Phase transitions in superconductor/ferromagnet bilayer driven by spontaneous supercurrents

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(Received 6 November 2020; revised 22 December 2020; accepted 26 January 2021; published 8 February 2021)

We investigate the superconducting phase transition in a superconductor (S)/ferromagnet (F) bilayer with Rashba spin-orbit interaction at the S/F interface. This spin-orbit coupling produces spontaneous supercurrents flowing inside the atomic-thickness region near the interface, which are compensated by the screening Meissner currents [Mironov and Buzdin, Phys. Rev. Lett. **118**, 077001 (2017)]. In the case of a thin superconducting film the emergence of the spontaneous surface currents causes an increase of the superconducting critical temperature, and we calculate the actual value of the critical temperature shift. We also show that in the case of a type-I superconducting film this phase transition can be of the first order. In the external magnetic field the critical temperature depends on the relative orientation of the external magnetic field and the exchange field in the ferromagnet. Also we predict the in-plane anisotropy of the critical current, which may open an alternative way for the experimental observation of the spontaneous supercurrents generated by the spin-orbit coupling.

DOI: 10.1103/PhysRevB.103.064504

I. INTRODUCTION

Superconducting states carrying spontaneous current in systems with broken time reversal symmetry have been a subject of interest for more than 20 years [1–10]. Such spontaneous supercurrents were predicted for d-wave [1,4,5,10] or chiral p-wave [7,8] superconductors, for a mesoscopic normal metal film in contact with a superconductor [11], and at the interface between a superconductor and a ferromagnet [6]. These currents are typically carried by Andreev edges states [1–10] and appear at temperature T well below the superconducting critical temperature T_c.

Recently, spontaneous supercurrents were predicted to appear at the interface between an s-wave superconductor (S) and a ferromagnetic (F) insulator [12]. Contrary to the spontaneous supercurrents carried by Andreev bound states [1–10], these currents appear at the superconducting transition, i.e., at $T = T_c$. The crucial condition for the emergence of these currents is the presence of Rashba [13] spin-orbit coupling (SOC) at the S/F interface [12]. Indeed, this SOC produces the additional term $\propto (\sigma \times \mathbf{p}) \cdot \mathbf{n}$ in the effective Hamiltonian of a conducting electron ($\mathbf{n}$ is the unit vector perpendicular to the S/F interface). As a result, spin and momentum appear to be coupled, which produces the nontrivial "helicity" of the electronic energy bands. Since the exchange field makes the spin-up state energetically more favorable than the spin-down one, one may expect the emergence of the electric current. Note that the helical states [14,15] also play an important role in the emergence of Majorana modes [16], the formation of Josephson φ_0 junctions with spontaneous nonzero phase difference across the junction in the ground state [17–22], and the appearance of Fulde-Ferrell-Larkin-Ovchinnikov-like states with finite Cooper-pair momentum [23].

Since spontaneous supercurrents flowing at the S/F interface with SOC appear at $T = T_c$, these currents can affect the parameters of the phase transition (the superconducting critical temperature, the phase diagram in the external magnetic field, the critical current, etc.). Although the spontaneous supercurrents result in the local enhancement of the superconductivity near the S/F interface, it was shown [12] that for a large thickness of the superconductor they do not affect the superconducting transition temperature, and thus, with an increase in temperature, the superconductivity is destroyed in the whole bulk of the sample. This situation is in contrast to the well-known phenomena arising in superconductors containing twinning planes which locally increase the critical temperature and favor the emergence of localized superconducting states above the bulk critical temperature (see, e.g., Ref. [24] for a review).

In the present paper we study the effect of the spontaneous supercurrents on the superconducting phase transition in the S/F bilayer with finite thickness of the superconducting layer and SOC of the Rashba type at the S/F interface. We show that in the case of a thin superconducting film these currents cause an increase of the superconducting critical temperature T_c, and we calculate the corresponding critical temperature shift. Surprisingly, in type-I superconductors the superconducting phase transition is of the first order even in the absence of external magnetic field. At the same time, in external magnetic field the critical temperature strongly depends on the relative orientation between external magnetic field and the exchange field in the ferromagnet. Note that in the case of a positive

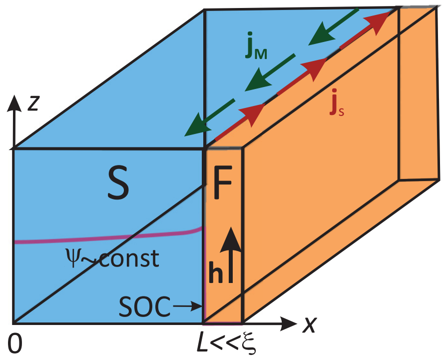

FIG. 1. Sketch of a superconducting film placed in contact with a thin ferromagnetic layer. The spin-orbit coupling at the S/F interface produces a spontaneous supercurrent j_s, causing the increase of the superconducting order parameter inside the superconductor.

