Edelstein effect in pseudospin Dirac systems

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Edelstein effect of spin polarization means an electric field could induce a magnetization in nonmagnetic materials. This effect can be well understood by a Zeeman-like term due to an effective magnetic field in spinorbit coupling systems. We propose a mechanism of current-induced spin polarization, where the real spin is not directly related to the effective magnetic field induced by an electric field. This effective field only results in the polarization of a general spin. Then the real spin polarization is generated through the coupling between the real spin and the general spin in pseudospin Dirac systems with low symmetry. The values of our spin polarization and the conventional one may have the same order of magnitude though ours involves two processes, which has been further verified by the numerical calculation in the 1T'-WTe₂ distorted monolayer. It will be helpful for the deeper understanding of the current-induced spin polarization in pseudospin Dirac materials.

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I. INTRODUCTION

One of the central goals in spintronics is to realize the efficient generation and manipulation of the spin in especially nonmagnetic materials, which remains a crucial challenge [\[1,2\]](#page-4-0). The Edelstein effect, also called inverse spin galvanic effect or current-induced spin polarization, refers to the generation of nonequilibrium spin polarization in the presence of an electric field in solids without structure or bulk inversion symmetry [\[3,4\]](#page-4-0). This effect offers us a viable pathway towards all-electric spintronics devices. Hence, continuous attention has been paid by researchers all over the world $[5-14]$ $[5-14]$. It was first observed in GaAs heterojunctions [\[15–18\]](#page-5-0). In addition to these traditional semiconductors with spin-orbit couplings, more recently, the Edelstein effect has also been studied in topological insulators [\[14,19–23\]](#page-5-0), van der Waals heterostructures [\[24,25\]](#page-5-0), Weyl semimetals [\[26,27\]](#page-5-0), and superconductors [\[28,29\]](#page-5-0).

The conventional Edelstein effect can be understood from the effective magnetic field induced by the spin-orbit interaction [Fig. $1(a)$]. For example, the two-dimensional electron gas with Rashba spin-orbit coupling is described by

$$
H = \frac{k^2}{2m} + \alpha (k_y \sigma_x - k_x \sigma_y), \qquad (1)
$$

with *m* being the effective mass, $\mathbf{k} = (k_x, k_y)$ the wave vector, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ the Pauli matrices, and α the coupling constant. In the presence of an electric field along the *y* direction $E = E\hat{y}$, the spin-orbit coupling of a diffusive system can be viewed as a spin Zeeman term $\alpha \langle k_v \rangle \sigma_x$ from an effective magnetic field $B_{\text{eff}} \propto \alpha \langle k_v \rangle$ along the *x* direction. This effective Zeeman interaction results in different occupation of different spin orientation, leading to the net magnetization *M* along the effective magnetic field, i.e., the *x* direction.

In contrast to the relatively simple structure of spinmomentum locking in traditional semiconductors, an additional degree of freedom, pseudospin, usually exists in topological materials, providing a new platform for studying the Edelstein effect. In this paper, we propose a mechanism of current-induced spin polarization in this system with low crystal symmetry [Fig. $1(b)$]. The external electric field first induces an effective magnetic field \tilde{B}_{eff} applied on the general spin, not the real spin. Then the net real spin polarization or magnetization is generated due to the coupling between the general spin and the real spin (GSSC). It can be viewed as the second-order Edelstein effect. Though this mechanism relates to two processes, surprisingly, the magnitude of the generated spin polarization is comparable to, or even larger than, the conventional one. This mechanism can be realized widely in low-symmetry pseudospin Dirac systems, where both the real spin and the pseudospin are involved. There exists a large class of low-symmetry pseudospin Dirac systems, including twodimensional transition-metal dichalcogenides. Among them, the $1T'$ phase monolayer WTe₂ is established as a quantum spin Hall insulator [\[30,31\]](#page-5-0). The 1T'-WTe₂ distorted monolayer, being applied to a normal electric field, can induce both in-plane spin orientation and out-of-plane spin orientation arising from the out-of-plane asymmetry [\[32\]](#page-5-0). Further, the electrically tunable Edelstein effect has been observed up to room temperature [\[27\]](#page-5-0). Taking the $1T'$ -WTe₂ distorted monolayer as an example, we demonstrate the mechanism of this type of the Edelstein effect proposed here.

