Effect of vortex annihilation lines on magnetic relaxation in high-temperature superconductors

L. Burlachkov* and S. Burov
Department of Physics, Bar-Ilan University, Ramat-Gan 5290002, Israel

(Received 16 August 2020; accepted 11 December 2020; published 13 January 2021)

We show that in superconductors of type II, where Abrikosov vortices with different polarities are present, the areas where they meet and annihilate each other ($B = 0$ lines) "attract" a significant part of the magnetization current j. This leads to redistribution of j over the sample, and as a result, the rate of magnetic relaxation is reduced. This effect is significant in the case of weak dependence of the activation energy U on j, particularly for the flux flow ($U = 0$) and at early stages of flux creep ($U \gtrsim kT$). The slowdown of the relaxation is mostly pronounced in the remanent state, where the $B = 0$ lines are located at the edges of the sample. In the case of flux flow, the remanent magnetization decays as $m \propto 1/t$ instead of the usual "field-on" exponential dependence $m \propto \exp(-t/\tau)$. The effect is important and observable in the magnetization measurements, for instance, in $La_{2-x}Sr_xCuO_4$ crystals and other novel superconducting materials.

DOI: 10.1103/PhysRevB.103.024511

I. INTRODUCTION

Starting from the pioneer work on giant flux creep [1], it has become clear that high-temperature superconductors (HTSCs) cannot be described by the classic Bean model [2] in which the magnetization is determined by the critical current j_c and magnetic relaxation that results from vortex flow or creep is totally neglected. Except for the region of low temperatures that varies from compound to compound, say, $T \lesssim 40$ K for $YBa_2Cu_3O_{7-x}$ (YBCO) and $T \lesssim 20$ K for $Bi_2Sr_2Ca_2Cu_3O_{10+x}$ (BSCCO), the relaxation processes in these compounds are very fast. Therefore, the experimentally measured magnetization current j is usually significantly less than j_c and even than the depinning current j_{depin} (see Refs. [3,4] as reviews). Instead, j appears to be a function of the experimental waiting time, sweeping rate of the magnetic field, the "history" of the sample (field cooled or zero field cooled), and other circumstances.

Whereas the relaxation in the field-on measurements has been successfully described theoretically for long cylinders and slabs by both analytic studies [4–6] and numeric simulations [6,7], a consistent description of the relaxation in the remanent state has not been developed yet. The main difficulty, as pointed out already in Ref. [5], is the definition of the flow of vortices, $D = Bv$ (where v is the vortex velocity) at the edges of a long sample. Since $B = 0$ at the edges in the remanent state, D seems to vanish there as well. This contradicts the obvious fact that D should remain finite at the edges and even reach its maximal value over the whole sample there. The explanation for this contradiction was provided in Refs. [6,8], where it was proved that v diverges near the edges, which compensates the vanishing B. For instance, it was shown that $B \propto \sqrt{x}$ in the flux flow regime, where x is the distance from the edge. At the same time, v proves to diverge as $1/\sqrt{x}$; thus, $D = Bv$ remains finite and reveals no singularity. However, a consistent study of the relaxation in the remanent state, including a comprehensive analysis of the $B(x,t)$ dependence in the remanent state, has not been performed yet.

In this paper we show that the peculiar nature of magnetic relaxation in the remanent state in long superconducting samples is a particular manifestation of a more general phenomenon, namely, a dramatic effect of the annihilation lines, where $B = 0$, on the kinetics of Abrikosov vortices. At these lines vortices of different polarities encounter and annihilate each other. We show that in long samples (slabs, cylinders), where vortices are almost straight, v and, in turn, the density of the magnetization current j diverge at the annihilation lines. Of course, the magnetic penetration depth λ serves as a cutoff for such a divergence. This effect results in redistribution of j over the sample, which, in turn, crucially slows down the relaxation of the total magnetic moment. The qualitative explanation for such an effect (later, we perform a quantitative analysis) could be as follows. The total current $I = \int j(x)dx$, where x is the coordinate across the sample width, is determined by the boundary conditions for $B(x)$. In symmetrical relaxation problems I flowing in one half of the sample is proportional to $B_{center} - B_{edge}$, where B_{center} and B_{edge} are the values of the magnetic field at the center of the sample and its edge, respectively. Thus, the total "amount" (integral) of j is finite, whereas the driving force for vortex motion (the Lorentz force) is proportional to j. An annihilation line "consumes" a lot of j since the latter diverges in its vicinity. As a result, the average values of j significantly decrease in the rest of the sample, and the total relaxation rate slows down. We will show below that the most dramatic deceleration of relaxation should be expected in the remanent state where $B_{edge} = 0$. In this case the annihilation line is located at the edge of the sample, and vortices annihilate with their mirror images, as described by Bean and Livingston [9].

