


Analysis of landscape hierarchy during coarsening and aging in Ising spin glassesStefan Boettcher  and Mahajabin Rahman*Department of Physics, Emory University, Atlanta, Georgia 30322, USA*

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We use record dynamics (RD), a coarse-grained description of the ubiquitous relaxation phenomenology known as “aging,” as a diagnostic tool to find universal features that distinguish between the energy landscapes of Ising spin models and the ferromagnet. According to RD, a nonequilibrium system after a quench relies on fluctuations that randomly generate a sequence of irreversible record-sized events (quakes or avalanches) that allow the system to escape ever-higher barriers of metastable states within a complex, hierarchical energy landscape. Once these record events allow the system to overcome such barriers, the system relaxes by tumbling into the following metastable state that is marginally more stable. Within this framework, a clear distinction can be drawn between the coarsening dynamics of an Ising ferromagnet and the aging of the spin glass, which are often put in the same category. To that end, we interpolate between the spin glass and ferromagnet by varying the admixture p of ferromagnetic over antiferromagnetic bonds from the glassy state (at 50% each) to wherever clear ferromagnetic behavior emerges. The accumulation of record events grows logarithmically with time in the glassy regime, with a sharp transition at a specific admixture into the ferromagnetic regime where such activations saturate quickly. We show this effect for both the Edwards-Anderson model on a cubic lattice as well as the Sherrington-Kirkpatrick (mean-field) spin glass. While this transition coincides with a previously observed zero-temperature equilibrium transition in the former, that transition has not yet been described for the latter.

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The morphology of complex energy landscapes [1,2], and the parameters that control it, are of continuing interest in a large variety of scientific endeavors, from protein folding and evolutionary landscapes in biology [3–7], the design of amorphous materials [8–18], to the hardness of combinatorial optimization problems [5,19–23]. The challenges encountered in describing the geometry of the extremely high-dimensional space of attainable configurations are enormous [1,2,12,23–28]. The structure of such energy landscapes hugely impacts the dynamics of statistical systems evolving through them. While relaxation in simple, smooth landscapes is rapid, like the exponential cooling of a cup of coffee [29], relaxation in complex energy landscapes can possess a myriad of metastable states to temporarily or permanently trap any dynamic process. In turn, simple relaxation processes can serve as diagnostic tools to explore features of landscapes [19,24,25,27,30–32]. It is particularly enticing when it is possible to discover universal aspects of such landscapes that allow us to categorize those features and, ultimately, predict and control dynamic behavior.

Variations in temperature can be used to take a full measure of landscapes. At high temperature correspondingly higher echelons in energy get explored, while annealing or quenching is used to trace out a descent through the landscape toward configurations of lower energy. A conceptually simple protocol consists of preparing a system at a high temperature, where it equilibrates easily, and then instantaneously quenching it down to a fixed, low temperature, to explore how it relaxes

toward equilibrium thereafter. Such an “aging” protocol [33], when applied to systems in a complex energy landscape, elicits quite subtle relaxation behaviors which, unlike for the coffee mentioned above, keep the system far from a new equilibrium for very long times. Anomalously slow relaxation and full aging in a complex landscape ensues when downward paths are obstructed by barriers, energetic or entropic, that trap the system in neighborhoods with many local minima.

The aging phenomenology is associated with memory effects by which the current activity is imprinted by a dependence on the waiting time t_w since the quench. For a wide class of systems, generally considered to be glassy, it is found that correlations, instead of being time translationally invariant, $G(t, t_w) \sim f(t - t_w)$, roughly depend on a ratio, $G \sim f(t/t_w)$. Although memory effects in out-of-equilibrium systems are generally of interest, that fact alone is not sufficient to categorize its energy landscape as complex or glassy. To emphasize this fact, and to provide a deeper insight into the relation between landscape morphology and aging dynamics, we investigate here the aging in families of models that interpolate between a well-known spin glass [34,35] and the corresponding ferromagnet [36]. Albeit glass and ferromagnet exhibit similar scaling with age t , it stands to reason that the aging dynamics of a homogeneous ferromagnet differs significantly from that of a glass. In contrast to the hierarchical, multimodal energy landscape of a glass [37–41], that of a ferromagnet is smooth. Yet, much of the literature leads to the impression that relaxation via coarsening in a ferromagnet and glassy aging are synonymous [42–44]. Technically, one could argue that the fact that in either extreme a growing length scale emerges

is indicative of coarsening domains. We posit that the process by which those length scales grow with age, logarithmically in the glassy case and with a power law for a ferromagnet, is fundamentally different.