(negative) SOC parameter T_c is significantly higher (lower) for the parallel magnetic configuration in comparison with the antiparallel one. At the same time, in the case of a type-II superconducting layer the emergence of the spontaneous supercurrents also increases the superconducting critical temperature in the absence of external magnetic field and makes it sensitive to the relative orientation of external magnetic field and the exchange field if the sample is placed into the field but the phase transition is of the second order. All the described phenomena can serve as hallmarks of the spontaneous supercurrents and can be used for experimental detection of these currents.

This paper is organized as follows. In Sec. II we introduce the model and study the phase transition in the S/F bilayer with a type-I superconductor. In Sec. III we calculate the dependence of the critical temperature on the external magnetic field. In Sec. IV we consider a type-II superconductor. In Sec. V we analyze the anisotropy of the critical current. In Sec. VI we summarize our results.

II. FIRST-ORDER PHASE TRANSITION IN THE S/F BILAYER WITH A THIN TYPE-I SUPERCONDUCTOR

We consider a thin superconducting film of thickness L placed in contact with a ferromagnetic insulator (see Fig. 1). We assume the presence of Rashba spin-orbit coupling at the S/F interface. The free-energy functional of the system under consideration is given by the following expression [12,25,26]:

$$F = \iiint dV \left\{ \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{4m}|\hat{\boldsymbol{D}}\psi|^2 + \frac{(\mathrm{rot}\boldsymbol{A})^2}{8\pi} + [\boldsymbol{n}\times\boldsymbol{h}]\epsilon(r)(\psi^*\hat{\boldsymbol{D}}\psi + \psi\hat{\boldsymbol{D}}^\dagger\psi^*) \right\}. \quad (1)$$

Here $\alpha = a(T - T_{c0})$ and β are the standard Ginzburg-Landau (GL) coefficients, T_{c0} is the critical temperature of the bulk superconductor, $\hat{\boldsymbol{D}} = -i\hbar\nabla + \frac{2e}{c}\boldsymbol{A}$, $\mathbf{A}$ is the vector potential, $\boldsymbol{n}$ is the unit vector perpendicular to the S/F interface and directed from the S to F layer, $\boldsymbol{h}$ is the exchange field in the ferromagnet, ψ is the order parameter of the superconductor, and ϵ is the spin-orbit constant, which is nonzero only in the atomically thin area of the width l_{so} near the S/F interface. Without loss of generality, let us assume that the exchange field is directed along the z axis, i.e., $\boldsymbol{h} = h\mathbf{e}_z$, the x axis coincides with $\mathbf{n}$, and the S/F interface is located at $x = L$.

In such a system spontaneous supercurrents flow near the S/F interface [12]. These currents cause an increase of the superconducting order parameter in the area of with a width of $\sim\xi$ (superconducting coherence length) near the S/F interface. If the thickness L of the superconducting film is much smaller than ξ, i.e., $L \ll \xi$, one can expect an increase of the superconducting critical temperature T_c. Here we find the actual temperature shift for a type-I superconductor, i.e., for the case $\lambda \ll L \ll \xi$, where λ is the London penetration depth. Also we show that the phase transition is of the first order. For this purpose, let us calculate the free energy for such a system. Due to the condition $L \ll \xi$ we may take $\psi(x) \approx \mathrm{const}$. Let us introduce the dimensionless order parameter $\varphi = \sqrt{\beta/|\alpha_0|}\psi$, where $\alpha_0 = -aT_{c0}$.

Since the spontaneous supercurrents are fully compensated by the Meissner currents, the magnetic field $\mathbf{B} = (0, 0, B_z)$ is absent outside the superconductor and has the following form:

$$B_z(x) = \begin{cases} B_0\exp\left(\frac{x-L}{\lambda}\right), & 0 < x < L, \\ 0, \quad x > L, & x \leqslant 0. \end{cases} \quad (2)$$

