Nonanalytic momentum dependence of spin susceptibility for Heisenberg magnets in the paramagnetic phase and its effect on critical exponents

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(Received 14 October 2020; revised 2 January 2021; accepted 25 January 2021; published 8 February 2021)

We study the momentum dependence of static magnetic susceptibility $\chi(q)$ in the paramagnetic phase of Heisenberg magnets and its relation to critical behavior within the nonlinear sigma model (NLSM) at arbitrary dimension $2 < d < 4$. In the first order of $1/N$ expansion, where N is the number of spin components, we find $\chi(q) \propto \{q^2 + \xi^{-2}[1 + f(q\xi)]\}^{-1+\eta/2}$, where ξ is the correlation length, q is the momentum, measured from the magnetic wave vector, and the universal scaling function $f(x)$ describes the deviation from the standard Landau-Ginzburg momentum dependence. In agreement with previous studies at large x we find $f(x \gg 1) \simeq (2B_4/N)x^{4-d}$; the absolute value of the coefficient B_4 increases with d at $d > 5/2$. Using NLSM, we obtain the contribution of the "anomalous" term $\xi^{-2} f(q\xi)$ to the critical exponent ν, comparing it to the contribution of the nonanalytical dependence, originating from the critical exponent η (the obtained critical exponents ν and η agree with previous studies). In the range $3 \leqslant d < 4$ we find that the former contribution dominates and fully determines the $1/N$ correction to the critical exponent ν in the limit $d \to 4$.

DOI: 10.1103/PhysRevB.103.054415

I. INTRODUCTION

The spatial or momentum dependence of response functions plays an important role in physical properties. The energy, corresponding to the spatial dependence of the order parameter field $\mathbf{n}_\mathbf{r}$, weakly changing in space, is proportional to $(\nabla \mathbf{n}_\mathbf{r})^2$ (as in the Ginzburg-Landau theory [1]). At the critical point this yields $S_R \sim R^{-(d-2)}$ decay of the correlation function $S_R = \langle n_0^\alpha n_\mathbf{R}^\alpha \rangle$ with the distance R, with d being the dimensionality of the system. At the same time, the interaction results in the appearance of an anomalous critical exponent η, which determines the long-range behavior of correlation functions $R^{-(d-2+\eta)}$ (see, e.g., Ref. [2]). The exponent η can vary from a rather small value for the three-dimensional (3D) Heisenberg model ($\eta \approx 0.04$) to a substantial value for the two-dimensional Ising model ($\eta = 1/4$); substantial values of $\eta = 0.2$–0.4 were recently also obtained for deconfined spinon theories [3,4].

The scaling considerations away from the critical point predict the spatial dependence of the correlation function $S_R = R^{-(d-2+\eta)} f(R/\xi)$, where $f(x)$ is some function and ξ is the correlation length. In momentum space the corresponding dependence reads $S_q = q^{-2+\eta} g(q\xi)$. The simplest function which fulfills this form is $S_q = A/(q^2 + \xi^{-2})^{1-\eta/2}$ (cf. Ref. [5]). This dependence generalizes the Ornstein-Zernike result to include an anomalous exponent η.

However, the Ornstein-Zernike form (even with the exponent η) was argued to be not sufficient to explain experimental data. In this respect, nonanalytic subleading corrections to the scaling functions were proposed within the Fisher-Langer theory [6,7] to explain the anomalies of the resistivity of transition metals near the magnetic phase transition [8–10]. These corrections were also invoked to explain the peculiarities of the density-density correlation function near the gas-liquid critical point [11,12].

Theoretically, the corrections to the correlation functions were obtained [13–20] in the large-momentum q limit within the linear sigma model (LSM). In the case of the specific heat critical exponent $\alpha < 0$, the corresponding leading nonanalytical term in the spin correlation function reads $S_q \propto q^{-2+\eta-1/\nu}$ [13–20], where ν is the critical exponent of the correlation length. This momentum dependence implies that the magnon self-energy, defined by $S_q = A/(q^2 + \overline{\Sigma}_q + \xi^{-2})^{1-\eta/2}$, acquires the nonanalytic contribution $\overline{\Sigma}_\mathbf{q} \sim q^{2-1/\nu}$. This result can also be confirmed by the renormalization group (RG) approach of Refs. [21–23] in $d = 2 + \varepsilon$ dimensions, where $\nu = 1/(d-2) + O(1)$ and therefore $\overline{\Sigma}_\mathbf{q} \sim q^{4-d}$. The RG analysis [23,24] and $1/N$ expansion [25] of $d = 2$ Heisenberg magnets also agree with the above result for the self-energy since they yield the nonanalytical momentum dependence $\overline{\Sigma}_\mathbf{q} \sim q^2 \ln^{-1/(N-2)}(q\xi)$ at $q \gg \xi^{-1}$ (N is the number of spin components) and $1/\nu \to 0$ in two dimensions (the critical exponent $\eta = 0$ for $d = 2, N > 2$).

For the number of order parameter components $N > 1$, the momentum dependence $\overline{\Sigma}_\mathbf{q} \sim q^{4-d}$, discussed above for d close to 2, is identical to that obtained for the self-energy, corresponding to the longitudinal correlation function deeply in the ordered phase [26]. The latter dependence is produced by a pair of spinons and therefore reflects spinon deconfinement in the presence of the long-range magnetic order [27–29]. On approaching the magnetic transition temperature the q^2 term in the inverse Green's function becomes progressively more important (see, e.g., Refs. [30,31]). Identical form of the above discussed corrections to the self-energy in the paramagnetic and ordered phase stresses a possible relation between the nonanalytical terms and the spinon (de)confinement. The nonanalytical terms, obtained within LSM, also reminiscent of nonanalytic contributions to the spin susceptibility $\chi_q \sim q^{d-1}$ in itinerant systems [32,33].

