Quantum theory of spin-torque driven magnetization switching

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Magnetization dynamics driven by the current-induced spin torque is conventionally determined by the classical Landau-Lifshitz-Gilbert-Slonczewski equation in which the spin (magnetization) fluctuation at finite temperature is modeled by a white-noise random field. We propose a quantum approach for current driven magnetization switching that explicitly includes the spin fluctuation by the quantum statistics of magnon excitations. We find that the spin fluctuation substantially reduces the critical spin torque at high temperatures. Since the spin fluctuations are fundamentally stronger in lower-dimensional systems, this reduction is stronger in two-dimensional (2D) than in three-dimensional magnets. The result implies that the 2D magnets may have an advantage in terms of electrically manipulating magnetization states for spintronic applications.

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I. INTRODUCTION

Efficient magnetization switching by electric means is highly desirable for modern magnetic storage devices such as magnetic random access memory [1–8]. The spin torque, generated by either the spin angular momentum transfer [9,10] or spin-orbit coupling in a magnetic layered structure [11–15], is a leading method for electrically switching the direction of magnetization in various magnetic films and magnetic multilayers. The essential physics of the spin-torque driven switching in conventional three-dimensional (3D) magnetic films is well understood: when the spin torque exceeds a critical value that overcomes the damping torque, the system becomes a negative damping system, and the original magnetization state at a local energy minimum is no longer stable [10,16–19]. More quantitatively, based on the Landau-Lifshitz-Gilbert-Slonczewski (LLGS) equation in the macrospin approximation in which all magnetic moments are aligned with each other and the spin fluctuation is ignored, the critical spin torque (CST) $a_J^c = \alpha H_K$ for the magnet with a simple uniaxial anisotropy, and $a_J^c = \alpha(H_K + 2\pi M_s)$ with both an in-plane uniaxial anisotropy and an out-of-plane demagnetizing field, where α is the damping parameter, H_K is the uniaxial anisotropy field, and M_s is the saturation magnetization [17,20]. To model the spin fluctuations at finite temperature, a classical frequency-independent (white-noise) stochastic field is often used to augment the LLGS equation [21–23]. While the above approach provides a simple and efficient method to model the current driven dynamics of magnetization in conventional 3D magnets, such a classical treatment is not expected to properly account for the spin fluctuations in low-dimensional ($d \leqslant 2$) magnets, where the long-wavelength (low-energy) excitations (magnons) play an essential role in the magnetic instability such that the classical model fails.

In this paper, motivated by the recent experimental discovery of new classes of two-dimensional van der Waals magnets [24–29], we develop a quantum approach that explicitly takes into account the spin fluctuation and allows us to establish an analytical self-consistent equation for the nonequilibrium magnetization state in the presence of spin torque, from which the CST for magnetization switching can be deduced. We find that the CST for two-dimensional (2D) magnets at finite temperatures could be substantially lower than 3D counterparts.

This paper is organized as follows. In Sec. II, we describe a simple heuristic approach to solve the anisotropic Heisenberg model at the equilibrium. A more rigorous proof of this approach is shown in Appendix A. In Sec. III, we extend the calculation to the nonequilibrium condition where a spin torque is injected in the magnet, with the detailed formulation given in Appendixes B and C. We present the numerical solutions of the critical spin torque for the magnetization switching for various parameters and temperatures in Sec. IV. We discuss and summarize our results in Sec. V.

II. MODEL AND ITS EQUILIBRIUM SOLUTIONS

We consider the standard Heisenberg Hamiltonian with an anisotropic exchange interaction,

$$\hat{\mathcal{H}} = -J\sum_{\langle i,j\rangle}\hat{\mathbf{S}}_i\cdot\hat{\mathbf{S}}_j - A\sum_{\langle i,j\rangle}\hat{S}_i^z\hat{S}_j^z - H_{\rm ex}\sum_i \hat{S}_i^z, \qquad (1)$$

where $\hat{\mathbf{S}}_i$ is the spin operator at lattice site $\mathbf{R}_i$, J and A are the nearest-neighbor isotropic and anisotropic exchange integrals

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($J,\ A > 0$), respectively, and $H_{\rm ex}$ is the external magnetic field along the z axis. We will use the above Hamiltonian for both 2D and 3D magnets. Nonzero A is essential for 2D magnets since the isotropic exchange interaction alone is unable to form long-range magnetic ordering [30]. Since the above Hamiltonian has no exact solutions even in the equilibrium state, one has to either rely on numerical simulations or make certain approximations for analytical solutions.

We shall first present a heuristic approach to solve the above Hamiltonian, and the justification for this approach will be given in Appendix A. Consider the commutator relation $[\hat{S}_i^+, \hat{S}_j^-] = 2\delta_{ij}\hat{S}_i^z$, where $\hat{S}_i^\pm = \hat{S}_i^x \pm \hat{S}_i^y$ are the spin-raising and -lowering operators, respectively, and we set $\hbar = 1$. If $\hat{S}_i^z$ on the right-hand side is replaced by its thermal average, $\langle \hat{S}_i^z\rangle = M(T)$, where $M(T)$ is defined as the magnetization, we then introduce the magnon creation and annihilation operators $\hat{a}_i^\dagger \equiv \hat{S}_i^-/\sqrt{2M}$ and $\hat{a}_i \equiv \hat{S}_i^+/\sqrt{2M}$, respectively, which satisfy the standard boson commutator relation, i.e., $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$. In Appendix A, we show that this treatment of the longitudinal component of spin operator is equivalent to the random phase approximation (RPA); that is, the correlation between the longitudinal and transverse spin fluctuations is neglected, within the framework of the Green's function approach. The excitation energy of the single magnon $\epsilon_{\mathbf{q}}$ is then determined by $[\hat{a}_{\mathbf{q}}, \hat{\mathcal{H}}] = \epsilon_{\mathbf{q}}\hat{a}_{\mathbf{q}}$, where $\hat{a}_{\mathbf{q}} = (1/\sqrt{N})\sum_j \hat{a}_j \exp(-i\mathbf{q}\cdot\mathbf{R}_j)$ is the Fourier transformation of $\hat{a}_i$, with N being the number of lattice sites. By using Eq. (1) and the above definition of the magnon operators, we have

$$\epsilon_{\mathbf{q}} = 2N_0 M(T)[A + J(1-\gamma_{\mathbf{q}})] + H_{\rm ex}, \tag{2}$$

where $\gamma_{\mathbf{q}} = (1/N_0)\sum_\delta e^{i\mathbf{q}\cdot\mathbf{R}_\delta}$ and the summation is over the nearest-neighbor sites, with $\mathbf{R}_\delta$ being the position vector with respect to the original site. The magnetization $M(T)$ appearing in the above magnon energy represents the "softening" of the magnon dispersion relation at finite temperatures.