As discussed in Ref. [45], energy barriers that scale with the size of a domain to be flipped imply that further growth in those domains is curtailed to be merely logarithmic in time. Such a feedback does not emerge in the coarsening of a ferromagnetic Ising system, where energy barriers remain insensitive to the size of the domain to be flipped. Accordingly, the landscape of a glassy system has a *hierarchical* structure in that the lower an energy it has reached, the higher the barriers get, and thus, the harder it becomes to escape local minima [46]. In a homogeneously coarsening system, energy barriers remain largely independent of the depth reached within the landscape, providing some roughness and metastability but of bounded scale beyond which the structure is relatively smooth. Within the aging process, this difference manifests itself dramatically in the manner that the relaxing system responds to fluctuations, as illustrated in Fig. 2. In a ferromagnet, average fluctuations in energy, beyond some low, fixed threshold, suffice to cross typical barriers, often followed by disproportionately large expulsions of heat. In contrast, to advance glassy systems with diverging energy barriers, mere average fluctuations become ineffective. To be able to relax, those fluctuations have to produce ever new records in their size to overcome ever steeper barriers, which is the basis of what is called record dynamics (RD) [47]. Such record production, decorrelated by a wide separation in time, is known to unfold only on a logarithmic scale [48,49].

Although irrelevant for the cases studied in Ref. [45] (and here), it should be noted that entropic effects can become dominant and may entail diverging free-energy barriers with domain size, even in an otherwise homogeneous system. One example is a 3-spin Ising ferromagnet [50]. Systems driven by entropic barriers, such as the free volume in a hard-core colloidal system, are referred to as “structural” glasses. In those systems, a hierarchical free-energy landscape emerges dynamically [17].

In the following, we define a simple coarse-graining procedure, counting the number of “valleys” traversed in the energy landscape, that effectively probes the impact of fluctuations on the aging dynamics. Our focus on landscape morphology reveals the nature of the irreversible, intermittent events that allow the expulsion of excess energy from the system. It shows a dynamical transition between a glassy and a ferromagnetic relaxation regime based on this measure. In that, we reproduce similar findings using scaling exponents of two-time correlation functions in the thermodynamic limit [36], which indicated that this dynamical transition is closely related with a zero-temperature equilibrium transition between a glass and a ferromagnet [51]. That this transition transcends into the nonequilibrium realm highlights the fact that aging in a glassy system is a distinct process from what is found in homogeneous systems, characteristic of a distinct, hierarchical landscape.

Our paper is organized as follows: In Sec. II, we introduce the families of Ising spin models we employ in our study. In Sec. III, we will discuss RD and the measures we will apply

to detect record fluctuations. In Sec. IV, we present the results of our investigation, and we conclude in Sec. V.

II. MODELS

Ising spin systems, consisting of spin variables $\sigma_i = \pm 1$, have been widely used, first of all as ferromagnets, to model spontaneous symmetry breaking and continuous phase transitions [52]. With the random admixture of antiferromagnetic bonds, they have also served as models for disordered materials and glasses generally [34,35,53,54]. The relevance of such spin models reaches far beyond physics, into biological and sociological applications, for example [55]. Here, we are employing families of such spin models that interpolate between the randomly disordered spin glass on a cubic lattice, called the Edwards-Anderson model (EA) [34], as well as its mean-field version, the Sherrington-Kirkpatrick model (SK) [35], on one side and the respective homogeneous ferromagnetic systems [52] on the other. Each system consists of a random mixture of ferromagnetic and antiferromagnetic bonds J between neighboring spins σ_i and σ_j that are drawn from a distribution $P(J)$ we have chosen to be bimodal, i.e., $J_{ij} = \pm J_0$, with energy units such that $J_0 = 1$ in 3D and $J_0 = 1/\sqrt{N}$ in the mean-field case. A fraction p of ferromagnetic bonds is balanced out with a fraction $1 - p$ of antiferromagnetic bonds such that

$$P(J) = p\delta(J - J_0) + (1 - p)\delta(J + J_0). \quad (1)$$

For each, the Hamiltonian (without external field) reads

$$H = - \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j, \quad (2)$$

where $\langle ij \rangle$ refers to all extant bonds between neighboring spins σ_i and σ_j , either on a cubic lattice for EA or all mutual pairs of spins for SK.