Previous theoretical studies of the momentum dependence of the susceptibility of the $d > 2$ Heisenberg model concentrated mainly on the large-momentum asymptotics $q \gg \xi^{-1}$ of correlation functions and the interpolation formulas between the Ornstein-Zernike and nonanalytic dependences [34]. To study the universal properties of the Heisenberg model, in particular the momentum dependence of correlation functions in the long-wavelength limit, this model can be mapped to the nonlinear sigma model (NLSM). The classical version of this model describes well the thermodynamic and statistical properties of Heisenberg magnets at finite, but not too low, temperature [21–23,25,30,35]. The NLSM has certain advantages over the linear sigma model, previously used to calculate the asymptotics of correlation functions, since it is applicable outside the critical regime. Also, in the NLSM the universal part of the magnon self-energy is directly related to the correlation length via the constraint equation, reflecting a fixed spin value. This allows us to study the effect of nonanalytical terms on the critical exponents.

In the present paper we consider the derivation of nonanalytic contributions to the momentum dependence of the spin susceptibility in the paramagnetic phase of Heisenberg magnets within the NLSM and study in detail their structure with varying dimensionality and their effect on the critical behavior. We determine a closed analytical expression for the coefficient of the leading nonanalytical term q^{4-d} in the self-energy to first order in $1/N$ in the arbitrary dimension $2 < d < 4$. The absolute value of the coefficient of the anomalous term becomes larger with the increase of the system dimensionality d, which is related to stronger spinon confinement with increasing dimensionality. We also argue that the nonanalytic term yields a substantial contribution to the critical exponent ν and therefore, via scaling relations, all other critical exponents, except the exponent η, which is shown to be independent of the presence of the term.

II. 1/*N* EXPANSION IN THE NONLINEAR SIGMA MODEL

We consider the classical nonlinear $O(N)$ sigma model

$$Z[\mathbf{h}] = \int D\sigma D\lambda \exp\left\{-\frac{1}{2t}\int d^d\mathbf{r}[(\nabla\boldsymbol{\sigma})^2 + i\lambda(\sigma^2-1) - 2t\mathbf{h}\boldsymbol{\sigma}]\right\}, \tag{1}$$

where $\boldsymbol{\sigma}(\mathbf{r})$ is the N-component field, d is the space dimensionality, $t = T/\rho_s$ is the coupling constant, and ρ_s is the spin stiffness. The constraint condition $\sigma^2 = 1$ is taken into account by introducing the auxiliary field $\lambda(\mathbf{r})$. To calculate the correlation functions we also introduce the external nonuniform magnetic field $\mathbf{h}(\mathbf{r})$. The model (1) is applicable to classical and quantum ferro- and antiferromagnets at finite temperatures (in the quantum case the temperature should not be too low: $JS\xi^{-1} \ll T$, where J is the exchange integral and S is the spin value; see Refs. [23,25,30,35]). The applicability of the classical model (1) to quantum ferro- and antiferromagnets at finite, but not very low, temperatures is related to the fact that quantum renormalizations at finite temperatures can be absorbed by the spin stiffness ρ_s. The model (1) is also applicable to quantum antiferromagnets in the ground state, in which case $t \sim 1/S$ and d is the space-time dimensionality.

To study nonanalytical terms in the self-energy, we use $1/N$ expansion, which is performed in the standard way [15,25,36,37]. In contrast to the (self-consistent) spin-wave theory [38] and $2+\epsilon$ renormalization group approach [21–23] this method allows us to study systems with dimensionality d not close to 2 at not too low temperatures. After integrating over $\boldsymbol{\sigma}$ the partition function takes the form

$$Z[\mathbf{h}] = \int D\lambda \exp(S_{\text{eff}}[\lambda, h]) \tag{2}$$

$$S_{\text{eff}}[\lambda, h] = \frac{N}{2}\ln\det\widehat{G} + \frac{1}{2t}\text{Sp}(i\lambda) + \frac{t}{2}\text{Sp}[h\widehat{G}h], \tag{3}$$

where

$$\widehat{G} = [-\nabla^2 + i\lambda]^{-1}.$$

Since N enters (2) only as a prefactor in the exponent, expanding near the saddle point generates a series in $1/N$. Below we treat the paramagnetic phase, where the value of $\lambda = \lambda_0$ at the saddle point is determined by the sum rule (constraint) $\langle\sigma^2\rangle = 1$, which takes the form

$$1 = Nt\int \frac{d^d\mathbf{k}}{(2\pi)^d} G^{nn}(k), \tag{4}$$

where $n = 1, \dots, N$ and we account for the fact that the Green's function of the field σ,

$$G^{nn'}(k) = \frac{1}{tZ[0]}\left[\frac{\partial^2 Z[h]}{\partial h^n(\mathbf{k})\partial h^{n'}(-\mathbf{k})}\right]_{h=0}, \tag{5}$$

depends only on $k = |\mathbf{k}|$ due to rotational symmetry in the considered long-wavelength limit and $h(\mathbf{k})$ is the Fourier transform of $h(\mathbf{r})$. Note that only diagonal elements $G^{nn'}$ are nonzero. We use the cutoff $k < \Lambda$ of momentum integrations. The Green's function represents the rescaled (staggered) spin susceptibility $\chi^{nn'}(k) = (S^2/\rho_s)G^{nn'}(k)$ and may be expressed within the $1/N$ expansion as

$$G^{nn}(k) = [k^2 + \Sigma(k) + m^2]^{-1}, \tag{6}$$

where $\Sigma(k)$ is the bosonic self-energy, defined such that $\Sigma(0) = 0$, and m is the renormalized mass of spin excitations to first order in $1/N$. We split the mass as $m^2 = m_0^2 + \delta m^2$, where we define m_0 in such a way that it absorbs all nonuniversal (Λ-dependent) contributions, except logarithmic terms (the latter contribute to critical exponents and are included, along with regular terms, in δm^2; see below). The terms included in m_0 determine the value of the magnetic phase transition temperature (or critical coupling constant), which is defined by vanishing m_0 (the quantity δm^2 vanishes simultaneously; see below).

