_y^3}\bigg|_{t=0} m^3. \tag{28}$$

Here, we neglected by the temperature dependence of the second and the third order terms. Taking into account that $\frac{\partial H_y}{\partial M_y}|_{t=0} = \frac{\partial^2 H_y}{\partial M_y^2}|_{t=0} = 0$ we obtain

$$h = bt + 2atm + 4Bm^3, \tag{29}$$

which obviously corresponds to the expansion of pressure $p = P - P_{\rm cr}$ in powers of density $\eta = n - n_{\rm cr}$ near the Van der Waals critical point [17].

At $t < 0$ according to the Maxwell rule the magnetization densities of two phases in equilibrium with each other are:

$$m_2 = -m_1 = \sqrt{\frac{-at}{2B}}. \tag{30}$$

The line of phase equilibrium between the two phases below and above the transition is given by the equation

$$h = bt,\ t < 0. \tag{31}$$

1. Specific heat

The specific heat at fixed external field (see Ref. [17]) is

$$C_h \propto T \frac{\left(\frac{\partial h}{\partial t}\right)_m^2}{\left(\frac{\partial h}{\partial m}\right)_t}. \tag{32}$$

Then, using Eq. (29) we obtain

$$C_h \propto \frac{b^2 T}{2at + 12Bm^2}. \tag{33}$$

Thus, the contribution to heat capacity according to the equation of state (29) near the critical point grows so long m^2 decreases until to m_1^2 and then begins to fall when m^2 increases starting from m_2^2 (see Fig. 3). This is the contribution to the specific heat of the whole system and cannot be directly attributed to the specific heat of itinerant electrons proportional to the electron effective mass.

The low temperature behavior of the URhGe specific heat in magnetic field has not been established by a direct measurement but was derived [6] by the application of the Maxwell relation $(\frac{\partial S}{\partial H_y})_T = (\frac{\partial M_y}{\partial T})_{H_y}$ from the temperature dependence of the magnetization $M_y(T, H_y)$ in the fixed field. The changes

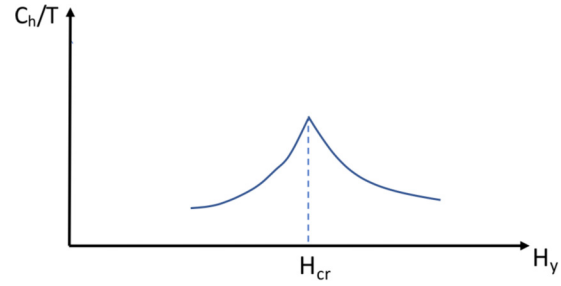

FIG. 3. Schematic behavior C_h/T (see the main text).

of the ratio $C(T)/T$ have been ascribed to the electron effective mass dependence from magnetic field [6,18]. This was done in the assumption that URhGe is a weak itinerant ferromagnet, in other words, all the low temperature degrees of freedom in this material belong to the itinerant electron subsystem. As we already mentioned above, the strong magnetic anisotropy of this material [10] points on the importance of the magnetic degrees of freedom localized on the uranium ions and related with crystal field levels [2,12].

2. Resistivity

The magnetic field dependence of effective mass was also found [18,19] by the application of the Kadowaki-Woods relation $A(H_y) \propto (m^\star)^2$ where coefficient A is a prefactor in the low-temperature dependence of resistivity $\rho = \rho_0 + AT^2$. The $A(H_y)$ behavior is determined by the processes of inelastic electron-electron scattering which in the multiband metals interfere with scattering on impurities (see Refs. [20–24]) and on magnetic excitations with field dependent spectrum. The nonspherical shape of the Fermi surface sheets and the screening of el-el Coulomb interaction can introduce deviations from T^2 resistivity dependence. So, the physical meaning of the coefficient $A(H_y)$ behavior is not so transparent and its relationship with the electron effective mass is questionable.

One can also note that the temperature fit of the experimental data was done in a very narrow temperature interval and the T^2 temperature dependence claimed in Ref. [19] seems somewhat unreliable. Compare with the results reported in Refs. [13,25].

3. Correlation function

The correlation function of fluctuations of the magnetization density m near the critical point at $t < 0$ behaves similar to the specific heat [17]

$$\varphi(\mathbf{k}) = \frac{T}{2(at + 6Bm^2 + \gamma_{ij}k_i k_j)}. \tag{34}$$

This is in correspondence with a marked increase of the NMR relaxation rate $1/T_2$ with field H_y increasing toward 12 T reported in Refs. [26,27].