We point out the difference between the magnon operator $\hat{a}$ ($\hat{a}^\dagger$) defined here and the boson operator $\hat{b}$ ($\hat{b}^\dagger$) defined in the conventional Holstein-Primakoff (HP) transformation, $\hat{S}^+ = (\sqrt{2S - \hat{b}^\dagger\hat{b}})\hat{b}$ and $\hat{S}^- = b^\dagger(\sqrt{2S - \hat{b}^\dagger\hat{b}})$. If one is to expand the square root in Taylor series of the boson operators, the second-order terms will also produce an energy term similar to Eq. (2) [31]. However, there are two differences. The HP transform is useful for low temperature where $(\hat{b}^\dagger\hat{b})/S \ll 1$ since one can take into account only several leading terms. For high temperature, the HP transform requires us to include all high-order nonlinear interactions between magnons, which are analytically unfeasible. In contrast, our definition of magnon operators allows us to deal with the case of any finite temperatures, and all these nonlinear interactions are absorbed by the magnetization that needs to be self-consistently determined. The second difference is that, in the HP transformation, the magnetization is directly related to the number of the magnons since $\hat{S}^z = S - \hat{b}^\dagger\hat{b}$, or, equivalently, $M = S - \langle\hat{b}^\dagger\hat{b}\rangle$. In our case, the relation between the number of magnons and the magnetization becomes more complicated. To see this, we use the spin operator identity $\hat{S}_i^z = S(S+1) - (\hat{S}_i^z)^2 - \hat{S}_i^-\hat{S}_i^+$ [32], and by substituting the spin-raising and -lowering operators with $\hat{a}$ and $\hat{a}^\dagger$, respectively, and taking the thermal average, we have

$$M = S(S+1) - \langle(\hat{S}_i^z)^2\rangle - \frac{2M}{N}\sum_{\mathbf{q}}\langle\hat{a}_{\mathbf{q}}^\dagger\hat{a}_{\mathbf{q}}\rangle. \tag{3}$$

For the spin-1/2 case, $S = 1/2$, $(\hat{S}_i^z)^2 = 1/4$, and the above relation becomes

$$M = \frac{1}{2} - \frac{2M}{N}\sum_{\mathbf{q}}\langle\hat{a}_{\mathbf{q}}^\dagger\hat{a}_{\mathbf{q}}\rangle. \tag{4}$$

For any other spin numbers, one needs an additional decoupling scheme to express $\langle(\hat{S}_i^z)^2\rangle$ in terms of $\langle\hat{S}_i^z\rangle$ and $\langle\hat{a}_{\mathbf{q}}^\dagger\hat{a}_{\mathbf{q}}\rangle$; we will discuss the case of an arbitrary spin number toward the end of the paper. In the following, we shall focus on only $S = 1/2$ such that a simple relation, Eq. (4), holds.

If one applies Eq. (4) to the thermal equilibrium, the number of magnons is given by the Bose-Einstein distribution, and one can immediately rewrite Eq. (4) as a self-consistent equation for the magnetization,

$$M = \frac{1}{2} - \frac{2M}{(2\pi)^d}\int_{BZ} d^d\mathbf{q}\left(\frac{1}{e^{\beta\epsilon_{\mathbf{q}}} - 1}\right), \tag{5}$$

where d is the spatial dimension of the system, $\beta = 1/k_BT$, the summation over $\mathbf{q}$ was replaced by an integral over the first Brillouin zone, and we set the volume of the unit cell to be unity. Note that the magnetization $M(T)$ appears in three places since the magnon energy $\epsilon_{\mathbf{q}}$ [see Eq. (2)] is also proportional to $M(T)$. Equation (5) was also previously obtained using the Green's function formalism within the RPA scheme [33].

III. MAGNETIZATION INSTABILITY WITH SPIN TORQUES

Now we consider a spin-polarized current injected with the spin polarization along the z axis, which is done to switch the magnetization through the induced spin torque. When a spin torque is present, the system is no longer in equilibrium, and the Bose-Einstein distribution for the number of magnons in Eq. (4) is no longer valid. In general, there are two types of spin torques. The fieldlike torque is equivalent to an effective magnetic field along the z axis, and one can thus continue to use Eq. (5) as long as one includes the fieldlike torque as an external field. The dampinglike torque for magnetization switching describes the antidamping process that competes with the magnetic dissipation, and one is unable to include it as an effective magnetic field. Below we use the Green's function approach to describe the nonequilibrium properties of the magnetization driven by the dampinglike torque. In the presence of both the dissipation and dampinglike torque, the retarded Green's function (propagator) for the magnon can be written as (see the derivation in Appendix B)

$$G^r(\mathbf{q}, \omega) = \frac{1}{\omega - \epsilon_{\mathbf{q}} - \Sigma^r(\omega)}, \tag{6}$$

where Σ^r is the retarded self-energy. In the Gilbert representation, $\Sigma^r(\omega) = -i(\alpha\omega - a_J)$, where a phenomenological damping constant α accounts for the dissipation of magnons and a_J is the strength of the dampinglike torque. Since the system is not in equilibrium, there is no direct connection between the imaginary part of the above retarded Green's function and the expectation of the magnon number.

