# **Metallic silicon subhydrides at high pressures studied by** *ab initio* **calculations**

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High pressure properties of silicon subhydrides  $SH$ ,  $SH<sub>2</sub>$ , and  $SH<sub>3</sub>$  are investigated and compared with those of SiH4 by using *ab initio* methods. Although the subhydrides are thermodynamically unstable at lower pressures, SiH, SiH<sub>2</sub>, and SiH<sub>3</sub> are found to be stabilized above 486, 386, and 430 GPa, respectively. The predicted phases of SiH, SiH<sub>2</sub>, and SiH<sub>3</sub> have  $Fm\overline{3}m$ ,  $I4/mmm$ , and  $I4/m$  symmetries, respectively, and are metallic. The density of states suggests that the electronic states at the Fermi energy are more delocalized in the subhydrides than in  $SiH<sub>4</sub>$ . This fact is well reflected in the superconducting transition temperature  $(T<sub>c</sub>)$  estimated for  $SiH<sub>2</sub>$  and  $SiH<sub>3</sub>$ . Indeed,  $T_c$  reaches 83 K in SiH<sub>2</sub> at 400 GPa and 88 K in SiH<sub>3</sub> at 450 GPa while that is 28 K in SiH<sub>4</sub> at 400 GPa. Although  $T_c$  of SiH<sub>4</sub> is expected to exceed 100 K above 503 GPa, the current capability of diamond anvil cells suggests that high-*Tc* silicon hydride is more likely to be attained experimentally in subhydrides.

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### **I. INTRODUCTION**

Dense metallic hydrogen was predicted to become a superconductor in 1968 [\[1\]](#page-5-0), and the superconducting transition temperature  $(T_c)$  is expected to be quite high. The superconductivity in dense hydrogen is supposed to be driven by phonon mediated mechanism, where the suggested high  $T_c$  is attributable to the high Debye temperature originating from the light hydrogen mass and the strong electron-ion interaction owing to the lack of core electrons. After this theoretical work, it was pointed out that the same mechanism of high- $T_c$ superconductivity should also be applicable to dense hydrides [\[2\]](#page-5-0). The validity of the predictions has been confirmed recently. In 2015, superconductivity of sulfur hydride was observed experimentally, where  $T_c$  reaches 203 K at 155 GPa [\[3\]](#page-5-0), and the composition of the superconducting phase was reported to be  $SH_3$  [\[4,5\]](#page-5-0). Then, in 2019, it was found that lanthanum hydride shows superconductivity with  $T_c$  of 260 K at 180 GPa [\[6\]](#page-5-0) and 250 K at 170 GPa [\[7\]](#page-5-0). The superconducting phase is very hydrogen rich and thought to have the compo-sition of LaH<sub>10</sub> [\[8,9\]](#page-5-0). In both SH<sub>3</sub> and LaH<sub>10</sub>, the  $T_c$  values estimated by theoretical calculations [\[4,8\]](#page-5-0) are consistent with the experiments  $[3,5-7]$ . This agreement suggests that the superconductivity indeed originates from phonon mediated mechanism.

Recent extensive studies of dense metallic hydrides, which started around the middle of the 2000s, were initiated chiefly by the works of Ashcroft  $[10,11]$ . The works suggested that metallization pressures of some hydrides actually lie within the reach of diamond anvil cells, proposing group-IV hydrides as promising candidate compounds. In particular, silane (SiH4) attracted great interest and was investigated in detail theoretically  $[12-17]$  and experimentally  $[18-21]$ . According to the experiment by Eremets *et al.* [\[20\]](#page-5-0), SiH<sub>4</sub> is transformed into a metal at 50 GPa, and shows superconductivity with  $T_c = 17$  K both at 96 GPa and at 120 GPa. Contrastingly, Sun *et al.* [\[18\]](#page-5-0) and Strobel *et al.* [\[21\]](#page-5-0) reported that SiH<sub>4</sub>

remains nonmetallic even above ∼150 GPa. Although there exists such a disagreement between the experiments, one thing which is clear about  $SiH_4$  is that very high  $T_c$  is not observed yet. In fact, most theoretical works suggest that  $T_c$  does not reach 40 K in SiH<sub>4</sub> [ $13,15,17$ ]. One exceptional phase is the  $C2/m$  SiH<sub>4</sub> proposed by Zhang *et al.* [\[16\]](#page-5-0), where  $T_c$  was estimated to be 106.31 K at 610 GPa.