*leonid@biu.ac.il

Our analysis proves that the deceleration (slowdown) effect in the presence of annihilation lines is particularly pronounced in the flux flow regime, where there are no potential barriers and vortex motion is impeded only by viscous friction [10]. Generally speaking, the same should happen in the flux creep regime, but with no or weak dependence of U on j. In the field-on case, where $B \neq 0$ in the whole sample (no annihilation lines), the magnetization depends on time exponentially: $|m| \propto \exp(-t/\tau)$, as shown in Ref. [6]. Here τ is the characteristic time of flux flow in the sample. However, as we show below, in the remanent state m is described by a power-law dependence $m \propto (\tau/t)^{\alpha}$, with $\alpha = 1$ in the remanent state. This is, of course, much slower than the exponential dependence in the field-on case.

Before starting our analysis, let us outline its applicability. Most popular HTSC crystals (YBCO, BSCCO, and others) usually have a platelet shape with a very small aspect ratio $r_c/r_{ab} \ll 1$, where r_c and r_{ab} are the characteristic sample sizes along the c axis and in the ab plane, respectively. In this case the vortices are curved, especially in the remanent state; the magnetization current j is determined by both components of the magnetic induction, B_c and B_{ab}, and both components v_c and v_{ab} of the vortex velocity are important [3,11]. In such platelets the vortex annihilation in the remanent state is implemented by formation of closed vortex loops that collapse at the $B_c = 0$ line. Neither divergence of the vortex velocity nor any effects of dramatic slowdown of the relaxation have been observed in platelet samples with the magnetic field directed along the c axis. Of course, it is possible to study the relaxation rates in these compounds at $H \perp c$, where the slowdown effects could be pronounced, but this requires a quite fine experimental technique since the corresponding magnetic moments will be small.

It is not a problem to create long samples of conventional (low-T_c) superconductors, like Nb-based materials. However, as we show below, the slowdown of the relaxation process due to presence of the $B = 0$ lines is most significant at $U = 0$ (flux flow) or at least at $U \lesssim kT$. In low-T_c superconductors the activation energies are usually high, $U/kT \gg 1$, and crucial deceleration of the magnetic relaxation related to the annihilation lines effect is unlikely to be found.

The most promising materials to observe the annihilation lines' effect on flux dynamics are the $La_{2-x}Sr_xCuO_4$ samples with $T_c \approx 32$ K that combine very high relaxation rates with aspect ratios $r_c/r_{ab} \gtrsim 1$ (see Ref. [12]). This is an intermediate case between a platelet ($r_c/r_{ab} \ll 1$) and an infinite slab or a cylinder ($r_c/r_{ab} = \infty$). The analysis of flux penetration into and exit from finite slabs [13] proves that, say, for $c/a = 2$ the vortices are quite straight even near the sample edge, except for the very "top" and "bottom" of the sample. Therefore, one can neglect B_{ab} and apply the one-dimensional analysis where $B = B_c$. We compare our theoretical analysis with experimental results obtained in $La_{2-x}Sr_xCuO_4$ samples and show that the dramatic slowdown explains the anomalies found in magnetization curves obtained experimentally in these compounds.

II. FLUX RELAXATION IN THE REMANENT STATE

Let us compare flux penetration into an infinite (along y and z directions) superconducting slab with width $2d$ ($0 < x < 2d$) when the external field H is switched on (field-on relaxation) with flux exiting from the same slab when the external field is removed: $H = 0$ (remanent relaxation). The magnetic field $B(x,t)$ is parallel to z, vortices move along x, the magnetization current $j(x,t)$ flows along y, and both are related by Maxwell's law:

$$j = -\frac{c}{4\pi}\frac{\partial B}{\partial x}. \tag{1}$$

The field-on case corresponds to the zero-field-cooled experiment, where $H \| z$ is switched on when there are no vortices in the sample: $B(x, t = 0) = 0$. In the "remanent" case, initially, $B(x, t = 0) = H$, and then H is abruptly switched off. In both these cases the dependence of B on x and t is determined by the one-dimensional flux "diffusion" equation [6,14,15]:

$$\frac{\partial B}{\partial t} = -\nabla(Bv) = -\frac{\partial}{\partial x}\left(\mathcal{A}\frac{\phi_0}{c\eta}Bj\exp(-U/kT)\right), \tag{2}$$

where ϕ_0 is the unit flux, η is the Bardeen-Stephen drag coefficient (viscous friction) [10], c is the light velocity, U is the activation energy for the vortex motion, k is the Boltzmann constant, T is temperature, and $\mathcal{A} \simeq 1$ is a numerical factor [16]. Generally, the activation energy U depends on both B and j; see Ref. [4] for a review. We focus on the case where the annihilation line $B = 0$ is present in the sample; thus, the magnetic field is relatively small, and it is possible to neglect the dependence of U on B that becomes important at high fields in the regime of collective flux creep [4]. We choose the logarithmic dependence of U on j:

$$U(j) = U_0 \ln(j_c/j), \tag{3}$$

which has been reported in numerous experiments [17] at low fields since it enables us to get (in some cases) analytic solutions of Eq. (2) along with numerical results. It is worth mentioning that Eq. (3) corresponds to the power-law E-j characteristics: $E \propto j^{u+1}$, where $\vec{E} = -(1/c)[\vec{v} \times \vec{B}]$ is the electric field inside a superconductor and $u = U_0/kT$. This particular choice of the $U(j)$ dependence does not confine the universality of our results, as we discuss later. Obviously, the case $u = 0$ corresponds to flux flow.

