Boosting the thermoelectric performance of Fe₂VAl-type Heusler compounds by band engineering

F. Garmroudi, * A. Riss, M. Parzer, N. Reumann, H. Müller, and E. Bauer Institute of Solid State Physics, Technische Universität Wien, A-1040 Vienna, Austria

S. Khmelevskyi

Research Center for the Computational Materials Science and Engineering, Technische Universität Wien, A-1040 Vienna, Austria

R. Podloucky

Institute of Physical Chemistry, Universität Wien, A-1090 Vienna, Austria

T. Mori

International Center for Materials Nanoarchitectonics (WPI-MANA), National Institute for Materials Science, Tsukuba 305-0044, Japan

K. Tobita, Y. Katsura, and K. Kimura

Department of Advanced Materials Science, The University of Tokyo, Kashiwa, Chiba 277-8561, Japan

(Received 20 November 2020; revised 8 January 2021; accepted 11 January 2021; published 1 February 2021)

Linking the fundamental physics of band structure and scattering theory with macroscopic features such as measured temperature dependencies of electronic and thermal transport is indispensable to a thorough understanding of thermoelectric phenomena and ensures more targeted and efficient experimental research. Regarding Fe₂VAl-based compounds, experimental work has seen mostly qualitative and often speculative interpretations, preventing this class of materials from tapping their full potential when it comes to applications. In this paper, the temperature-dependent Seebeck coefficient and electrical resistivity of a set of p-type and n-type samples with the composition $Fe_2V_{1-x}Ta_xAl_{1-y}Si_y$ are presented from 4 K up to 800 K as well as the Hall mobility and carrier concentration from 4 K to 520 K. We attempt a quantitative analysis of our data using a parabolic twoand three-band model and compare the model results with those from density functional theory calculations. Our findings show an increase of the band gap $E_{\rm g}$ from almost zero in undoped Fe₂VAl toward $E_{\rm g} \approx 0.1 \, {\rm eV}$ with increasing Ta substitution, consistent with results from first-principles calculations. Due to the resulting enhancement of the Seebeck coefficient, the maximum power factor is boosted up to 10.3 mW/mK², which is, to the best of our knowledge, the highest value among *n*-type bulk semiconductor systems reported near room temperature up until now. We further show that for the p-type $Fe_2V_{1-x}Ta_xA1$ compounds, the dominant scattering mechanism of electrons is intrinsically different compared to the *n*-type samples, for which acoustic phonon scattering can well describe the temperature-dependent Hall mobility in a broad range of temperatures.

DOI: 10.1103/PhysRevB.103.085202

I. INTRODUCTION

Designing advanced functional materials with distinct properties is a necessity for facing future problems such as the global energy crisis. Thermoelectric materials, which can directly convert waste heat into electricity via the Seebeck effect, are especially attractive for sustainable high-end technological applications. The dimensionless figure of merit $ZT = \frac{S^2}{\rho(\lambda_e + \lambda_{latt})}T$ determines the thermoelectric conversion efficiency, with *S* being the Seebeck coefficient, ρ the electrical resistivity, λ_e and λ_{latt} the electronic and lattice thermal conductivity, and *T* the absolute temperature, respectively. Among various material classes, Fe₂VAl-based full-Heusler compounds are being investigated due to a large peak of the power factor $PF = S^2/\rho$ at 300–400 K, where most waste

heat is released to the environment [1–7]. Additionally, those materials consist of ubiquitous elements and have chemical and mechanical long-term stability [8,9].

The electronic structure of stoichiometric Fe₂VAl has mostly been reported as a narrow pseudogap system, with the band gap E_g ranging from -0.2 eV to -0.1 eV, together with a large differential density of states (DOS) at either side of the Fermi energy E_F . This was predicted by several density functional theory (DFT) studies [10–15] and indeed confirmed by numerous experimental works [16–18]. However, recent studies have brought into question whether an indirect narrow-band-gap semiconductor is a more accurate description of this compound [19,20]. Kato *et al.* found that Si doping at the Al site in Fe₂VAl can effectively optimize the charge carrier concentration through a rigid-band-like shift of E_F , which already allows for large values of $PF \approx 5.5$ mW/mK² [1]. Even higher power factors of up to 6.7 mW/mK² [21] and 6.8 mW/mK²

^{*}Corresponding author: fabian.garmroudi@tuwien.ac.at



FIG. 1. Calculated electronic band structures at equilibrium volume (VASP-GGA) of (a) Fe₂VAl and (b) Fe₂TaAl with and without spin-orbit coupling (SOC). Energy-derivative of the Fermi-Dirac distribution is plotted for T = 300 and 600 K.

[22] were found in the off-stoichiometric compounds $Fe_{2-x}V_{1+x}Al_{1-y}Si_y$ and $Fe_2V_{1+x}Al_{1-x}$ respectively. The recent focus of studies has therefore primarily been on the reduction of the large lattice thermal conductivity with encouraging results obtained by heavy elemental substitution [3,23–25], thin film deposition [26–29], or microstructural tailoring [30]. In the case of heavy elemental substitution, Terazawa et al. reported on the effect of a V/Ta substitution with respect to the reduction of λ_{latt} through increased phonon scattering [23,24]. However, the effect of this substitution with respect to the electronic structure and Seebeck coefficient is only scarcely and inconsistently reported in the literature [31,32]. Ab initio calculations in the present paper suggest a semimetal-insulator-like transition in the case of a full substitution of V by Ta due to its higher d-orbital energies compared to that of V (see Figs. 1 and 2). This should be an effective strategy for enhancing the Seebeck coefficient. In fact, a recent DFT study on Fe_2TaZ (Z =Al, Ga, In) by Khandy et al. predicts maximum Seebeck coefficients of almost 700 μ V/K at T = 600 K for optimally doped Fe₂TaAl due to the larger band gap [33]. Even for smaller doping concentrations, a partial band-gap increase is noticeable in the calculated DOS of this paper. We aim to experimentally demonstrate such an opening of the band gap, causing an enhancement of the



FIG. 2. Energy-dependent electronic density of states (DOS) of (a) Fe_2VA1 and (b) Fe_2TaA1 at equilibrium volume (VASP-GGA). V contributes delocalized states at the Fermi energy while the Ta conduction band states are located far above E_F .

Seebeck coefficient and thermoelectric power factor. This is expected to be realized in $Fe_2V_{1-x}Ta_xAl_{1-y}Si_y$.

II. EXPERIMENTAL METHODS AND COMPUTATIONAL DETAILS

Bulk elements of high purity (Fe 99.99%, V 99.93%, Ta 99.95%, Al 99.999%, Si 99.9999%) were stoichiometrically weighed and melted by high-frequency induction heating. Polycrystalline samples of around 5 g were obtained, with a mass loss of less than 0.1%, and evacuated in a quartz tube at $\approx 10^{-5}$ mbar for annealing at 1073 K for 168 h, followed by furnace cooling. The crystal structure was probed with Cu-K α radiation in a Bragg-Brentano geometry. Rietveld refinements x-ray diffraction (XRD) performed and the lattice parameter was obtained. For the high-temperature resistivity and Seebeck measurements, a conventional four-probe dc method (ZEM3-ULVAC) was used, while for the low-temperature Seebeck coefficient measurements from 4 K up to 300 K, an ac method similar to the one in Ref. [34], using a toggled heating technique (seesaw heating), was applied. The results for the undoped Fe₂VAl compound were taken from our pre-



FIG. 3. XRD powder patterns of $Fe_2V_{1-x}Ta_xAl_{1-y}Si_y$ (upper panel) and concentration-dependent lattice parameters taken at room temperature (lower panel). The inset sketches the crystal structure of Fe_2VAl .

vious study, where the electronic transport properties have been extensively discussed and compared with DFT-derived data using the BOLTZTRAP package [35]. Hall resistance was measured using the van der Pauw technique with fields up to 10 T. Alloy-averaged DOS of samples discussed in this paper were calculated using the Kohn-Korringa-Rostoker (KKR) Green's function formalism under the coherent potential approximation (CPA). The experimentally obtained lattice constants were taken into account for the calculations. Furthermore, the VIENNA AB INITIO SIMULATION PACKAGE (VASP [36]) was applied for calculating the band structure and DOS of relaxed Fe₂VAl and Fe₂TaAl using the standard GGA-PBE functional for the exchange-correlation term. Regarding Fe₂TaAl, a standard nonrelativistic calculation of the band structure and one taking into account spin-orbit coupling were performed. A plane-wave basis cutoff energy of 500 eV for Fe₂VAl and 600 eV for Fe₂TaAl were taken into consideration for the calculations. After structural relaxation, lattice parameters of 5.691 Å for Fe₂VAl and 5.903 Å for Fe₂TaAl were obtained, which were used for the band structure and DOS calculations. The DOS was calculated on a grid point density of 1000 eV^{-1} using the tetrahedron method with Blöchl corrections.