In the family EA of models we study on the cubic lattice [36,51], we change the admixture of bonds by varying p between $\frac{1}{2} \leq p \leq 1$, from the pure glass with an equal mix of bonds ($p = \frac{1}{2}$) to a pure ferromagnet when all bonds are ferromagnetic ($p = 1$). The situation is more complicated for SK, where already a subextensive excess of ferromagnetic bonds, away from the pure glass, results in ferromagnetic behavior. Specifically, since all N spins are mutually connected, there are $\frac{1}{2}N(N - 1)$ bonds, and it only takes an imbalance between either type of bond, merely of order $\sim \sqrt{N}$, to achieve ferromagnetic ordering. Thus, we define a family of mean-field models parametrized by α with $p = \frac{1}{2} + \frac{\alpha}{\sqrt{N}}$, varying between $0 \leq \alpha \leq 2$ to explore the full range of behaviors [56].

III. AGING AND RECORD DYNAMICS

A. Simulation of quenches in spin glasses

The distinction between slow relaxation in glassy versus homogeneous systems is succinctly analyzed in the simplest conceivable protocol of a hard quench from an easily equilibrated high-temperature state into an ordered phase, whether glassy or ferromagnetic, crossing a phase transition in the process. Such a pure aging protocol has been studied exten-

sively in the last 40 years [33,44,48,57,58]. In this process, the system is thrown far out of equilibrium, left with an enormous amount of excess heat to be released to the bath to be able to descend deeper into its energy landscape to reach states with the appropriate (equilibrium) energy.

To facilitate such a quench for the family of Ising spin models considered in our study, for each instance at time $t = 0$, we initiate with randomly assigned spins, either $\sigma_i = \pm 1$, which corresponds to $T = \infty$, and run the simulation for $t > 0$ at a low, finite temperature. For our family of models on the cubic lattice, the critical temperature for a transition into an ordered state varies from $T_c \approx 1.1J_0$ in EA [59], to about $T_c \approx 4.5J_0$ for the ferromagnet [60]. In our Monte Carlo simulations, we quench to $T_q = 0.7J_0$ for all p , similarly to Ref. [36], and monitor the aging process for about 10^5 sweeps. For the family of mean-field models, we only vary the admixture of ferromagnetic bonds minutely, so that the transition temperature does not deviate much from that of SK, which is known to be $T_c = J_0$ [35]. Here, we also quench to $T_q = 0.7J_0$ throughout. For each value of p in our study, we have averaged results over at least 10^4 realizations.

B. Valleys in an energy landscape

Key to our analysis of coarsening versus glassy relaxation is the definition of a measure that can serve to distinguish the effect of fluctuations on the irreversible events by which a system relaxes. One such measure has been provided by Dall and Sibani in Ref. [30]. There, the internal energy of an entire system of finite size is monitored to observe its time trace for the ensuing quench. Since the system is expelling energy into the bath to relax, on average, the energy gradually decreases, albeit via localized, intermittent events [61], in line with experimental observations of glassy systems [43,62–64]. In particular, record-sized fluctuations are needed for a glassy system to relax, according to RD [46,47].

As illustrated in Fig. 1, Dall and Sibani defined the “valley” production as an observable. [30] in the following way: Let E be the up-to-now lowest energy value encountered up to time t and let $E(t)$ be the instantaneous energy. In turn, let the “barrier” B be the up-to-now highest energy attained, *relative* to the most recent E , i.e., $B = E(t) - E$. An energy trace then maps into a random sequence of symbols, like in Fig. 1, $\dots E_1 B_1 B_2 B_3 E_2 E_3 B_4 B_5 E_4 E_5 \dots$. Note that the trace can generate a subsequence of records in the lowest energy, i.e., multiple E 's in a row, before it encounters its next higher (record) barrier B , and also a subsequence of such B 's before it meets the next E , and so on. Clearly, it is the latest E or B in either type of subsequence that is significant: Each prior one is merely transitory, while the last one supersedes each prior one as record that reaches its ultimate significance *only after* a new record fluctuation in the opposite direction is attained. Thus, we squash the entire sequence into a strict alternation between E and B , as the stricken letters in Fig. 1 imply, which then yield $\dots E_1 B_3 E_3 B_5 E_5 \dots$. Then, a “valley” is defined as the part of the trace between two consecutive record barrier crossings, as indicated by vertical dashed lines there. If the ground state were to be reached, i.e., no further energy minimum could be found, the sequence would terminate, of course.