Substituting Eqs. (1) and (3) into Eq. (2), we get

$$\frac{\partial B}{\partial t} = \frac{\mathcal{A}\phi_0}{4\pi\eta}\left(\frac{c}{4\pi j_c}\right)^u \frac{\partial}{\partial x}\left[B\left(\frac{\partial B}{\partial x}\right)^{u+1}\right]. \tag{4}$$

In the field-on case the boundary conditions for Eq. (4) are $B(0) = B(2d) = H$, and in the remanent state we have $B(0) = B(2d) = 0$. A method of semianalytical solutions of similar equations, provided $B \neq 0$, was developed in Ref. [6]. However, the logarithmic dependence of U on j [see Eq. (3)] was not considered there.

Let us start with relaxation of the remanent magnetization. In this case Eq. (4) can be solved exactly by separation of variables: $B(x,t) = B_0 b(x) f(t)$. The boundary conditions are $b(0) = 0$, $b(d) = 1$, and the initial condition is $f(0) = 1$. Correspondingly, B_0 is the magnetic field at the slab's center $x = d$ at $t = 0$. We confine our analysis to the area $0 \leqslant x \leqslant d$ since, apparently, the field profiles are symmetric at any time:

$b(x) = b(2d - x)$. Then Eq. (4) acquires the form

$$b\frac{df}{dt} = sf^{u+2}\frac{\partial}{\partial x}\left[b\left(\frac{db}{dx}\right)^{u+1}\right], \tag{5}$$

where $s = (cB_0/4\pi j_c)^u \mathcal{A}\phi_0 B_0/(4\pi\eta)$. Note that the dimension of s is $\mathrm{cm}^{u+2}\,\mathrm{s}^{-1}$. Thus, we obtain two separate equations for the spatial and temporal dependencies of the field:

$$\frac{1}{b}\frac{d}{dx}\left[b\left(\frac{db}{dx}\right)^{u+1}\right] = -C, \tag{6}$$

$$\frac{1}{sf^{u+2}}\frac{df(t)}{dt} = -C, \tag{7}$$

where C is a positive constant.

Equation (6) can be transformed into an elementary differential equation,

$$g + \frac{u+1}{u+2}b\frac{dg}{db} + bC = 0, \tag{8}$$

by the substitution $g = (db/dx)^{u+2}$, treating b as a variable and $g(b)$ as a function. After solving Eq. (8), we find from Eq. (6) that at $0 \leqslant x \leqslant d$,

$$\int_0^b \frac{\xi^{\frac{1}{u+1}}d\xi}{\left(1-\xi^{\frac{2u+3}{u+1}}\right)^{\frac{1}{u+2}}} = \frac{Q}{d}x, \tag{9}$$

where

$$Q(u) = (u+1)\int_0^1 \frac{\xi^{u+1}d\xi}{(1-\xi^{2u+3})^{\frac{1}{u+2}}}. \tag{10}$$

Note that the constant C in Eqs. (6) and (7) is related to Q as $C = (Q/d)^{u+2}(2u+3)/(u+2)$. The temporal dependence $f(t)$ can be found from Eq. (7):

$$f(t) = \left(\frac{1}{1+(u+1)t/\tau(u)}\right)^{\frac{1}{u+1}}, \tag{11}$$

where

$$\tau(u) = \frac{1}{s}\frac{u+2}{2u+3}\left(\frac{d}{Q}\right)^{u+2}. \tag{12}$$

Equation (9) implicitly determines the spatial dependence of the field $b(x)$, which is shown in Fig. 1 for $0 \leqslant x \leqslant d$ (left half of the sample) at different values of u. Obviously, the greater u is, the more straight profiles we get since the spatial variations of the activation energy $U = kTu\ln(j_c/j)$ should be of order kT, as discussed in Ref. [6]. It is worth mentioning that if the activation energy depends on field, $U = U(B, j)$, then the condition of its approximate constancy is violated in the remanent state (see Ref. [6]). In this work, however, we focus on the case where U depends only on j and will consider $U(B)$ dependence elsewhere. Note also that the spatial dependence of b [determined by Eq. (9) and shown in Fig. 1] is of the *self-organized* type [6,8]. This means that for any arbitrary initial distribution $B(x, 0)$ the profile of the magnetic field approaches $b(x)$ at $t \gg \tau$.