III. RESULTS

A. Crystal structure

X₂YZ full-Heusler systems crystallize in the fully ordered Cu₂MnAl-type crystal structure which can be interpreted as four interpenetrating fcc sublattices. However, antisite disorder is a common phenomenon in Fe₂VAl-based compounds and might change the cubic crystal structure with respect to their space groups [17,37,38]. As can be seen in Fig. 3, all samples show the typical Heusler-type $(L2_1)$ structure pattern with no impurity peaks being visible. A more detailed study on the solubility limit and XRD patterns in this series of Heusler compounds can be found elsewhere [39]. The diminished intensity of the (111) Bragg peak in the diffractogram is a strong indication for a CsCl-type disorder between V and Al atoms, which share a similar surrounding in the crystal lattice. The nominal composition of the undoped sample is therefore better described as $Fe_2V_{1-x_d}Al_{x_d}Al_{1-x_d}V_{x_d}$. Rietveld refinements allow us to obtain a rough approximation for the fractional disorder concentration, $x_d = 0.3 \pm 0.1$. The lattice parameters in Fig. 3 match previously reported values and decrease with increasing the Al/Si substitution but increase linearly with the V/Ta substitution [2,4,40].

B. Electronic structure

Conventional DFT-based calculations implementing a local density approximation (LDA) or general gradient approximation (GGA) exchange correlation functional have predominantly referenced the electronic band structure of Fe₂VAl as a semimetal with an indirect negative band gap, meaning an overlap of bands at different high symmetry points in k-space. As illustrated in Fig. 1(a), a threefold degenerate hole pocket lies near $E_{\rm F}$ at the Γ point and a sixfold degenerate electron half pocket is located at the X point of the first Brillouin zone. Those bands show a highly dispersive character. While the valence band is dominated by Fe states, the conduction band is controlled by V states. GGA calculations using VASP were also conducted for a complete substitution of Ta on the V site of the L2₁ Heusler structure. The electronic band structure and DOS are shown in Figs. 1(b) and 2(b), respectively. A complete opening of the band gap can be observed, as the dispersive Ta states are located well above the Fermi energy. Taking into account spin-orbit coupling for Fe2TaAl reveals slight modifications in the electronic band structure with the most notable being a sizeable splitting of the X valence bands. The energy derivative of the Fermi-Dirac distribution, added in Fig. 1, shows that the energy range which heavily contributes to electronic transport is confined around the highly dispersive Γ valence bands and the X conduction band in Fe₂VA1. Decreasing the bipolar conduction by opening the pseudogap should thus have a very promising effect on the Seebeck coefficient. To understand the partial substitution effect on the electronic structure, alloy-averaged DOS were calculated for all $Fe_2V_{1-x}Ta_xAl_{1-y}Si_y$ -type samples synthesized and measured in this paper. The Kohn-Korringa-Rostoker coherent potential approximation method (KKR-CPA) was applied, taking into account the experimentally derived room-temperature lattice parameters. Figure 4(a) shows the rigid-band-like shift of E_F from the valence band edge toward the steep slope of the conduction band caused by the Al/Si substitution. Figure 4(b) shows a magnified image of the band gap, where the DOS is governed by the previously mentioned dispersive bands. It should be noted that for Fe₂V_{1-x}Ta_xAl the Fermi energy is located at the tail of the Γ valence bands but much farther in the conduction band for Fe₂V_{1-x}Ta_xAl_{1-y}Si_y, where the total DOS has a steep slope and is about two orders of magnitude larger.

C. Electronic transport

1. Seebeck coefficient

Figures 5(a) and 5(b) show the temperature-dependent Seebeck coefficient S from 4 K up to 800 K for various concentrations x of $Fe_2V_{1-x}Ta_xAl$ (p-type samples) and of $Fe_2V_{1-x}Ta_xAl_{0.9}Si_{0.1}$ (*n*-type samples), respectively. The sign of S changes upon substituting Si on the Al site due to the rigid-band-like shift of $E_{\rm F}$ into the conduction band [see Fig. 4(a)], resulting in a change of predominant charge carrier type from holes to electrons. It can be further seen that the maximum values of the Seebeck coefficient S_{max} in Fig. 4(a) increase with Ta concentration as a consequence of the band gap increase. Moreover, the low-temperature slope of S(T), which corresponds inversely to the carrier concentration in the single band picture, remains almost the same for all *p*-type samples but gradually increases with x in the *n*-type samples. This is in accordance with the opening of the band gap as demonstrated in Figs. 4(b) and 4(c). The experimental data are analyzed within a parabolic two-band (2PB) and three-band (3PB) model (dashed and solid lines), which will be reviewed in detail in Sec. V.

2. Electrical resistivity

Figure 6 summarizes the electrical resistivity of all samples discussed in this paper. A drastic change upon Al/Si substitution toward a more metalliclike nature at low temperatures is observed, which is overlain by a semiconducting contribution at higher temperatures. This is in accordance with previous observations on $Fe_2VAl_{1-\nu}Si_{\nu}$ [1,4]. As the isoelectronic substitution of V/Ta does not add any excess valence electrons, a semiconductorlike behavior of Fe₂VAl that has been reported in the literature [5,41] remains for the *p*-type samples. It can be seen that the residual resistivity ρ_{4K} strongly decreases in Fe₂V_{1-x}Ta_xAl as x goes from 0 toward 0.1, whereas ρ_{4K} even slightly increases with Ta doping in the *n*-type samples $Fe_2V_{1-x}Ta_xAl_{0.9}Si_{0.1}$. To investigate this peculiar effect, we examined the Hall carrier concentration and mobility in a temperature range from 4 K up to 520 K. Such study can help to microscopically understand the resistivity behavior and is still lacking in the literature for this class of materials.



FIG. 4. Energy-dependent DOS (KKR-CPA) of $Fe_2V_{1-x}Ta_x$ Al_{1-y}Si_y at the respective experimental volume. The first panel (a) demonstrates a rigid-band-like shift of the Fermi energy caused by the Al/Si substitution, while panels (b) and (c) show an opening of the pseudo band gap toward the conduction band side, which is caused by the V/Ta substitution.



FIG. 5. Temperature-dependent Seebeck coefficient of (a) $Fe_2 V_{1-x}Ta_xAl$ and (b) $Fe_2V_{1-x}Ta_xAl_{0.9}Si_{0.1}$ for various concentrations of *x*. Dashed and solid lines represent least-squares fits calculated within a parabolic two-band (2PB) and three-band (3PB) model, respectively. Details are explained in the text.

3. Hall mobility and carrier concentration

The Hall mobility $\mu_{\rm H}$ and carrier concentration $n_{\rm H}$ were evaluated from measurements of the Hall resistivity $\rho_{\rm xy}$ in the linear response regime (low magnetic field limit). The magnetic field dependence of the Hall resistivity of Fe₂V_{0.95}Ta_{0.05}Al and Fe₂V_{0.95}Ta_{0.05}Al_{0.9}Si_{0.1} is shown in the Supplemental Material [42]. With some exceptions, a linear magnetic field dependence was observed for all temperatures. While, for Fe₂V_{1-x}Ta_xAl, $\rho_{\rm xy}$ is always positive, which implies hole-dominant transport, Si-doped samples show a negative Hall effect, corresponding to electron dominant transport. This is consistent with our measurements of the



FIG. 6. Temperature-dependent electrical resistivity of Fe_2V_{1-x} Ta_xAl_{1-y}Si_y. Dashed lines represent least-squares fits as explained in the text.