II. MODEL AND GENERAL ARGUMENT

We consider an effective four-band $\mathbf{k} \cdot \mathbf{p}$ description of the $1T'$ -WTe₂ distorted monolayer as an example to

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FIG. 1. Schematic of (a) the conventional Edelstein effect and (b) the type proposed in this paper. Note that the effective magnetic field B_{eff} in (a) is not normal to the plane of the electron gas with Rashba spin-orbit coupling, actually. Here the effective magnetic field plotted perpendicular to the plane is just for convenience. GSOC is the general-spin–orbit coupling. GSSC is the coupling between the general spin and the real spin. E_F is the Fermi energy. $D_{\uparrow/\downarrow}$ and $\mu_{\uparrow/\downarrow}$ are the density of states and the magnetic moment for spin up or down.

discuss this Edelstein effect proposed here. Under the basis ${\psi_{c\uparrow}, \psi_{v\uparrow}, \psi_{c\downarrow}, \psi_{v\downarrow}}$, the Hamiltonian has the form [\[32\]](#page-5-0)

$$
H = H_0 + H_{\text{SOC}}.\tag{2}
$$

In the basis, *c* and *v* represent the conduction and valence orbits, and \uparrow and \downarrow mean spin up and down. Here H_0 refers to a Hamiltonian describing the electronic behavior without the electric-field-induced spin-orbit coupling:

$$
H_0 = \epsilon + m\tau_0 \otimes \sigma_z + v_x k_x \tau_z \otimes \sigma_x - v_y k_y \tau_0 \otimes \sigma_y, \quad (3)
$$

with $\epsilon = (\epsilon_c + \epsilon_v)/2$, $m = (\epsilon_c - \epsilon_v)/2$, and $\tau = (\tau_x, \tau_y, \tau_z)$ being the Pauli matrices, τ_0 the identity matrix, and ϵ_c and ϵ _v denoting the diagonal parts for the conduction and valence bands. The spin-orbit coupling H_{SOC} can be decomposed into two parts $H_{\text{SOC}} = H_Z + H_R$. Here

$$
H_Z = \lambda k_y \tau_z \otimes \sigma_0 - \delta_z \tau_z \otimes \sigma_y, \qquad (4)
$$

$$
H_R = -\delta_x \tau_y \otimes \sigma_x + \alpha_x k_x \tau_y \otimes \sigma_0 + \alpha_y k_y \tau_x \otimes \sigma_0 \qquad (5)
$$

represent the Zeeman-like and Rashba-type spin-orbit coupling terms, respectively. λ , $\delta_{x/z}$, and $\alpha_{x/y}$ are coupling constants. In this paper, we set $\hbar = 1$. We emphasize that the Zeeman-like term H_Z leads to the out-of-plane spin polarization, while the Rashba-type one H_R gives rise to the in-plane spin polarization. Due to the order of the spin up and down in the basis, the spin operators are not $\frac{1}{2}\tau_0 \otimes \sigma_a$, and should have the form [\[33\]](#page-5-0)

$$
\hat{s}_a = \frac{1}{2}\tau_a \otimes \sigma_0,\tag{6}
$$

with $a = (x, y, z)$.