For boron [\[22\]](#page-5-0), carbon [\[23\]](#page-5-0), aluminium [\[24\]](#page-5-0), and germanium [\[25,26\]](#page-5-0), which are adjacent to silicon on the Periodic Table, several subhydrides are predicted to be stabilized thermodynamically at high pressures; here, subhydrides are supposed to mean the compounds whose hydrogen concentrations are lower than the typical compositions, namely,  $BH<sub>3</sub>$ ,  $CH<sub>4</sub>$ ,  $AlH<sub>3</sub>$ , and  $GeH<sub>4</sub>$ . Among the predicted subhydrides, BH, AlH,  $Al<sub>2</sub>H$ , GeH<sub>3</sub>, Ge<sub>2</sub>H, and Ge<sub>3</sub>H are metallic. Moreover, AlH  $[24]$  and GeH<sub>3</sub>  $[25]$  are expected to show superconductivity with higher  $T_c$  than AlH<sub>3</sub> and GeH<sub>4</sub>, respectively. These findings tempt one to examine whether silicon subhydrides likewise become stable and show superconductivity at megabar pressures.

Thus, in this paper,  $SiH$ ,  $SiH_2$ , and  $SiH_3$  are investigated by using *ab initio* methods. One of the well-known silicon subhydrides is disilane  $(Si<sub>2</sub>H<sub>6</sub>)$ , and the high-pressure properties of disilane (or, strictly,  $SiH<sub>3</sub>$ ) were already investigated by using *ab initio* calculations [\[27,28\]](#page-5-0). The theoretical works on SiH<sub>3</sub> proposed  $P\overline{1}$ ,  $Pm\overline{3}m$ ,  $C2/c$ , and *Cmcm* structures, and the *Pm* $\overline{3}$ *m* phase was shown to have the highest  $T_c$  (139 K at 275 GPa). As already suggested by Cui *et al.* [\[17\]](#page-5-0), all these SiH3 phases are actually unstable to decomposition into Si and SiH4 and should be considered as possible metastable phases. Yet, in this paper, it is demonstrated that a SiH<sub>3</sub> phase with *I*4/*m* symmetry is stabilized from 430 to 578 GPa. Also, SiH2 and SiH turn out to be stable in structures with *I*4/*mmm* and *Fm*3*m* symmetries above 386 and 486 GPa, respectively. Although the estimated  $T_c$  is not so high in the  $Fm\overline{3}m$  SiH (14 K at 500 GPa) owing to the low hydrogen concentration,  $T_c$  reaches 83 K in the  $I4/mmm$  SiH<sub>2</sub> at 400 GPa and 88 K in <span id="page-1-0"></span>the  $I4/m$  SiH<sub>3</sub> at 450 GPa. Interestingly, these  $T_c$  values are rather high in comparison with those estimated for stable SiH4 phases below ∼500 GPa.

## **II. TECHNICAL DETAILS**

*Ab initio* calculations based on the density functional theory are performed within the generalized gradient approximation [\[29\]](#page-5-0), where the functional form of Perdew, Burke, and Ernzerhof is chosen [\[30\]](#page-5-0). Plane-wave basis sets and ultrasoft pseudopotentials [\[31\]](#page-5-0) are utilized with the use of QUANTUM ESPRESSO [\[32\]](#page-5-0). The cutoff radius of the Si pseudopotential is 0.953 Å (with 3*s* and 3*p* electrons treated as valence ones) and that of the H pseudopotential is 0.423 Å. The cutoff energy of the plane-wave basis sets is set to 816 eV. The number of *k* points is chosen to be greater than  $(40 \text{ Å})^3/v_{cell}$ , where  $v_{cell}$ is the volume of the simulation cell. As a smearing function for the occupation around the Fermi energy, a Fermi-Dirac distribution with a temperature of  $k_B T = 0.1$  eV is used. Electron-phonon coupling is analyzed by using the density functional perturbation theory [\[33\]](#page-5-0). The numbers of *k* points and *q* points (namely, phonon wave vectors) are chosen to be larger than  $(50 \text{ Å})^3/v_{cell}$  and  $(13 \text{ Å})^3/v_{cell}$ , respectively, in the calculations of electron-phonon interaction.