It is an interesting question how the field profile, i.e., the function $b(x)$, behaves near the edge of the sample. As shown in Ref. [6] by numerical methods, in the flux flow regime the field vanishes near the edge as a square root: $b(x) \propto \sqrt{x}$. Now

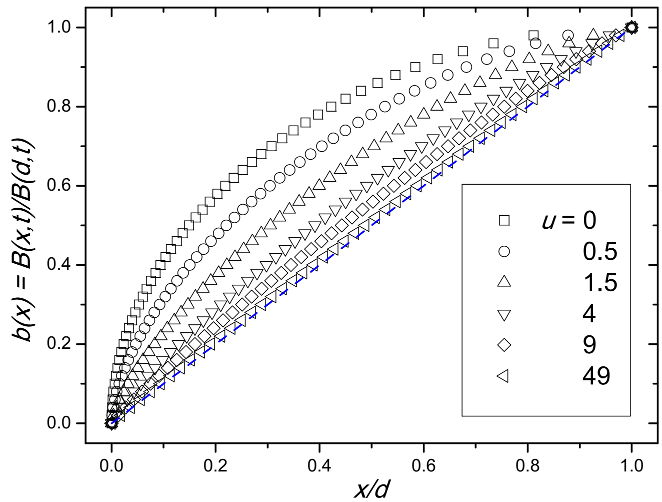

FIG. 1. Spatial dependence of the magnetic field $b(x)$ shown for $0 \leqslant x \leqslant d$ (one half of the slab) and different values of the parameter $u = U_0/kT$. The profiles approach the straight line (dashed) as u grows; see discussion in the text.

we can prove analytically that this is a particular case of a more general result. One easily finds from Eq. (9) that

$$b(x) \propto x^\beta, \quad \beta = \frac{u+1}{u+2} \tag{13}$$

at $x/d \ll 1$. The exponent β in Eq. (13) ranges between 0.5 at $u = 0$ (flux flow, square-root field profile) and 1 at $u \to \infty$ (high activation energies, straight field profile). The corresponding $b(x)$ profiles are shown in Fig. 1. Moreover, the power-law dependence described by Eq. (13) appears to hold not only close to the edge $x = 0$ but for a significant part of the sample $x \lesssim d$ except its very center (see Fig. 2). The magnetization current $j \propto \partial B/\partial x$ diverges as $x^{\beta-1}$ near the edges of the sample. The strongest divergence is achieved at $u = 0$ (flux flow), where $\beta = 1/2$ and $j \propto x^{-1/2}$, whereas at $u \to \infty$ we get $\beta = 1$, and the divergence of j fades out.

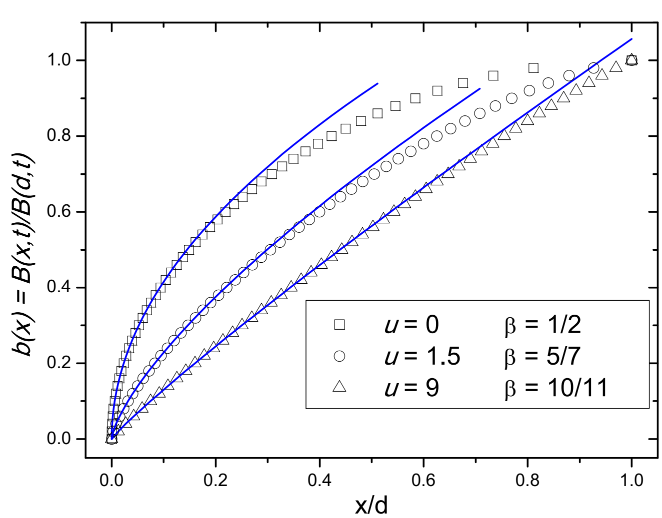

FIG. 2. Power-law approximation for the spatial dependence of the field, $b(x) \approx x^\beta$ [see Eq. (13)], at different values of u.

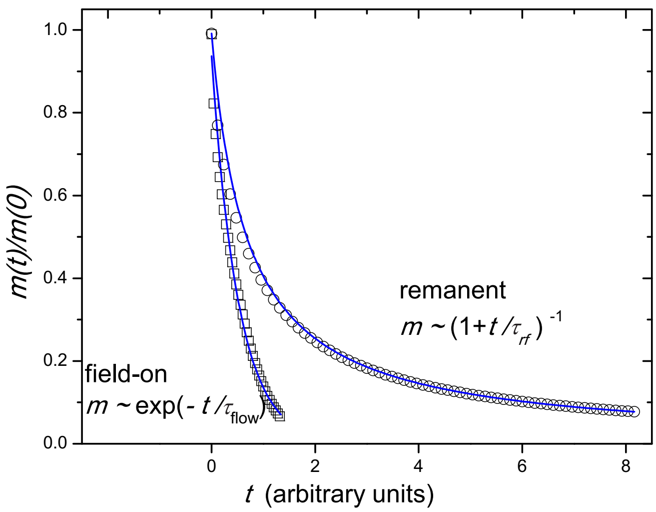

FIG. 3. Comparison of the relaxation of the magnetic moment $m(t)$ at $u = 0$ (flux flow) for field-on measurements where the relaxation is exponentially fast [see Eq. (16)] and in the remanent state where $m \propto 1/t$ [see Eq. (15)]. We assume $H = B_0$, so $\tau_{rf} = 1.79\tau_{\rm flow}$. The analytic results (solid lines) are shown together with the numeric data obtained for the flux penetration into an empty sample $B(x, 0) = 0$ after its instantaneous switching on (squares) and for the flux exit from a completely filled sample $B(x, 0) = B_0$ after H is instantaneously switched off (circles).