The self-energy $\Sigma(k)$ in the first order of the $1/N$ expansion is given by [25]

$$\Sigma(k) = \frac{2}{N}\int \frac{d^d\mathbf{q}}{(2\pi)^d}\frac{G_0(|\mathbf{k}+\mathbf{q}|) - G_0(q)}{\Pi(q)}, \tag{7}$$

where

$$\Pi(q) = \int \frac{d^d \mathbf{p}}{(2\pi)^d} G_0(p) G_0(|\mathbf{p}+\mathbf{q}|) \tag{8}$$

and $G_0(k) = (k^2 + m_0^2)^{-1}$. To first order in $1/N$ the sum rule (4) takes the form

$$1 = Nt \int \frac{d^d \mathbf{k}}{(2\pi)^d} G_0(k) - Nt \int \frac{d^d \mathbf{k}}{(2\pi)^d} G_0^2(k)[\Sigma(k) + \delta m^2]. \tag{9}$$

III. RESULTS

A. Three dimensions

In three dimensions the polarization operator (8) reads

$$\Pi(q) = \frac{1}{4\pi q} \arctan \frac{q}{2m_0}. \tag{10}$$

Using this expression, we find from Eq. (7) (see details in Sec. A 1)

$$\Sigma(k) = \eta k^2 \ln \frac{\Lambda}{\left(k^2 + m_0^2\right)^{1/2}} + \frac{2}{N} m_0^2 F(k/m_0), \tag{11}$$

where Λ is the momentum cutoff, $\eta = 8/(3\pi^2 N)$ is the standard exponent determining the correlation function decay to first order in $1/N$ for the 3D $O(N)$ model (cf. Refs. [25,30,39]), and we have introduced a universal function,

$$F(x) = \frac{1}{\pi} \int_0^\infty q^2 dq \left\{ \left[\frac{1}{2qx} \ln \frac{(x+q)^2+1}{(x-q)^2+1} - \frac{2}{q^2+1} \right] \times \frac{q}{\arctan(q/2)} - \frac{4x^2}{3\pi q^3} \theta\left(q - \sqrt{x^2+1}\right) \right\}. \tag{12}$$

Evaluating the asymptotics of this function, we find

$$F(x) \simeq \begin{cases} \frac{4x^2}{9\pi^2} - \frac{2x}{\pi} - \frac{4}{\pi^4}(16-\pi^2)\ln x + 1.10334 & x \gg 1, \\ -0.24553x^2 & x \ll 1. \end{cases} \tag{13}$$

One can see that at $k \gg m_0$ apart from quadratic term Ak^2 the self-energy contains also subleading nonanalytical terms, proportional to k and $\ln(k/m_0)$ with the coefficients, which agree with Ref. [15], but expressed in terms of elementary functions. These terms are not related to the exponent η, introduced by the first term in Eq. (11). The plot of the function $F(x)$ together with its asymptotes is shown in Fig. 1. Note that although the function $F(x)$ is not positively defined, it is quadratic at small x, and therefore, the leading term k^2 in the propagator overcomes the negative contribution in the second line of Eq. (13), and the whole spectrum is positively defined at large N (including $N = 3$).

The transition temperature (or critical coupling constant) to first order in $1/N$ is obtained from Eq. (9) by putting $m_0 = \delta m = 0$. We find

$$t_c = \frac{2\pi^2}{N\Lambda}\left(1 + \frac{32}{9\pi^2 N}\right). \tag{14}$$

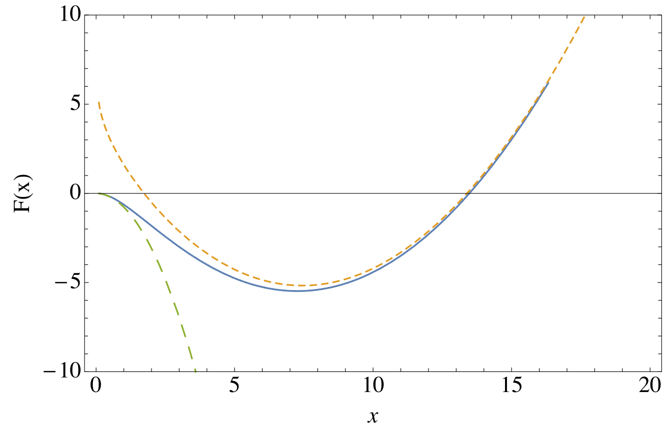

FIG. 1. Plot of the function $F(x)$ (solid line) and its large- and small-x asymptotes (dashed lines).

The details of the calculation of the mass terms m_0 and δm are presented in Sec. A 1. The mass m_0 behaves near the phase transition as [see Eq. (A3)]

$$m_0 \propto \frac{1}{t_c} - \frac{1}{t}. \tag{15}$$

Summation of logarithmic contributions to Eq. (9) yields

$$\delta m^2 = \delta m_\eta^2 + \delta m_k^2 + \cdots = \left(3\eta + \frac{16}{\pi^2 N}\right) m_0^2 \ln \frac{\Lambda}{m_0} + \cdots, \tag{16}$$

where δm_η^2 and δm_k^2 (as well as the respective terms on the right-hand side) denote the contribution of the first (proportional to η) term in the self-energy Σ_k, Eq. (11), and linear in k term in Σ_k, originating from the second term in Eq. (11) [the other terms in Eq. (13), apart from the linear one, do not contribute to the singular term in Eq. (16)]; the dots stand for the nonsingular terms proportional to m_0^2. Collecting all logarithmic contributions, which are of the order of $1/N$, to the Green's function and transforming them to the respective powers to introduce $1/N$ corrections to critical exponents, we obtain (see Sec. A 1)

$$G(k) = \frac{1}{\{k^2 + \xi^{-2}[1 + f(k\xi)]\}^{1-\eta/2}}, \tag{17}$$

where $f(x) = (2/N)\{F(x) - [2/(3\pi^2)] \ln[1/(x^2+1)]\}$, $\xi = m_0^{-\nu} \propto (t - t_c)^{-\nu}$ is the correlation length, and $\nu = 1 - \eta - \nu_k$ is the corresponding critical exponent. The contribution η originates again from the first term in the self-energy Σ_k, Eq. (11), while $\nu_k = 8/(\pi^2 N)$ originates from the linear in k term in Σ_k. Although the sum of the two terms yields the standard result $\nu = 1 - 32/(3\pi^2 N) \simeq 0.64$ $(N = 3)$ [25,30,39], our result allows us to discriminate the contribution of nonanalytic terms originating from the anomalous exponent η and the linear in k term in the self-energy. One can see that the latter is three times larger than the former; that is, the main contribution to the $1/N$ correction to the critical exponent ν originates from the linear in k term of the self-energy. Indeed, excluding, at $N = 3$, the term related to η yields $\nu = 0.73$, but excluding the linear in k term we get $\nu = 0.9$, which is far from the $1/N$ result. This shows the importance of the