The investigation of stable structures is carried out by random structure searches [\[34,35\]](#page-5-0), where many initial structures are randomly generated and then optimized in order to find out low-enthalpy structures. The number of randomly generated structures ranges from 400 to 3000, depending on the number of atoms and the composition. The *k*-point number is decreased to about  $(20 \text{ Å})^3/v_{cell}$  in the random searches. After several low-enthalpy structures are picked out as candidate structures from the random searches, their structural parameters and enthalpies are refined with the *k*-point number increased. Then, for those candidate structures, the zero-point energy of nuclei (ZPE) is estimated to finally determine the lowest-enthalpy structure. The ZPE is calculated within the harmonic approximation, the frozen phonon method being used by making use of PHONOPY [\[36\]](#page-5-0). The supercells for the frozen-phonon calculations are constructed such that the number of atoms contained is 64 or larger.

#### **III. STABLE PHASES**

Random structure searches have been performed for SiH,  $SiH<sub>2</sub>$ , and  $SiH<sub>3</sub>$  at 200, 300, and 400 GPa. Search cells with 2, 3, 4, 5, and 6 formula units were investigated for all the compositions. The resulting low-enthalpy structures are summarized in the Supplemental Material (Ref. [\[37\]](#page-6-0)).

It should be mentioned that the above choice of search cells turned out not to be large enough for  $SiH<sub>3</sub>$ . In fact, the low-enthalpy structures of  $SiH<sub>3</sub>$  predicted by the random searches have imaginary-frequency phonons and are dynamically unstable. Therefore, in SiH3, dynamically stable structures are further looked for manually, where the unstable structures are modified by displacing atoms according to the imaginary-frequency modes with the use of larger cells and then optimized. As a result, there are found out two dynamically stable structures of SiH3, which have rather large primitive cells: one with 16 formula units below ∼340 GPa



FIG. 1. Relative enthalpy per atom of  $Si_{1-x}H_x$  as a function of *x*: (a) without ZPE; (b) with ZPE. Stable compositions are connected with solid lines. See the Supplemental Material (Ref. [\[37\]](#page-6-0)) for the candidate structures and the structural parameters in  $SiH$ ,  $SiH_2$ ,  $SiH_3$ , and SiH4. The enthalpy of Si is calculated for the fcc structure [\[38\]](#page-6-0).

which has  $P2_1/c$  symmetry and the other with 8 formula units above ∼340 GPa which has *I*4/*m* symmetry. This fact might seem to indicate that the above search cells are not large enough even in SiH and SiH2. However, it should be stressed that the SiH and SiH2 phases obtained from the random searches have very simple structures as shown later. Usually, such simple structures are not predicted by random searches when search cells are not large enough to accommodate the stable structure. This suggests that the random searches are appropriately performed with the above cell choice in SiH and  $SiH<sub>2</sub>$ .

In Fig. 1, the relative enthalpy per atom of  $Si_{1-x}H_x$  is presented as a function of the hydrogen concentration *x*. The stable compositions are connected with solid lines which correspond to the lower convex envelopes. Without the ZPE, all the subhydrides are unstable to decomposition into Si and  $SiH<sub>4</sub>$  at least up to 450 GPa [Fig. 1(a)], though SiH is nearly stabilized around 500 GPa. However, the consideration of the ZPE makes SiH,  $SiH<sub>2</sub>$ , and  $SiH<sub>3</sub>$  possible above 486, 386, and 430 GPa, respectively [Fig. 1(b)]. Actually, as will be discussed later, H atoms are surrounded by more Si atoms in the subhydrides than in SiH4. This structural property indicates that, in the subhydrides, the Si-H bonding is chemically loose and therefore the average phonon frequency should be low.