Seebeck coefficient. Despite the linear Hall effect, bipolar transport should be expected in these compounds, as suggested by the temperature-dependent behavior of both the Seebeck coefficient and electrical resistivity. Therefore, the mobilities and carrier concentrations shown in this paper merely represent the interplay of electron and hole contributions, much like the Seebeck coefficient.

Inspecting Figs. 7 and 8, it can be seen that both $n_{\rm H}(T)$ and $\mu_{\rm H}(T)$ display very different temperature-dependent behaviors for the *p*-type and *n*-type samples. While $n_{\rm H}(T)$ is almost constant up to 300 K in Si-doped samples, typical for metals, a steep increase of the carrier concentration at low temperatures is found for Fe₂V_{1-x}Ta_xAl, presumably a measurement artifact due to the anomalous Hall effect. The absolute values of the carrier concentration are about one order of magnitude higher in the *n*-type samples, causing the drastic decrease of ρ mentioned above.

The Hall mobilities shown in Fig. 8 demonstrate distinct differences between *p*-type and *n*-type samples. While for the latter a monotonic decrease with increasing temperatures is observed over the whole temperature range, $\mu_{\rm H}(T)$ becomes almost constant for the former and even increases from 100 to 300 K. In addition, $\mu_{\rm H}(T)$ increases with Ta doping for the *p*-type samples but is reduced for the *n*-type samples. We aim to rationalize this utterly different behavior in Sec. V, using the 2PB parameters, as derived from the Seebeck coefficient modeling, to account for multiband contributions.



FIG. 7. Temperature-dependent charge carrier concentration of (a) $Fe_2V_{1-x}Ta_xAl$ and (b) $Fe_2V_{1-x}Ta_xAl_{0.9}Si_{0.1}$.

In conclusion, the temperature-dependent electrical resistivity of the *n*-type samples is mostly dictated by $\mu(T)$ up to 300 K, resulting in a metalliclike behavior, due to the almost constant charge carrier density $n_{\rm H}(T)$. The distinct increase of the carrier concentration above room temperature explains the semiconductorlike contribution to $\rho(T)$ in Si-doped samples at higher temperatures. Contrarily, Fe₂V_{1-x}Ta_xAl shows a weak temperature dependence of the mobility above 100 K, indicating that electron scattering might be fundamentally different in these compounds compared to the former.

IV. MODELING OF ELECTRONIC TRANSPORT

The measured experimental data in Sec. III will be analyzed by means of different phenomenological multicarrier transport models. Since the temperature-dependent Seebeck



FIG. 8. Temperature-dependent Hall mobility of $Fe_2V_{1-x}Ta_x$ $Al_{1-y}Si_y$.

coefficient is more sensitive toward the electronic band structure than the electrical resistivity or mobility, which also heavily depends on the carrier scattering, we started our analysis by least squares fitting S(T) in a wide temperature range.

The total Seebeck coefficient with contributions from an arbitrary number of bands can be expressed by its respective single-band contributions S_i , which have to be weighted by the respective single-band electrical conductivities σ_i ,

$$S_{\text{tot}} = \frac{\sum_{i} N_{v,i} S_i \sigma_i}{\sum_{i} N_{v,i} \sigma_i},\tag{1}$$

where $N_{v,i}$ are the band degeneracies and *i* is the parabolic band index. In the case of two bands, and by making the substitution $\sigma_i^* := m_i \sigma_i$, the total Seebeck coefficient can be written as

$$S_{\text{tot}} = \frac{S_1 \sigma_1^* + \varepsilon S_2 \sigma_2^*}{\sigma_1^* + \varepsilon \sigma_2^*},\tag{2}$$

where we defined ε as

$$\varepsilon := \frac{N_{v,2}}{N_{v,1}} \frac{m_1}{m_2}.$$
(3)

In this notation, both $S_{1,2}$ and $\sigma_{1,2}^*$ only depend on the reduced chemical potential $\eta_i = E_{\mathrm{F},i} - E_{\mathrm{g},i}$ (i = 1, 2). Furthermore, it can be quickly seen that the total temperaturedependent Seebeck coefficient does neither depend on the absolute values of the degeneracies nor the effective masses, but only on their mutual relationship ε . The distance of the Fermi level toward the first band ΔE_{F} , the band gap E_{g} as well as ε were set as the fitting parameters of our model. We presumed $m_1 = m_e$ and $N_{v,1} = N_{v,2}$. These *ad hoc* assumptions will not influence the fitted band gap and Fermi level, but only the conduction band mass m_2 , which can be quickly recalculated through ε when different *ad hoc* parameters are guessed. All of the fitting parameters have a unique effect on S(T). Thus, when modeling S(T) in a wide temperature range, unambiguous results are assured. After using our experimental Seebeck data in this spectroscopic way, we proceeded by attempting to model the electrical resistivity and Hall mobility, taking into account the very same band parameters to test the validity of the model and extract further microscopic information regarding carrier scattering mechanisms. The total Hall mobility for carrier transport with multiband contributions can be modeled by weighting the single-band mobilities μ_i with the respective carrier densities n_i and charges q_i ($q = \pm e$ for holes and electrons, respectively):

$$\mu_{\rm H,tot} = \frac{\sum_{i} N_{v,i} q_i |n_i| \mu_i^2}{\sum_{i} N_{v,i} q_i |n_i| \mu_i}.$$
(4)

The single-band mobilities μ_i are negative for electrons and positive for holes like the Seebeck coefficient, following the usual notation. The following section will discuss the model results and aim to rationalize the experimental data, presented in the previous section.

V. DISCUSSION

A. Seebeck coefficient and band gap

The Seebeck coefficient heavily depends on the energy dependence of the charge carrier mobility $\mu(E)$ and the carrier density n(E) = N(E)f(E), where N(E) is the electronic DOS and f(E) is the Fermi-Dirac distribution. As an approximation, this relationship between *S*, μ , and *n* can be expressed by the well-known Mott formula [43]:

$$S = \frac{\pi^2}{3} \frac{k_{\rm B}^2}{-e} T\left(\frac{1}{n(E)} \frac{\partial n(E)}{\partial E} + \frac{1}{\mu(E)} \frac{\partial \mu(E)}{\partial E}\right)\Big|_{E_{\rm F}}.$$
 (5)

Band-structure calculations predict that the substitution of V by Ta leads to slight modifications of the DOS above $E_{\rm F}$, resulting in an opening of the pseudogap toward the conduction band side (see Fig. 4). To substantiate this claim, we employed the parabolic band (PB) model, whose applicability for this class of materials was demonstrated in a recent study by Anand et al. [19]. While the analyses in terms of the 2PB and 3PB model (see the Supplemental Material, Sec. 3 [42]) of the temperature-dependent Seebeck coefficient of Fe₂VAl reveal a small band gap of $E_g = -0.03 - 0.002$ eV, E_g increases up to 0.08 - 0.1 eV in Fe₂V_{0.9}Ta_{0.1}Al. In the following, we will demonstrate that the enhancement of S due to the V/Ta substitution is best described by this increase of the band gap. Therefore, by simulating the band structure with parabolic bands, a relation between the temperature-dependent behavior of S and E_g is established.

At low temperatures, where S(T) remains almost linear and is mostly governed by the influence of a single parabolic band, one can write

$$|S| = \frac{\pi^2 k_{\rm B}^2}{2e} \frac{T}{E_{\rm F}^{\rm sb}}.$$
 (6)



FIG. 9. Schematic sketch of the V/Ta substitution effect on the electronic band structure and band gap of Fe₂VAl-based (a) *p*-type and (b) *n*-type samples. The energy scaling is in arbitrary units.