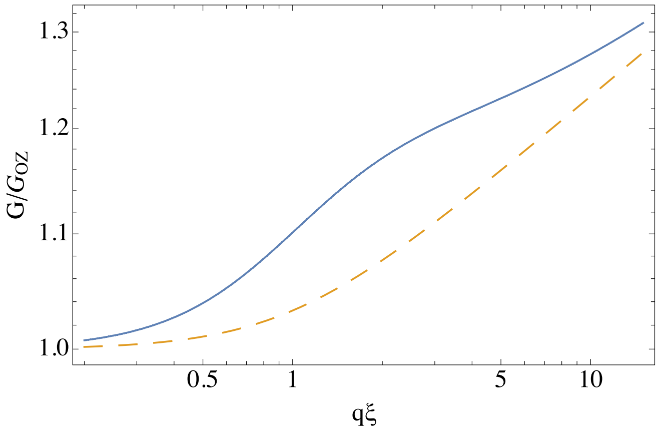

FIG. 2. The dependence of the ratio of Green's functions G/G_{OZ} on $q\xi$ at $N=3$ (solid line) in comparison to the correction $G_{\text{OZ}}^{-\eta/2}$ according to the modified Ornstein-Zernike dependence (dashed line; see text).

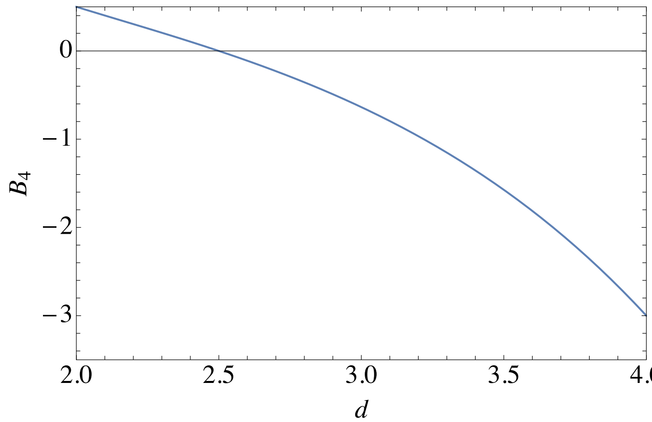

FIG. 3. The dependence of the coefficient B_4 at the leading power k^{4-d} of the momentum dependence of the self-energy on the dimensionality d.

nonanalytic contribution to the self-energy for critical exponents in three dimensions.

To estimate the deviation from the Ornstein-Zernike form we introduce the Green's function $G_{\text{OZ}} = 1/(\xi^{-2} + \varkappa k^2)$, where the coefficient $\varkappa = 1 + 8/(9\pi^2 N)$ takes into account renormalization of the coefficient at k^2 by the first-order $1/N$ expansion [see Eq. (13)]. The momentum dependence of the ratio of the Green's function (17) to the Ornstein-Zernike one is shown in Fig. 2. One can see that the obtained Green's function $G(k)$ essentially differs from both the Ornstein-Zernike $G_{\text{OZ}}(k)$ and modified dependence $G_{\text{OZ}}^{1-\eta/2}(k)$. In particular, in comparison to the $G_{\text{OZ}}^{1-\eta/2}(k)$ dependence two flection points appear. Interestingly, these flection points can be seen on the experimental data near the liquid-gas critical point [12], although the present theory, based on $1/N$ expansion, is not applicable directly to the $N = 1$ case.

B. Arbitrary $2 < d < 4$

Let us now generalize the obtained results to an arbitrary dimension $2 < d < 4$. Performing integration in Eq. (7), we find in this case (see Sec. A 2)

$$\Sigma(k) = \eta k^2 \ln \frac{\Lambda}{\left(k^2 + m_0^2\right)^{1/2}} + \frac{2}{N} m_0^2 F_d(k/m_0), \tag{18}$$

with $\eta = -(2/N)(4-d)\sin(\pi d/2)\Gamma(d-1)/[\pi d \Gamma(d/2)^2]$ being the value of the exponent for the correlation function to first order in $1/N$ [39] and the function $F_d(x)$ being given by Eq. (A9). The expansion of this function at $x \gg 1$ reads

$$F_d(x) \overset{x\gg 1}{\cong} B_0 + B_0' \ln x + B_2 x^2 + B_4 x^{4-d} + B_6 x^{6-2d} + B_8 x^{8-3d} + \cdots, \tag{19}$$

where the coefficients at $\ln x$ and at the highest power of x (apart from the quadratic term) are given by

$$B_0' = \frac{2(2d-5)\sin^2(\pi d/2)\Gamma\left(2-\frac{d}{2}\right)\Gamma(d-2)}{\pi^2 \Gamma(d/2)},$$
$$B_4 = \frac{(5-2d)\Gamma(d-1)}{2\Gamma(d/2)^2}. \tag{20}$$

The first terms of the expansion (19) were considered within the LSM in Ref. [15]; the obtained coefficients B_0' and B_4 coincide numerically with those obtained in LSM [40], although here we present a simple analytical expression for B_4 instead of the series obtained in Ref. [15]. A plot of the dependence of B_4 on dimensionality d is shown in Fig. 3. The coefficient B_4 decreases with increasing dimensionality and becomes negative for $d > 5/2$. We note the following peculiarities of the function $F_d(x)$.

(i) At $d \to 2$ we have $B_0' = 0$, while all the powers of $x = k/m_0$ in Eq. (19) approach 2. This yields the nonanalytic momentum dependence obtained in Ref. [25], $F_2(x) \sim x^2 \ln\ln x$. Moreover, as argued in Ref. [25], in this limit the summation of an infinite series of the $1/N$ expansion is required, which yields $k^2[1 - (1/N)\ln\ln(k/m_0)] \to k^2 \ln^{-1/(N-2)}(k/m_0)$. Therefore, in the limit $d \to 2$ one cannot restrict oneself to the finite number of terms either in the expansion (19) or in the $1/N$ expansion (the latter problem can, however, be solved by the replacement $N \to N-2$ in the lowest-order $1/N$ term and transforming logarithmic contributions into powers; cf. Ref. [25]).