Figure [2](#page-2-0) shows the comparison of enthalpy between candidate structures in SiH,  $SiH_2$ ,  $SiH_3$ , and  $SiH_4$ . In  $SiH_2$ , the phase-separated system consisting of Si and  $SiH<sub>4</sub>$  is transformed into the homogeneous  $I4/mmm$  SiH<sub>2</sub> above 386 GPa. In SiH, the homogeneous phase (namely, the *Fm*3*m* SiH) appears at a little higher pressures than in  $SiH<sub>2</sub>$ ; this is partly

<span id="page-2-0"></span>

FIG. 2. Comparison of enthalpy between candidate structures in  $SiH$ ,  $SiH_2$ ,  $SiH_3$ , and  $SiH_4$ , where the ZPE is taken into consideration. The enthalpy of Si is calculated for the fcc structure [\[38\]](#page-6-0), and that of H for the *C*2/*c* [\[39\]](#page-6-0), the *Cmca*-12 [\[39\]](#page-6-0), the *Cmca* [\[40\]](#page-6-0), and the Cs-IV (*I*4<sub>1</sub>/*amd* with  $c/a > \sqrt{2}$ ) [\[41\]](#page-6-0) structures. The thin dotted lines mean that the structures have imaginary-frequency phonons at the pressures, and that the ZPE is calculated by neglecting the unstable phonon modes. See the Supplemental Material (Ref. [\[37\]](#page-6-0)) for the structural parameters of the SiH,  $SiH<sub>2</sub>$ ,  $SiH<sub>3</sub>$ , and  $SiH<sub>4</sub>$  phases.

due to the stability of the *I*4/*mmm* SiH2, which forms part of the disproportionated SiH system above 386 GPa. In SiH3, the homogeneous *I*4/*m* phase becomes stable above 430 GPa, and the system is again decomposed into  $SiH<sub>2</sub>$  and  $SiH<sub>4</sub>$ above 578 GPa. Note that there have been so far proposed several SiH<sub>3</sub> phases, which are the  $P\overline{1}$ ,  $C2/c$ ,  $Pm\overline{3}m$ , and *Cmcm* structures [\[27,28\]](#page-5-0). However, as already pointed out in Ref. [\[17\]](#page-5-0), these structures turn out to be thermodynamically unstable to decomposition into Si and  $SiH<sub>4</sub>$ . Regarding  $SiH<sub>4</sub>$ , eight structures are compared in Fig.  $2(d)$ : the  $I4_1/a$  [\[13\]](#page-5-0), the



FIG. 3. Structures of the high-pressure phases in SiH,  $SiH<sub>2</sub>$ , and SiH3. Solid lines indicate the conventional unit cells. See the Supple-mental Material (Ref. [\[37\]](#page-6-0)) for the structural parameters.

*Pbcn* [\[15\]](#page-5-0), the *C*2/*m*-4 [\[17\]](#page-5-0), the *C*2/*c* [\[13\]](#page-5-0), the *P*21/*c* [\[16\]](#page-5-0), the  $\overline{P_3}$  [\[17\]](#page-5-0), the  $\overline{P_1}$ , and the  $C2/m-6$  [\[16\]](#page-5-0) structures, where the  $C2/m$  structure with 4 (6) formula units per primitive cell is denoted by *C*2/*m*-4 (*C*2/*m*-6) for the purpose of clarification. In Ref. [\[17\]](#page-5-0), the *Pbcn* structure was shown to be stable at a very small pressure range from 169 to 175 GPa; in the present calculations, the *Pbcn* structure does not appear as a stable phase. In fact, it is rather difficult to finally determine whether the *Pbcn* structure is possible because the enthalpy of the *Pbcn* structure is quite close to those of the *I*41/*a* and the  $P\overline{3}$  structures around 170 GPa. In Ref. [\[16\]](#page-5-0), the  $C2/m-6$ structure is predicted to be stable above ∼600 GPa. Yet, there are two factors which were not considered in the analysis in Ref. [\[16\]](#page-5-0): one is the effect of the ZPE and the other is the *P*3 structure [\[17\]](#page-5-0) as a stable phase. By taking account of these two factors, the *C*2/*m*-6 structure is found to be possible even below 600 GPa; to be precise, the  $C2/m-6$  SiH<sub>4</sub> is stable above about 550 GPa and its slight modification (the  $P\overline{1}$  SiH<sub>4</sub>) is stable from 503 to 550 GPa. The*C*2/*m*-6 SiH4 has imaginaryfrequency phonons below  $\sim$ 550 GPa [\[42\]](#page-6-0), and the analysis of the imaginary-frequency modes leads to finding the  $P\overline{1}$  SiH<sub>4</sub>, which has 12 formula units per primitive cell. Actually, the *P*1 structure is quite close to the *C*2/*m*-6 structure, so that the difference of their enthalpies (without the ZPE) is of the order of 0.1 meV per atom around 500 GPa.