Equation (6) infers that the slope $\partial S/\partial T$ is inversely proportional to the effective single-band Fermi level $E_{\rm F}^{\rm sb}$, where $E_{\rm F}^{\rm sb}$ denotes the distance from the conduction band bottom E_{\min}^{c} or from the valence band top E_{\max}^{v} to the Fermi energy $E_{\rm F}$ [compare Figs. 9(a) and 9(b)]. Figure 5(a) shows the maximum of S(T) of $\text{Fe}_2 V_{1-x} \text{Ta}_x \text{Al}$ is shifted toward lower temperatures, as x increases from 0 toward 0.05, corresponding to a reduction of $E_{\rm F}^{\rm sb}$. This effect can easily be understood when looking at the calculated band structures of Fe₂VAl and Fe₂TaAl in Fig. 1; it is also illustrated in the left panel of Fig. 9(a). The *d* states of the dispersive Ta conduction band are located well above $E_{\rm F}$. Therefore, the V/Ta substitution causes electrons to occupy free states in the dispersive Fe valence band, temporarily raising $E_{\rm F}$ and lowering $E_{\rm F}^{\rm sb}$. This causes a shift of S_{max} toward lower temperatures as the Ta content x goes from 0 toward 0.05. However, the slope $\partial S/\partial T$ remains almost equal up to S_{max} for Fe₂V_{1-x}Ta_xAl as x goes from 0.05 toward 0.1. This is another strong indication for the postulated band gap increase, where the conduction band bottom surpasses $E_{\rm F}$ [sketched in the middle and right panel of Fig. 9(a)]. With the Fermi level below the conduction band edge, an increase of $E_{\rm g}$ will no longer cause a significant variation of $E_{\rm F}^{\rm sb}$, thus yielding a similar slope of S(T).

For Fe₂V_{1-x}Ta_xAl_{0.9}Si_{0.1} (*n*-type samples) shown in Fig. 5(b), the temperature-dependent Seebeck coefficient is more complicated and will be discussed in the following. Opposed to the *p*-type samples, the Fermi level is now located close to a flat nonparabolic Fe band [Fig. 9(b)], characterized by a steep incline in the electronic DOS [Fig. 4(a)]. Least-squares fits in terms of the 2PB model yield band gaps



FIG. 10. Temperature and band gap dependence of the Seebeck coefficient for a fixed position of the Fermi energy $E_{\rm F}$ inside the valence and conduction band; calculated within a parabolic two-band model.

 $E_g = 0.12$, 0.14, 0.16 eV for x = 0, 0.025, 0.05, respectively, contradicting the sometimes claimed rigid-band-like shift of E_F in this series of compounds [1,4]. Even though the absolute value of E_g may differ from the proper value for the *n*-type samples, the tendency of the band gap increase with Ta substitution is still apparent. Furthermore, the maxima of S(T), $(T_{\text{max}}^S, S_{\text{max}})$ shift toward lower temperatures, in contrary to what one would intuitively expect for an increasing band gap. The explanation to that effect requires a combination of two distinct phenomena [see Fig. 9(b)]:

(i) $E_{\rm F}$ is almost fixed in its position.

(ii) $E_{\rm g}$ increases between the two parabolic bands.

While (ii) is effectively caused by the Ta substitution as previously established, (i) is caused by the presence of the strongly localized flat Fe band that provides an abundance of free states for the electrons, which have been removed from below $E_{\rm F}$. Accordingly, the shift of $E_{\rm F}$ is significantly slowed down by this barrier and $E_{\rm F}^{\rm sb}$ decreases. This explains the continuously increased slope of S(T) with increasing the Ta content as observed in Fig. 5(b).

Employing the PB model, the temperature-dependent Seebeck coefficient is sketched in Fig. 10, highlighting the dependence of S_{max} and T_{max}^S if the band gap E_g is varied $(0.04 \le E_g \le 0.13 \text{ eV})$. Here, the model calculations assume a fixed position of E_F inside the valence and conduction band for *p*- and *n*-type samples, respectively. To summarize, the values of S_{max} are shifted toward higher temperatures in the case of *p* type and lower temperatures for *n*-type samples; curves calculated in terms of the PB model reveal convincing agreement with the experimental data.

Values of the band gap have also been approximated using the Goldschmid-Sharp formula [44],

$$E_{\rm g}^{\rm GS} = 2e|S_{\rm max}|T_{\rm max}^S,\tag{7}$$

and are compared to the values obtained by the PB model in Table I.

TABLE I. Band gaps and maximum Seebeck coefficients of $Fe_2V_{1-x}Ta_xAl_{1-y}Si_y$.

	x	$E_{\rm g}^{\rm GS}$ [eV]	$E_{\rm g}^{\rm 2PB}$ [eV]	S [µV/K]
y = 0	0 0.05 0.1	0.02 0.04 0.07	0.002 0.02 0.1	75 90 120
y = 0.1	0 0.025 0.05	0.08 0.09 0.1	0.12 0.14 0.16	$-120 \\ -140 \\ -170$

Figure 11 shows the Seebeck coefficient at T = 300 K versus the chemical carrier concentration for the majority of substitution studies from literature [1-5,7,23,25,45-50], together with data derived in the present paper. The opening of the band gap as shown in this paper leads to the largest obtained *p*-type and *n*-type values of the Seebeck coefficient for stoichiometric Fe₂VAl-based samples so far. Besides the band-gap modifications, an increase of the effective mass is likely in Fe₂V_{1-x}Ta_xAl and in Fe₂V_{1-x}Ta_xAl_{0.9}Si_{0.1} due to the band convergence in the conduction band, resulting in a shift of *S*_{max} toward higher carrier concentrations.

Figure 12 summarizes the opening of the band gap with increasing Ta content in Fe₂V_{1-x}Ta_xAl, consistently predicted by various phenomenological models and *ab initio* calculations. The band gap extracted from the VASP calculations is in good agreement with the parabolic band model if the standard GGA functional is replaced by an LDA+U approach (U = 2.145 eV) in the calculations [35].

In conclusion, the effect of the V/Ta substitution on thermoelectric transport is best described by band convergence in the conduction band and an increase of the band gap, which

FIG. 11. Seebeck coefficient versus carrier concentration for different concentrations of Ta in $Fe_2V_{1-x}Ta_xAl$, compared with data from literature. The solid lines are calculated within a parabolic two-band model for varying band gaps and effective masses.

FIG. 12. Concentration-dependent increase of the band gap of $Fe_2V_{1-x}Ta_xAl$ obtained from various phenomenological models and DFT calculations.

has a unique impact on the temperature-dependent Seebeck coefficient. The various features observed are well accounted for within a PB model. This model also yields good quantitative agreement with *ab initio* calculations based on the KKR-CPA method as well as the VASP code if an enhanced exchange correlation (LDA+U with U = 2.145 eV [35]) is considered.

B. Electrical resistivity

In general, the electrical resistivity within the parabolic band model is related to band-specific and band-independent parameters and can be written as

$$\rho(T) = \frac{1}{\sum_{i} \sigma_{i}(T)} = \frac{1}{KT^{\beta+3/2} \sum_{i} \frac{1}{m_{i}^{*}} F_{0}(\eta_{i}, T)},$$
 (8)

where *K* is a constant prefactor and β is a coefficient for the temperature-dependent mobility, independent of the band structure. In the case of acoustic phonon scattering $\beta = -\frac{3}{2}$, whereas $\beta = -\frac{1}{2}$ for alloy-disorder scattering. $F_0(\eta_i, T)$ is the Fermi integral of zeroth order for each band depending on the reduced chemical potential. Both have been obtained from the above-mentioned Seebeck coefficient modeling and are characteristic band parameters. Varying only the bandindependent parameters *K* and β in Eq. (8), the fits obtained in Fig. 13 show fairly good agreement with the measured high-temperature resistivity. This corroborates the previously drawn picture of band structure modifications rendered in Fig. 9.