It can be easily checked that the magnetic flux current $D(x) \propto B(\partial B/\partial x)^{u+1}$ [see Eqs. (2) and (4)] does not even remain finite at $x = 0$ (and, correspondingly, at $x = 2d$) but reaches its maximal value over the sample there, as apparently should be the case in the remanent state. Note that $D(x)$ is proportional to the electric field $E(x)$ in the sample.

Obviously, the magnetic moment per unit volume $m(t) = (4\pi d)^{-1}\int_0^d (B(x,t) - H)dx$ obeys the same dependence on time as $f(t)$ [see Eq. (11)]:

$$m_{\rm rem}(t) = m_0\left(\frac{1}{1 + (u+1)t/\tau(u)}\right)^{\frac{1}{u+1}}. \qquad (14)$$

In the case of flux flow, where $u = 0$, Eq. (14) reads

$$m_{\rm rem}(t) = \frac{m_0}{1 + t/\tau_{rf}}, \qquad (15)$$

with $\tau_{rf} = \tau(0) = 3.58\pi\eta d^2/(\mathcal{A}\phi_0 B_0)$ [see Eq. (12)]; note that $Q(0) = 0.863$. The subscript rf means "remanent flow." Thus, we see that in the flux flow regime the remanent magnetization relaxes as $1/t$. At the same time, as proved in Ref. [6], in the field-on case where the external magnetic field H penetrates into a zero-field-cooled ("empty") sample, the relaxation is described by the exponential function:

$$|m(t)| = m_0 \exp(-t/\tau_{\rm flow}), \qquad (16)$$

where $\tau_{\rm flow} = 2\pi\eta d^2/(\mathcal{A}\phi_0 H) = 0.559(B_0/H)\tau_{rf}$. This manifests the *crucial slowdown* of the flux flow in the remanent state when compared to that in the field-on case. In Fig. 3 we show the temporal dependence of the normalized magnetic moment $m(t)/m_0$, obtained both analytically [see Eqs. (15) and (16)] and numerically, for remanent and field-on relaxation. Note that the slight difference between the analytic and numerical results at $\tau \lesssim \tau_{\rm flow}$ originates from the fact that Eqs. (15) and (16) describe the relaxation of self-organized field profiles, whereas for our initial field distributions it takes time of the order of $\tau_{\rm flow}$ to approach the self-organized shapes.

As we show below, the same result holds for $u \simeq 1$, and only in the limit $u \gg 1$ do the relaxation rates for flux entry and exit become similar. Such a difference gives rise to a crucial role of the flux flow and the early creep stages in the remanent state. Really, for $\eta \simeq 10^{-5}\,{\rm g\,cm^{-1}\,s^{-1}}$, which is typical for high-T_c compounds at T not too close to T_c (see Ref. [18]), one gets $\tau_{\rm flow} \simeq [d^2/H]$s, where d is measured in millimeters and H is in gauss. Thus, for typical samples and fields we have $\tau_{\rm flow} \sim 10^{-3}/10^{-1}$ s. Thus, most field-on experiments, where the waiting time between the changing of the field and the first measurement of the field (Hall probes or magneto-optics) or the magnetic moment [superconducting quantum interference device (SQUID)] is at least of the order of several seconds, i.e., much greater than $\tau_{\rm flow}$, "miss" the short exponential flux flow stage described by Eq. (16) and deal exclusively with the deep creep stage where $U \gg kT$. On the contrary, in the remanent state the long-lasting $1/t$ "tail" [see Eq. (15)] affects the field configuration $B(x, t)$ and, in turn, the magnetic moment $m(t)$ at times that considerably exceed $\tau_{\rm flow}$, which could be found experimentally.

In order to get a general expression for the $m(t)$ dependence in the case of field penetration into a zero-field cooled sample with the $U(j)$ dependence described by Eq. (3), one can apply the semianalytic approach developed in Ref. [6]. It is based on the fact that the activation energy U is almost constant spatially at any stage of the relaxation process: $U(x,t) \simeq U(t)$. Since U is determined by j, this means that the field profiles are almost straight during all the relaxation process, i.e., $j \propto |dB/dx| =$ const, and depends only on time. Then, integrating Eq. (2) over x and using Eq. (1), we find

$$\frac{dj}{dt} = -\frac{j}{\tau_{\rm flow}} \exp[-U(j)/kT]. \qquad (17)$$

Substituting the logarithmic $U(j) = U_0 \ln(j_c/j)$ dependence [see Eq. (3)] into Eq. (17) and taking into account that $m \propto j$ within our semianalytic approach where j is supposed to be independent of x, we get

$$|m(t)| = m_0\left(\frac{1}{1 + ut/\tau_{\rm flow}}\right)^{\frac{1}{u}}. \qquad (18)$$

Of course, Eq. (18) transforms into Eq. (16) at $u \to 0$ (flux flow). Equation (18) coincides up to numerical factor in $\tau_{\rm flow}$ with the result obtained in Ref. [15], where flux dynamics was analysed in terms of self-organized criticality.