We first give a general argument of the current-induced spin polarization for this distorted low-symmetry monolayer. If a dc electric field is applied along the *y* direction, there will exist a nonzero average wave vector $\langle k_{v} \rangle$ in the diffusive monolayer, resulting in an effective magnetic field $B_{\text{eff}} \propto$ $\lambda \langle k_v \rangle$ acting on the \hat{s}_z from the first term of H_Z . Therefore,

the spin polarization along the *z* direction occurs due to this conventional Edelstein effect. This spin polarization is proportional to the coupling λ . Moreover, it is found that the nonzero *k_y*) also brings about an effective magnetic field $\tilde{B}_{\text{eff}} \propto v_y \langle k_y \rangle$ coupling to a general spin component $\frac{1}{2}\tau_0 \otimes \sigma_y$ from the last term of spin-orbit coupling independent H_0 . Since $\delta_z \tau_z \otimes \sigma_y =$ $\delta_z(\tau_0 \otimes \sigma_y)(\tau_z \otimes \sigma_0)$, the GSSC term in H_Z also leads to a part of the *z* component of spin polarization proportional to $\delta_z v_y$. This is the proposed type of the Edelstein effect mentioned in the Introduction. Here the Fermi velocity v_y is usually large in the Dirac system. Hence, although the type proposed here is a secondary process, the spin polarizations due to two types of Edelstein effects will have the same order of magnitude when the coupling constant δ_z/k_F is comparable to the λ with k_F being the Fermi wave vector. For the in-plane component of spin polarization, the electric field along the *y* direction only induces the in-plane *x*-component spin $(\alpha \alpha_v)$ due to the conventional Edelstein effect. When the dc electric field is along the *x* direction, the out-of-plane spin polarization vanishes, completely. For the in-plane case, the conventional Edelstein effect contributes the *y* direction spin polarization $(\alpha \alpha_{x})$, while the type proposed here leads to the *x*-component one ($\alpha \delta_x v_x$). In the above process, we realize the electrical control of the spin orientation in the whole three-dimensional space.

III. ANALYTICAL CALCULATION OF SPIN POLARIZATION

To demonstrate the above argument, we analytically evaluate the spin polarization based on the Kubo formula in this section. The general expression for the spin polarization that appears as a response to the external electric field along the *b* direction $\mathbf{E} = E_b \hat{b}$ can be written in the following form [\[34,35\]](#page-5-0):

$$
S_a(\omega) = -\frac{eE_b}{\omega} \int \frac{d\varepsilon}{2\pi} n_F(\varepsilon) \sum_k \text{Tr}\{\hat{v}_b G^A(\varepsilon - \omega)\hat{s}_a
$$

$$
\times [G^R(\varepsilon) - G^A(\varepsilon)] + \hat{v}_b [G^R(\varepsilon) - G^A(\varepsilon)]\hat{s}_a
$$

$$
\times G^R(\varepsilon + \omega)].
$$
 (7)

Here $G^{R/A}$ are the retarded/advanced Green's functions, $n_F(\varepsilon)$ is the Fermi distribution function, and \hat{v}_b is the *b*th component of the velocity operator:

$$
\hat{\nu}_b = \frac{\partial H}{\partial k_b},\tag{8}
$$

with $b = (x, y, z)$. We focus on the dc electric-field limit, where the Green's functions $G^A(\varepsilon - \omega)$ and $G^R(\varepsilon + \omega)$ can be expanded to the first order of frequency ω . At zero temperature $T \to 0$, $-\frac{\partial n_F(\varepsilon)}{\partial \varepsilon}$ $\to \delta(\varepsilon - E_F)$ with E_F being the Fermi energy. Further, the terms relating to the production of two retarded or advanced Green's functions are negligibly small comparing to the one $G^A G^R$. Therefore, the Kubo formula for the spin polarization S_a in the dc limit is

$$
S_a = \frac{eE_b}{2\pi} \sum_{k} \text{Tr}[\hat{v}_b G^A(E_F) \hat{s}_a G^R(E_F)]. \tag{9}
$$