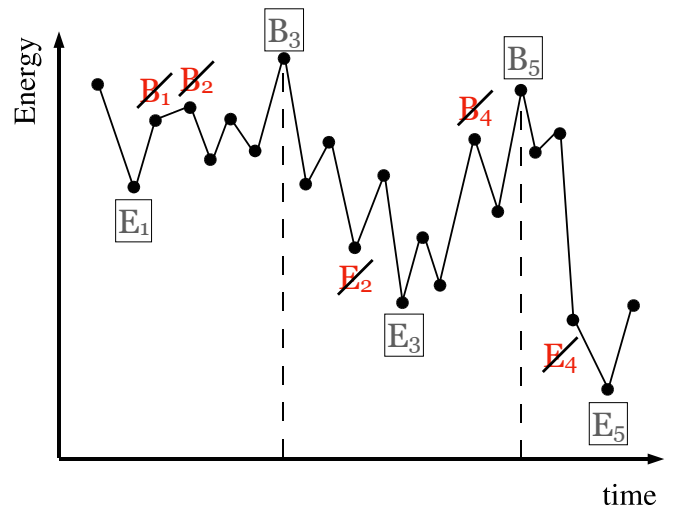


FIG. 1. Illustration of the definition of valleys. The trace through an energy landscape produces a time sequence of energy records (E_i) and of barrier records (B_j), relative to the most recent “ E_i ” [30,31]. Only the highest and lowest records of the “ E_i ” and “ B_j ” are kept to give a strictly alternating sequence “ $\dots E_{i_0} B_{j_1} E_{i_1} B_{j_2} E_{i_2} \dots$ ”. Then, any sequence “ $B_{j_1} E_{i_1} B_{j_2}$ ” demarcates a valley (vertical lines).

To focus truly on locally correlated record barrier crossings, it would be useful to refine this definition of valley [65]. However, unless a system gets too large, with too many simultaneous but spatially distant quakes, by considering a small enough system these events become sufficiently rare to dominate the fluctuations in the entire system trace, instead of being “washed out” by overlapping ones. This point illustrates also that, to understand a thermodynamic system *out-of-equilibrium*, it is often *not* helpful to take the thermodynamic limit.

Examples of a valley sequence from our simulations is shown for single energy traces in Fig. 2 for the EA spin glass (top) and the corresponding ferromagnetic system (bottom) on a cubic lattice. These plots exemplify the stark difference in the effect of fluctuations on either type of system that we discuss in the following.

C. Dynamics driven by record-sized fluctuations

As alluded to in the introduction, glassy and otherwise homogeneous systems such as a ferromagnet distinguish themselves in the manner fluctuations affect their relaxation dynamics. In the latter, barriers are comparably low and remain invariant independently of the depth within the landscape and, thus, of the age of the process. As Fig. 2 exemplifies, large releases of energy are preceded by typical fluctuations at any stage of the process. Fewer events, like the evaporation of a domain in coarsening, happen not because individual events become so much harder but rather because so many fewer events can happen when only a few domains are left. Larger domains may take a little more time to evaporate, as meandering interfaces need to find each other and collide, but such an entropy barrier does not dominate the otherwise domain-size-independent energetic barriers [45]. Yet, ordinary fluctuations suffice to bring those interfaces together.

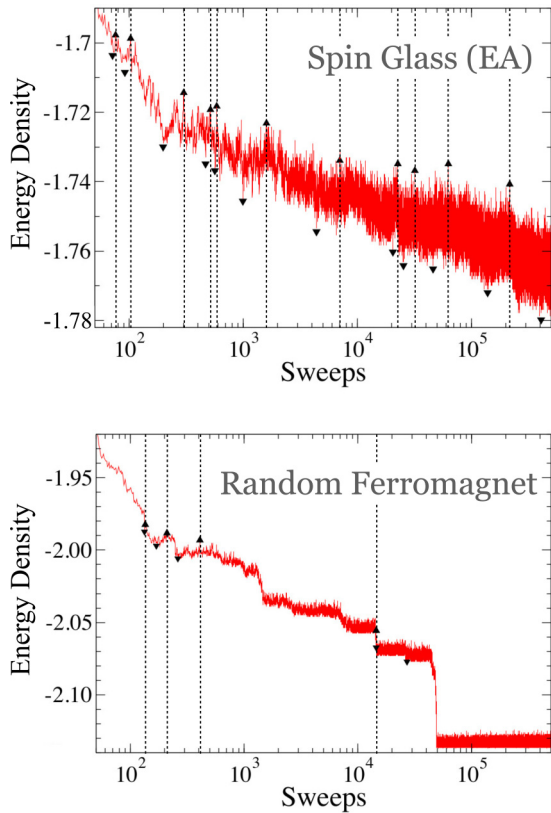


FIG. 2. Typical trajectories of an aging process through the energy landscape of the spin glass model on a $3d$ lattice with $L^3 = 16^3$ spins and a fraction p of ferromagnetic bonds and $1 - p$ antiferromagnetic bonds, here with $p = 0.5$ (top) and $p = 0.85$ (bottom). Energy (\blacktriangledown) and barrier (\blacktriangle) records, as defined in Fig. 1, are marked along each trajectory, where the vertical dashed lines indicate the transition between consecutive valleys. While the energy decreases, on average, gradually as a logarithm in time with an ongoing but random production of further records in the glassy case ($p = 0.5$), the more ferromagnetic system ($p = 0.85$) expels energy in a few large events which appear to be triggered by typical fluctuations; record-sized fluctuations are seemingly irrelevant.