It is worth noting just the "mathematical" reason for such a dramatic change (power law instead of exponential time dependence) in the solution of Eq. (4) when we consider remanent relaxation, particularly in the case of flux flow ($u = 0$). At $H \neq 0$ the magnetic field B approaches H with time, so the term $B(\partial B/\partial x)$ can be approximately substituted by $H(\partial B/\partial x)$. Thus, Eq. (4) becomes linear at $u = 0$, and its solution should be of the exponential type. On the contrary, at $H = 0$ the flux flow equation remains significantly nonlinear at all t; therefore, its solution appears to be power law instead of exponential.

At $u \neq 0$ the above arguments concerning the quasilinearity of Eq. (4) no longer hold. Although B still approaches H with time, the term $(\partial B/\partial x)^{u+1}$ remains nonlinear, and the relaxation rate appears to be as far from exponential as u is greater than zero [see Eq. (18)]. Correspondingly, the slowdown effect fades out, and the relaxation rates for the field-on and remanent states become similar [compare Eq. (14) with Eq. (18)].

Finally, it should be emphasized that the dramatic slowdown of the remanent relaxation occurs due to *weak dependence* of the activation energy U on j rather than due to the smallness of U. In the situation where the activation energy is prominent, $U \gg kT$, but weakly depends on j, the almost constant factor $\exp(-U/kT)$ just "renormalizes" time in Eq. (2). Therefore, all the results obtained for the flux flow regime still hold for such a "constant-U" creep, but with the substitution $t \to t\exp(U/kT)$. As a result, the remanent relaxation appears to be still crucially slower than the field-on one.

III. EFFECT OF SLOWED-DOWN RELAXATION ON MAGNETIZATION CURVES

An experimental confirmation of the dramatic slowdown of the relaxation in the remanent state by its direct comparison with the field-on relaxation is quite possible. However, the weak dependence of U on j that is the condition for such a slowdown can usually be achieved only at small $U \gtrsim kT$ even in clean samples since at very small $j \ll j_c$ the activation energy U diverges and its dependence on j becomes more steep. Thus, the time delay between the stabilization of magnetic field (after H is instantaneously switched on or off) and starting magnetization measurements should be very short (of the order of τ_{flow}) if we are going to measure j_c or at least estimate it. Both τ_{flow} and τ_{rf} usually do not exceed 0.1 s even for large samples with $d \simeq 1$ cm, which practically rules out the SQUID technique and requires Hall probes or magneto-optics methods.

Besides direct measurements of flux relaxation we propose an indirect but more convenient method for observation of substantial slowdown in magnetic relaxation using the analysis of magnetization loops $m(H)$. Consider the case where the external magnetic field H varies periodically between H_0 and $-H_0$ with a constant (by absolute value) sweeping rate $dH/dt = H_0/t_0$, where t_0 is, obviously, one quarter of the total period. For low-T_c superconductors, where the flux creep is extremely slow, the magnetization curve $m(H)$ reflects the dependence of j_c on B, and sweeping rate has a minor effect on $m(H)$. For high-T_c compounds the relaxation processes are crucial, and the shape of the magnetization curve results from the interplay between the sweeping rate of the external field, dH/dt, and the flux dynamics in the sample. One could expect the dramatic slowdown discussed in the previous section to affect the shape of the $m(H)$ curve in the vicinity of $H = 0$ that corresponds to the remanent state. In fact, j and, in turn, m are determined by the condition of dynamic equilibrium between the sweeping rate of H and the relaxation of flux in the sample. The faster dH/dt is, the greater $|m|$ we should expect.

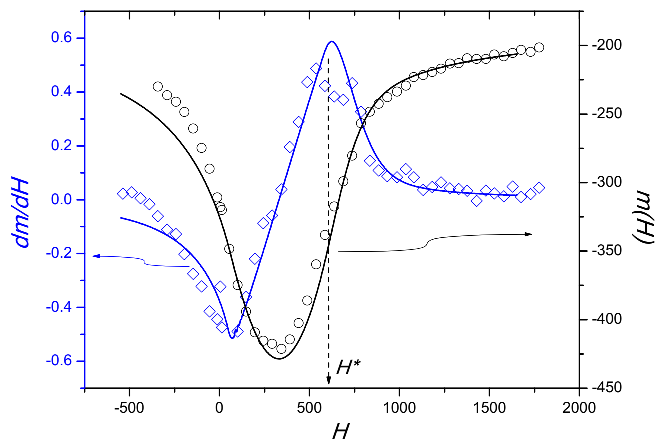

FIG. 4. Results of the numerical solution of Eq. (4) at $u = 0$ (flux flow) for the magnetization curve $m(H)$ and dynamic relaxation rate dm/dH. The external field H is swept on with a constant rate dH/dt from negative values to positive ones. For large negative and positive H we get $dm/dH \to 0$ (see discussion in the text). One can see a clear anomaly in dm/dH in the field range $0 \lesssim H \lesssim H^*$, where the annihilation lines are present in the sample. The numerical data are compared with the experimental results [12] obtained in a clean $La_{2-x}Sr_x CuO_4$ crystal (diamonds and circles).