It was observed earlier that the case of d close to 2 is described well by the $d = 2 + \varepsilon$ expansion [21,22] and that for $N = 3$ is described well by the $1/M$ expansion of the noncompact CP^{M-1} model [27]. Since the latter model was argued to be applicable near deconfined quantum critical points [4,41], the limit $d \to 2$ can be viewed as corresponding to the weakly confined spinons (cf. Ref. [28]). Fully deconfined spinons are characterized by k^{d-4} dependence of the spin correlation function (obtained as a convolution of two spinon Green's functions, each with the dependence $1/q^2$) [27–29], similar to the longitudinal correlation function in the ordered phase [26]. Therefore, the obtained leading nonanalytical term in Eq. (19), $B_4 x^{4-d}$, for d close to 2 can be considered a "trace" of this tendency to deconfinement. This term, however, does not dominate over the k^2 dependence, and the tendency to confinement dominates.

(ii) With decreasing dimensionality from $d = 4$ at the set of dimensions $d_i = 2i/(i-1)$, where $i > 2$ is an integer, i.e., at $d = 3, 8/3, 5/2, \ldots$, the term proportional to B_{2i} in the expansion (19) becomes relevant since the corresponding power changes sign. The dimensions d_i coincide with those at which the operators $(\boldsymbol{\phi}^2)^i$ in the LSM (which can be viewed as a soft constraint version of the NLSM) become relevant and corresponding new fixed points in the renormalization group flow appear. Since the fixed-point structures of the LSM and NLSM are expected to be the same, one can consider the nonanalytical terms to be related to these fixed points. One can verify that the corresponding coefficients B_{2i} are logarithmically singular in these special dimensions d_i, providing an additional contribution to B_0'. From this point of view, the leading nonanalytical term k^{4-d} is always relevant for $d < 4$, and it is related to the non-Gaussian Wilson-Fisher fixed point (cf. Refs. [13,14,16]). However, in contrast to the other coefficients B_{2i} at the dimensions $d \to d_i$, the coefficient B_4 contains at $d \to 4$ the ratio of two logarithms $\ln(k/m)/\ln(\Lambda/k)$ (see Sec. A 3): the numerator reflects the logarithmic divergence of the integral in Eq. (7), while the denominator appears because of the logarithmic divergence of $\Pi(q)$ in four dimensions.

(iii) For d not too close to 2 only the leading terms $B_2x^2 + B_4x^{4-d}$ are important; the latter provides the nonanalytical contribution to the self-energy, which, as we will see below, yields the contribution to the critical exponents, similar to the $d = 3$ case. We note that neither the results of the $2 + \varepsilon$ expansion nor the $1/M$ expansion of the noncompact CP^{M-1} model become applicable for the $d \gtrsim 3$ nonlinear sigma model (see, e.g., the discussion in Ref. [27]). This is in line with the suggestion of Ref. [42] (see also Ref. [43]) that a sharp change in critical exponents occurs somewhere in the range $2 < d < 3$ and may imply stronger spinon confinement at $d \gtrsim 3$.

Following the same strategy as for $d = 3$, we obtain the correction to the magnetic transition temperature (or critical coupling constant)

$$t_c = \frac{d-2}{NA_d\Lambda^{d-2}}\left[1 + \frac{\eta}{d-2} + B_2\right]. \tag{21}$$

The calculation of the masses m_0 and δm, as well as the critical exponent ν, is performed in the same way as for $d = 3$ and detailed in Sec. A 2. We find $m_0 \propto (t - t_c)^{1/(d-2)}$, and the $1/N$ correction to the mass reads

$$\delta m^2 = \frac{\eta d + (8B_4/\pi N)\sin(\pi d/2)}{d-2} m_0^2 \ln\frac{\Lambda}{m_0} + \dots, \tag{22}$$

where the first and second terms in the numerator correspond to the contribution of the first term in Eq. (18) and the x^{4-d} term in the asymptotic $F_d(x)$ and the dots stand for the nonlogarithmic terms. From these contributions, we obtain

$$\nu = \frac{1}{d-2}\left[1 - \frac{\eta + (4B_4/\pi N)\sin(\pi d/2)}{d-2}\right]. \tag{23}$$

This critical exponent also coincides with the earlier known result of the $1/N$ expansion [39], but we again individuate here contributions of two different effects: the anomalous exponent η and the k^{4-d} term in the self-energy. The contributions of these two effects to the critical exponent ν at $N = 3$ and various dimensionality d are plotted in Fig. 4. One can see that while for $d \to 2$ these two effects almost cancel each other, with increasing dimensionality d the k^{4-d} term of the self-energy gives a progressively larger contribution; for $d \to 4$, where the anomalous exponent η vanishes, the k^{4-d} term gives the major contribution to the $1/N$ correction to the critical exponent ν. For completeness we also show in Fig. 4 the result of the $1/N$ expansion with the replacement $N \to N - 2$, which, according to Refs. [25,27], should be performed in the limit $d \to 2$, allowing us, e.g., to achieve agreement with $2 + \varepsilon$ expansion and recent results of the functional renormalization group approach of Ref. [43]. One can see that at $d \sim 5/2$ the exponent ν is expected to sharply change from the result of the $1/N$ expansion with $N \to N - 2$ to that without the replacement, which seems to correspond to the transition predicted in Ref. [42] and/or the crossover discussed recently in Ref. [43]. Following the discussion in points (i) and (iii) above, this transition (or crossover) would correspond to the change from weak to strong spinon confinement. Interestingly, at the dimension $d = 5/2$ the coefficient B_4 changes sign, which may be related to the weak-strong confinement transition since vanishing of this coefficient reflects full confinement of spinons.