In the glassy system, however, it is the barrier height growing with domain size that decelerates the event rate. Although many domains remain available even after a long aging time, few muster the chance fluctuation required to break up. In a landscape with those barriers, ordinary fluctuations become ineffective to drive the relaxation process. They merely “rattle” the system during increasingly longer quasiequilibrium interludes. Only rare, extraordinary large, in fact, record-sized fluctuations manage to scale such barriers to expel excess heat, advance the relaxation, and grow domain size, minutely.

These features, widely shared across many disordered materials, have inspired the phenomenological description at the basis of RD [47]. The definition of valleys in the preceding section provides an especially adapt measure for this phenomenology. In RD, the relaxation process of a nonequilibrium system after a hard quench is determined by large (i.e., record-sized), irreversible fluctuations which move the system from one metastable state to the next (usually only marginally more stable than the last one) within its complex

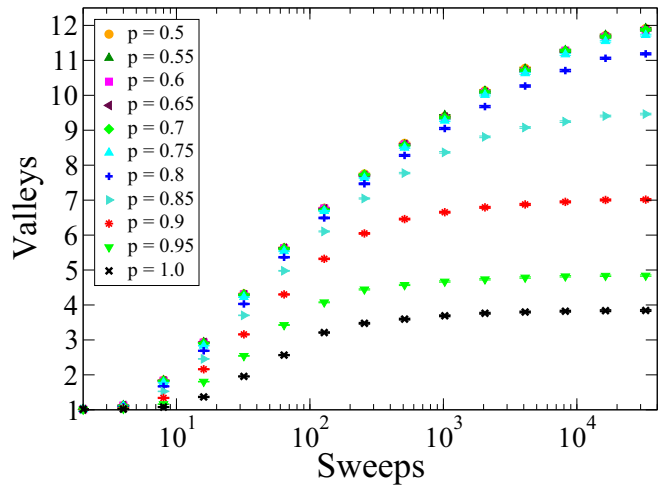


FIG. 3. Average number of valleys in EA, as defined in Fig. 1, that are traversed with time after a quench to $T = 0.7J_0$ in a $L^d = 16^3$ spin glass with a fraction p of ferromagnetic bonds and $1 - p$ antiferromagnetic bonds. For $p \leq 0.75$, the generation of valleys evolves essentially independently of p , while for a larger admixture of ferromagnetic bonds valley generation progresses to cease ever more rapidly and the number of valleys reaches plateaus.

energy landscape [46,66]. This can be thought of as the system overcoming energy barriers in a hierarchical energy landscape [37–41]. The rate $\lambda(t)$ of such record events, also termed “quakes,” decelerates with time as $1/t$. Hence, the expected number of events in a time interval $[t, t_w]$ is

$$\langle n(t, t_w) \rangle \propto \int_t^{t_w} \lambda(t') dt' \propto \ln\left(\frac{t_w}{t}\right), \quad (3)$$

implying that the dynamics of the system is self-similar in the logarithm of time. That time homogeneity is a common feature of many aging systems [44,57,67]. In our studies here, we are more concerned with the rate of events $\lambda(t)$ and the logarithmic growth of observables in time. The dependence on waiting time t_w has been the focus elsewhere [46,61,66].

IV. NUMERICAL RESULTS

A. Edwards-Anderson model

Applying the measure of a valley number defined in Sec. III B to the cubic Ising spin model introduced in Sec. II provides a notable distinction between glassy and homogeneous coarsening behavior, as Fig. 3 shows. For all $p < p_c \approx 0.77$, the critical threshold found in Ref. [51], we find that the valley count progresses logarithmically in time (in fact, like the root of that logarithm [66]), consistent with Eq. (3). For larger values of p , the valley count slows ever more significantly to eventually plateau at a finite value, apparently. All the results shown here were obtained for systems with $N = 16^3 = 8096$ spins, using periodic boundaries, since we found very little variation with system size for larger N .