The structures of the high-pressure phases in  $SiH$ ,  $SiH<sub>2</sub>$ , and SiH3 are illustrated in Fig. 3. The *Fm*3*m* SiH takes the NaCl-type structure. Since pure Si has the fcc structure at high pressures  $[38]$ , the  $Fm\overline{3}m$  SiH can be seen as the fcc Si with H atoms absorbed in the octahedral sites. The *I*4/*mmm* SiH2 also takes a simple structure, which contains only 2 formula units per primitive cell. On the other hand, the  $I4/m$  SiH<sub>3</sub> has a large primitive cell with 8 formula units as mentioned earlier.

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FIG. 4. Number of Si atoms within the distance *r* from an H atom,  $N_{\rm Si}(r|H)$ , which is averaged all over H atoms (see text). The broadening parameter of  $N_{\rm Si}(r|H)$  is set at 0.02 Å. The data for  $Si<sub>5</sub>H<sub>18</sub>$  (400 GPa,  $\overline{P3}$ ) is calculated simply by ignoring the relatively isolated H pairs in the  $P\overline{3}$  SiH<sub>4</sub> [\[17\]](#page-5-0).

In SiH4 at lower pressures, each H atom forms a bridging bond between two Si atoms, and each pair of neighboring Si atoms has the two H-bridging bonds [\[13,14\]](#page-5-0). This bonding rule is clearly observed in the  $I4_1/a$  SiH<sub>4</sub> and the *Pbcn* SiH<sub>4</sub>, though the *Pbcn* structure consists of two-dimensional SiH4 layers and looks very different from the *I*41/*a* structure. The bonding rule is changed at higher pressures above ∼165 GPa (namely, in the  $\overline{P3}$  SiH<sub>4</sub>) such that an H atom begins to form a bridge between more than two Si atoms. The trend can be found in  $N_{\text{Si}}(r|H)$  in Fig. 4, where  $N_{\text{Si}}(r|H)$  is the number of Si atoms within the distance *r* from an H atom being averaged all over H atoms. In other words, by using the radial distribution function between the  $\alpha$ th and the  $\beta$ th elements  $g_{\alpha\beta}(r)$ ,  $N_{\alpha}(r|\beta)$  is defined by

$$
N_{\alpha}(r|\beta) = \int_0^r dr' 4\pi r'^2 n_{\alpha} g_{\alpha\beta}(r'),
$$

where  $n_{\alpha}$  is the density of the  $\alpha$ th element. The value of  $N_{\text{Si}}(r|H)$  at the first plateau indicates how many Si atoms surround a single H atom on average. For instance, the first plateau appears at  $N_{Si}(r|H) = 2.0$  in the  $I4_1/a$  SiH<sub>4</sub> (134 GPa) and the *Pbcn* SiH<sub>4</sub> (165 GPa), where every H atom forms a bridge between two Si atoms. As pressure is raised in  $SiH<sub>4</sub>$ , the value of  $N<sub>Si</sub>(r|H)$  at the first plateau is increased. Indeed, the first plateau can be seen at  $N_{Si}(r|H) = 3.0$  in the  $P_3$  SiH<sub>4</sub> (400 GPa) and at  $N_{Si}(r|H) = 3.167$  in the  $C2/m-6$  $SiH<sub>4</sub>$  (600 GPa). The  $\overline{P3}$  SiH<sub>4</sub> contains an H pair in the primitive cell [\[17\]](#page-5-0), and the H pair is in fact irrelevant to the H-bridging bonds between Si atoms. Therefore,  $N_{\text{Si}}(r|H)$ is recalculated by ignoring the H pair and shown as  $Si<sub>5</sub>H<sub>18</sub>$  $(400 \text{ GPa}, P\overline{3})$  in Fig. 4. This treatment increases the value of  $N_{\text{Si}}(r|H)$  at the first plateau from 3.0 to 3.333.