In the following analytical calculation, we concentrate on the *z* component of spin polarization S_z since it is fascinating that an in-plane electric field results in the spin out of the two-dimensional monolayer. Further, this out-of-plane spin polarization is particularly useful for its potential high-density magnetic application [\[36\]](#page-5-0). In this case, the electric field is assumed along the *y* direction $\mathbf{E} = E_y \hat{y}$ and we only consider the spin-orbit coupling term H_Z for simplicity. Since these coupling terms are generated by the vertical electric field *E*⊥, one can properly tune E_{\perp} to make δ_x , α_x , and α_y vanish [\[32\]](#page-5-0). In the scenario, the considered Hamiltonian is $H = H_0 + H_Z$. Hence, the related velocity component is \hat{v}_y , which is written as

$$
\hat{v}_y = -v_y \tau_0 \otimes \sigma_y + \lambda \tau_z \otimes \sigma_0. \tag{10}
$$

The energies of the system are

$$
E_{\mu\nu} = \mu \varepsilon_0 + \nu \varepsilon_\mu,\tag{11}
$$

with μ , $\nu = \pm 1$, and

$$
\varepsilon_0 = \lambda k_y,\tag{12}
$$

$$
\varepsilon_{\mu} = \sqrt{v_x^2 k_x^2 + (v_y k_y + \mu \delta_z)^2 + m^2}.
$$
 (13)

Here we have set $\epsilon = 0$.

A. Bare bubble approximation

We first calculate the spin polarization in the bare bubble approximation. The retarded and advanced Green's functions are

$$
G^{R/A} = \frac{1}{E_{\rm F} - H \pm i\eta}.\tag{14}
$$

Here $\eta = 1/(2\tau)$ with τ being the relaxation time. By introducing 16 matrices $\tau_i \otimes \sigma_j$ with $i, j = (0, x, y, z)$, the Green's functions can be obtained as

$$
G^{R/A} = \sum_{i,j} G_{ij}^{R/A} \tau_i \otimes \sigma_j,
$$
 (15)

with the nonzero coefficients of the retarded one:

$$
G_{00}^{R} = \frac{1}{4} \sum_{\mu\nu} \tilde{G}_{\mu\nu}^{R},
$$
 (16)

$$
G_{0x}^{R} = \frac{v_{x}k_{x}(v_{y}\delta_{z} + \lambda \tilde{E}_{F})}{2\left[\lambda\left(\Delta_{k}^{2} - \lambda^{2}k_{y}^{2} + \tilde{E}_{F}^{2}\right) + 2v_{y}\tilde{E}_{F}\delta_{z}\right]}\sum_{\mu\nu}\mu\tilde{G}_{\mu\nu}^{R}, \quad (17)
$$

$$
G_{0y}^{R} = \frac{v_{y}(\Delta_{k}^{2} - \lambda^{2} k_{y}^{2} - \tilde{E}_{F}^{2} - 2\delta_{z}^{2}) - 2\lambda \tilde{E}_{F}\delta_{z}}{4[\lambda(\Delta_{k}^{2} - \lambda^{2} k_{y}^{2} + \tilde{E}_{F}^{2}) + 2v_{y}\tilde{E}_{F}\delta_{z}]} \sum_{\mu\nu} \mu \tilde{G}_{\mu\nu}^{R},
$$
\n(18)

$$
G_{0z}^{R} = \frac{-m(\Delta_{k}^{2} - \lambda^{2}k_{y}^{2} - \tilde{E}_{F}^{2})}{4k_{y}[\lambda(\Delta_{k}^{2} - \lambda^{2}k_{y}^{2} + \tilde{E}_{F}^{2}) + 2v_{y}\tilde{E}_{F}\delta_{z}]} \sum_{\mu\nu} \mu \tilde{G}_{\mu\nu}^{R}, (19)
$$

$$
G_{z0}^{R} = \frac{1}{4} \sum_{\mu\nu} \mu \tilde{G}_{\mu\nu}^{R},
$$
 (20)