III. PHASE TRANSITION TO SUPERCONDUCTING STATE

The superconducting state in URhGe is completely suppressed by the magnetic field $H_{c2}(T = 0) \approx 2$ T in the y direction due to the orbital depairing effect. Then superconductivity recovers in the field interval 9–13 T around the

critical field $H_{cr} \approx 12$ T of the transition of the first order from the ferromagnetic state with spontaneous magnetization along the z direction to the state with induced magnetization along the y direction. Evidently such type behavior is possible if the magnetic field somehow stimulates the pairing interaction surmounting the orbital depairing effect.

In numerous publications starting from the paper by A. Miyake *et al.* [18] the treatment of this phenomenon was related with the assumption of an enhancement of electron effective mass $m^\star = m(1+\lambda)$ leading to the enhancement of pairing interaction and consequently of the temperature of transition to superconducting state according to the McMillan-like formula [28]

$$T_{sc} \approx \epsilon \exp\left(-\frac{1+\lambda}{\lambda}\right) \tag{35}$$

derived in the paper [29] for the superconducting state with p pairing in an itinerant isotropic ferromagnetic metal. Similar to the liquid He-3 in this model there are two independent phase transition to the superconducting state in the subsystems with spin-up and spin-down electrons. The constant λ determined by the Hubbard four-fermion interaction [29,30] increases as we approach but not get too close to ferromagnetic instability. In the frame of this model the question of why the growth of the magnetic field H_y approaches the ferromagnetic transition remains unanswered.

The following development of this type approach has been undertaken by Yu. Sherkunov and co-authors [31]. The reentrant superconductivity and mass enhancement have been associated with the Lifshitz transition [32] which occurs in one of the bands in a finite magnetic field stimulating the splitting of spin-up and spin-down bands. There was established modest enhancement of the transition critical temperature in the field about 10 T. Thus, the model can claim to the qualitative explanation of the superconducting state reentrance. However, it should be noted that the measured [32] quasiparticle mass in the corresponding band does not increase but decreases and remains finite, implying that the Fermi velocity vanishes due to the collapse of the Fermi wave vector. The cross section of the Fermi surface of this band corresponds to 7% of the Brillouin zone area. Thus, the reentrance of superconductivity hardly could be associated with the observed Lifshitz transition.

The models [29,31] describe the physics of pure itinerant electron subsystem. Such a treatment is approved in application to the ^{3}He Fermi liquid. The measurements by x-ray magnetic circular dichroism [12] point to the local nature of the URhGe ferromagnetism. Namely, the comparison of the total uranium moment μ^U_{tot} to the total magnetization M_{tot} at different magnitude and direction of magnetic field indicates that the uranium ions dominate the magnetism of URhGe. The same is true also in the parent compound UCoGe [33]. So, the magnetic susceptibility $\chi_{ij}(\mathbf{q},\omega)$ is mostly determined by the localized moments subsystem. Hence, an approach based on the exchange interaction between conduction electrons and magnetic moments localized on uranium atoms seems more appropriate. This type theory has been developed in the paper by Hattori and Tsunetsugu [34]. Here, there will be undertaken another approach allowing explicitly taking into account the enhancement of magnetic susceptibility near the metamagnetic transition from the ferromagnet state with spontaneous magnetization along the c axis to the magnetic state polarized along the b axis.

Using the standard functional-integral representation of the partition function of the system (see Ref. [35]), we obtain the following term in the fermionic action describing an effective two-particle interaction between electrons:

$$S_{int} = -\frac{1}{2}I^2 \int dx\, dx' S_i(x)\mathcal{D}_{ij}(x-x')S_j(x'), \tag{36}$$

where $\mathbf{S}(\mathbf{r}) = \psi^\dagger_\alpha(\mathbf{r})\boldsymbol{\sigma}_{\alpha\beta}\psi_\beta(\mathbf{r})$ is the operator of the electron spin density, $x = (\mathbf{r},\tau)$ is a shorthand notation for the coordinates in real space and the Matsubara time, $\int dx(...) = \int d\underline{r}\int_0^\beta d\tau(...)$, I is the exchange constant of interaction of itinerant electrons with localized magnetic moments, $\mathcal{D}_{ij}(x-x')$ is the spin-fluctuation propagator expressed in terms of the dynamical spin susceptibility $\chi_{ij}(\mathbf{q},\omega)$.

Making use of the interaction (36) one can calculate the electron self energy and find the dependence of the electron effective mass from magnetic field as well the temperature of transition to the superconducting state with triplet pairing. The energy of electronic excitations in the temperature region where the superconducting state is realized is much smaller than typical energy of magnetic excitations. Hence, in calculation of the superconducting properties one can neglect the frequency dependence of susceptibility.