Yet, more significant change in  $N_{\text{Si}}(r|H)$  is observed when the composition is changed. In the  $I4/m$  SiH<sub>3</sub> (450 GPa),  $N_{\text{Si}}(r|H)$  is 4.0 at the first plateau. In the *I4/mmm* SiH<sub>2</sub> (400 GPa),  $N_{\text{Si}}(r|H)$  is likewise 4.0 at the first plateau, but the second plateau with  $N_{Si}(r|H) = 4.5$  appears at a relatively small *r* (∼1.8 Å). Then, in the *F m3m* SiH (500 GPa),  $N_{Si}(r|H)$ 



FIG. 5. DOS and PDOS in SiH, SiH<sub>2</sub>, and SiH<sub>3</sub>, where those of the fcc Si [\[38\]](#page-6-0), the  $P\overline{3}$  SiH<sub>4</sub> [\[17\]](#page-5-0), and the  $C2/m-6$  SiH<sub>4</sub> [\[16\]](#page-5-0) are also shown for comparison. The PDOS of Si (H) is defined as the sum of the contributions from all the Si (H) spheres divided by the number of all the valence electrons.

is 6.0 at the first plateau because H atoms are located on the octahedral sides of the fcc Si. These findings mean that the atoms are arranged more evenly in SiH,  $SiH<sub>2</sub>$ , and  $SiH<sub>3</sub>$ than in SiH4. Apparently, these structural properties in the subhydrides work to lower the ZPE and are crucial for sta-bilizing them (see Fig. [1\)](#page-1-0). Moreover, in such phases where the atoms are distributed evenly, the electronic structures tend to be delocalized and highly metallic. Hence the structural properties observed in  $N_{Si}(r|H)$  also imply that high- $T_c$  superconductivity is expected in the subhydrides.

### **IV. ELECTRONIC STRUCTURES AND SUPERCONDUCTING PROPERTIES**

The density of states (DOS) and the partial DOS (PDOS) of the SiH,  $SiH<sub>2</sub>$ , and  $SiH<sub>3</sub>$  phases are presented in Fig. 5, where those of Si and SiH4 are also shown for comparison. In the *P*3 SiH4, the DOS shows a clear decrease around the Fermi energy [Fig.  $5(e)$ ]. The decrease of the DOS in SiH<sub>4</sub> persists up to 503 GPa, where the  $\overline{P3}$  SiH<sub>4</sub> is transformed into the *P*<sup>1</sup> SiH<sub>4</sub>, whose DOS and PDOS are almost the same as those of the  $C2/m-6$  SiH<sub>4</sub> [Fig. 5(f)]. The PDOS in the  $P\overline{3}$  SiH<sub>4</sub> shows that the electronic states just below the Fermi energy largely come from H orbitals, while those just above the Fermi energy from Si orbitals. Thus, in SiH4, the charge is transferred to some extent from Si atoms to H atoms. The charge



FIG. 6. Phonon DOS (left panels) and Eliashberg function  $\alpha^2 F$ multiplied by  $2/\omega$  (right panels) in SiH, SiH<sub>2</sub>, and SiH<sub>3</sub>. See Ref. [\[43\]](#page-6-0) for the details of the computational conditions for the *I*4/*m* SiH3 at 450 GPa.

transfer was previously pointed out for the  $I4_1/a$  SiH<sub>4</sub> by Pickard and Needs [\[13\]](#page-5-0), and is as well seen in the *Pbcn* SiH4. Curiously, while the charge transfer is weakened at higher pressures in the *C*2/*m*-6 phase, it is likewise weakened at lower hydrogen concentrations in the  $I4/m$  SiH<sub>3</sub> [Fig.  $5(d)$ ] and the  $I4/mmm$  SiH<sub>2</sub> [Fig.  $5(c)$ ]. Accordingly, the electronic states are delocalized in these subhydrides as conjectured from the structural properties observed in  $N_{\rm Si}(r|H)$ . Indeed, the DOS curves are relatively close to that of the nearly free electron model in the  $I4/mmm$  SiH<sub>2</sub> and the  $I4/m$  SiH<sub>3</sub>. The  $Fm\overline{3}m$  SiH is also a good metal, but the DOS is similar to that of the fcc Si around the Fermi energy. In SiH, because of the low hydrogen concentration, the PDOS of H atoms is a little small at the Fermi energy, and the property implies that electron-phonon coupling is not so large in this system.