In nonequilibrium, the spin fluctuation is described by the lesser Green's function, defined as $G^<(\mathbf{q},t) \equiv -i\langle \hat{a}^\dagger_{\mathbf{q}} \hat{a}_{\mathbf{q}}(t)\rangle$ such that its Fourier transform determines the nonequilibrium magnon numbers, $\langle \hat{a}^\dagger_{\mathbf{q}} \hat{a}_{\mathbf{q}}\rangle = i/(2\pi)\int d\omega\, G^<(\mathbf{q},\omega)$. In the steady-state condition, the lesser Green's function and its corresponding lesser self-energy $\Sigma^<(\omega)$ satisfy the general relation [34,35]

$$G^<(\mathbf{q},\omega) = G^r(\mathbf{q},\omega)\Sigma^<(\omega)G^a(\mathbf{q},\omega), \tag{7}$$

where $G^a(\mathbf{q},\omega) = G^r(\mathbf{q},\omega)^*$ is the advanced Green's function. Physically, the lesser self-energy represents the correlation of the magnetic fluctuation field h_f associated with the magnetic dissipation (see the details in Appendix C). In equilibrium, the fluctuation-dissipation theorem demands $\Sigma^<(\omega) = 2i\mathrm{Im}[\Sigma^r(\omega)]n(\omega) = -2i\alpha\omega n(\omega)$ [i.e., $\langle h_f(\mathbf{x},\omega)h_f^*(\mathbf{x}',\omega')\rangle = 2\alpha\omega n(\omega)\delta(\mathbf{x}-\mathbf{x}')\delta(\omega-\omega')$], where $n(\omega) = 1/[e^{\beta\omega}-1]$ is the Bose-Einstein distribute function. In the classical limit where $k_BT \gg \omega$, $\langle h_f(\mathbf{x},\omega)h_f^*(\mathbf{x}',\omega')\rangle = 2\alpha k_BT\delta(\mathbf{x}-\mathbf{x}')\delta(\omega-\omega')$ restores the well-known classical white-noise fluctuation field, which has been extensively used in the stochastic Landau-Lifshitz-Gilbert equation to capture the spin fluctuations [36]. As shown below, it is, however, essential to include the frequency dependence of fluctuation field in order to derive the proper magnon occupation number.

In the presence of the spin torque, we assume that the fluctuation field does not change; that is, the spin torque is treated as a deterministic driving force which leads to an effective damping but does not affect the randomness of the fluctuation field. This assumption was previously used to study the thermal effect of the spin torque where the temperature of the magnetic system is kept the same with and without the spin torque [21,37]. By using the above self-energy along with Eq. (7), we have

$$\langle \hat{a}^\dagger_{\mathbf{q}} \hat{a}_{\mathbf{q}}\rangle = \frac{1}{\pi}\int d\omega \frac{\alpha\omega n(\omega)}{(\omega-\epsilon_{\mathbf{q}})^2 + (\alpha\omega - a_J)^2} \approx \frac{Z_{\mathbf{q}}}{e^{\beta\epsilon_{\mathbf{q}}}-1}, \tag{8}$$

where we have used the limit of small α in the second equality and we have defined $Z_{\mathbf{q}} = (1-a_J/\alpha\epsilon_{\mathbf{q}})^{-1}$. By placing Eq. (8) into Eq. (4), we obtain

$$M = \frac{1}{2} - \frac{2M}{(2\pi)^d}\int_{BZ} d^d\mathbf{q} \frac{Z_{\mathbf{q}}}{e^{\beta\epsilon_q}-1}. \tag{9}$$

Equation (9) is our main result for the magnetization in the presence of the spin torque. By comparing it with Eq. (5), we immediately see that the role of the spin torque is to renormalize the number of magnons by a factor of $Z_{\mathbf{q}}$. For the antidamping process ($a_J > 0$, $Z_{\mathbf{q}} > 1$), the spin torque enhances the spin fluctuations, leading to an increase in the number of magnons, which was observed using Brillouin light scattering [38]. As the spin torque increases beyond a critical value such that Eq. (9) no longer has a solution, consequently, the magnetization switching occurs. Below we analyze the detailed solutions.

IV. CRITICAL SPIN TORQUE AT FINITE TEMPERATURE

At nearly zero temperature, the magnon number is vanishingly small, and the magnetic instability occurs only when $Z_{\mathbf{q}}$ approaches its divergent singularity, i.e., $a_J \to \alpha\epsilon_{\mathbf{q}}$. Since the minimum of $\epsilon_{\mathbf{q}}$ is at $\mathbf{q}=0$, we recover the classical result of the CST $a_J^c \to \alpha(\Delta + H_{\mathrm{ex}})$, where $\Delta = 2N_0M(T)A$ is the effective anisotropy or the magnon gap. As the magnon number increases with increasing temperature, it is expected that the CST will decrease and the instability will occur before the divergence of $Z_{\mathbf{q}}$. At high temperature near the Curie temperature, a small spin torque could destabilize the magnetization ordering.

The most interesting temperature range is the one in which the magnetization $M(T)$ remains substantial; that is, the temperature is high enough that a significant number of magnons are excited, and meanwhile, the temperature is only a fraction of the Curie temperature with the magnetization not far away from the low-temperature value. To obtain quantitative values of the CST, we rewrite Eq. (9) by $m = F(m, a_J)$, with

$$F^{-1}(m,a_J) = 1 + \frac{2}{(2\pi)^d}\int_{BZ} d^d\mathbf{q} \frac{\alpha\epsilon_{\mathbf{q}}}{(e^{\beta\epsilon_{\mathbf{q}}}-1)(\alpha\epsilon_{\mathbf{q}} - a_J)}, \tag{10}$$

where $m \equiv M/S = 2M$ is the normalized magnetization. Note that the magnon energy $\epsilon_{\mathbf{q}}$ depends linearly on m, given by Eq. (2). In Fig. 1(a), we take the 2D magnet with a square lattice as an example and plot the curve $F(m, a_J)$ as a function of m for several typical values of a_J. The solutions of Eq. (9) are given by the intersection points of the curves of function $F(m, a_J)$ and the direct proportion function with a slope of 1 [the dashed straight line in Fig. 1(a)]. For zero spin torque, there are two solutions: one is at $m = 0$, representing the paramagnetic solution, and the other is at the finite value. The finite one is the desired ferromagnetic solution. When the spin torque is turned on, there are two solutions of magnetization as well. The solution with the smaller m evolves from the paramagnetic solution in which the magnetization is small but nonzero since the spin torque induces a net magnetization (similar to the magnetic field induced magnetization). The solution with the larger m is slightly smaller than that without the spin torque, resulting from the enhancement of spin fluctuations by the spin torque. As the spin torque reaches a critical value, there is only one solution; this gives the CST at which the magnetization starts to become unstable against the dramatic spin fluctuations, i.e., the magnetization switching occurs. Beyond the critical value, there is no longer a solution for the positive magnetization since the magnetization has been switched to the opposite ($-z$) direction, which the spin torque helps the magnetization to stabilize (note that, after the magnetization switching, $a_J < 0$ and thus $Z_{\mathbf{q}} < 1$ since the relative direction of the magnetization and the spin polarization of current has changed). Quantitatively, the CST can be determined by the condition