The fact that the underlying ordered state is either glassy or ferromagnetic affords us to also measure the increase in magnetization with time, as demonstrated in Fig. 4. This measure actually exhibits a more dramatic transition between the glassy and the ferromagnetic case, as consecutive snapshots

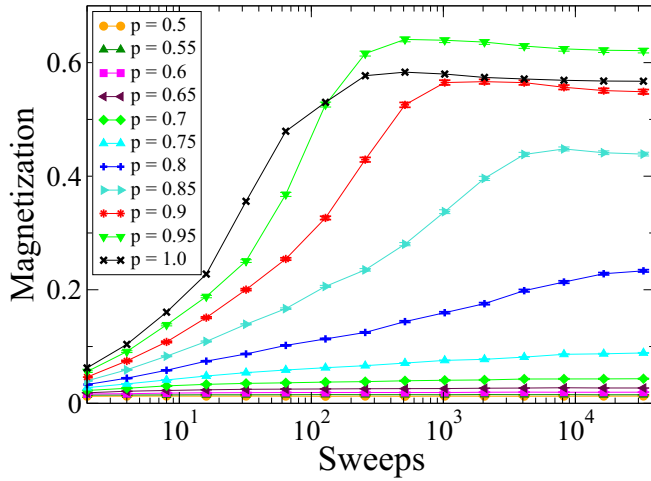


FIG. 4. Average magnetization per spin in EA, $\langle m \rangle$, observed with time after a quench during the ensuing aging process, as described in Fig. 3. Like there, systems with $p \leq 0.75$ behave glassy in a p -independent manner with little discernible magnetic ordering, while the more ferromagnetic systems become increasingly more ordered.

of both, the valley count as well as the magnetization, are shown in Fig. 5 for a progression of times that increases by a factor of 8. In these plots, we have also marked the zero-temperature transition at $p_c \approx 0.77$, which proves consistent asymptotically with the transition out of the glassy relaxation behavior.

Finally, we can also look at the instantaneous rate of barrier crossing events (Fig. 6), effectively the derivative of the valley production, i.e., inverting the integral in Eq. (3). Indeed, throughout the glassy regime, the rate decelerates roughly hyperbolically, in accordance with the RD predictions. [Note

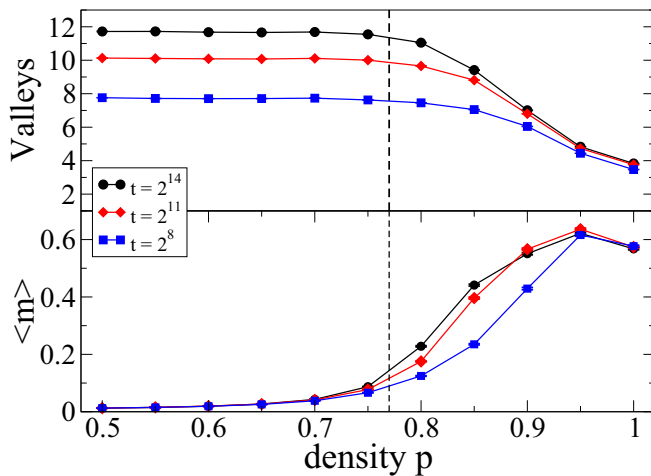


FIG. 5. Finite-time snapshots for EA of the numbers of valleys generated (top) and the corresponding magnetization per spin, $\langle m \rangle$ (bottom), as a function of ferromagnetic bond fraction p for three different times, taken from the data at $T = 0.7J_0$ shown in Fig. 3 and Fig. 4, respectively. The vertical line at $p_c = 0.77$ indicates the zero-temperature transition found in Ref. [51] between a glassy and a ferromagnetic phase.

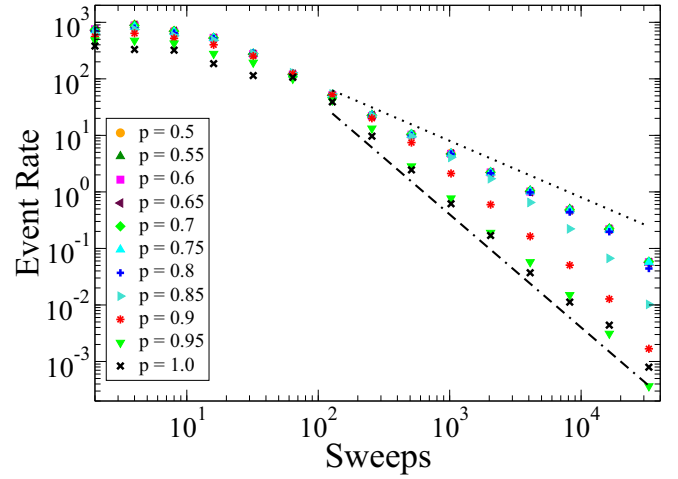


FIG. 6. Instantaneous rate of record barrier crossing events in EA, as defined in Fig. 1, with time after the quench, as described in Fig. 3. Asymptotically, for larger times, that rate varies as a power law with a seemingly hyperbolic decline, $\sim 1/t$ (dotted line), for all $p < 0.75$ to an almost quadratic decline, $\sim 1/t^2$ (dash-dotted line), for larger p .