Figure 6 presents the phonon DOS and the Eliashberg function  $\alpha^2 F$  multiplied by  $2/\omega$ , which is the integrand of electron-phonon coupling parameter  $\lambda$  ( $\lambda = \int d\omega 2\alpha^2 F/\omega$ ). In the *Fm*3*m* SiH, the phonon modes consisting mostly of H motions give rise to very large peaks in  $2\alpha^2 F/\omega$  just below 2000 cm−1. However, the electron-phonon coupling does not seem to be large as a whole. On the other hand, in the *I*4/*mmm* SiH<sub>2</sub> and the *I*4/*m* SiH<sub>3</sub>,  $2\alpha^2 F/\omega$  is appreciable over a wide range of pressures. In these two phases, the role of the low-frequency peaks below about 700 cm<sup>-1</sup> is somewhat important in increasing electron-phonon coupling. The contributions of such low-frequency peaks to electron-phonon coupling are sometimes affected by anharmonicity. For example, anharmonicity is found to drastically suppress the electron-phonon coupling in PdH at one atmosphere [\[44\]](#page-6-0) and

TABLE I. Superconducting properties of Si, SiH, SiH<sub>2</sub>, SiH<sub>3</sub>, and  $SiH_4$ : electron-phonon coupling parameter  $\lambda$ , logarithmic average phonon frequency  $\omega_{\text{log}}$ , and superconducting  $T_c$  by solving the isotropic Eliashberg equation [\[48\]](#page-6-0) with  $\mu^* = 0.13$ . See Ref. [\[43\]](#page-6-0) for the details of the computational conditions for the *I*4/*m* SiH3 at 450 GPa and the  $P\overline{3}$  SiH<sub>4</sub> at 400 GPa.

Phase		$P$ (GPa)	λ	$\omega_{\text{log}}$ (K)	$T_c$ (K)
Si	fcc	400	0.137	991	
<b>SiH</b>	Fm3m	500	0.484	1602	14
SiH <sub>2</sub>	I4/mmm	400	0.885	1461	83
SiH <sub>3</sub>	I4/m	450	0.934	1413	88
SiH <sub>4</sub>	$P\overline{3}$	400	0.616	1291	28
SiH <sub>4</sub>	$C2/m-6$	600	1.125	1436	127

PtH at 100 GPa [\[45\]](#page-6-0). It should be noted, however, that the PdH and PtH phases have imaginary-frequency phonons in the harmonic approximation and anharmonicity is essential to their dynamical stability. When a system is already stable in the harmonic approximation, it seems that anharmonic effects on  $T_c$  are moderate. This can be seen, for instance, in the  $Im3m$  SH<sub>3</sub> and the  $Fm3m$  LaH<sub>10</sub>, where theoretical estimates of  $T_c$  in the harmonic approximation  $[4,8]$  are in good agreement with experimental observations [\[3,5–7\]](#page-5-0). As an exceptional case, one might think of the  $Pm\overline{3}n$  AlH<sub>3</sub>, where anharmonicity is predicted to decrease  $\lambda$  considerably [\[46\]](#page-6-0). Yet, in the  $Pm\overline{3}n$  AlH<sub>3</sub>, the electron-phonon coupling comes only from some specific phonon modes around the *X* points with low frequencies [\[47\]](#page-6-0). Therefore, the electron-phonon coupling strongly depends on how the specific phonon modes react to anharmonicity in the  $Pm\overline{3}m$  AlH<sub>3</sub>. In contrast, in the  $SiH<sub>2</sub>$  and  $SiH<sub>3</sub>$  phases, various phonons (including highfrequency phonons) contribute to  $\lambda$ ; that is to say, anharmonic effects on the electron-phonon coupling are averaged all over the Brillouin zone and all over the branches. Thus, by considering the dynamical stability in the harmonic approximation and the significant contributions to  $\lambda$  from various phonon modes, the superconducting properties are expected to be less sensitive to anharmonicity in the silicon subhydrides.