Here $\lambda = \lambda_0/\varphi$, where $\lambda_0^2 = mc^2\beta/(8\pi e^2|\alpha_0|)$ is the zero-temperature London penetration depth for the bulk superconductor. Note that inside the superconducting slab we neglect the second exponent [see the first line in Eq. (2)] since we assume $\lambda \ll L$. The resulting magnetic field (2) is continuous at $x = 0$ and experiences a jump by the value B_0 at $x = L$ due to the surface spontaneous supercurrents flowing along the S/F interface. To find the actual value of the jump we substitute the magnetic field to the free energy and minimize it with respect to B_0. We obtain

$$A_0 = \Delta H\varphi^2. \quad (3)$$

Here $\Delta H = 4\sqrt{2}H_{cm}k_{so}l_{so}(\xi_0/\lambda_0)$ is the jump of the magnetic field at the S/F interface due to spontaneous surface supercurrents, where $H_{cm} = \sqrt{4\pi\alpha_0^2/\beta}$ is the thermodynamic critical magnetic field, $k_{so} = mh\epsilon/\hbar$, and $\xi_0 = \sqrt{\hbar^2/4m|\alpha_0|}$.

The resulting free energy reads

$$F = V\left(\frac{\alpha|\alpha_0|}{\beta}\varphi^2 + \frac{|\alpha_0|^2}{2\beta}\varphi^4\right) - S\frac{\Delta H^2}{8\pi}\lambda_0\varphi^3, \quad (4)$$

where V is the volume of the superconducting slab and S is the surface area of the S/F boundary.

We find the value of the critical temperature T_c and the order parameter φ_{cr} at $T = T_c$ by minimizing F with respect to φ and using the condition $F = 0$, which is fulfilled at the critical temperature. We find

$$\varphi_{cr} = \frac{1}{2}\frac{\lambda_0}{L}\left(\frac{\Delta H}{H_{cm}}\right)^2, \quad (5)$$

$$\frac{T_c}{T_{c0}} = 1 + \frac{1}{8}\left(\frac{\lambda_0}{L}\right)^2\left(\frac{\Delta H}{H_{cm}}\right)^4. \quad (6)$$

We see that the critical temperature, indeed, increases due to the spin-orbit interaction at the S/F interface. Moreover, since the order parameter does not equal zero at the transition temperature, in the structure under consideration the phase transition is of the first order. Note that the critical temperature (6) is not divergent when the slab thickness L tends to zero since we consider the situation $\lambda \ll L$.

The obtained result is valid only if the assumption $\lambda \ll L$ is fulfilled at $T = T_c$. To check this, we find the London penetration depth at the critical temperature:

$$\lambda_{cr} = \frac{2L}{\left(\frac{\Delta H}{H_{cm}}\right)^2}. \tag{7}$$

Since $\Delta H/H_{cm} \sim k_{so}l_{so}(\xi_0/\lambda_0)$ and $(\xi_0/\lambda_0) \gg 1$ the assumption $\lambda_{cr} \ll L$ is valid if $k_{so}l_{so}$ is not very small.

Let us estimate $\Delta T_c = T_c - T_{c0}$. Since $\epsilon = v_{so}/E_F$, where v_{so} is the spin-orbit velocity and E_F is the Fermi energy, the jump in the magnetic field at $x = L$ due to the spontaneous supercurrents can be estimated as $\Delta H \approx 4\sqrt{2}H_{cm}(h/E_F)(v_{so}/v_F)(\xi_0/\lambda_0)$, where v_F is the Fermi velocity. It is reasonable to take $h/E_F \sim 0.1$, $v_{so}/v_F \sim 0.1$. If we also assume $\xi_0/\lambda_0 \sim 50$, then we find $\Delta H \approx 3H_{cm}$. Taking $\lambda_0/L \sim 0.1$, we obtain $\Delta T_c/T_{c0} \approx 0.1$. Since $\Delta T \ll T_{c0}$ the Ginzburg-Landau approach is applicable.

Note that the first-order phase transition in the S/F bilayer with Rashba-type spin-orbit interaction and $\lambda_0 \ll L \ll \xi$ can serve as a hallmark of the spontaneous supercurrents flowing at the S/F interface [12].

A suitable system for the observation of the discussed effects may be based on thin epitaxially grown layers of extreme type-I superconductors. For example, recently, the epitaxial growth of high-quality single-crystalline aluminum films was demonstrated [27].