The experimentally obtained low temperature resistivity data, however, can hardly be understood within this model, since a more detailed consideration of the electron-phonon interaction is necessary. As successfully applied in several previous studies [25,51], the Bloch-Grüneisen formula can be combined with a temperature-dependent carrier density for modeling $\rho(T)$ of such materials. Results of least-squares

FIG. 13. Temperature-dependent electrical resistivity above 300 K for (a) $Fe_2V_{1-x}Ta_xAl$ and (b) $Fe_2V_{1-x}Ta_xAl_{0.9}Si_{0.1}$ modeled with parabolic band parameters obtained from fitting the temperature-dependent Seebeck coefficient.

fits are shown as dashed lines in Fig. 6, revealing a reasonable estimation of parameters such as the Debye temperature $\Theta_{\rm D}$. The latter can be used to calculate the mean sound velocity v_s . The values obtained for β , Θ_D , and v_s are summarized in Table II. Considering the values of β as well as the temperature-dependent behavior of the Hall mobility, we suggest that the carrier scattering mechanism in the ptype compounds is inherently different than for the *n*-type samples and cannot be understood by simple electron-phonon scattering. Thus, absolute values for the Debye temperatures and sound velocities are questionable and shown in brackets. For the *n*-type samples, where a considerable contribution of acoustic phonon scattering can be expected, more meaningful results are obtained. However, the Debye temperatures are still slightly overestimated compared to the directly measured values from literature [52,53], which showcases the limits

TABLE II. Parameters derived from modeling the electrical resistivity of $\text{Fe}_2\text{V}_{1-x}\text{Ta}_x\text{Al}_{1-y}\text{Si}_y$. The absolute values for the fitting parameters shown in brackets are questionable, as explained in the text.

	x	β	$\Theta_{\rm D} [{\rm K}]$	<i>v</i> _s [m/s]
y = 0	0 0.05	-0.5 -0.4	(803) (700)	(6250) (5510)
	0.1	-0.3	(598)	(4760)
y = 0.1	0 0.025 0.05	-1.8 -1.7 -1.6	704 637 514	5540 5015 4050

of the phenomenological models in the presence of anti-site defects and disorder, which notably influence carrier transport even in the extrinsic *n*-type samples.

The sound velocities and Debye temperatures decrease and the lattice becomes less stiff with Ta doping. The tendency of a softening of the crystal lattice is further substantiated from an evolution of the deformation potential. Considering again the parabolic band model, it can be seen that the fit parameter β is closer to the alloy-disorder scattering limit in Fe₂V_{1-x}Ta_xAl (*p* type), while it is closer to acoustic phonon scattering in Fe₂V_{1-x}Ta_xAl_{0.9}Si_{0.1} (*n* type). Even in undoped Fe₂VAl, disorder scattering seems to play an important role, which might be due to intrinsic antisite defects, as suggested by the low x-ray intensity of the (111) Bragg peak in Fig. 3 and also a large negative magnetoresistance at low temperatures (Supplemental Material).

C. Mobility and scattering potentials

The impact of the charge carrier interaction with longwavelength acoustic phonons is proportional to the deformation potential Ξ_{ph} , which, in first approximation, describes the electronic band structure modifications due to collective lattice vibrations. Here, we treated Ξ_{ph} as a band-independent fitting parameter that relates to the single-band mobility for acoustic phonon scattering [54]

$$\mu_{\mathrm{ph},i} = \frac{\sqrt{2\pi}e\hbar^4}{3(k_{\mathrm{B}}T)^{3/2}} \frac{v_1^2 d_{\mathrm{m}}}{\Xi_{\mathrm{ph}}^2} \frac{F_0(\eta_i)}{F_{1/2}(\eta_i)m_i^{*5/2}},\tag{9}$$

where $d_{\rm m}$ is the material density and $v_{\rm l}$ is the longitudinal sound velocity, which was reasonably approximated to the mean sound velocity $v_{\rm s}$ by $v_{\rm l} = 1.53 v_{\rm s}$. On the other hand, the alloy-disorder scattering constant $U_{\rm d}$ denotes the local potential perturbations caused by atomic disorder and neutral impurities. It relates to the mobility $\mu_{\rm dis}$ via the Brooks formula [55],

$$\mu_{\text{dis},i} = \frac{16e\hbar^4}{9\sqrt{2}(k_{\text{B}}T)^{1/2}} \frac{N_0}{(x_{\text{d}} - x_{\text{d}}^2)U_{\text{d}}^2} \frac{F_0(\eta_i)}{F_{1/2}(\eta_i)m_i^{*5/2}}, \quad (10)$$

with N_0 being the number of atoms per unit volume. For the *p*-type samples, we considered another scattering mechanism with ionized impurity scattering, where the mobility increases

with temperature:

$$\mu_{\text{ion},i} \propto T^{3/2} \frac{F_0(\eta_i)}{F_{1/2}(\eta_i)}.$$
(11)

The Mathiessen rule can be used for multiple scattering mechanisms by summing up the reciprocal values of independent scattering times. Thus, the temperature-dependent mobility for combined scattering mechanisms is given by

$$\mu_{\text{comb},i}(T) = \frac{\mu_{\text{ph},i}(T)\,\mu_{\text{dis},i}(T)\,\mu_{\text{ion},i}(T)}{\mu_{\text{ph},i}(T) + \mu_{\text{dis},i}(T) + \mu_{\text{ion},i}(T)}.$$
(12)

For the total mobility from two or more bands, the mobility has to be weighted with the respective carrier densities n_i as described in Sec. IV. This allows us to model the temperaturedependent Hall mobility and extract information about the dominant scattering mechanisms that limit carrier transport.

In Fig. 14(a), the temperature-dependent mobility is modeled using Eqs. (9) and (10). For Fe₂VAl, both acoustic phonon and disorder scattering can describe the data reasonably well only above room temperature. As the Ta concentration in $Fe_2V_{1-x}Ta_xA1$ is increased from 0 to 0.05, only disorder scattering traces the temperature-dependent mobility fairly well, while pure phonon scattering misses the respective temperature dependence. Below room temperature, both of these interactions fail to account for the experimentally obtained mobility data. This suggests that the scattering mechanism in these samples might be fundamentally different and further interaction processes need to be taken into consideration. A recent investigation of the charge transfer in undoped Fe₂VAl, derived by Bader's approach, suggests that all three atoms in the ternary compound become ionized, as a significant charge transfer from Al and V to the neighboring Fe atoms occurs [35]. Hence, additionally taking ionized impurity scattering into account, Eq. (11) indeed allows for a much better fitting in the whole temperature range above 100 K [solid lines, Fig. 14(a)]. Therefore, we presume that ionized and/or magnetic antisite defects might be the cause for the total failure of Eqs. (9) and (10) to describe the experimental mobility data below 300 K. As the exact influence of antisite disorder and phonon scattering is unclear in the present *p*-type samples, a quantitative description about the scattering potentials U_d , Ξ_{ph} and their evolution with V/Ta substitution is hardly feasible. Despite the softening of the lattice as well as the induced atomic disorder by heavy Ta atoms, the electron mobility consistently increases in $Fe_2V_{1-x}Ta_xAl$ for $x = 0 \rightarrow 0.05$ and even for the sample with x = 0.1, which is not shown here.

The *n*-type samples $Fe_2V_{1-x}Ta_xAl_{0.9}Si_{0.1}$, on the other hand, exhibit an opposite behavior and the Ta-doped sample shows lower values of μ_H in the whole temperature range. We suppose that the band convergence and increase of effective mass mentioned in Fig. 9 are most likely responsible for this effect, together with the increased atomic disorder upon alloying.