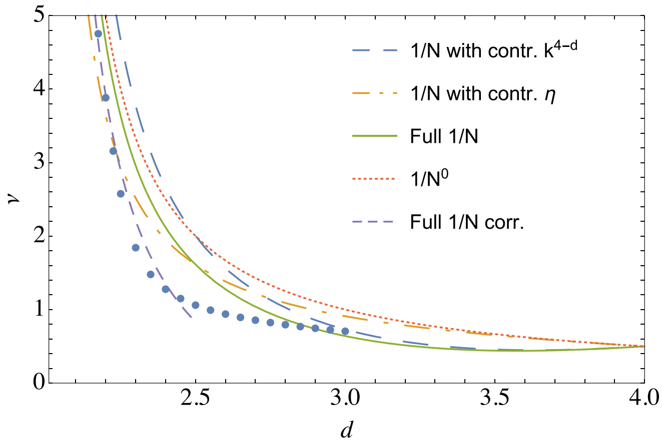

FIG. 4. The dependence of the critical exponent ν of the $N = 3$ nonlinear sigma model on dimensionality d in various approaches: zeroth-order $1/N$ result (dotted line), first-order $1/N$ approach neglecting the exponent η (long-dashed line), first-order $1/N$ approach with only correction from the exponent η included (dash-dotted line), full first-order $1/N$ result (solid line), and first-order $1/N$ result with the replacement $N \to N - 2$, which has to be performed for $d \to 2$ (short-dashed line). Dots show the results of the functional renormalization group approach of Ref. [43].

IV. CONCLUSIONS

In summary, we have obtained the momentum dependence of the self-energy of spin excitations in the Heisenberg model in the first-order $1/N$ expansion. The obtained dependence contains the nonanalytic contribution B_4k^{4-d}; the coefficient B_4 decreases from positive values at $d \to 2$ to negative values at $d \to 4$. We have argued that the nonanalytical term likely originates from the Wilson-Fisher non-Gaussian fixed point. In the general dimension d, there are also subleading terms $B_{2i}k^{2i-(i-1)d}$ with integer values $i > 2$ which correspond to the new fixed points, appearing at dimensions $d_i = 2i/(i-1)$ and related also to the relevance of the operators $(\boldsymbol{\phi}^2)^i$ in the linear sigma model.

We have shown that the critical exponent ν in the first order in $1/N$ is determined by two contributions. The first contribution originates from the exponent η, while the second is proportional to the coefficient B_4 of the nonanalytic term. While at $d \to 2$ the two contributions almost compensate each other, at $3 \lesssim d < 4$ the second contribution dominates and fully determines the value of the $1/N$ correction to the exponent ν for $d \to 4$. The change in sign of the coefficient B_4 at $d = 5/2$ is associated with the transition (or crossover) from weak to strong spinon confinement. This is also in line with the predicted sharp change in critical exponents at $d \sim 5/2$ [42,43].

Apart from the importance of the obtained results for the interpretation of numerical and experimental data for Heisenberg magnets, they may have some importance for itinerant antiferromagnets. Indeed, at half filling the Hubbard model can be effectively reduced to the nonlinear sigma model for arbitrary on-site Coulomb repulsion U (see Ref. [44]). Therefore, one can expect the appearance of the nonanalytical terms in the susceptibility in itinerant half-filled antiferromagnets as well. These terms may be rather hard to obtain from purely fermionic approaches since they correspond to greater than four-point fermion interaction vertices. Although the numerically correct value of the critical exponent ν was obtained previously within the dynamic vertex approximation (DΓA) [45] and dual-fermion approach [46] (see also the review [47]), the conclusions drawn in these studies possibly need to be reexamined in light of the results of the present paper, as well as of the most recent DΓA calculations [48].

Investigation of the connection between spinon (de)confinement and the nonanalytic term k^{4-d} in the self-energy of spin excitations represents another important topic for future studies.

ACKNOWLEDGMENTS

The author acknowledges A. Toschi and G. Rohringer for stimulating discussions on itinerant magnets, which led to the formulation of the problem considered in the paper, and P. Jakubczyk for providing the data from Ref. [43]. This work is partly supported by the theme “Quant” AAAA-A18-118020190095-4 of Minobrnauki, Russian Federation.

APPENDIX: EVALUATION OF SELF-ENERGY AND MASS CORRECTIONS

1. Dimension $d = 3$

The self-energy is obtained from Eqs. (7) and (10) and reads

$$\Sigma(k) = \frac{2}{N}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{1}{k^2 + 2\mathbf{kq} + q^2 + m_0^2} - \frac{1}{q^2 + m_0^2}\right]\frac{4\pi q}{\arctan[q/(2m_0)]}$$
$$= \frac{2}{\pi N}\int_0^\Lambda q^2 dq\left[\frac{1}{2kq}\ln\frac{(k+q)^2 + m_0^2}{(k-q)^2 + m_0^2} - \frac{2}{q^2 + m_0^2}\right]\frac{q}{\arctan[q/(2m_0)]}. \tag{A1}$$

By picking out the singular contribution

$$\Sigma_1(k) = \eta k^2 \ln\left[\Lambda/\left(k^2 + m_0^2\right)^{1/2}\right]$$

the integral can be made convergent. Taking the limit $\Lambda \to \infty$ in the remaining part and rescaling the variable of the integration by m_0, we obtain Eq. (11) of the main text. In the following we denote $\Sigma = \Sigma_1(k) + \Sigma_2(k) + \Sigma_3(k) + \Sigma_4(k)$, where

$$\Sigma_2(k) = \frac{8k^2}{9\pi^2}, \quad \Sigma_3(k) = -4km_0/(\pi N),$$

and $\Sigma_4(k) = (2/N)m_0^2 F(k/m_0) - 8k^2/(9\pi^2) + 4km_0/(\pi N)$ is the remaining part, obtained by subtracting and adding the asymptotic value of the integrand at large q, where the function $F(x)$ is given by the Eq. (13). Evaluation of the integrals which enter Eq. (9) yields