III. PHASE TRANSITION IN THE EXTERNAL MAGNETIC FIELD

In the external magnetic field $\mathbf{H}_0$ the critical temperature of the superconducting phase transition depends on the relative orientation of $\mathbf{H}_0$ and the exchange field $\mathbf{h}$. For simplicity, let us restrict ourselves to the case $\mathbf{H}_0 = H_0\mathbf{e}_z$, where H_0 can be both positive and negative. To find the actual $T_c(H_0)$ dependence let us write down the Gibbs free energy of the system:

$$G = \iiint dV\left\{\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{4m}|\hat{\boldsymbol{D}}\psi|^2 + \frac{(\boldsymbol{B}-\boldsymbol{H}_0)^2}{8\pi} + [\boldsymbol{n}\times\boldsymbol{h}]\epsilon(r)(\psi^*\hat{\boldsymbol{D}}\psi + \psi\hat{\boldsymbol{D}}^\dagger\psi^*)\right\}. \tag{8}$$

As before, we consider the case $L \ll \xi$ and assume $\psi \approx$ const. Since we also assume $\lambda \ll L$, the magnetic field $\mathbf{B} = B\mathbf{e}_z = \mathrm{rot}\boldsymbol{A}$ inside the superconducting slab reads

$$B = H_0\exp\left(-\frac{x}{\lambda}\right) + (H_0 + \Delta H\varphi^2)\exp\left(\frac{x-L}{\lambda}\right). \tag{9}$$

Calculating the resulting Gibbs free energy, we obtain

$$\frac{G}{V} = \frac{\alpha|\alpha_0|}{\beta}\varphi^2 + \frac{|\alpha_0|^2}{2\beta}\varphi^4 + \frac{H_0^2}{8\pi} - \frac{H_0\Delta H}{4\pi}\frac{\lambda_0}{L}\varphi - \frac{\Delta H^2}{8\pi}\frac{\lambda_0}{L}\varphi^3. \tag{10}$$

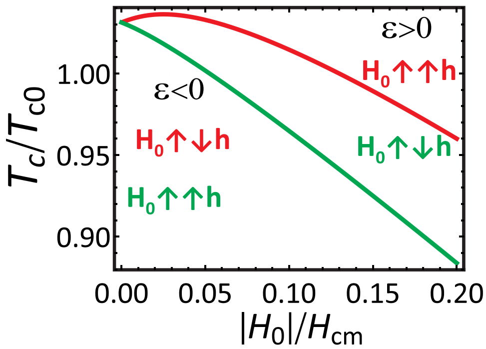

FIG. 2. The dependence of the superconducting critical temperature T_c of the S/F bilayer with $\lambda_0 = 0.02L$ and $\Delta H = 5H_{cm}$ on the external magnetic field $\mathbf{H_0} = H_0\mathbf{e}_z$. Here ϵ is the spin-orbit coupling constant.

Note that the surface contribution to the free energy caused by spontaneous supercurrents results in the increases(decreases) of the free energy if $\mathbf{H}_0 \uparrow\uparrow \mathbf{h}$ ($\mathbf{H}_0 \uparrow\downarrow \mathbf{h}$).

The critical temperature and the order parameter at the critical point φ_{cr} can be found from the system of equations $G = 0$ and $\partial_\varphi G = 0$, which are fulfilled at $T = T_c$. From the second equation we find the critical temperature as a function of φ_{cr}:

$$\frac{T_c}{T_{c0}} = 1 - \varphi_{cr}^2 + \frac{3}{4}\left(\frac{\Delta H}{H_{cm}}\right)^2\frac{\lambda_0}{L}\varphi_{cr} + \frac{1}{2\varphi}\frac{H_0}{H_{cm}}\frac{\Delta H}{H_{cm}}\frac{\lambda_0}{L}. \tag{11}$$

At the same time, the order parameter obeys the equation

$$\varphi_{cr}^4 - \frac{1}{2}\left(\frac{\Delta H}{H_{cm}}\right)^2\frac{\lambda_0}{L}\varphi_{cr}^3 - \left(\frac{H_0}{H_{cm}}\right)^2 + \frac{H_0}{H_{cm}}\frac{\Delta H}{H_{cm}}\frac{\lambda_0}{L}\varphi_{cr} = 0. \tag{12}$$

Solving the equations, we indeed find that the critical temperature of the superconducting phase transition strongly depends on the relative orientation of the external magnetic field and the exchange field: for a positive (negative) SOC parameter ϵ, T_c is higher for the parallel (antiparallel) orientation compared to the antiparallel (parallel) one (see Fig. 2). This finding provides a tool for the experimental observation of the predicted effect, although the correction to T_c due to SOC is small. Changing the direction of the external magnetic field, one can observe the variation of the superconducting critical temperature. Note that the dependence of the critical temperature on the magnetic configuration also can be used for the experimental detection of the sign of the spin-orbit parameter. Indeed, the critical temperature is higher for the parallel orientation between the external magnetic field and the exchange field in comparison with the antiparallel one only if the spin-orbit constant is positive, while for the negative SOC parameter the situation is reversed.