For the SiH,  $SiH_2$ , and  $SiH_3$  phases,  $T_c$  is estimated by directly solving the isotropic Eliashberg equation. The values of  $T_c$  are presented in Table I together with  $\lambda$  and logarithmic average phonon frequency  $\omega_{\text{log}}$ , where the data of Si and SiH<sub>4</sub> are also shown for comparison. Note that the fcc Si does not show superconductivity at all because  $\lambda$  is very small  $(\lambda = 0.137$  at 400 GPa). This clearly shows that the electronphonon coupling in the hydrides owes a lot to the existence of H atoms, whose 1*s* states strongly interact with proton motions. The estimated  $T_c$  is still small in the  $Fm\overline{3}m$  SiH (14 K at 500 GPa). Yet, the values of  $T_c$  in the  $I4/mmm$  SiH<sub>2</sub> and the  $I4/m$  SiH<sub>3</sub> are high, reaching 83 K at 400 GPa and 88 K at 450 GPa, respectively. These  $T_c$  values are higher than those in  $SiH_4$  at corresponding pressures:  $T_c$  in the P3  $SiH_4$  is 28 K at 400 GPa or 35 K at 300 GPa according to Ref. [\[17\]](#page-5-0). When pressure is raised so that SiH4 is in the *C*2/*m*-6 phase,  $T_c$  is increased to 127 K at 600 GPa  $[16]$ . Such a high  $T_c$  is similarly expected above 503 GPa since the structure of the *P*1 SiH4 is very close to that of the *C*2/*m*-6 SiH4. However, <span id="page-5-0"></span>KAZUTAKA ABE PHYSICAL REVIEW B **103**, 134118 (2021)

the current capability of diamond anvil cells being considered, it seems to be still difficult to experimentally attain such high pressures though it is not impossible. Thus, as a possible way for achieving high-*Tc* superconductivity in silicon hydride, it might be worth trying to compress silicon and hydrogen in silicon excess conditions.

## **V. SUMMARY**

Dense silicon hydrides have been investigated by using *ab initio* methods, with chief attention being paid to SiH,  $SiH<sub>2</sub>$ , and  $SiH<sub>3</sub>$ . The prominent stability of  $SiH<sub>4</sub>$  prohibits the other compositions to be thermodynamically stable at lower pressures. Yet the *Fm*3*m* SiH, the *I*4/*mmm* SiH2, and the  $I4/m$  SiH<sub>3</sub> become possible above 486, 386, and 430 GPa, respectively.

The phases of silicon subhydrides predicted here are good metals. While the DOS in SiH4 shows a clear decrease around the Fermi energy up to 503 GPa, such a decrease of DOS is much smaller in the subhydrides. The estimated  $T_c$  values in  $SiH<sub>2</sub>$  and  $SiH<sub>3</sub>$  reach 83 K at 400 GPa and 88 K at 450 GPa, respectively, which are higher than that in  $SiH<sub>4</sub>$  (28 K at 400 GPa). These results in silicon hydrides are similar to those in aluminium and germanium hydrides, where subhydrides AlH  $[24]$  and GeH<sub>3</sub>  $[25]$  are predicted to have higher  $T_c$ than  $AH<sub>3</sub>$  and  $GeH<sub>4</sub>$ , respectively. All the findings suggest that some other subhydrides are also capable of bringing about high- $T_c$  superconductivity. For example, it might be interesting to investigate subhydrides of gallium, which is close to silicon, germanium, and aluminium on the Periodic Table.

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the  $C2/m-6$  SiH<sub>4</sub> likewise has a large primitive cell, the H pseudopotential with  $r_c^H = 0.423$  Å is used for the system because the target pressure is quite high (600 GPa).] The accuracy of this computational condition is checked by comparing  $T_c$  in the  $I4/m$  SiH<sub>3</sub> at 450 GPa between the smaller- $r_c^H$  (namely, higher- $E_c$ ) and the larger- $r_c^H$  (namely, lower- $E_c$ ) calculations, where coarse *k*-point and *q*-point grids are used:  $n_k \sim (45 \text{ Å})^3/v_{cell}$ and  $n_q \sim (10 \text{ Å})^3/v_{cell}$ . The resulting  $T_c$  calculated with the smaller  $r_c^H$  is 96 K and that with the larger  $r_c^H$  is 97 K. The small difference in  $T_c$  shows that the error brought about by the choice of  $r_c^H = 0.529$  Å and  $E_c = 626$  eV is almost negligible even at such high pressures.

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