The slowdown in relaxation should result in accumulation of m when H changes sign and, correspondingly, annihilation ($B = 0$) lines are present in the sample. As $B = 0$ lines that crucially decelerate flux motion exit the sample, one should expect much faster relaxation dm/dt.

In Fig. 4 we show the magnetization curve $m(H)$ and the "dynamic" relaxation rate dm/dH obtained by numerical solution of Eq. (4) at $u = 0$ (flux flow). The magnetic field is swept from the negative values to positive ones with a constant sweeping rate: $H \propto t$. The corresponding field profiles $b(x)$ are shown in Fig. 5. The dynamic relaxation rate dm/dH, which is proportional to dm/dt since $H(t)$ depends linearly on time, shows a peculiar behavior around $H = 0$. In order to understand it let us note that far from $H = 0$ (for both signs of H) the dynamic relaxation rate $dm/dH \approx 0$, i.e., $m(H) \approx$ const, since in the case of flux flow (and, generally speaking, for U independent of B) the magnetization m is a function of the sweeping rate provided $B \neq 0$ everywhere in the sample (compare profiles 1 and 5 in Fig. 5). As H approaches zero, the flux dynamics becomes slower since $\tau_{\text{flow}} \propto 1/H$ [see Eq. (16)], the profiles $b(x)$ become more convex (see profile 2 in Fig. 5), and negative magnetic moment m gets accumulated. Thus, dm/dH drops and reaches a sharp minimum at $H = 0$. At this point two annihilation lines enter the sample from both sides ($x = 0$ and $x = 2d$) and start to move towards its center $x = d$. The "deepening" of m and, in turn, the growth of the mean magnetization current $|j|$ continue as dm/dH remains negative. This lasts until the annihilation lines pass (approximately) half way between the edge of the sample and its center. At this point $m(H)$ reaches its minimum (see Fig. 4). Then the flux dynamics, increased due to increased j, "surpasses" the sweeping rate of H, the field profile $b(x)$ starts to lose its additional convexity and to overtake with the growing H. As a result dm/dH changes sign and reaches its

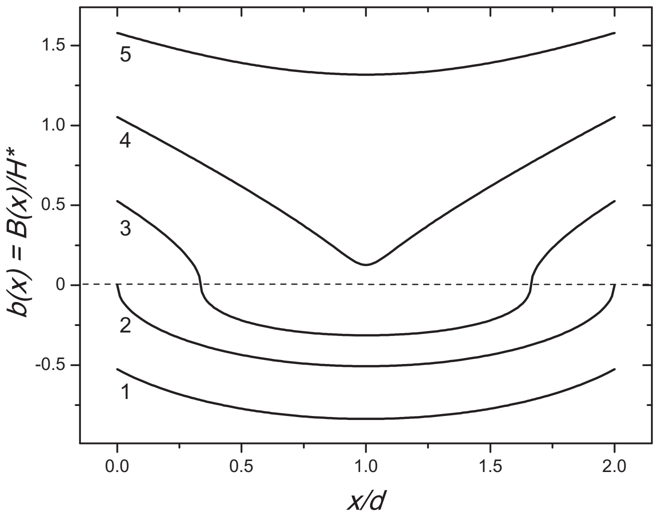

FIG. 5. Field profiles $b(x)$ corresponding to the numerical magnetization curve shown in Fig. 4 as H is swept on from negative to positive values. The annihilation ($b = 0$) lines are present starting from profile 2 ($H = 0$), where they enter the slab from both sides, to profile 4 (H is just above H^*), where they reach the center of the sample ($x = d$) and disappear. One can note a clear square-root behavior of $b(x)$ around the annihilation lines (see profiles 2 and 3).

maximum at $H = H^*$. The latter is defined as the field where the annihilation lines reach the center $x = d$ and disappear (see profile 4 in Fig. 5). As H grows further, the initial field profile that results from the dynamic equilibrium between the sweeping rate dH/dt and the normal flux dynamics in the sample (without annihilation lines) is restored (look at profiles 1 and 5 in Fig. 5, which are almost identical).

In Fig. 4 our theoretical predictions are compared with experimental data for $m(H)$ (circles) and $\partial m/\partial H$ (squares), obtained in the $La_{2-x}Sr_xCuO_4$ sample [12] at $dH/dt \approx 20$ G/s (one experimental point was taken per 2 s). The similarity between our numerical curves obtained for an infinite slab with $U = 0$ and the experimental results obtained for the sample with $r_c/r_{ab} \simeq 1$ and finite (although not large) $U \gtrsim kT$ is quite promising. It is worth mentioning that the pronounced dependence of U on B, which gives rise to fishtail features in $m(H)$, starts in $La_{2-x}Sr_xCuO_4$ at $|B| \gtrsim 1000$ G and does not affect the area of small H where the above-described peculiar behavior of $m(H)$ takes place.