$$
G_{zx}^R = \frac{-v_x k_x \left(\Delta_k^2 - \lambda^2 k_y^2 - \tilde{E}_{\rm F}^2\right)}{4k_y \left[\lambda \left(\Delta_k^2 - \lambda^2 k_y^2 + \tilde{E}_{\rm F}^2\right) + 2v_y \tilde{E}_{\rm F} \delta_z\right]} \sum_{\mu\nu} \mu \tilde{G}_{\mu\nu}^R, \tag{21}
$$

$$
G_{zy}^{R} = \frac{\delta_z \left(\Delta_k^2 - \lambda^2 k_y^2 - \tilde{E}_{\rm F}^2 - 2v_y^2 k_y^2\right) - 2\lambda v_y k_y^2 \tilde{E}_{\rm F}}{4k_y \left[\lambda \left(\Delta_k^2 - \lambda^2 k_y^2 + \tilde{E}_{\rm F}^2\right) + 2v_y \tilde{E}_{\rm F} \delta_z\right]}
$$

$$
\times \sum_{\mu\nu} \mu \tilde{G}_{\mu\nu}^{R}, \tag{22}
$$

$$
G_{zz}^R = \frac{m(v_y \delta_z + \lambda \tilde{E}_{\rm F})}{2\left[\lambda\left(\Delta_k^2 - \lambda^2 k_y^2 + \tilde{E}_{\rm F}^2\right) + 2v_y \tilde{E}_{\rm F} \delta_z\right]} \sum_{\mu\nu} \mu \tilde{G}_{\mu\nu}^R. \quad (23)
$$

Here $\tilde{E}_{\rm F} = E_{\rm F} + i\eta$, $\Delta_k^2 = v_x^2 k_x^2 + v_y^2 k_y^2 + m^2 + \delta_z^2$, and

$$
\tilde{G}^R_{\mu\nu} = \frac{1}{E_{\rm F} - E_{\mu\nu} + i\eta}.\tag{24}
$$

Likewise the advanced Green's function G^A could be obtained by replacing η with $-\eta$ in G^R .

From the Kubo formula [\(9\)](#page-1-0), we find that the spin polarization S_z can be decomposed into two parts: $S_z = S_{z1} + S_{z2}$ with

$$
S_{z1} = \lambda \frac{eE_y}{\pi} \sum_{k} \left[G_{00}^A G_{00}^R + G_{0x}^A G_{0x}^R + G_{0y}^A G_{0y}^R \right. \\
\left. + G_{0z}^A G_{0z}^R + G_{z0}^A G_{z0}^R + G_{zx}^A G_{xy}^R + G_{zy}^A G_{zy}^R + G_{zz}^A G_{zz}^R \right],
$$
\n
$$
S_{z2} = -v_y \frac{eE_y}{\pi} \sum_{k} \left[G_{00}^A G_{zy}^R + G_{0y}^A G_{z0}^R + G_{z0}^A G_{0y}^R + G_{zy}^A G_{00}^R \right].
$$
\n(25)

(26) The coefficient
$$
\lambda
$$
 before the S, means it originates from the

k

The coefficient λ before the S_{z1} means it originates from the conventional Edelstein effect.

In the following calculation, we consider the electronic conduction, that is, the Fermi energy is positive, $E_F > m > 0$. The coupling constants λ and δ_z in Green's functions of the conventional S_{z1} could be omitted since only the leading order of the spin-orbit coupling is taken into account. After some direct but tedious derivations, the S_{z1} due to the conventional Edelstein effect is written as

$$
S_{z1} = \lambda \frac{3eE_y}{8} D_{\rm F} \tau.
$$
 (27)

Here D_F is the density of states in the absence of spin-orbit couplings at Fermi energy:

$$
D_{\rm F} = 2\sum_{k} \delta(E_{\rm F} - \xi_k). \tag{28}
$$

with $\xi_k = \sqrt{v_x^2 k_x^2 + v_y^2 k_y^2 + m^2}$ being the dispersion of electrons without any spin-orbit couplings. The density of states could be calculated as $D_F = E_F/(\pi v_x v_y)$.