$$m_c = F\big(m_c,\, a_J^c\big), \tag{11}$$

$$\partial_m F\big(m,\, a_J^c\big)|_{m=m_c} = 1, \tag{12}$$

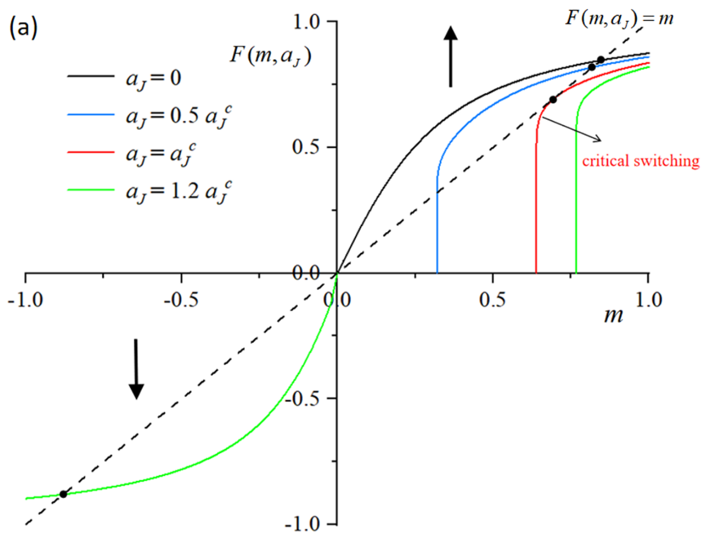

FIG. 1. (a) The graphic solutions of the magnetization as a function of spin torque. Four solid curves of the function $F(m, a_J)$, Eq. (10), are shown for four typical values of the spin torque $a_J = 0, 0.5a_J^c, a_J^c, 1.2a_J^c$, where a_J^c is the critical spin torque (CST). When the spin torque is smaller than a_J^c, the function $F(m, a_J)$ has two intersection points with the dashed straight line, whose slope is 1, representing two solutions of Eq. (9) in which the larger m is the desired ferromagnetic solution. As the spin torque a_J increases to a critical value, $a_J = a_J^c$, there is only one solution (red line). When the spin torque exceeds a_J^c, there is no solution for the positive magnetization, indicating the magnetization has switched to the opposite $(-z)$ direction, where one finds the solution with a negative magnetization (see the green line). (b) The magnetization solution as a function of the spin torque, which is normalized by $\alpha\Delta_0$, where $\Delta_0 = 2N_0SA$ is the magnon gap at zero temperature. The numerical parameters for (a) and (b) are $T = 0.4T_c$, $A = 0.01J$, and $H_{ex} = 0$ for a 2D square lattice with $\gamma_{\mathbf{q}} = (\cos k_xa + \cos k_ya)/2$ in Eq. (2), where a is the lattice constant and T_c is the Curie temperature in the absence of spin torque.

where m_c is the (normalized) magnetization at the CST of a_J^c. In Fig. 1(b), we show the desired ferromagnetic solution as a function of the spin torque. Before the magnetization switching, the magnetization decreases slowly with increasing spin torque. At the CST, the magnetization switches to the opposite direction, with its magnitude slightly larger than that before the switching.

We next compare the CST for the 2D and 3D magnets. The fundamental differences between 2D and 3D come from the effective number of magnons. In the 3D case, the long-wavelength magnon number is finite and scales with temperature even if there is no gap in the magnon spectrum, and the short-wavelength (high-energy) magnons dominate the magnetic instability. In contrast, the 2D magnetization is unstable at any finite temperatures without the magnetic anisotropy due to the divergence of magnon numbers, and the long-wavelength (low-energy) magnons control the magnetic instability. In Fig. 2, we plot the CST as a function of the temperature for 2D and 3D magnets. Figure 2 shows that the CST for the 3D magnets is nearly equal to its classical value when the temperature is not too high relative to the Curie temperature T_c, while the reduction of the CST for the 2D magnets is much more significant even at low temperature. Consequently, the 3D magnetization switching dynamics could be approximately modeled by coherent magnetization (macrospin) rotation as long as the temperature is not close to the Curie temperature, and thus, the micromagnetic model [17,18] is appropriate to simulate the dynamic trajectory of the magnetization switching. Yet the dynamic modeling of the 2D magnetization switching would go beyond the classical micromagnetics. Further study is required for the 2D magnetization dynamics.

V. DISCUSSION AND SUMMARY

Until now, we have used Eq. (9), which is based on the model in the case of $S = 1/2$. For higher spin numbers, $\langle(\hat{S}_i^z)^2\rangle$ is no longer a constant. Instead, an additional step is required