Since $\lambda_0 \ll L$, we can find approximate analytical expressions for $T_c^{\uparrow\uparrow}$ and $T_c^{\uparrow\downarrow}$, expanding the results obtained from Eqs. (11) and (12) over λ_0/L. To have good agreement between the exact results and the approximate one, we should expand these expressions up to third order over λ_0/L. If $\Delta H > 0$ ($\Delta H < 0$) for the parallel (antiparallel) magnetic configuration, we obtain

$$\frac{T_c^{\uparrow\uparrow(\uparrow\downarrow)}}{T_{c0}} = 1 - \frac{|H_0|}{H_{cm}} + \frac{1}{2}\sqrt{\frac{|H_0|}{H_{cm}}\frac{\Delta H}{H_{cm}}}\left(2 + \frac{\Delta H}{H_{cm}}\right)\frac{\lambda_0}{L} + \frac{1}{32}\left(\frac{\Delta H}{H_{cm}}\right)^2\left[\left(\frac{\Delta H}{H_{cm}}\right)^2 - 2\right]^2\left(\frac{\lambda_0}{L}\right)^2. \quad (13)$$

At the same time, if $\Delta H > 0$ ($\Delta H < 0$) for the antiparallel (parallel) magnetic configuration, we find

$$\frac{T_c^{\uparrow\downarrow(\uparrow\uparrow)}}{T_{c0}} = 1 - \frac{|H_0|}{H_{cm}} - \sqrt{\frac{|H_0|}{H_{cm}}\frac{\Delta H}{H_{cm}}}\frac{\lambda_0}{L} + \frac{1}{\sqrt{3}}\left(\frac{\Delta H}{H_{cm}}\right)^2\left(0.9\frac{H_0}{H_{cm}} + \frac{\sqrt{3}}{8}\right)\left(\frac{\lambda_0}{L}\right)^2. \quad (14)$$

Note that the above expressions are invalid when H_0 tends to zero. Assuming $H_0 \ll \Delta H$ from Eqs. (11) and (12), we find

$$\frac{T_c^{\uparrow\uparrow(\uparrow\downarrow)}}{T_{c0}} = 1 + \frac{1}{8}\left(\frac{\Delta H}{H_{cm}}\right)^4\left(\frac{\lambda_0}{L}\right)^2 \pm \frac{2|H_0|}{\Delta H}. \quad (15)$$

Note that the dependence of the critical temperature on the in-plane magnetic field orientation is another hallmark of spontaneous supercurrents flowing at the S/F interface [12] (in addition to the appearance of the stray magnetic field near the S/F interface and the anisotropy of the upper critical field predicted in Ref. [12]). This fact can be used for the experimental detection of the spontaneous supercurrents.

IV. PHASE TRANSITION IN THE CASE OF A THIN TYPE-II SUPERCONDUCTING LAYER

In this section we analyze the peculiarities of the superconducting phase transition in the S/F bilayer (see Fig. 1) for the case when the superconducting layer is type II. We show that in this case the critical temperature also increases in the absence of the external magnetic field $\mathbf{H}_0$ and depends on the relative orientation between the external magnetic field and the exchange field. Let us consider the temperatures close to the superconducting transition temperature and the external magnetic field directed along the z axis so that $\mathbf{H}_0 = (0, 0, H_0)$. At the point of the phase transition the superconducting order parameter is small, which allows us to neglect the term $\propto\psi|\psi|^2$ and the term associated with the spontaneous supercurrents in the GL equation. Choosing the gauge of the vector potential in the form $\mathbf{A} = (0, H_0x, 0)$ and searching for a solution in the form $\psi = e^{ik_yy}\psi(x)$, we obtain the following GL equation:

$$\alpha\psi(x) - \frac{\hbar^2}{4m}\partial_{xx}\psi(x) + \frac{(\hbar k_y + 2eH_0x/c)}{4m}\psi(x) = 0, \quad (16)$$

with the boundary conditions

$$\partial_x\psi(0) = 0, \quad \partial_x\psi(L) = \frac{8mh\epsilon l_{so}}{\hbar^2}\left(\hbar k_y + \frac{2eH_0L}{c}\right)\psi(L). \quad (17)$$

In the absence of the external magnetic field the order parameter reads $\psi(x) = A\cosh qx$, where $q^2 = (4m\alpha/\hbar^2 + k_y^2)$. Calculating the free energy, we find

$$\frac{F}{S} = \frac{A^2L}{2}\left(\alpha + \frac{\hbar^2k_y^2}{4m}\right)\left(\frac{\sinh 2ql}{2qL} + 1\right) + A^2\frac{\hbar^2q^2}{4m} \times \frac{L}{2}\left(\frac{\sinh 2ql}{2qL} - 1\right) - 2A^2h\epsilon l_{so}\hbar k_y\cosh^2 qL. \quad (18)$$

Assuming $qL \ll 1$ and minimizing the free energy with respect to k_y, we find the optimal modulation vector $k_y = 4k_{so}l_{so}/L$. Since at the critical point $F = 0$, we obtain the increase of the superconducting critical temperature:

$$\frac{T_c - T_{c0}}{T_{c0}} = 16k_{so}^2l_{so}^2\left(\frac{\xi_0}{L}\right)^2. \quad (19)$$

Note that the effect is absent in the limit $L \to \infty$, i.e., for a thick superconducting slab.