Figure 14(b) reveals a monotonic decrease of $\mu_{\rm H}$ with temperature for both *n*-type samples. Most convincing least-squares fits are revealed, assuming only phonon scattering to be the limiting scattering mechanism for the mobility. This allows us to evaluate the phonon deformation potential. Using a valence band mass of am_e ($a \in \mathbb{R}$), the deformation po-

FIG. 14. Temperature-dependent Hall mobility of (a) $Fe_2V_{1-x}Ta_xAl$ and (b) $Fe_2V_{1-x}Ta_xAl_{0.9}Si_{0.1}$; the data was least squares fitted within a parabolic two-band model assuming a valence band mass of $|m_{VB}| = m_e$. Various scattering mechanisms were considered for the *p*-type samples (a). A quantitative analysis is only possible for the *n*-type samples (b), where a much simpler behavior of the mobility is observed that can be well described by acoustic phonon scattering in a wide temperature range 100 < T < 500 K.

tential decreases from $\Xi_{\rm ph} = 3.6|a|^{5/2}$ to $1.9|a|^{5/2}$ eV upon increasing the Ta concentration from x = 0 up to 0.05. This is a substantial reduction of the electron-phonon interaction by more than 47%. The decrease in $\Xi_{\rm ph}$ almost compensates the decrease of the sound velocities and the disorder from

alloying as well as the increasing conduction band mass, yielding comparable values of $\mu_{\rm H}$ for the *n*-type samples above room temperature. In a recent work on Fe₂V_{1-x}W_xAl bulk systems, we showed that the experimentally obtained electron mobility increases significantly as *x* goes from 0.05

FIG. 15. Temperature-dependent power factor $S^2\sigma$ times temperature for several high performance *n*-type materials from literature and the present paper.

up to 0.2, against all common expectations that the mobility should decrease due to the decreased sound velocities and substantial disorder caused by the V/W substitution [25]. Consistent with our current interpretation, this might also be attributed to a reduction of the phonon deformation potentials. If similar tendencies can be likewise observed for other 5*d* heavy elements, this might pave the way for a new strategy of increasing the thermoelectric performance in Fe₂VA1-based compounds.

D. Thermoelectric performance

Recently, it has been outlined that a larger value of the thermoelectric power factor (PF) is more important for its contribution to a given value of the figure of merit, ZT, than a reduction of thermal conductivity [56]. For the latter, many efforts like nanostructuring, providing scattering centers of all length scales, or other approaches have already been undertaken [30,57–60]. Hence, it is of great interest to find materials with high power factors. Full-Heusler compounds are already known for their intrinsically large PF. In this paper, we established a significant improvement of the Seebeck coefficient due to an increase of the band gap upon an increasing Ta content in $Fe_2V_{1-x}Ta_xAl$ and $Fe_2V_{1-x}Ta_xAl_{0.9}Si_{0.1}$ as shown in Fig. 12. Furthermore, the decreased deformation potentials and Debye temperatures suggest a weakened electron-phonon interaction. This allows for high values of the mobility, despite the disorder created by elemental substitution. Ultimately, the power factor is further enhanced up to 10.3 mW/mK² for $Fe_2V_{0.95}Ta_{0.05}Al_{0.9}Si_{0.1}$, which is the highest value ever reported in this class of bulk materials. We compare our two best performing samples with various *n*-type materials from literature reported for high thermoelectric performance [61-71]. As suggested by Dehkordi et al. [72], it is advantageous to multiply PF with the absolute temperature, when comparing PF for different materials. Such a plot is shown in Fig. 15. It can be seen that samples from the present paper significantly outperform all other materials around room temperature and are only surpassed by the intermediate valent system YbAl₃, which has the highest power factor due to its very low resistiv-

- H. Kato, M. Kato, Y. Nishino, U. Mizutani, and S. Asano, Effect of silicon substitution on thermoelectric properties of Heuslertype Fe₂VAl alloy, Nippon Kinzoku Gakkaishi (1952) 65, 652 (2001).
- [2] M. Vasundhara, V. Srinivas, and V. Rao, Low-temperature electrical transport in Heusler-type Fe₂V(AlSi) alloys, J. Phys.: Condens. Matter 17, 6025 (2005).
- [3] Y. Nishino, S. Deguchi, and U. Mizutani, Thermal and transport properties of the Heusler-type $\text{Fe}_2 \text{VAI}_{1-x} \text{Ge}_x$ ($0 \le x \le 0.20$) alloys: Effect of doping on lattice thermal conductivity, electrical resistivity, and Seebeck coefficient, Phys. Rev. B 74, 115115 (2006).
- [4] C. S. Lue, C. Chen, J. Lin, Y. Yu, and Y. Kuo, Thermoelectric properties of quaternary Heusler alloys Fe₂VAl_{1-x}Si_x, Phys. Rev. B **75**, 064204 (2007).
- [5] M. Vasundhara, V. Srinivas, and V. Rao, Electronic transport in Heusler-type Fe₂VAl_{1-x}M_x alloys (M= B, In, Si), Phys. Rev. B 77, 224415 (2008).

ity of $\rho_{300K} \approx 45 \,\mu\Omega$ cm. Compared to the archetypal Bi₂Te₃ compound, for example, $S^2 \sigma T$ is about 3–4 times higher in our samples. Estimations on the global waste heat potential [73] have suggested that the majority of waste heat (~ 63%) arises at temperatures below 100 °C, which makes the material presented in this paper extremely attractive for thermoelectric energy applications.

VI. CONCLUSION

In summary, we investigated the V/Ta substitution effect with respect to the electronic structure and temperaturedependent electronic transport properties such as the electrical resistivity, Seebeck coefficient, mobility, and carrier concentration in Fe₂V_{1-x}Ta_xAl_{1-y}Si_y-based samples. DFT calculations and phenomenological models with respect to the experimental data unambiguously conclude an opening of the pseudo or small indirect band gap in terms of converging conduction bands. This results in an improvement of the maximum Seebeck coefficients from 75 up to 120 and from -120up to $-170 \ \mu$ V/K for *p*- and *n*-type samples, respectively.

Additionally, the mobility of the predominant charge carriers remains high, despite the reduced sound velocities, increased conduction band mass, and disorder introduced by alloying. We suggest this results from the weakened electron-phonon interaction in terms of decreasing deformation potentials, caused by the heavy elemental substitution of Ta. It is argued that this peculiar behavior of the scattering potentials can retain low values of the electrical resistivity. Ultimately, an enhancement of the maximum power factor is achieved with PF reaching up to $PF_{max} \approx 10.3 \text{ mW/mK}^2$ near room temperature in Fe₂V_{0.95}Ta_{0.05}Al_{0.9}Si_{0.1}, which is among the highest ever reported values for any bulk material.

ACKNOWLEDGMENTS

Research in this paper was supported by the Japan Science and Technology Agency (JST) programs MIRAI, JPMJMI19A1. The authors also thank the Vienna Scientific Cluster (VSC) for providing their computational capabilities.

- [6] M. Mikami, S. Tanaka, and K. Kobayashi, Thermoelectric properties of Sb-doped Heusler Fe₂VAl alloy, J. Alloys Compd. 484, 444 (2009).
- [7] M. Mikami, Y. Kinemuchi, K. Ozaki, Y. Terazawa, and T. Takeuchi, Thermoelectric properties of Tungsten-substituted Heusler Fe₂VAl alloy, J. Appl. Phys. **111**, 093710 (2012).
- [8] T. Graf, C. Felser, and S. S. Parkin, Simple rules for the understanding of Heusler compounds, Prog. Solid State Chem. 39, 1 (2011).
- [9] Heusler Alloys, Properties, Growth, Applications, edited by C. Felser and A. Hirohata, Springer Series in Material Science (Springer-Verlag, Berlin, 2016), Vol. 222.
- [10] G. Guo, G. Botton, and Y. Nishino, Electronic structure of possible 3d 'heavy-fermion' compound, J. Phys.: Condens. Matter 10, L119 (1998).
- [11] D. Singh and I. Mazin, Electronic structure, local moments, and transport in Fe₂VAl, Phys. Rev. B 57, 14352 (1998).