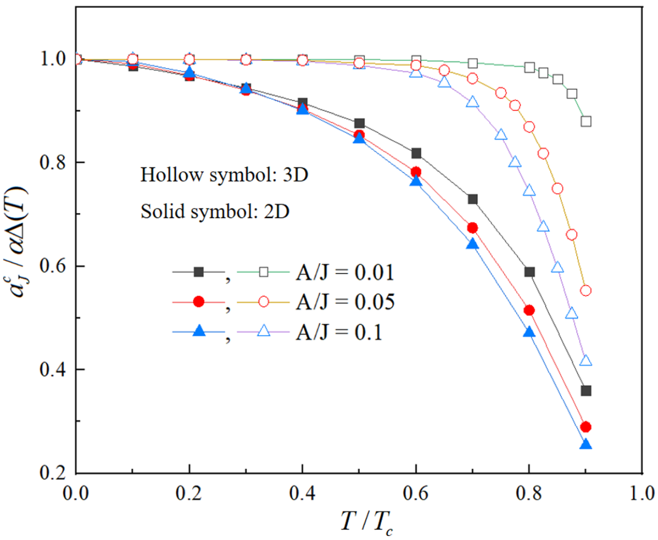

FIG. 2. The temperature dependence of the CST normalized by its classical value, $\alpha\Delta(T)$, where $\Delta(T) = N_0m_c(T)A$ is the effective magnon gap, with m_c being the magnetization (normalized by S) at a_J^c, for the 2D square and 3D cubic lattice magnets with several different anisotropies. Here, we take $H_{ex} = 0$ and $\gamma_{\mathbf{q}} = (\cos k_xa + \cos k_ya)/2$ for the 2D square lattice and $\gamma_{\mathbf{q}} = (\cos k_xa + \cos k_ya + \cos k_za)/3$ for the 3D cubic lattice.

to reduce Eq. (3) to a self-consistent equation for the magnetization. In Ref. [39], Callen developed a decoupling scheme that can express $\langle(\hat{S}_i^z)^2\rangle$ as a function of the magnetization $\langle\hat{S}_i^z\rangle$ and the magnon number $\langle\hat{a}_{\mathbf{q}}^\dagger\hat{a}_{\mathbf{q}}\rangle$, resulting in a more complex expression for self-consistently determining the magnetization $M(T)$ at the thermal equilibrium. In Appendix D, we extend Callen's procedure to the nonequilibrium case and arrive at

$$M = \frac{(S-N_m)(1+N_m)^{2S+1} + (S+1+N_m)N_m^{2S+1}}{(1+N_m)^{2S+1} - N_m^{2S+1}}, \quad (13)$$

where $N_m = (1/N)\sum_{\mathbf{q}} Z_{\mathbf{q}} n(\epsilon_{\mathbf{q}})$ is the magnon density in the presence of spin torque. It is straightforward to verify that Eq. (13) reduces to Eq. (9) when $S = 1/2$.

Finally, we comment on the possible experimental observation of the temperature dependence of the CST. The material parameters such as the magnetic anisotropy and damping constant are highly temperature dependent. The effects of these parameters were already included in the modeling of spin-torque driven switching before taking into account the spin fluctuation. The present study shows a significant reduction of the CST from the spin fluctuation without altering the temperature dependence from other sources. Another possible uncertainty is the measuring time since the longer the measurement time is, the greater the chance is for thermally assisted magnetization switching when the duration of current pulse lasts more than a few nanoseconds, as it has been shown both experimentally and theoretically that the observed CST might be much reduced in the thermal activation regions [21,40–42]. In this thermal picture, the magnetic layer was treated as a macrospin, and the thermal agitation is to overcome the magnetic energy barrier, known as superparamagnetism [43], in contrast to the deterministic magnetization switching dynamics in our present study. A further complication might be the difficulty in determining the quantitative relation between the spin-torque parameter a_J and the applied electric current for particular physical systems and materials.

In summary, we proposed a quantum theory for the current driven magnetization switching, which allowed us to properly address the influence of spin fluctuation on the magnetization switching. Particularly, we showed that the CST for 2D magnets can be considerably reduced at finite temperature owing to the fundamentally strong spin fluctuations compared to their 3D counterparts. Our work indicated that the 2D magnets are highly suitable for energy-efficient magnetization manipulation by electric means, despite the reduced thermal stability for information storage. A realistic device design would balance the two sides of the coin, i.e., the reduced switching current and the enhanced instability of two-dimensional magnets.

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APPENDIX A: EQUIVALENCE TO THE RANDOM PHASE APPROXIMATION WITHIN THE FRAMEWORK OF THE GREEN'S FUNCTION APPROACH

In the main text, we introduced the boson (magnon) operators, $a_i^\dagger \equiv (2M)^{-1/2}\hat{S}_i^-$ ($a_i \equiv (2M)^{-1/2}\hat{S}_i^+$), by taking the averaging over the longitudinal component spin in the commutator, $[\hat{S}_i^+, \hat{S}_j^-] = 2\delta_{ij}\hat{S}_i^z \approx 2\delta_{ij}M$, where $M = \langle\hat{S}_i^z\rangle$ is the magnetization. In this Appendix, we show that this treatment is equivalent to the random phase approximation (RPA) within the framework of the Green's function approach in which the correlation of the longitudinal and transverse fluctuations of spin at different sites is discarded. To see this, we first define the following retarded spin Green's function:

$$\begin{aligned} G_S^r(i,t;j,t') &\equiv \langle\langle S_i^+(t); S_j^-(t')\rangle\rangle \\ &\equiv -i\theta(t-t')\langle[\hat{S}_i^+(t), \hat{S}_j^-(t')]\rangle, \end{aligned} \quad (A1)$$

which is related to the retarded magnon Green's function in the main text through $G_S^r(i,t;j,t') = 2MG^r(i,t;j,t')$, where $G^r(i,t;j,t') \equiv -i\theta(t-t')\langle[\hat{a}_i(t), \hat{a}_j^\dagger(t')]\rangle$ is the retarded Green's function of the magnon. The equation of motion is given by

$$\begin{aligned} i\frac{d}{dt}G_S^r(i,t;j,t') &= \langle[\hat{S}_i^+(t), \hat{S}_i^-(t')]\rangle\delta(t-t')\delta_{ij} + \langle\langle[\hat{S}_i^+(t), \hat{H}(t)]; \hat{S}_j^-(t')\rangle\rangle \\ &= 2\langle\hat{S}_i^z\rangle\delta(t-t')\delta_{ij} + H_{\mathrm{ex}}G_S^r(i,t;j,t') \\ &\quad + 2A\sum_\delta \langle\langle\hat{S}_{i+\delta}^z(t)\hat{S}_i^+(t); \hat{S}_j^-(t')\rangle\rangle \\ &\quad + 2J\sum_\delta \langle\langle[\hat{S}_{i+\delta}^z(t)\hat{S}_i^+(t) - \hat{S}_i^z(t)\hat{S}_{i+\delta}^+(t)]; \hat{S}_j^-(t')\rangle\rangle, \end{aligned} \quad (A2)$$

where the summation is over the nearest neighbor of site i. In order to make the above equation of motion closed, it is necessary to use an approximation to break the higher-order Green's functions. The simplest decoupling scheme, the RPA which neglects the correlation of longitudinal and transverse components of the spin operators at different sites, is used below:

$$\langle\langle\hat{S}_l^z(t)\hat{S}_i^+(t); \hat{S}_j^-(t')\rangle\rangle \approx \langle\hat{S}_l^z\rangle\langle\langle\hat{S}_i^+(t); \hat{S}_j^-(t')\rangle\rangle \ (i \neq l). \quad (A3)$$