Similarly, for the S_{z2} , we consider the diagonal productions of Green's functions and expand it to the first order of the coupling constants: $S_{z2} = S_{z2}' + S_{z2}''$. Here S_{z2}' and S_{z2}'' are proportional to $v_y \delta_z$ and λ , respectively. To the leading order of scattering, S'_{z2} is written as

$$
S_{z2}' = \frac{\delta_z}{k_{\rm F}} \frac{v_y k_{\rm F}}{E_{\rm F}} \frac{eE_y}{2} \tilde{D}_{\rm F} \tau, \tag{29}
$$

with $k_F = E_F / \sqrt{v_x v_y}$ and

$$
\tilde{D}_{\rm F} = D_{\rm F} + 2 \sum_{k} \frac{v_{\rm y}^2 k_{\rm y}^2}{E_{\rm F}} \delta'(E_{\rm F} - \xi_k). \tag{30}
$$

The $S_{z2}^{\prime\prime}$ has the form

$$
S_{z2}^{\prime\prime} = -\lambda \frac{eE_y}{4} \tilde{D}_{\rm F}^{\prime} \tau, \tag{31}
$$

with

$$
\tilde{D}_{\rm F}' = D_{\rm F} - 4 \sum_{k} \frac{v_{\rm y}^2 k_{\rm y}^2}{E_{\rm F}} \delta'(E_{\rm F} - \xi_k). \tag{32}
$$

Hence, according to the argument in Sec. II , the conventional spin polarization proportional to λ is $S_z^I = S_{z1} + S_{z2}^{\prime\prime}$ and the one related to $v_y \delta_z$ is due to this type of Edelstein effect proposed here: $S_z^{\text{II}} = S_{z2}^{\prime}$. They can be calculated explicitly:

$$
S_z^{\rm I} = -\lambda \frac{eE_y \tau}{8\pi v_x v_y E_F} \left(2E_F^2 - m^2\right),\tag{33}
$$

$$
S_{z}^{\text{II}} = -\frac{\delta_{z}}{k_{F}} \frac{v_{y} k_{F}}{E_{F}} \frac{e E_{y} \tau}{4 \pi v_{x} v_{y} E_{F}} (E_{F}^{2} - m^{2}). \tag{34}
$$

The expression of the conventional spin polarization is in accordance with the usual one $[3,5-9]$ $[3,5-9]$, proportional to the spin-orbit coupling constant. If the values of coupling constants λ and δ_z/k_F are comparable, proposed type S_z^{II} and the conventional S^I_z will have the same order of magnitude since $v_y k_F/E_F \sim 1$. It is amazing that S_z^{II} can be very large although it relates to a combined process. The effective magnetic field first induces a general spin relating to the Fermi velocity *v^y* in the part of the Hamiltonian having nothing to do with the spin-orbit coupling. Then the real spin is generated through the GSSC in the spin-orbit coupling part of the Hamiltonian. Hence, this type of spin polarization is a phenomenon in pseudospin Dirac systems.

B. Vertex correction

The impurity vertex correction usually has a significant impact on various physical quantities. For example, it completely cancels the spin Hall conductivity of the bare bubble approximation in two-dimensional electron gas with linear spin-orbit couplings [\[37–41\]](#page-5-0). The renormalized spin vertex function \tilde{s}_z by considering the noncrossing ladder diagrams is determined by the following equation:

$$
\tilde{s}_z = \hat{s}_z + n_i u_0^2 \sum_{\mathbf{k}} G^A(E_F) \tilde{s}_z G^R(E_F). \tag{35}
$$

Then the spin polarization S_z could be obtained by replacing \hat{s}_z with \tilde{s}_z in the Kubo formula [\(9\)](#page-1-0). Here n_i is the impurity density and u_0 is the scattering strength, satisfying $n_i u_0^2 =$ $1/(2\pi D_F \tau)$.