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} G_0(k) = \frac{1}{2\pi^2}\left(\Lambda - \frac{\pi m_0}{2}\right),$$
$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} G_0^2(k)\Sigma_1(k) = \frac{\eta}{2\pi^2}\left[\Lambda - \frac{3\pi}{4} m_0 \ln\frac{\Lambda}{m_0}\right],$$
$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} G_0^2(k)\Sigma_2(k) = \frac{1}{2\pi^2}\frac{8\Lambda}{9\pi^2 N},$$
$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} G_0^2(k)\Sigma_3(k) = -\frac{1}{2\pi^2}\frac{4m_0}{\pi N}\ln\frac{\Lambda}{m_0},$$
$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} G_0^2(k)\Sigma_4(k) = \frac{1}{2\pi^2}\frac{2m_0 I}{\pi N},$$
$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} G_0^2(k) = \frac{1}{8\pi m_0}, \tag{A2}$$

where $I = \int_0^\infty \frac{k^2 dk}{(k^2+1)^2}(F(k) - \frac{4k^2}{9\pi} + 2k)$ is a universal number. Collecting contributions to the sum rule (9), which are linear in Λ or m_0 and do not contain logarithmic terms, we find

$$m_0 = \frac{4\pi}{N}\left(1 - \frac{4}{\pi^2 N} I\right)\left(\frac{1}{t_c} - \frac{1}{t}\right), \tag{A3}$$

where t_c is defined according to Eq. (14). The remaining contributions to Eq. (9) taking into account the last integral in Eqs. (A2) lead to the mass correction (16) in the main text. The resulting Green's function reads

$$G(k) = \frac{1}{k^2 + m^2 + \eta k^2 \ln\left[\Lambda/\left(k^2+m_0^2\right)^{1/2}\right] + \left(2m_0^2/N\right)F(k/m_0)}. \tag{A4}$$

Transforming log to a power, which is usual in the $1/N$ expansion, and neglecting higher-order terms in $1/N$, we obtain

$$G(k) = \frac{1}{\{k^2 + m^2\left[1 - \eta\ln\left[\Lambda/\left(k^2+m_0^2\right)^{1/2}\right]\right] + \left(2m_0^2/N\right)F(k/m_0)\}^{1-\eta/2}}; \tag{A5}$$

the remaining log contributes to ν (see below), and the function $F(k/m)$, obtained above, describes the nonanalytic contribution to the expression in square brackets. Using $m^2 = m_0^2 + \delta m^2$ and the expression for the mass correction δm (16), we obtain for the Green's function

$$G(k) = \frac{1}{\left\{k^2 + m_0^2[1 + 2(\eta + 8/(\pi^2 N))\ln(\Lambda/m_0)] + \left(2m_0^2/N\right)\widetilde{F}(k/m_0)\right\}^{1-\eta/2}}, \tag{A6}$$

where $\widetilde{F}(x) = F(x) - [2/(3\pi^2)]\ln[1/(x^2+1)]$. Transforming again the logarithmic term into the power $m_0^{2(\nu-1)}$, denoting $\xi = m_0^{-\nu}$, and neglecting the terms of the higher order of $1/N$, we obtain Eq. (17).

2. Arbitrary $2 < d < 4$

In this case we find the polarization operator

$$\Pi(q) = m_0^{d-4}\widetilde{\Pi}(q/m_0),$$
$$\widetilde{\Pi}(x) = \frac{2^{-\frac{d}{2}}\pi A_d}{x}\csc(\pi d/2)(4+x^2)^{\frac{d}{4}-1}$$
$$\times\left[(\sqrt{4+x^2} - x)^{d/2-1}\,{}_2F_1\left(2 - \frac{d}{2}, \frac{d}{2} - 1, \frac{d}{2}; \frac{1}{2} - \frac{x}{2\sqrt{4+x^2}}\right)\right.$$
$$\left. - (\sqrt{4+x^2} + x)^{d/2-1}\,{}_2F_1\left(2 - \frac{d}{2}, \frac{d}{2} - 1, \frac{d}{2}; \frac{1}{2} + \frac{x}{2\sqrt{4+x^2}}\right)\right], \tag{A7}$$

where ${}_2F_1(a, b, c; z)$ is the hypergeometric function and $A_d = 2^{1-d}\pi^{-d/2}/\Gamma(d/2)$. For the self-energy we obtain

$$\Sigma(k) = \frac{4A_d}{N}\frac{\Gamma\left(\frac{d}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)}\int\frac{q^{d-1}dq\sin^{d-2}\theta\, d\theta}{\Pi(q)}\left(\frac{1}{k^2 + 2kq\cos\theta + q^2 + m_0^2} - \frac{1}{q^2+m_0^2}\right)$$
$$= \frac{2A_d}{N}\int_0^\Lambda q^{d-1}dq\left[\frac{{}_2F_1\left(1, \frac{d-1}{2}, d-1; -\frac{4kq}{(k-q)^2+m_0^2}\right)}{(k-q)^2+m_0^2} - \frac{1}{q^2+m_0^2}\right]\frac{1}{\Pi(q)}. \tag{A8}$$

By subtracting and adding the asymptotic of the integrand at $q \to \infty$, taking the limit $\Lambda \to \infty$ in the convergent integral, and rescaling again the variable of integration by m_0, the result can be put in the form of Eq. (11), with

$$F_d(x) = A_d\int_0^\infty q^{d-1}dq\left\{\left[\frac{{}_2F_1\left(1, \frac{d-1}{2}, d-1; -\frac{4xq}{(x-q)^2+1}\right)}{(x-q)^2+1} - \frac{1}{q^2+1}\right]\frac{1}{\widetilde{\Pi}(q)}\right.$$
$$\left. - \frac{2^{d-2}(d-4)\pi^{\frac{d}{2}-1}\sin\left(\frac{\pi d}{2}\right)\Gamma(d-1)}{\Gamma(d/2+1)}x^2 q^{-d}\theta(q - \sqrt{x^2+1})\right\}, \tag{A9}$$

which yields Eq. (18) in the main text. The lowest-order coefficients in the expansion of $F_d(x)$ at large x are given by Eq. (20) and

$$B_2 = \frac{N\eta}{2}\int_0^\infty t\,dt\left\{\frac{d}{4-d}\left[\frac{t^2}{(t-1)^2}\,{}_2F_1\left(1, \frac{d-1}{2}, d-1; -\frac{4t}{(t-1)^2}\right) - 1\right] - \frac{1}{t^2}\theta(t-1)\right\}.$$