Assuming the system is translationally invariant (the magnetization is thus uniform), we can write

$$G_S^r(i,t;j,t') = \frac{1}{N}\sum_{\mathbf{q}}\int\frac{d\omega}{2\pi}G_S^r(\mathbf{q},\omega)e^{i\mathbf{q}\cdot\mathbf{R}_{ij} - i\omega(t-t')}, \quad (A4)$$

and then we find from Eq. (A2) that

$$G_S^r(\mathbf{q},\omega) = \frac{2M}{\omega - \epsilon_{\mathbf{q}} + i0^+}, \quad (A5)$$

where

$$\epsilon_{\mathbf{q}} = 2N_0M[A + J(1-\gamma_{\mathbf{q}})] + H_{\mathrm{ex}} \quad (A6)$$

is the magnon spectrum as in the main text, with N_0 being the nearest-neighbor number and $\gamma_{\mathbf{q}} = (1/N_0)\sum_\delta e^{i\mathbf{q}\cdot\mathbf{R}_{ij}}$. According to the spectrum theorem (i.e., the fluctuation-dissipation theorem), the *equilibrium* correlation function $\langle \hat{S}_i^- \hat{S}_i^+ \rangle$ is related to the imaginary part of the above spin Green's function through

$$\langle \hat{S}_i^- \hat{S}_i^+ \rangle = -\frac{1}{\pi N}\sum_{\mathbf{q}}\int d\omega \frac{\mathrm{Im}G_S^r(\mathbf{q},\omega)}{e^{\beta\omega}-1} = \frac{1}{N}\sum_{\mathbf{q}}\frac{2M}{e^{\beta\epsilon_{\mathbf{q}}}-1}, \tag{A7}$$

with $\beta = 1/k_B T$. For the simplest case of $S = 1/2$, the operator identity $\hat{S}_i^z = 1/2 - \hat{S}_i^- \hat{S}_i^+$ leads to

$$M = 1/2 - \langle \hat{S}_i^- \hat{S}_i^+ \rangle = 1/2 - \int_{BZ}\frac{d^d\mathbf{q}}{(2\pi)^d}\frac{2M}{e^{\beta\epsilon_{\mathbf{q}}}-1}, \tag{A8}$$

where the summation over the momentum was replaced by an integral over the first Brillouin zone. The above equation is exactly Eq. (5) in the main text for the equilibrium magnetization. When the spin torque is present, the same procedure as in the main text can be implemented with the replacement of $G_S^<(\mathbf{q},\omega) = 2MG^r(\mathbf{q},\omega)$ and $\Sigma_S^r(\mathbf{q},\omega) = \Sigma^r(\mathbf{q},\omega)/(2M)$, where $\Sigma_S^r(\mathbf{q},\omega)$ and $\Sigma^r(\mathbf{q},\omega)$ are the retarded self-energies of the spin and magnon, respectively.

Thus, we have proven that the approach using an effective magnon operator in the main text is equivalent to the RPA approximation in the Green's function formalism.

APPENDIX B: THE RETARDED MAGNON GREEN'S FUNCTION IN THE PRESENCE OF MAGNETIC DISSIPATION AND SPIN TORQUE

In this section, we show the retarded magnon Green's function, Eq. (6) in the main text, can be derived by the classical Landau-Lifshitz-Gilbert (LLG) equation. The LLG equation with the spin torque reads

$$\dot{\mathbf{m}} = -\mathbf{m}\times(\mathbf{h}+\mathbf{h}_f) + \alpha\mathbf{m}\times\dot{\mathbf{m}} + a_J\mathbf{m}\times(\mathbf{m}\times\hat{\mathbf{z}}), \tag{B1}$$

where we omitted the space and time coordinates $(\mathbf{x}, t)$ for notation simplicity; $\mathbf{m}$ is the unit vector of magnetization; $\mathbf{h} = -\delta U/\delta\mathbf{m}$ represents the effective field, with $U = 1/2\int d\mathbf{r}[\mathcal{J}(\nabla\mathbf{m})^2 - \mathcal{A}m_z^2]$ being the magnetic energy, where $\mathcal{J}$ and $\mathcal{A}$ are the exchange stiffness and out-of-plane anisotropy constant, respectively; $\mathbf{h}_f$ is the random field; α is the Gilbert damping constant; and a_J is the parameter of the (dampinglike) spin torque with its spin polarization along the z direction.

The magnetization without the random field is in the positive z direction for the ground state. With the random field, the magnetization deviates from the z direction. We may define the complex field (magnon wave function) $\psi = \delta m_x + i\delta m_y$, where δm_x and δm_y are the small transverse deviations caused by $\mathbf{h}_f$. Up to the first order in $\mathbf{h}_f$, Eq. (B1) becomes

$$\{i(1+i\alpha)\partial_t + \mathcal{J}\nabla^2 - \mathcal{A} - ia_J\}\psi(\mathbf{x},t) = h_f(\mathbf{x},t), \tag{B2}$$

where $h_f(\mathbf{x},t) = h_f^x(\mathbf{x},t) + ih_f^y(\mathbf{x},t)$. The retarded Green's function of the magnon is then readily identified as

$$[i(1+i\alpha)\partial_t + \mathcal{J}\nabla^2 - \mathcal{A} - ia_J]G^r(\mathbf{x},t;\mathbf{x}',t') = \delta(\mathbf{x}-\mathbf{x}')\delta(t-t'). \tag{B3}$$

In the momentum and frequency space,

$$G^r(\mathbf{q},\omega) = \frac{1}{\omega - \epsilon_{\mathbf{q}} + i(\alpha\omega - a_J)}, \tag{B4}$$

where $G^r(\mathbf{q},\omega) = \iint d(\mathbf{x}-\mathbf{x}')d(t-t')G^r(\mathbf{x},t;\mathbf{x}',t')e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{x}')+i\omega(t-t')}$ and $\epsilon_{\mathbf{q}} = \mathcal{J}\mathbf{q}^2 + \mathcal{A}$ is the magnon spectrum in the continuity limit. For the discrete spin Hamiltonian, the magnon spectrum is replaced by Eq. (A6), and the above retarded magnon Green's function becomes Eq. (6) in the main text.