When the sample is placed in the external magnetic field, it is convenient to choose the origin of the x axis to be at the center of the S film, so the S/F boundary is located at $x = L/2$, and the other boundary is at $x = -L/2$. Following the procedure described in Ref. [28], we introduce the dimensionless coordinate $X = 2x/L$, the modulation vector $K_y = k_yL/2$, and the parameters $\tilde{H}_0 = eH_0L^2/(2\hbar c)$ and $\epsilon_0 = -m\alpha L^2/\hbar^2$ and rewrite the GL equation in the following form:

$$\partial_{XX}\psi(X) + (K_y + \tilde{H}_0X)^2\psi(X) = \epsilon_0\psi(X). \quad (20)$$

At the same time, the boundary conditions read

$$\partial_X\psi(1) = s(K_y + \tilde{H}_0)\psi(1), \quad \partial_X\psi(-1) = 0, \quad (21)$$

where $s = 8k_{so}l_{so}$.

Next, it is useful to introduce the new variable $t = \sqrt{2|\tilde{H}_0|}(X + K_y/h_0)$. The resulting GL equation and the boundary conditions are as follows:

$$-\partial_{tt}\psi(t) + \frac{1}{4}t^2\psi(t) = \frac{\epsilon_0}{2|\tilde{H}_0|}\psi(t), \quad (22)$$

$$\partial_t\psi|_{\sqrt{2|\tilde{H}_0|}(-1+K_y/\tilde{H}_0)} = 0, \quad (23)$$

$$\left(\frac{\partial_t\psi}{\psi}\right)\Bigg|_{\sqrt{2|\tilde{H}_0|}(1+K_y/\tilde{H}_0)} = \frac{s(K_y + \tilde{H}_0)}{\sqrt{2|\tilde{H}_0|}}. \quad (24)$$

The solution of Eq. (22) has the form of the linear combination of the Weber functions $\psi(t) = A_\nu D_\nu(t) + B_\nu D_\nu(-t)$, where $2\nu + 1 = \epsilon_0/|\tilde{H}_0|$. Substituting it into the boundary conditions, we obtain the following equation, which implicitly defines the function $T_c(H_0)$:

$$D_\nu'(\alpha_+)D_\nu'(-\alpha_-) - D_\nu'(-\alpha_+)D_\nu'(\alpha_-) = \frac{s(K_y + \tilde{H}_0)}{\sqrt{2|\tilde{H}_0|}}[D_\nu(\alpha_+)D_\nu'(-\alpha_-) - D_\nu(-\alpha_+)D_\nu'(\alpha_-)], \quad (25)$$

where $\alpha_\pm = \sqrt{2|\tilde{H}_0|}(\pm 1 + K_y/\tilde{H}_0)$.

The maximal value of T_c at fixed magnetic field corresponds to the minimal value of ν, which satisfies Eq. (25). At fixed K_y and $\tilde{H}_0$ this equation has an infinite but discrete

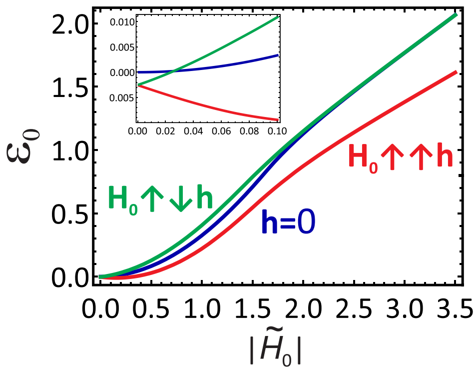

FIG. 3. The dependence of the critical temperature T_c on the external magnetic field $\mathbf{H}_0 = (0, 0, H_0)$. Here $\epsilon_0 = (1 - T_c/T_{c0})L^2/(4\xi_0^2)$, and $\tilde{H}_0 = eH_0L^2/(2\hbar c)$.

number of solutions for ν, and we find the minimal one. Then we minimize it with respect to K_y and find the minimal value ν_0 for fixed $\tilde{H}_0$. As a result, we obtain the dependence $\epsilon_0(\tilde{H}_0)$ [i.e., $T_c(H_0)$] in the form $\epsilon_0 = (2\nu_0 + 1)|\tilde{H}_0|$ (see Fig. 3). The critical temperature is higher for the parallel orientation of the external magnetic field and the exchange field in comparison with the antiparallel one. Here we assume that the spin-orbit coupling parameter ϵ is positive.