A. Upper critical field parallel to the c axis in UCoGe

In application to UCoGe in magnetic field parallel to direction of spontaneous magnetization this program has been accomplished in the paper [36]. There has been considered transition into the equal-spin pairing superconducting state in two-band (spin-up, spin-down) orthorhombic ferromagnetic metal. According to this paper in the simplified case of a single-band (say spin-up) equal-spin pairing superconducting state the critical temperature without including the orbital effect of the field is

$$T_{sc} = \epsilon\, \exp\left(-\frac{1+\lambda}{\langle N_0(\mathbf{k})\chi^u_{zz}\rangle I^2}\right), \tag{37}$$

where, as in the McMillan formula, $1+\lambda$ corresponds to the effective mass renormalization, whereas the pairing amplitude expressed through the odd in momentum part of static susceptibility

$$\chi^u_{zz} = \tfrac{1}{2}[\chi_{zz}(\mathbf{k}-\mathbf{k}') - \chi_{zz}(\mathbf{k}+\mathbf{k}')],$$

which is the main source of the critical temperature dependence from magnetic field. Here,

$$\chi_{zz}(\mathbf{k}) = \frac{1}{\chi_z^{-1} + 2\gamma_{ij}k_ik_j}, \tag{38}$$

and $\chi_z = \chi_z(H_z)$ is the z component of susceptibility in the finite field H_z. Its magnitude at $H_z \to 0$, and we will denote χ_{z0}. The angular brackets denote averaging over the Fermi surface and $N_0(\mathbf{k})$ is the angular dependent density of electronic states on the Fermi surface,

$$\langle N_0(\mathbf{k})\chi^u_{zz}(H_z)\rangle \approx \frac{2\langle N_0(\mathbf{k})\hat{k}_z^2\rangle k_F^2\chi_z}{(2\chi_z)^{-1} + 4\gamma_{zz}k_F^2}. \tag{39}$$

The denominator in the exponent of Eq. (37) can be expressed through its value at $H_z \to 0$

$$\frac{\langle N_0(\mathbf{k})\chi^u_{zz}(H_z)\rangle}{\langle N_0(\mathbf{k})\chi^u_{zz}(H_z\to 0)\rangle}=\frac{\chi_z}{\chi_{z0}}\frac{1+4(\xi_m k_F)^2}{\frac{\chi_{z0}}{\chi_z}+4(\xi_m k_F)^2}. \tag{40}$$

Here the product $2\gamma_{zz}k_F^2\chi_{z0}=(\xi_m k_F)^2$ is expressed through the magnetic coherence length ξ_m which near the zero temperature is of the order of several interatomic distances.

In assumption $(\xi_m k_F)^2 \gg 1$ one can rewrite Eq. (40) as

$$\langle N_0(\mathbf{k})\chi^u_{zz}(H_z)\rangle \approx \frac{\chi_z(H_z)}{\chi_{z0}}\langle N_0(\mathbf{k})\chi^u_{zz}(H_z\to 0)\rangle. \tag{41}$$

This very rough estimation presents the qualitative dependence of exponent in equation (37) from magnetic field. The longitudinal susceptibility drops with the augmentation of magnetic field parallel to the spontaneous magnetization (see Fig. 3 in the paper [37]) leading to the suppression of the temperature of transition to the superconducting state without including the orbital effect according to Eq. (37).

Taking into account the orbital effect one can write the field dependence of critical temperature of transition to the superconducting state in the Ginzburg-Landau region

$$T_{sc}^{orb}(H)=T_{sc}(H)-\frac{H}{AT_{sc}(H)}, \tag{42}$$

where A is a constant. Thus, the decreasing of $T_{sc}(H)$ with magnetic field causes not only faster drop but also the peculiar upward curvature in the critical temperature $T_{sc}^{orb}(H)$ dependence from magnetic field in correspondence with the experimental data reported in Ref. [38].

B. Reentrant superconductivity in URhGe

In the field perpendicular to the spontaneous magnetization the similar approach applied to the simplified single band model in weak coupling approximation yields (see Eq. (169) in the review [2]) the critical temperature

$$T_{sc}\approx\epsilon\,\exp\left(-\frac{1}{\left[\langle N_0(\mathbf{k})\chi^u_{zz}\rangle\cos^2\varphi+\langle N_0(\mathbf{k})\chi^u_{yy}\rangle\sin^2\varphi\right]I^2}\right), \tag{43}$$

where $\tan\varphi = H_y/h$ and h is the exchange field acting on the electron spins. This is the critical temperature of transition to the superconducting state without including the orbital effect.