APPENDIX C: THE PHYSICAL MEANING OF THE LESSER SELF-ENERGY $\Sigma^<$

In this section, we will show that the lesser self-energy represents the correlation of the random fluctuation field, and thus, it is reasonable to assume in the main text that it depends only on the properties of the thermal bath and remains unchanged even when the spin torque is present, i.e., $\Sigma^<(\omega) = -2i\alpha\omega n(\omega)$.

First, we consider the wave function of the magnon $\psi(\mathbf{x},t)$ in the presence of the fluctuation field $\mathbf{h}_f(\mathbf{x},t)$. From Eqs. (B2) and (B3) in Appendix B, $\psi(\mathbf{x},t)$ is related to the fluctuation field through

$$\psi(\mathbf{x},t) = \iint d\mathbf{x}'dt'G^r(\mathbf{x},t;\mathbf{x}',t')h_f(\mathbf{x}',t'), \tag{C1}$$

where $G^r(\mathbf{x},t;\mathbf{x}',t')$ is the retarded Green's function of the magnon given by Eq. (B3) and $h_f(\mathbf{x},t) = h_f^x(\mathbf{x},t) + ih_f^y(\mathbf{x},t)$. On the other hand, the lesser Green's function $G^<(\mathbf{x},t;\mathbf{x}',t')$ related to the magnon density [i.e., $|\psi(\mathbf{x},t)|^2$] is given by the correlation between magnon wave functions, i.e.,

$$G^<(\mathbf{x},t;\mathbf{x}',t') \sim -i\langle\psi^*(\mathbf{x}',t')\psi(\mathbf{x},t)\rangle, \tag{C2}$$

where $\langle\cdots\rangle$ represents the average over the configuration of the fluctuation field. Substituting Eq. (C1)) into Eq. (C2), we find that

$$\begin{aligned}G^<(\mathbf{x},t;\mathbf{x}',t') &\sim -i\iiint\int d\mathbf{x}_1dt_1d\mathbf{x}_2dt_2G^r(\mathbf{x},t;\mathbf{x}_1,t_1)\\ &\quad\times[G^r(\mathbf{x}',t';\mathbf{x}_2,t_2)]^*\langle h_f(\mathbf{x}_1,t_1)h_f^*(\mathbf{x}_2,t_2)\rangle\\ &\sim \iiint\int d\mathbf{x}_1dt_1d\mathbf{x}_2dt_2G^r(\mathbf{x},t;\mathbf{x}_1,t_1)\\ &\quad\times\Sigma^<(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2)G^a(\mathbf{x}_2,t_2;\mathbf{x}',t'),\end{aligned} \tag{C3}$$

where

$$\Sigma^<(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2) \sim -i\langle h_f(\mathbf{x}_1,t_1)h_f^*(\mathbf{x}_2,t_2)\rangle \tag{C4}$$

is the lesser self-energy and $G^a(\mathbf{x},t;\mathbf{x}',t') = [G^r(\mathbf{x}',t';\mathbf{x},t)]^*$ is the advanced Green's function of the magnon. Note that the above relation is exactly the Keldysh formalism of Eq. (7) in the main text using the time and space coordinates, with $\Sigma^<$ representing the correlation of the fluctuation field from the thermal bath. Thus, $\Sigma^<$ is supposed to be not relevant to the spin torque even though both G^r and G^a are.

In the main text, we used the lesser self-energy, $\Sigma^<(\omega) = -2i\alpha\omega n(\omega)$, obtained from the equilibrium fluctuation-dissipation relation in the absence of the spin torque. From

Eq. (C4), the corresponding correlation of the fluctuation field is $\langle h_f(\mathbf{x},\omega)h_f^*(\mathbf{x}',\omega')\rangle \sim 2\alpha\omega n(\omega)\delta(\mathbf{x}-\mathbf{x}')\delta(\omega-\omega')$, where $h_f(\mathbf{x},\omega)=\int dt\, h_f(\mathbf{x},t)\exp(i\omega t)$.

APPENDIX D: THE SELF-CONSISTENT EQUATION OF NONEQUILIBRIUM MAGNETIZATION WITH AN ARBITRARY SPIN NUMBER

In this Appendix, we shall generalize Callen's decoupling scheme for the self-consistent equation of the equilibrium magnetization with an arbitrary spin number to the nonequilibrium case. First, we define the following retarded Green's function:

$$G^r_\chi(i,j,t) = -i\theta(t)\big\langle\big[\hat{a}_i(t), e^{\chi\hat{S}^z_j}\hat{a}^\dagger_j\big]\big\rangle, \tag{D1}$$

where χ is the Callen parameter. Compared to the retarded Green's function of the magnon (i.e., $\chi=0$) in the main text, the above Green's function is given by

$$G^r_\chi(\mathbf{q},\omega) = \frac{\Theta(\chi)}{\omega-\epsilon_\mathbf{q}-\Sigma^r(\omega)} = \frac{1}{g^r_\chi(\mathbf{q},\omega)^{-1}-\Sigma^r_\chi(\omega)}, \tag{D2}$$

where $\Theta(\chi)\equiv\langle[\hat{a}_i, e^{\chi\hat{S}^z_i}\hat{a}^\dagger_i]\rangle$, $\Sigma^r(\omega)=-i(\alpha\omega-a_J)$ is the retarded self-energy of the magnon, $g^r_\chi(\mathbf{q},\omega)=\Theta(\chi)/(\omega-\epsilon_\mathbf{q}+i0^+)$ is the free retarded Green's function corresponding to $G^r_\chi(\mathbf{q},\omega)$, and $\Sigma^r_\chi(\omega)=\Sigma^r(\omega)/\Theta(\chi)$ is the retarded self-energy of G^r_χ. In the nonequilibrium steady state, we have the following relation:

$$G^<_\chi(\mathbf{q},\omega) = G^r_\chi(\mathbf{q},\omega)\Sigma^<_\chi(\omega)G^a_\chi(\mathbf{q},\omega), \tag{D3}$$

where $G^<_\chi(\mathbf{q},\omega) = -i\int dt\int d\mathbf{R}_{ij}e^{i\omega t-i\mathbf{q}\cdot\mathbf{R}_{ij}}\langle e^{\chi\hat{S}^z_j}\hat{a}^\dagger_j\hat{a}_i(t)\rangle$ is the lesser Green's function in the frequency and momentum space, $\Sigma^<_\chi(\omega)$ is the corresponding lesser self-energy, and $G^a_\chi(\mathbf{q},\omega)=G^r_\chi(\mathbf{q},\omega)^*$ is the advanced Green's function. As in the main text, we assume that $\Sigma^<_\chi$ remains unchanged when the spin torque is present, i.e., $\Sigma^<_\chi(\omega)=-2i\alpha n(\omega)/\Theta(\chi)$. Then Eq. (D3) leads to

$$\begin{aligned}\langle e^{\chi\hat{S}^z_i}\hat{a}^\dagger_i\hat{a}_i\rangle &= i\int_{BZ}\frac{d^d\mathbf{q}}{(2\pi)^d}\int\frac{d\omega}{2\pi}G^<_\chi(\mathbf{q},\omega)\\ &\approx \Theta(\chi)\int_{BZ}\frac{d^d\mathbf{q}}{(2\pi)^d}\frac{Z_\mathbf{q}}{e^{\beta\epsilon_\mathbf{q}}-1}\\ &= \Theta(\chi)N_m,\end{aligned} \tag{D4}$$

where in the second equality a small damping constant is assumed and $N_m = 1/(2\pi)^d\int_{BZ}d^d\mathbf{q}Z_\mathbf{q}/[e^{\beta\epsilon_\mathbf{q}}-1]$ is the nonequilibrium magnon density with $Z_\mathbf{q}=(1-a_J/\alpha\epsilon_\mathbf{q})^{-1}$. Following Callen's work, we define $\Omega(\chi)=\langle e^{\chi\hat{S}^z_i}\rangle$, and thereby, $\langle e^{\chi\hat{S}^z_i}\hat{a}^\dagger_i\hat{a}_i\rangle$ and $\Theta(\chi)$ of Eq. (D4) can be expressed in terms of the differential form of $\Omega(\chi)$ with respect to χ. Too see this, using the operator identities $[\hat{S}^+_i, e^{\chi\hat{S}^z_i}]=(e^{-\chi}-1)e^{\chi\hat{S}^z_i}\hat{S}^+_i$ and $\hat{S}^z_i = S(S+1)-(\hat{S}^z_i)^2-\hat{S}^-_i\hat{S}^+_i$, one has

$$\begin{aligned}\Theta(\chi) &= \frac{1}{2M}\big[S(S+1)(e^{-\chi}-1)\langle e^{\chi\hat{S}^z_i}\rangle + (e^{-\chi}+1)\langle e^{\chi\hat{S}^z_i}\hat{S}^z_i\rangle\\ &\quad - (e^{-\chi}-1)\langle e^{\chi\hat{S}^z_i}(\hat{S}^z_i)^2\rangle\big]\\ &= \frac{1}{2M}\big[S(S+1)(e^{-\chi}-1)\Omega(\chi)+(e^{-\chi}+1)\partial_\chi\Omega(\chi)\\ &\quad -(e^{-\chi}-1)\partial^2_\chi\Omega(\chi)\big],\end{aligned} \tag{D5}$$

where we have used the definitions $\hat{a}_i=\hat{S}^+_i/\sqrt{2M}$ and $\hat{a}^\dagger_i=\hat{S}^-_i/\sqrt{2M}$, and similarly,

$$\langle e^{\chi\hat{S}^z_i}\hat{a}^\dagger_i\hat{a}_i\rangle = \frac{S(S+1)\Omega(\chi)-\partial_\chi\Omega(\chi)-\partial^2_\chi\Omega(\chi)}{2M}. \tag{D6}$$

Substituting Eqs. (D5) and (D6) into Eq. (D4)), one arrives at

$$\partial^2_\chi\Omega(\chi)+\frac{(1+N_m)e^\chi+N_m}{(1+N_m)e^\chi-N_m}\partial_\chi\Omega(\chi)-S(S+1)\Omega(\chi)=0, \tag{D7}$$

with boundary conditions

$$\Omega(0)=1, \tag{D8a}$$

$$\prod_{p=-S}^{S}(\partial_\chi-p)\Omega(\chi)|_{\chi=0}=0, \tag{D8b}$$

where the second boundary condition comes from the operator identity $\prod_{p=-S}^S(\hat{S}^z_i-p)=0$. The above differential equation was previously obtained by Callen but for the equilibrium state, while we here extend it to the spin-torque driven nonequilibrium state. The solution of differential equation (D7) satisfying boundary conditions is given by

$$\Omega(\chi)=\frac{N_m^{2S+1}e^{-S\chi}-(1+N_m)^{2S+1}e^{(S+1)\chi}}{\big[N_m^{2S+1}-(1+N_m)^{2S+1}\big][(1+N_m)e^\chi-N_m]}, \tag{D9}$$

from which the magnetization is found with

$$\begin{aligned}M &\equiv \langle\hat{S}^z_i\rangle\\ &= \partial_\chi\Omega(\chi)|_{\chi=0}\\ &= \frac{(S-N_m)(1+N_m)^{2S+1}+(S+1+N_m)N_m^{2S+1}}{(1+N_m)^{2S+1}-N_m^{2S+1}},\end{aligned} \tag{D10}$$

which is the self-consistent equation of magnetization with an arbitrary spin number in the presence of spin torque.

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