This ladder equation for \tilde{s}_z can be solved by first finding the structure of the solution. The structure can be obtained iteratively by setting the zeroth-order $\tilde{s}_z^{(0)} = \frac{1}{2} \pi \otimes \pi_z$ then the first order $\tilde{s}_z^{(1)}$ satisfies $\tilde{s}_z^{(1)} = \tilde{s}_z + \tilde{s}_z^{(2)}$ $\frac{1}{2}\tau_z \otimes \sigma_0$, then the first-order $\tilde{s}_z^{(1)}$ satisfies $\tilde{s}_z^{(1)} = \hat{s}_z + \sigma_0$ $n_i u_0^2 \sum_{k} G^{A}(E_{F}) \tilde{s}_{z}^{(0)} G^{R}(E_{F})$. After integrating all the integrations

$$
I_{ij,i'j'} = n_i u_0^2 \int \frac{d^2 \mathbf{k}}{(2\pi)^2} G_{ij}^A G_{i'j'}^R,
$$
 (36)

with *i*, *j*, *i'*, *j'* = (0, *x*, *y*, *z*), the first-order $\tilde{s}_z^{(1)}$ is found $\tilde{s}_z^{(1)} \propto$ $\tau_z \otimes \sigma_0$ for small gap system $m \ll E_F$. Hence, the accurate solution \tilde{s}_z can be assumed as

$$
\tilde{s}_z = C \tau_z \otimes \sigma_0. \tag{37}
$$

With the help of the integrations $I_{ij,i'j'}$, we can easily obtain $C = 2/3$, and then the vertex correction of spin is

$$
\tilde{s}_z = \frac{2}{3}\tau_z \otimes \sigma_0. \tag{38}
$$

Hence, the spin polarizations by considering vertex correction can be obtained directly by multiplying 4/3 from the bare ones:

$$
S_z^{\rm I} = -\lambda \frac{eE_y \tau}{6\pi v_x v_y E_F} \left(2E_F^2 - m^2\right),\tag{39}
$$

$$
S_{z}^{\text{II}} = -\frac{\delta_{z}}{k_{F}} \frac{v_{y} k_{F}}{E_{F}} \frac{e E_{y} \tau}{3 \pi v_{x} v_{y} E_{F}} (E_{F}^{2} - m^{2}). \tag{40}
$$

The qualitative behavior of the spin polarization does not change when the vertex correction of the spin function is included.

IV. NUMERICAL RESULTS

To further demonstrate this type of Edelstein effect proposed here in this low-symmetry system, we numerically calculate the spin polarization from the Kubo formula [\(9\)](#page-1-0). In the evaluation, all the spin-orbit coupling terms are considered for the realistic $1T'$ -WTe₂ distorted monolayer. The parameters are the Fermi ve-locities [\[32\]](#page-5-0) $v_x = 15.5 \times 10^{-11}$ eV m and $v_y = 4 \times 10^{-11}$ eV m, the mass $m = 2.5$ meV, the other coupling constants $\alpha_x = 1 \times 10^{-11}$ eV m and $\delta_x = 10$ meV, the Fermi energy $E_F = 10$ meV, and the electric field $E_y = 1$ V/m. The out-of-plane spin polarization linearly depends on both λ and δ_z at small coupling constant, but is almost independent of the other coupling constants, in agreement with the above analytical discussion. At large coupling λ or δ_z , the deviation from the linearity appears. In this parameter regime, the conventional spin polarization and the type proposed here are of the same order of magnitude. The *y* component of spin polarization still vanishes completely even when all the spin-orbit couplings are considered for the field along the *y* direction. However, the *x* component is nonzero, which is proportional to the coupling constant α ^y. Hence, we can deduce that it is from the conventional Edelstein effect. It is also influenced by the δ_z and the λ , as shown in Fig. [2\(d\).](#page-4-0) This is due to the reason that the density of states depends on these coupling constants.