The orbital effect suppresses the superconducting state and near the upper critical field at zero temperature

$$H_{c2y}(T=0)=H_0=cT_{sc}^2 \tag{44}$$

the actual critical temperature is

$$T_{sc}^{orb}=a\sqrt{H_0-H_y}, \tag{45}$$

where $a\sqrt{c}$ is the numerical constant of the order of unity. This is the usual square root BCS dependence of the critical temperature from magnetic field in low temperature-high field region such that $T_{sc}^{orb}(H_y=H_0)=0$. However, in the present case the magnitude H_0 itself is a function of the external field H_y. Let us look on its behavior.

Similar to Eq. (41) we get

$$\begin{aligned}&\langle N_0(\mathbf{k})\chi^u_{zz}(H_y)\rangle\cos^2\varphi+\langle N_0(\mathbf{k})\chi^u_{yy}(H_y)\rangle\sin^2\varphi\\ &\approx\frac{\chi_z(H_y)}{\chi_{z0}}\langle N_0(\mathbf{k})\chi^u_{zz}(H_y\to 0)\rangle\cos^2\varphi\\ &+\frac{\chi_y(H_y)}{\chi_{y0}}\langle N_0(\mathbf{k})\chi^u_{yy}(H_y\to 0)\rangle\sin^2\varphi.\end{aligned} \tag{46}$$

Here, $\chi_z(H_y)$ and $\chi_y(H_y)$ are the z and y components of susceptibility in finite field H_y and χ_{z0} and χ_{y0} are the corresponding susceptibilities at $H_y \to 0$. Unlike Eq. (41) the field dependence of Eq. (46) is not so visible. One can note, however, the different field dependence of two summands in Eq. (46).

(i) The susceptibility along the z direction $\chi_z(H_y)$ increases with magnetic field H_y following to the decreasing of the Curie temperature according to Eq. (22). The growth of susceptibility along the z direction at the approaching field H_y to H_{cr} is confirmed by the field dependence of the NMR scattering rate $1/T_1$ reported in Refs. [26,27]. At the same time, the increase of $\chi_z(H_y)$ is limited by the decrease of $\cos^2\varphi$. We do not know how fast it is because the magnitude of the exchange field is not known.

(ii) As the field approaches to H_{cr} the low temperature susceptibility $\chi_y(H_y)$ has a high delta-function-like peak [7] with magnitude more than 10 times greater than it is at $H_y \to 0$. The factor $\sin^2\varphi$ is also increased. This indicates that in URhGe, more important is the second term connected with the metamagnetic transition.

Thus, in the vicinity of metamagnetic transition one can expect the increase of the critical temperature estimated without including the orbital effect according to Eq. (43). The radicand in equation (45) after being negative in some field interval acquires the positive value as the field approaches to H_{cr}. The critical temperature Eq. (45) reaches maximum in the vicinity of metamagnetic transition, see Fig. 2.

Similar arguments in favor of stimulation superconductivity near the metamagnetic transition in the field parallel to the b axis can be applied to the recently discovered other superconducting compound UTe_2 [39–41] isostructural with URhGe. However, in view of many particular properties of this material we leave this subject for future studies.

In the parent compound UCoGe the metamagnetic transition is absent (at least at $H_y < 40$ T) [42]. Hence, in this material the unusual temperature dependence of the upper critical field parallel to the b axis is probably mostly determined by the first term in Eq. (46).

Near $H_y = H_{cr}$ at temperatures $T < T_{cr}$ the NMR spectrum is composed of two components indicating that the transition is of the first order accompanied by the phase separation [26]. Thus, in almost whole interval near H_{cr} the superconductivity is developed in a mixture of ferromagnetic state with polarization along the z direction and the field polarized state with polarization along the y direction.

C. Upper critical field near metamagnetic transition in UGe_2

A peculiar example of superconductivity stimulation in the vicinity of metamagnetic transition is realized in the other ferromagnetic compound UGe_2. This material has

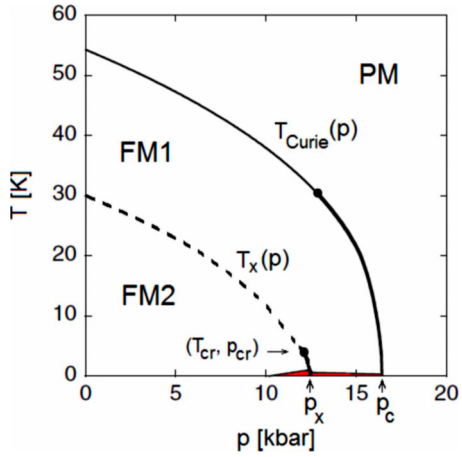

FIG. 4. The schematic P, T phase diagram of UGe_2 [45,46]. Thick lines represent first-order transitions and thin lines denote second-order transitions. The dashed line indicates a crossover while the dots mark the positions of critical points. The superconducting region is represented in the red area at the bottom.