- [12] R. Weht and W. Pickett, Excitonic correlations in the intermetallic Fe₂VAl, Phys. Rev. B 58, 6855 (1998).
- [13] M. Weinert and R. Watson, Hybridization-induced band gaps in transition-metal aluminides, Phys. Rev. B **58**, 9732 (1998).
- [14] A. Bansil, S. Kaprzyk, P. Mijnarends, and J. Toboła, Electronic structure and magnetism of Fe_{3-x}V_xX (X= Si, Ga, and Al) alloys by the KKR-CPA method, Phys. Rev. B 60, 13396 (1999).
- [15] G. Botton, Y. Nishino, and C. Humphreys, Microstructural evolution and stability of $(Fe_1 xV_x)_3Al$ alloys in relation to the electronic structure, Intermetallics 8, 1209 (2000).
- [16] H. Okamura, J. Kawahara, T. Nanba, S. Kimura, K. Soda, U. Mizutani, Y. Nishino, M. Kato, I. Shimoyama, H. Miura *et al.*, Pseudogap Formation in the Intermetallic Compounds (Fe_{1-x}V_x)₃Al, Phys. Rev. Lett. **84**, 3674 (2000).
- [17] Y. Feng, J. Rhee, T. Wiener, D. W. Lynch, B. Hubbard, A. Sievers, D. L. Schlagel, T. A. Lograsso, and L. Miller, Physical properties of Heusler-like Fe₂VAl, Phys. Rev. B 63, 165109 (2001).
- [18] H. Miyazaki, K. Soda, S. Yagi, M. Kato, T. Takeuchi, U. Mizutani, and Y. Nishino, Surface and bulk electronic structures of Heusler-type Fe₂VAl, J. Vac. Sci. Technol. A 24, 1464 (2006).
- [19] S. Anand, R. Gurunathan, T. Soldi, L. Borgsmiller, R. Orenstein, and G. J. Snyder, Thermoelectric transport of semiconductor full-Heusler VFe₂Al, J. Mater. Chem. C 8, 10174 (2020).
- [20] A. Berche, M. T. Noutack, M.-L. Doublet, and P. Jund, Unexpected band gap increase in the Fe₂VAl Heusler compound, Mater. Today Phys. 13, 100203 (2020).
- [21] Y. Nishino and Y. Tamada, Doping effects on thermoelectric properties of the off-stoichiometric Heusler compounds $Fe_{2-x}V_{1+x}Al$, J. Appl. Phys. **115**, 123707 (2014).
- [22] H. Miyazaki, S. Tanaka, N. Ide, K. Soda, and Y. Nishino, Thermoelectric properties of Heusler-type off-stoichiometric $Fe_2V_{1+x}Al_{1-x}$ alloys, Mater. Res. Express 1, 015901 (2013).
- [23] Y. Terazawa, M. Mikami, T. Itoh, and T. Takeuchi, Effects of heavy element substitution on electronic structure and lattice thermal conductivity of Fe₂VAl thermoelectric material, J. Electron. Mater. **41**, 1348 (2012).
- [24] T. Takeuchi, Y. Terazawa, Y. Furuta, A. Yamamoto, and M. Mikami, Effect of heavy element substitution and offstoichiometric composition on thermoelectric properties of Fe₂VAI-based Heusler phase, J. Electron. Mater. **42**, 2084 (2013).
- [25] B. Hinterleitner, P. Fuchs, J. Rehak, F. Garmroudi, M. Parzer, M. Waas, R. Svagera, S. Steiner, M. Kishimoto, R. Moser *et al.*, Stoichiometric and off-stoichiometric full Heusler $Fe_2V_{1-x}W_xAl$ thermoelectric systems, Phys. Rev. B **102**, 075117 (2020).
- [26] S. Choi, S. Hiroi, M. Inukai, S. Nishino, R. Sobota, D. Byeon, M. Mikami, M. Matsunami, and T. Takeuchi, Effects of partial substitution by 5d heavy elements on the thermal transport in Fe₂VAl thin films, Phys. Rev. B **101**, 104312 (2020).
- [27] S. Hiroi, S. Nishino, S. Choi, O. Seo, J. Kim, Y. Chen, C. Song, A. Tayal, O. Sakata, and T. Takeuchi, Phonon scattering at the interfaces of epitaxially grown Fe₂VAl/W and Fe₂VAl/Mo superlattices, J. Appl. Phys. **125**, 225101 (2019).
- [28] B. Hinterleitner, I. Knapp, M. Poneder, Y. Shi, H. Müller, G. Eguchi, C. Eisenmenger-Sittner, M. Stöger-Pollach, Y.

Kakefuda, N. Kawamoto *et al.*, Thermoelectric performance of a metastable thin-film Heusler alloy, Nature **576**, 85 (2019).

- [29] K. Kudo, S. Yamada, J. Chikada, Y. Shimanuki, T. Ishibe, S. Abo, H. Miyazaki, Y. Nishino, Y. Nakamura, and K. Hamaya, Significant reduction in the thermal conductivity of Sisubstituted Fe₂VAI epilayers, Phys. Rev. B **99**, 054201 (2019).
- [30] S. Masuda, K. Tsuchiya, J. Qiang, H. Miyazaki, and Y. Nishino, Effect of high-pressure torsion on the microstructure and thermoelectric properties of Fe₂VAl-based compounds, J. Appl. Phys. **124**, 035106 (2018).
- [31] H. Miyazaki, M. Inukai, and Y. Nishino, Effect of Ta substitution on the electronic structure of Heusler-type Fe₂VAl-based alloy, J. Appl. Phys. **120**, 125106 (2016).
- [32] H. Al-Yamani and B. Hamad, Thermoelectric properties of Fe₂VAl and Fe₂V_{0.75}M_{0.25}Al (M=Mo, Nb, Ta) alloys: Firstprinciples calculations, J. Electron. Mater. 45, 1101 (2016).
- [33] S. A. Khandy, I. Islam, D. C. Gupta, M. A. Bhat, S. Ahmad, T. A. Dar, S. Rubab, S. Dhiman, and A. Laref, A case study of Fe₂TaZ (Z=Al, Ga, In) Heusler alloys: Hunt for halfmetallic behavior and thermoelectricity, RSC Adv. 8, 40996 (2018).
- [34] R. Resel, E. Gratz, A. Burkov, T. Nakama, M. Higa, and K. Yagasaki, Thermopower measurements in magnetic fields up to 17 tesla using the toggled heating method, Rev. Sci. Instrum. 67, 1970 (1996).
- [35] B. Hinterleitner, F. Garmroudi, N. Reumann, T. Mori, E. Bauer, and R. Podloucky, The electronic pseudo band gap states and electronic transport of the full-Heusler compound Fe₂VAI, J. Mater. Chem. C (2021), doi:10.1039/D0TC05187J.
- [36] G. Kresse and J. Furthmüller, Efficient iterative schemes for ab initio total-energy calculations using a plane-wave basis set, Phys. Rev. B 54, 11169 (1996).
- [37] S. Maier, S. Denis, S. Adam, J.-C. Crivello, J.-M. Joubert, and E. Alleno, Order-disorder transitions in the Fe₂VAl Heusler alloy, Acta Mater. **121**, 126 (2016).
- [38] C. Venkatesh, V. Srinivas, V. Rao, S. Srivastava, and P. S. Babu, Effect of site disorder on the electronic properties of Fe₂VAI Heusler alloy, J. Alloys Compd. 577, 417 (2013).
- [39] F. Garmroudi, M. Parzer, A. Riss, N. Reumann, B. Hinterleitner, K. Tobita, Y. Katsura, K. Kimura, T. Mori, and E. Bauer, Solubility limit and annealing effects on the microstructure and thermoelectric properties of $Fe_2V_{1-x}Ta_xAl_{1-y}Si_y$ Heusler compounds, Acta Mater. (to be published, 2021).
- [40] K. Renard, A. Mori, Y. Yamada, S. Tanaka, H. Miyazaki, and Y. Nishino, Thermoelectric properties of the Heusler-type $Fe_2VTa_xAl_{1-x}$ alloys, J. Appl. Phys. **115**, 033707 (2014).
- [41] Y. Nishino, M. Kato, S. Asano, K. Soda, M. Hayasaki, and U. Mizutani, Semiconductorlike Behavior of Electrical Resistivity in Heusler-Type Fe₂VAl Compound, Phys. Rev. Lett. **79**, 1909 (1997).
- [42] See Supplemental Material at http://link.aps.org/supplemental/ 10.1103/PhysRevB.103.085202 for additional information regarding magnetotransport properties, Seebeck coefficient modeling as well as Rietveld refinements.
- [43] J. P. Heremans, V. Jovovic, E. S. Toberer, A. Saramat, K. Kurosaki, A. Charoenphakdee, S. Yamanaka, and G. J. Snyder, Enhancement of thermoelectric efficiency in PbTe by distortion of the electronic density of states, Science 321, 554 (2008).