We have also calculated the spin polarization by considering a lattice model, where the wave vector k_i is replaced by $\sin k_i a/a$ with *a* being the lattice constant [\[42\]](#page-5-0). Then the model is periodic in the wave vector. The results are shown in Fig. [2](#page-4-0) with the dots. It is found that the spin polarizations of this lattice model are in agreement with the low-energy model. In real materials, there may be multiple valleys. The total spin polarizations are the summation of the contributions

FIG. 2. Current-induced spin polarization as the function of coupling constants. The *z* component of spin polarization is plotted vs the λ for various δ_z (a) and the δ_z for various λ (b). The *x* component is plotted vs the α ^{*y*} for various δ ^{*z*} (c) and the δ ^{*z*} for various λ (d). The dots are the ones calculated from the lattice model. In (a), (b), and (d), $\alpha_y = 1.0 \times 10^{-11}$ eV m. In (c), $\lambda = 1.0 \times 10^{-11}$ eV m.

from all the valleys if the correlation between valleys could be neglected.

V. DISCUSSION AND CONCLUSION

Recently, the out-of-plane spin polarization was observed in the WTe 2 monolayer in the presence of a normal electric field [\[27,43\]](#page-5-0). Since the Te atoms are not perfectly aligned in this monolayer system, the normal electric field induces both out-of-plane and in-plane local potential gradients [\[32\]](#page-5-0). The in-plane one results in both the λ and the δ_z couplings, simultaneously [\[32,](#page-5-0)[44,45\]](#page-6-0). Therefore, if the out-of-plane spin polarization is observed in this system experimentally, the conventional and proposed contributions are both included in the measured value. Thus it is believed that the proposed one actually has been observed experimentally. The effective magnetic field of this type, which leads to the general spin, is proportional to the Fermi velocity v_y . However, the con-

ventional one is proportional to the coupling λ. Hence, we can distinguish two contributions from each other through their v_y -dependent relations. The intercept in the $v_x v_y S_z$ axis of the $v_x v_y S_z v_y$ straight line is just the conventional spin polarization. The present spin polarization at finite v_y could be obtained by subtracting the intercept from the total one. It has been demonstrated experimentally that, in artificial Dirac systems, such as the InAs/GaSb asymmetric quantum well and GaSb/InAs/GaSb symmetric quantum well, the Fermi velocity could be electrically tuned $[46,47]$.

A general 4×4 pseudospin Dirac system can be expanded by 16 linearly independent $\tau_i \otimes \sigma_j$, where the symmetries make some terms vanish. If the symmetry of the system is low enough, there will be many spin-polarization components induced by the conventional or proposed type of Edelstein effects. Hence, the symmetry breaking by strain, electric field, etc., will realize the control of the spin polarization or magnetization. Moreover, for a pseudospin Dirac system having more degrees of freedom, which are described by Pauli matrices, the situation will become more complicated. Several general spins rather than a single one may be involved in the final generation of the real spin polarization. Hence, our paper will be helpful for the deeper understanding of the Edelstein effect and its applications in pseudospin Dirac materials.

In conclusion, we demonstrate a type of the Edelstein effect for the spin polarization. This spin polarization induced by the external electric field cannot be explained by the Zeeman term due to an effective magnetic field arising from the spin-orbit coupling as the conventional Edelstein effect. It is produced by the coupling between the real spin and the general spin in pseudospin Dirac systems with low symmetry, where the polarization of the general spin is generated by the effective magnetic field. This type of Edelstein effect does not come directly from the Zeeman interaction between the effective magnetic field and the involved spin. Therefore, it may be neglected by researchers. We validate that the value of this proposed type of spin polarization can be comparable to the conventional one in the realistic $1T'$ -WTe₂ distorted monolayer.

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