that this could miss a minor logarithmic correction, such as $\lambda(t) \sim 1/(t\sqrt{\ln t})$, for instance, needed to get $\sqrt{\ln t}$ for the valley production in Fig. 3.] For $p > p_c$, in the ferromagnetic coarsening regime, we notice that the rate falls off increasingly sharper, ultimately about as $\sim 1/t^2$. Consequently, its integral stalls out into the plateaus seen in Fig. 3. Apparently, domain mergers occur more rapidly, on a power-law scale, in coarsening ferromagnets. Despite the rapid drop in the event rate, the average domain size manages to increase as a power law [45], because later mergers expel larger amounts of excess heat; see Fig. 2. In the case of the glass, each event expels on average a fixed amount of heat, roughly. Therefore, both valley production and domain growth proceed similarly (logarithmically), as an integral of the event rate, since each activation has the same impact.

B. Sherrington-Kirkpatrick model

Using the valley counts defined in Sec. III B as an order parameter, we find a clear transition from a glassy regime to a ferromagnetic one in the mean field as well. However, unlike for EA on a cubic lattice, extending the neighborhood of each spin to all others in the case of SK changes the dynamics, and we have to explore the critical threshold at which the spin glass to ferromagnetic transition takes place on a different scale. Mutual connections between all spins require the number of ferromagnetic bonds to only slightly exceed the number of antiferromagnetic bonds, in order to tip the system into becoming ordered. The transition to the ferromagnetic regime occurs almost immediate beyond a bond density of $p = 0.5$, with a strong system size dependence, forcing us to adapt a different scale to observe it. To properly describe the behavior of SK, we therefore reparametrize the bond density in terms of α via $p = \frac{1}{2} + \frac{\alpha}{\sqrt{N}}$. Then, within the range of $0 \leq \alpha \leq 2.0$, we can localize a transition that varies only slowly with size.

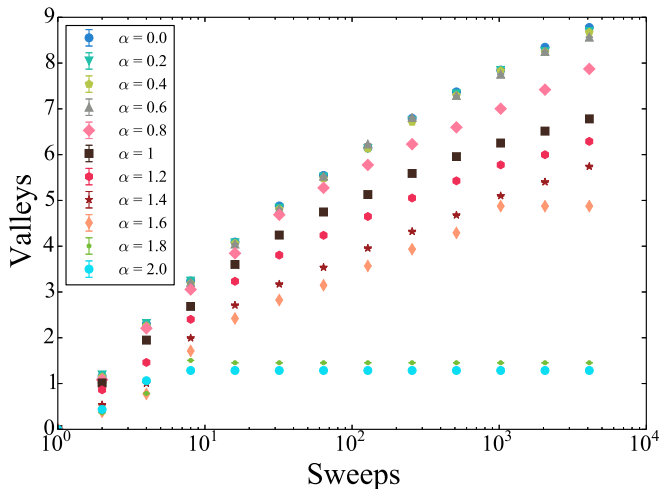


FIG. 7. Number of valleys traversed during relaxation ensuing after a quench of SK for different bond fractions α from a high temperature $T = \infty$ to $T = 0.7J_0$, averaged over an ensemble of trajectories for $N = 2048$ spins. In the range $0.0 \leq \alpha \leq 0.6$, the number of valleys traversed grows logarithmically and largely independently of α , indicating that the regime is glassy.

Similar to Fig. 3 for EA, in Fig. 7 we show the numbers of valleys found in a SK system with $N = 2048$ spins. There appears to be a critical threshold at $\alpha_c \approx 0.6$. For $\alpha \leq 0.6$, the valley production increases about as $\log(t)$, essentially uniform with bond density, given the nearly perfect overlap in the data. This is no longer case when $\alpha > 0.6$, where the production of valleys decreases gradually before plateauing completely. While domains in the sense of geometric regions of a certain length do not exist in a mean-field system with long-range interactions, individual spins develop clusters of increasingly ordered local fields with some of their neighbors that entrench the system into deeper valleys. It becomes increasingly more difficult for the system to overcome the energy barrier of flipping the entire cluster, causing the relaxation process to evolve logarithmically [45].