- [44] H. Goldsmid and J. Sharp, Estimation of the thermal band gap of a semiconductor from Seebeck measurements, J. Electron. Mater. 28, 869 (1999).
- [45] E. J. Skoug, C. Zhou, Y. Pei, and D. T. Morelli, High thermoelectric power factor near room temperature in full-Heusler alloys, J. Electron. Mater. 38, 1221 (2009).
- [46] Y. Nishino, Development of thermoelectric materials based on Fe₂VAl Heusler compound for energy harvesting applications, IOP Conf. Ser.: Mater. Sci. Eng. 18, 142001 (2011).
- [47] M. Mikami, K. Ozaki, H. Takazawa, A. Yamamoto, Y. Terazawa, and T. Takeuchi, Effect of Ti substitution on thermoelectric properties of W-doped Heusler Fe₂VAl alloy, J. Electron. Mater. 42, 1801 (2013).
- [48] H. Nakayama, N. Ide, and Y. Nishino, Thermoelectric properties of p-type Heusler compounds $(Fe_{2-x}Co_x)(V_{1-y}Ti_y)Al$, Mater. Trans. **49**, 1858 (2008).
- [49] Y. Sandaiji, N. Ide, Y. Nishino, T. Ohwada, S. Harada, and K. Soda, Off-stoichiometric effects on thermoelectric properties of Fe₂VAI-based compounds, Funtai Oyobi Funmatsuyakin 57, 207 (2010).
- [50] W. Lu, W. Zhang, and L. Chen, Thermoelectric properties of (Fe_{1-x}Co_x)₂VAl Heusler-type compounds, J. Alloys Compd. 484, 812 (2009).
- [51] I. Knapp, B. Budinska, D. Milosavljevic, P. Heinrich, S. Khmelevskyi, R. Moser, R. Podloucky, P. Prenninger, and E. Bauer, Impurity band effects on transport and thermoelectric properties of $Fe_{2-x}Ni_xVAI$, Phys. Rev. B **96**, 045204 (2017).
- [52] Y. Kawaharada, K. Kurosaki, and S. Yamanaka, High temperature thermoelectric properties of $(Fe_{1-x}V_x)_3Al$ Heusler type compounds, J. Alloys Compd. **349**, 37 (2003).
- [53] Y. Kawaharada, K. Kurosaki, and S. Yamanaka, Thermophysical properties of Fe₂VAl, J. Alloys Compd. 352, 48 (2003).
- [54] J. Bardeen and W. Shockley, Deformation potentials and mobilities in non-polar crystals, Phys. Rev. 80, 72 (1950).
- [55] S. Krishnamurthy, A. Sher, and A.-B. Chen, Generalized Brooks' formula and the electron mobility in Si_xGe_{1-x} alloys, Appl. Phys. Lett. **47**, 160 (1985).
- [56] W. Liu, H. S. Kim, Q. Jie, and Z. Ren, Importance of high power factor in thermoelectric materials for power generation application: A perspective, Scr. Mater. 111, 3 (2016).
- [57] X. Zhao, X. Ji, Y. Zhang, T. Zhu, J. Tu, and X. Zhang, Bismuth telluride nanotubes and the effects on the thermoelectric properties of nanotube-containing nanocomposites, Appl. Phys. Lett. 86, 062111 (2005).
- [58] C. Nethravathi, C. R. Rajamathi, M. Rajamathi, R. Maki, T. Mori, D. Golberg, and Y. Bando, Synthesis and thermoelectric behavior of copper telluride nanosheets, J. Mater. Chem. A 2, 985 (2014).
- [59] W. Xie, J. He, H. J. Kang, X. Tang, S. Zhu, M. Laver, S. Wang, J. R. Copley, C. M. Brown, Q. Zhang *et al.*, Identifying the specific nanostructures responsible for the high thermoelectric performance of (Bi, Sb)₂Te₃ nanocomposites, Nano Lett. **10**, 3283 (2010).

- [60] K. Biswas, J. He, I. D. Blum, C.-I. Wu, T. P. Hogan, D. N. Seidman, V. P. Dravid, and M. G. Kanatzidis, Highperformance bulk thermoelectrics with all-scale hierarchical architectures, Nature 489, 414 (2012).
- [61] B. Lenoir, A. Dauscher, M. Cassart, Y. I. Ravich, and H. Scherrer, Effect of antimony content on the thermoelectric figure of merit of $Bi_{1-x}Sb_x$ alloys, J. Phys. Chem. Solids **59**, 129 (1998).
- [62] J. Androulakis, I. Todorov, J. He, D.-Y. Chung, V. Dravid, and M. Kanatzidis, Thermoelectrics from abundant chemical elements: High-performance nanostructured PbSe-PbS, J. Am. Chem. Soc. 133, 10920 (2011).
- [63] G. Joshi, X. Yan, H. Wang, W. Liu, G. Chen, and Z. Ren, Enhancement in thermoelectric figure-of-merit of an Ntype half-Heusler compound by the nanocomposite approach, Adv. Energy Mater. 1, 643 (2011).
- [64] K. F. Hsu, S. Loo, F. Guo, W. Chen, J. S. Dyck, C. Uher, T. Hogan, E. Polychroniadis, and M. G. Kanatzidis, Cubic AgPb_mSbTe_{2+m}: Bulk thermoelectric materials with high figure of merit, Science **303**, 818 (2004).
- [65] X. Shi, J. Yang, J. R. Salvador, M. Chi, J. Y. Cho, H. Wang, S. Bai, J. Yang, W. Zhang, and L. Chen, Multiple-filled skutterudites: High thermoelectric figure of merit through separately optimizing electrical and thermal transports, J. Am. Chem. Soc. 133, 7837 (2011).
- [66] X. Wang, H. Lee, Y. Lan, G. Zhu, G. Joshi, D. Wang, J. Yang, A. Muto, M. Tang, J. Klatsky *et al.*, Enhanced thermoelectric figure of merit in nanostructured n-type silicon germanium bulk alloy, Appl. Phys. Lett. **93**, 193121 (2008).
- [67] M. Ohtaki, T. Tsubota, K. Eguchi, and H. Arai, Hightemperature thermoelectric properties of $(Zn_{1-x}Al_x)O$, J. Appl. Phys. **79**, 1816 (1996).
- [68] A. D. LaLonde, Y. Pei, and G. J. Snyder, Reevaluation of PbTe_{1-x}I_x as high performance n-type thermoelectric material, Energy Environ. Sci. **4**, 2090 (2011).
- [69] M. Zebarjadi, G. Joshi, G. Zhu, B. Yu, A. Minnich, Y. Lan, X. Wang, M. Dresselhaus, Z. Ren, and G. Chen, Power factor enhancement by modulation doping in bulk nanocomposites, Nano Lett. 11, 2225 (2011).
- [70] X. Yan, B. Poudel, Y. Ma, W. Liu, G. Joshi, H. Wang, Y. Lan, D. Wang, G. Chen, and Z. Ren, Experimental studies on anisotropic thermoelectric properties and structures of n-type Bi₂Te_{2.7}Se_{0.3}, Nano Lett. **10**, 3373 (2010).
- [71] W. Liu, X. Tan, K. Yin, H. Liu, X. Tang, J. Shi, Q. Zhang, and C. Uher, Convergence of Conduction Bands as a Means of Enhancing Thermoelectric Performance of n-type Mg₂Si_{1-x}Sn_x Solid Solutions, Phys. Rev. Lett. **108**, 166601 (2012).
- [72] A. M. Dehkordi, M. Zebarjadi, J. He, and T. M. Tritt, Thermoelectric power factor: Enhancement mechanisms and strategies for higher performance thermoelectric materials, Mater. Sci. Eng. R 97, 1 (2015).
- [73] C. Forman, I. K. Muritala, R. Pardemann, and B. Meyer, Estimating the global waste heat potential, Renew. Sustain. Energy Rev. 57, 1568 (2016).