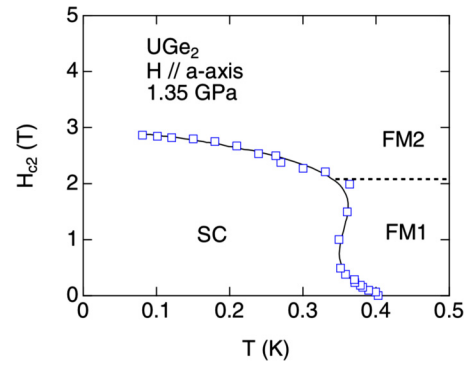

FIG. 5. Temperature dependence of H_{c2} for a field parallel to the a axis in UGe_2 at 1.35 GPa, which is just above P_x. The metamagnetic transition is detected at H_x between FM1 and FM2 [48,49].

orthorhombic structure with spontaneous magnetization directed along the a crystallographic direction. The magnetism in UGe_2 has an even more localized nature [2,43,44] than in related compounds URhGe and UCoGe. The superconductivity exists inside of the ferromagnetic state in the pressure interval shown in Fig. 4. Inside of this interval at $P = P_x$ there is a metamagnetic transition from ferromagnetic state FM1 to ferromagnetic state FM2 characterized by the jump of spontaneous magnetization from smaller to larger value [45]. At a bit higher pressure $P = P_x + \delta P$ the transition from FM1 to FM2 occurs in a finite magnetic field applied along the direction of spontaneous magnetization [47]. Near this transition in a finite field the magnetic susceptibility along the a-axis χ_a strongly increases. Hence, the critical temperature without including the orbital effect

$$T_{sc} = \epsilon \exp\left(-\frac{1+\lambda}{\langle N_0(\mathbf{k})\chi_a^{\mu}\rangle I^2}\right) \tag{47}$$

growths up. As a result the upper critical field in a crystallographic direction measured at $P = P_x + \delta P$ acquires nonmonotonic temperature dependence shown in Fig. 5 [48,49].

It is worth noting that at pressures far from metamagnetic transition the upper critical field parallel to the a direction does not reveal an upward curvature [48,49]. This important distinction from the upper critical field behavior in UCoGe considered in Sec. III A is related to the difference of susceptibility dependence from magnetic field along spontaneous magnetization in these two materials. Whereas in UCoGe the susceptibility χ_c along the c axis is strongly field dependent [37], in UGe_2 the susceptibility χ_a along the a axis is practically field independent [45,50].

IV. CONCLUSION

We have demonstrated that in the orthorhombic ferromagnet URhGe the ferromagnetic ordering along the c axis is suppressed in the process of increase of magnetization in the perpendicular b direction induced by the external magnetic field. This process is accelerated by the tendency to the metamagnetic transition which occurs at $H_y = H_{cr} = 12$ T. The transition of the first order is accompanied by the suppression of the ferromagnetic state with polarization along the c axis and the arising of magnetic state polarized along the b axis. The line of first order phase transition is finished at the critical end point with temperature $T = T_{cr} = 4$ K.

The uniaxial stress along the b axis causing moderate suppression of the Curie temperature in the absence of magnetic field accelerates the Curie temperature drop in finite magnetic field H_y and quite effectively decreases the critical field of metamagnetic transition. As a result, the superconducting state recovers itself in a much smaller field and can even be merged with the superconducting state in the small fields region. The superconducting pairing is determined by the exchange interaction between the conduction electrons and the magnetic moments localized on uranium atoms.

In UCoGe the upward curvature of the upper critical field along the c axis is mostly determined by the longitudinal magnetic susceptibility decrease along with the magnetization saturation. In URhGe the superconducting state suppressed in field $H_y \approx 2$ T is recovered in fields interval (9–13) T near the critical field. This phenomenon is related to the strong increase of the pairing interaction caused mostly by the strong augmentation of the magnetic susceptibility along the b direction in the vicinity of the metamagnetic transition. The nonmonotonous behavior of the upper critical field in UGe2 is explained by the strong increase of longitudinal magnetic susceptibility at the metamagnetic transition from FM1 to FM2.

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