Evaluating integrals entering Eq. (9), we find

$$\int \frac{d^d k}{k^2+m_0^2} = A_d\left[\frac{\Lambda^{d-2}}{d-2} + \frac{\pi}{2}\csc\left(\frac{\pi d}{2}\right)m_0^{d-2}\right],$$
$$\int \frac{d^d k}{\left(k^2+m_0^2\right)^2} k^2 \ln\frac{\Lambda}{\sqrt{k^2+m_0^2}} = A_d\left[\frac{\Lambda^{d-2}}{(d-2)^2} + \frac{\pi d}{4}\csc\left(\frac{\pi d}{2}\right)m_0^{d-2}\ln\frac{\Lambda}{m_0} + \cdots\right],$$
$$\int \frac{k^2 d^d k}{\left(k^2+m_0^2\right)^2} = \frac{A_d}{d-2}\Lambda^{d-2},$$
$$m_0^{d-2}\int \frac{d^d k}{\left(k^2+m_0^2\right)^2} k^{4-d} = A_d m_0^{d-2}\ln\frac{\Lambda}{m_0}, \quad \int \frac{d^d k}{\left(k^2+m_0^2\right)^2} = -A_d\frac{\pi(d-2)}{4}\csc\left(\frac{\pi d}{2}\right)m_0^{d-4}. \tag{A10}$$

Collecting the terms proportional to Λ^{d-2} or m_0^{d-2}, we find the equation for m_0,

$$1 = tA_d\frac{N\Lambda^{d-2}}{(d-2)}\left[1 - \frac{\eta}{d-2} - \frac{2}{N}B_2\right] + NtA_d\frac{\pi\csc(\pi d/2)}{2}m_0^{d-2}. \tag{A11}$$

By defining t_c according to Eq. (21) we obtain

$$m_0 = \left[-\frac{2}{NA_d\pi\csc(\pi d/2)}\left(\frac{1}{t_c} - \frac{1}{t}\right)\right]^{1/(d-2)}.$$

The correction δm^2 is obtained then straightforwardly from the remaining terms in the sum rule (9) and given by the Eq. (22). Repeating the calculation of the Green's function similarly to that for the $d = 3$ case, we find

$$G(k) = \frac{1}{\{k^2 + m_0^2\left[1 + [2\eta + (8B_4/\pi N)\sin(\pi d/2)]/(d-2)\ln(\Lambda/m_0)\right] + \left(2m_0^2/N\right)\widetilde{F}_d(k/m_0)\}^{1-\eta/2}}, \tag{A12}$$

where $\widetilde{F}_d(x) = F_d(x) - (N\eta/4)\ln[1/(x^2+1)]$. After transforming the logarithm into power we obtain again the result (17) with $\xi = m_0^{-\nu(d-2)} \propto (t-t_c)^{-\nu}$, where ν is given by Eq. (23).

3. Dimension $d = 4$

For completeness, let us also present some results in four dimensions. Performing integration in Eq. (8), we obtain

$$\Pi(q) = \frac{1}{8\pi^2}\left[\ln\left(\frac{\Lambda}{m_0}\right) - \frac{\sqrt{4m_0^2+q^2}}{2q}\tanh^{-1}\left(\frac{q\sqrt{4m_0^2+q^2}}{2m_0^2+q^2}\right) + \frac{1}{2}\right]. \tag{A13}$$

The corresponding contribution to the self-energy reads

$$\Sigma(k) = \frac{1}{4\pi^2 N}\int q^3 dq \frac{1}{\Pi(q)}\left[\frac{k^2+q^2+m_0^2 - \sqrt{k^4 + \left(q^2+m_0^2\right)^2 - 2k^2\left(q^2-m_0^2\right)}}{2k^2q^2} - \frac{1}{q^2+m_0^2}\right]. \tag{A14}$$

After evaluating the integral in the limit $k \gg m_0$ and neglecting terms of the order of k^2/l and m_0^2/l, where $l = \ln(\Lambda/k)$, $\ln(\Lambda/m_0)$, or $\ln(k/m_0)$, we obtain

$$\Sigma(k) = -\frac{6m_0^2}{N}\left[\frac{2\ln(k/m_0)}{1+2\ln(\Lambda/k)} - \ln\left(\frac{2\ln(\Lambda/m_0)}{1+2\ln(\Lambda/k)}\right)\right]. \tag{A15}$$

We note that the coefficient in front of the square brackets is equal to $2B_4(d\to 4)/N$. Performing integrations in Eq. (9), we obtain with logarithmic accuracy

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{\left(k^2+m_0^2\right)} = \frac{1}{8\pi^2}\left(\frac{\Lambda^2}{2} - m_0^2\ln\frac{\Lambda}{m_0}\right),$$
$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{\left(k^2+m_0^2\right)^2}\Sigma(k) = \frac{3m_0^2}{4\pi^2 N}\ln\frac{\Lambda}{m_0}\left(2 - \ln\ln\frac{\Lambda}{m_0}\right),$$
$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{\left(k^2+m_0^2\right)^2} = \frac{1}{8\pi^2}\ln\frac{\Lambda}{m_0}. \tag{A16}$$

Putting $m_0 = \delta m = 0$, we find the critical temperature $t_c = 16\pi^2/(\Lambda^2 N)$. Absorbing $\ln(\Lambda/m_0)$ contributions into the bare mass m_0, we find

$$m_0^2 \ln \frac{\Lambda}{m_0} = \frac{8\pi^2}{N+12}\left(\frac{1}{t_c} - \frac{1}{t}\right). \tag{A17}$$

Finally, the remaining contributions to Eq. (9) yield

$$\delta m^2 = \frac{6m_0^2}{N} \ln \ln \frac{\Lambda}{m_0}. \tag{A18}$$

In Eqs. (A17) and (A18) we recognize the zeroth- and first-order terms in the $1/N$ expansion of the one-loop renormalization group result (see, e.g., Ref. [49]) $m^2 \propto (t - t_c)/\ln^{(N+2)/(N+8)}(\Lambda/m)$.

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