As we can see from the inset in Fig. 3 the weak parallel magnetic field leads to the initial increase of the critical temperature, which is replaced by the usual decrease at higher magnetic field. Such peculiar behavior resembles the increase of the critical temperature in thin Pb films experimentally observed in Ref. [29]. We may speculate that the local SOC could be generated at the Pb/substrate interface, while the role of the exchange field is played by a Zeeman field.

V. CRITICAL CURRENT

In this section we show that the spin-orbit coupling makes the in-plane critical current of the S/F bilayer anisotropic. Although the total spontaneous superconducting current generated by the SOC is zero, it is nonuniformly distributed across the layers. As a result, for the fixed direction of the exchange field the local current density (and thus the local damping of the superconducting order parameter) at a certain point of the S film becomes dependent on the angle θ between the external current and the spontaneous current flowing along the S/F interface. Consequently, the maximal current which does not destroy the superconducting state (critical current) also becomes dependent on θ (diodelike effect).

To calculate the critical current of the S/F bilayer we again consider the system sketched in Fig. 1. First, we consider the situation when the external transport current of the linear density J is directed along the y axis. Since the magnetic field produced by both the current J and the spontaneous surface current due to SOC is directed along the z axis and depends only on the x axis, we may choose a vector potential in the form $\mathbf{A} = A(x)\hat{\mathbf{y}}_0$. Also we choose the order parameter φ to be real. Then the density of the free energy accounting for the nonuniform profile of the order parameter φ and the vector potential A across the structure can be written in the form

$$\frac{F}{V} = \frac{H_{cm}^2}{4\pi}\big(-\tau\varphi^2 + \varphi^4 + \xi_0^2\varphi'^2\big) + \frac{A'^2}{8\pi} + \frac{A^2\varphi^2}{8\pi\lambda_0^2} - \frac{\Delta H A\varphi^2}{4\pi}\delta(x-L), \tag{26}$$

where $\tau = 1 - T/T_{c0}$, $\varphi' \equiv \partial\varphi/\partial x$ and $A' \equiv \partial A/\partial x$. Varying the free energy with respect to $\varphi(x)$ and $A(x)$ inside the S layer, we derive the standard set of Ginzburg-Landau equations:

$$-\xi_0^2\varphi'' - \tau\varphi + \varphi^3 + A^2\varphi/\big(2H_{cm}^2\lambda_0^2\big) = 0, \tag{27}$$

$$-\lambda_0^2 A'' + \varphi^2 A = 0, \tag{28}$$

supplemented by the boundary conditions accounting for the surface energy contribution due to the spin-orbit coupling [the last term with the δ function in Eq. (26)] and the magnetic field $\pm(2\pi/c)J$ generated by the external transport current J at the outer boundaries $x = 0$ and $x = L$ of the superconducting film:

$$\varphi'(0) = 0,\ \ \varphi'(L) = \Delta H A(L)\varphi(L)/H_{cm}^2, \tag{29}$$

$$A'(0) = -(2\pi/c)J, \quad A'(L) = (2\pi/c)J + \Delta H\varphi^2(L). \tag{30}$$

The accurate solution of Eqs. (27) and (28) requires focusing on two features responsible for the anisotropy of the critical current. The first one is the nonuniform distribution of the screening Meissner current across the S film. The second one is the damping of the order parameter at the S/F interface by the transport current and the subsequent renormalization of the spontaneous surface current (and the screening Meissner one). Thus, the terms containing spatial derivatives of the order parameter and the vector potential cannot be neglected even in the case of a thin S layer.

In order to find the analytical solution of the Ginzburg-Landau equations we make several assumptions simplifying the calculations. First, we restrict ourselves to the most interesting case of the type-II superconductor and assume that the thickness L of the S film is much smaller than the superconducting coherence length so that $L \ll \xi \ll \lambda$. Second, we consider the limit of small spin-orbit coupling assuming the dimensionless parameter $\mu = \Delta H\lambda_0/(H_{cm}L)$ is small ($\mu \ll 1$). These assumptions allow us to expand the functions $A(x)$ and $\varphi(x)$ over x, keeping the terms up to $(L/\xi)^3$ in order to account for the nonuniform distribution of the superconducting current across the S film:

$$A = A_0 + A_1x + A_2x^2 + A_3x^3,$$
$$\varphi = \varphi_0 + \varphi_1x + \varphi_2x^2 + \varphi_3x^3.$$

Also in the resulting perturbation theory it is enough to consider the terms proportional to μ and neglect the higher order contributions.