IV. DISCUSSION

We have reported a crucial slowdown of the magnetic relaxation in the remanent state compared with the field-on relaxation. This is a particular case of a more general statement that the annihilation ($B = 0$) lines, if present in the sample, suppress and decelerate the flux motion. This effect is mostly pronounced in long slabs or cylinders where vortices are straight and can be described by the one-dimensional flux diffusion equation [see Eq. (2)]. In real experiments this effect should still be noticeable in samples with the aspect ratio $r_c/r_{ab} \gtrsim 1$, where the vortices are almost straight except for the vicinity of the top and bottom edges (see Ref. [13]). Another condition for observation of such a dramatic slowdown in the relaxation rate is weak dependence of the activation energy U on j. Thus, the effect should be most prominent in the flux flow regime or at early stages of flux creep where $U \gtrsim kT$. The reason for the dramatic slowdown in relaxation is the accumulation of the magnetization current j in the vicinity of the $B = 0$ line that results in considerable diminution of j in the rest of the sample. For flux flow, where $U = 0$, the effect is mostly striking and results in power-law relaxation with $j \propto t^{-1}$ instead of exponential time dependence $j \propto \exp(-t/\tau_{\text{flow}})$, which describes the flux flow in the absence of annihilation lines [6]. As shown in Sec. II, strong dependence of U on j lessens the divergence [see Eq. (13)]; thus, the slowdown effect fades out. In addition the slowdown effect should not be expected in platelet samples with $r_c/r_{ab} \ll 1$. In such thin samples flux motion is essentially two-dimensional where both the in-plane component of magnetic field (B_x in our geometry) and the out-of-plane one (B_z) are important [3,11]. Then the line $B_z = 0$ is different from the annihilation line in long samples since B_x remains finite there, the vortices form loops that disappear by collapsing, and this process does not lead to any divergence in j.

In the high-T_c family, clean $La_{2-x}Sr_xCuO_4$ crystals provide a good opportunity for such a prominent slowdown to be observed since they obey the condition $U \gtrsim kT$ at $T \lesssim T_c$ (vortex liquid) and have a nonplatelet shape of samples with $r_c/r_{ab} \gtrsim 1$ (see Ref. [12]). However, most popular HTSCs like YBCO and BSCCO could also be possible candidates for studying significant slowdown in the remanent state at high enough temperatures (flux flow regime) if magnetic field is directed perpendicular to the c axis.

In addition, we have described a simple method for observing the slowdown in the relaxation process by the analysis of the magnetization curves $m(H)$ obtained at a constant sweeping rate of the external field dH/dt. The latter should be chosen to be comparable to the mean relaxation rate $\partial B/\partial t$ in the sample. The numerical results (see Sec. III) show a clear anomaly in the dynamic relaxation rate dm/dH when the annihilation lines are present in the sample. The comparison with the experimental data [12] obtained in $La_{2-x}Sr_xCuO_4$ crystals looks quite promising.

ACKNOWLEDGMENTS

We thankfully acknowledge useful conversations with Y. Yeshurun, A. Shaulov, Y. Radzyner, and V. M. Vinokur.

[1] Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988).

[2] C. P. Bean, Phys. Rev. Lett. **8**, 250 (1962); Rev. Mod. Phys. **36**, 31 (1964).

[3] Y. Yeshurun, A. Shaulov, and A. P. Malozemoff, Rev. Mod. Phys. **68**, 911 (1996).

[4] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).

[5] M. V. Feigel'man, V. B. Geshkenbein, and V. M. Vinokur, Phys. Rev. B **43**, 6263 (1991).

[6] L. Burlachkov, D. Giller, and R. Prozorov, Phys. Rev. B **58**, 15067 (1998).

[7] C. J. van der Beek, G. J. Nieuwenhuys, and P. H. Kes, Phys. C (Amsterdam, Neth.) **185–189**, 2241 (1991).

[8] V. V. Bryksin and S. N. Dorogovtsev, Physica C **215**, 173 (1993).

[9] C. P. Bean and J. D. Livingston, Phys. Rev. Lett. **12**, 14 (1964).

[10] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965); see also M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (McGraw-Hill, 1996) p. 167.

[11] Y. Radzyner, A. Shaulov, and Y. Yeshurun, Phys. B (Amsterdam, Neth.) **284**, 689 (2000).

[12] Y. Radzyner, A. Shaulov, Y. Yeshurun, I. Felner, K. Kishio, and J. Shimoyama, Phys. Rev. B **65**, 214525 (2002).

[13] E. H. Brandt, Phys. Rev. B **58**, 6506 (1998).

[14] M. R. Beasley, R. Labusch, and W. W. Webb, Phys. Rev. **181**, 682 (1969).

[15] V. M. Vinokur, M. V. Feigel'man, and V. B. Geshkenbein, Phys. Rev. Lett. **67**, 915 (1991).

[16] Y. Abulafia, A. Shaulov, Y. Wolfus, R. Prozorov, L. Burlachkov, Y. Yeshurun, D. Majer, E. Zeldov, and V. M. Vinokur, Phys. Rev. Lett. **75**, 2404 (1995).

[17] E. Zeldov, N. M. Amer, G. Koren, and A. Gupta, Appl. Phys. Lett. **56**, 1700 (1990).

[18] M. Golosovsky, M. Tsindlekht, H. Chayet, and D. Davidov, Phys. Rev. B **50**, 470 (1994).