Substituting the expansion for A and φ into Eqs. (27) and (28) and boundary conditions (29) and (30), we find the dependence of the external current J on the dimensionless vector potential $a_0 = A_0/(H_{cm}\lambda_0)$:

$$J = cH_{cm}L/(8\pi\lambda_0)\big(2\tau a_0 - 2\tau\mu + 3\mu a_0^2 - a_0^3\big) \times \big[1 - L^2\big(2\tau + 4a_0\mu - 3a_0^2\big)/\big(8\lambda_0^2\big)\big]. \quad (31)$$

Then the critical current J_c of the S/F bilayer can be obtained as the maximum of the dependence $J(a_0)$ for $a_0 > 0$. This maximum corresponds to $a_0 = \mu$ and can be written in the form

$$J_c^{\pm} = \frac{\sqrt{2}LcH_{cm}\tau^{3/2}}{6\pi\sqrt{3}\lambda_0}\left(1 \pm \Delta H \frac{L\sqrt{\tau}}{2\sqrt{6}H_{cm}\lambda_0}\right). \quad (32)$$

Here the + (−) sign corresponds to the current flowing parallel (antiparallel) to the y axis.

Expression (32) clearly shows the anisotropy of the critical current, which differs for the two opposite directions of the current flow. The difference between the critical currents $J_c^{\pm}$ is proportional to the spontaneous magnetic field ΔH generated at the S/F interface due to the SOC. Note that the critical current is higher if the exchange field in the F layer is parallel to the magnetic field generated by the external current at $x = L$ and lower in the opposite case. Equation (32) can be straightforwardly generalized for the case of the arbitrary direction of the external current in the plane of the S/F structure. In this case the ± sign in the brackets should be replaced with $\cos\theta$, where θ is the angle between the direction of the current and the y axis.

The predicted diode effect provides an alternative way for the experimental observation of the spontaneous currents generated by the SOC. To protect the S/F bilayer from the distraction caused by the heating effects one may use pulse currents [30].

VI. CONCLUSION

To sum up, we developed a theory of the superconducting phase transition in a superconductor/ferromagnet bilayer with Rashba-type spin-orbit interaction at the S/F interface and $L \ll \xi$ (see Fig. 1) using the Ginzburg-Landau approach. In the case of $\lambda \ll L$ the phase transition is of the first order even in the absence of external magnetic field. Moreover, its critical temperature is higher than in the bulk superconductor. In the external magnetic field $\mathbf{H}_0$ the critical temperature depends on the mutual orientation of $\mathbf{H}_0$ and the exchange field inside the ferromagnet and the sign of the spin-orbit coupling parameter: for a positive (negative) SOC parameter it is higher (lower) for the parallel orientation in comparison with antiparallel one (see Fig. 2). Both these results are manifestations of the spontaneous supercurrents flowing at the S/F interface [12] and can serve as hallmarks of these currents. Moreover, the dependence of the critical temperature on the magnetic configuration can be used for the experimental detection of the sign of the SOC parameter. In the case of a type-II superconducting layer the phase transition is the second-order one; its critical temperature also increases in the absence of external magnetic field and depends on the relative orientation between the external magnetic field and the exchange field if the former is present. We also showed that the critical current of the S/F bilayer reveals anisotropy in the plane of the layers. The resulting diodelike effect may provide an alternative way for the experimental observation of the spontaneous superconducting currents generated by the SOC at the S/F interface.

ACKNOWLEDGMENTS

The authors thank A. S. Mel'nikov and I. V. Zagorodnev for useful discussions. S.M. acknowledges the funding from the Russian Science Foundation (Grant No. 20-12-00053, in part related to the calculations of the critical current). Zh.D. acknowledges the funding from the Russian Science Foundation (Grant No. 18-72-10118, in part related to the calculations of the critical temperature). A.I.B. acknowledges support by the Ministry of Science and Higher Education of the Russian Federation within the framework of state funding for the creation and development of World-Class Research Center "Digital biodesign and personalized healthcare", Grant No. N075-15-2020-92. This work was supported by the French ANR OPTOFLUXONICS, Grant No. EU COST CA16218 Nanocohybri, Foundation for the Advancement of Theoretical Physics and Mathematics BASIS (Grants No. 18-1-3-58-1 and No. 18-1-4-24-1) and Russian Presidential Scholarship (Grants No. SP-3938.2018.5 and No. SP-5551.2021.5).

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