That said, evidence of a critical threshold suggests that beyond α_c , the system changes its landscape dramatically. It exhibits an inclination to order rapidly, facilitated by the fact that local fields of individual spins immediately affect all others, as the evolution of magnetization in Fig. 8 suggests. Flat interfaces between such clusters, as they may exist between domains in low-dimensional lattices like EA, are absent here and any imbalance in size quickly erodes inferior clusters. Therefore, despite the quantitative differences pertaining to local structure between the Edwards-Anderson and Sherrington-Kirkpatrick spin glass, our results suggest that the glassy behavior in both can be attributed to the hierarchical nature of the energy landscape, and the lack of it beyond the transition to ferromagnetic order, seen both in Fig. 4 and Fig. 8.

We have also checked the evolution of valley counts across different system sizes and found only a minimal dependence of the transition on larger size, as shown in Fig. 9. While the relationship (or lack thereof) between the number of valleys encountered and the bond admixture exhibits time dependence, the critical threshold with regard to ordering already emerges after about two hundred sweeps. There is clearly an

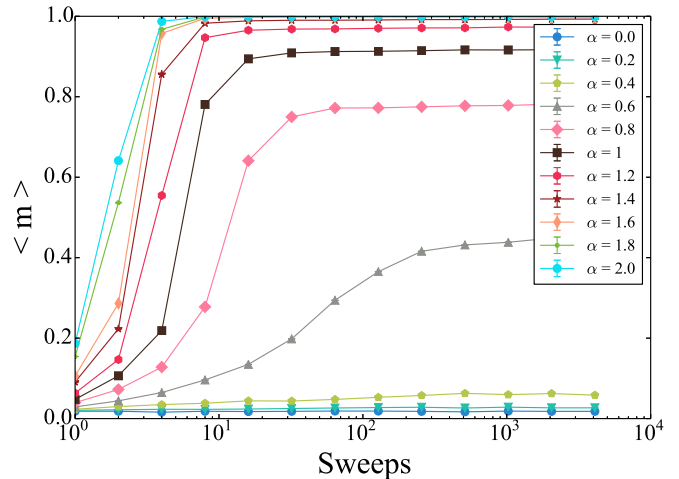


FIG. 8. Average magnetization for SK in the same simulations shown in Fig. 7. According to this measurement, the system begins to order at $\alpha_c \approx 0.6$, since a nonzero magnetization in the long-time limit indicates that the majority of the spins have ferromagnetically ordered. The transition in magnetization shown here is far more dramatic than in the valley counts, but nevertheless affirms the same critical threshold.

agreement between valley statistics and the ferromagnetic order parameter in suggesting $\alpha_c \approx 0.6$ as the critical threshold.

Lastly, we look at the deceleration of the rate of record barrier crossing events in Fig. 10. As shown in Fig. 6 for the Edwards-Anderson model, the rate decays with a power of time t . While there is a steeper deceleration in the barrier crossing events for larger α values, the difference between the exponents is quite subtle on this timescale within our simulations. In the glassy regime, $\alpha < \alpha_c \approx 0.6$, the rate clearly decays hyperbolically, whereas it falls off steeper above α_c . However, for values $\alpha > 1.6$, the falloff becomes so

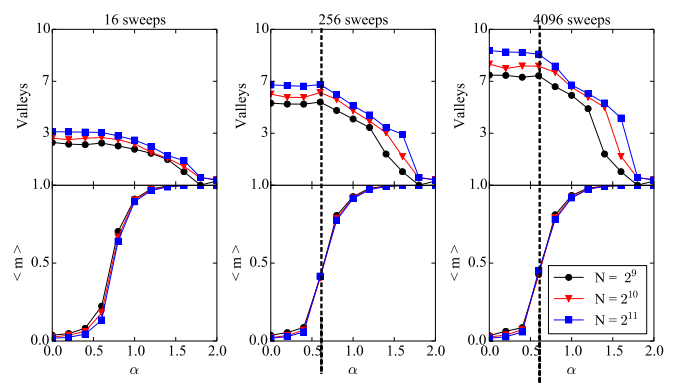


FIG. 9. Instantaneous average valley counts and magnetization in SK as function of α at different sweep times $t = 16, 256,$ and 4096 from left to right, each for three different system sizes indicated on the legend. The first row shows the average number of valleys, and the second row shows the average magnetization. According to these data, the valley production is time dependent as the sharpness of the transition becomes more pronounced in the later sweeps. In contrast, the magnetization appears to be saturated already early on, predicting the critical threshold within 16 sweeps. Additionally, we see no system size effects when using α as the parameter.

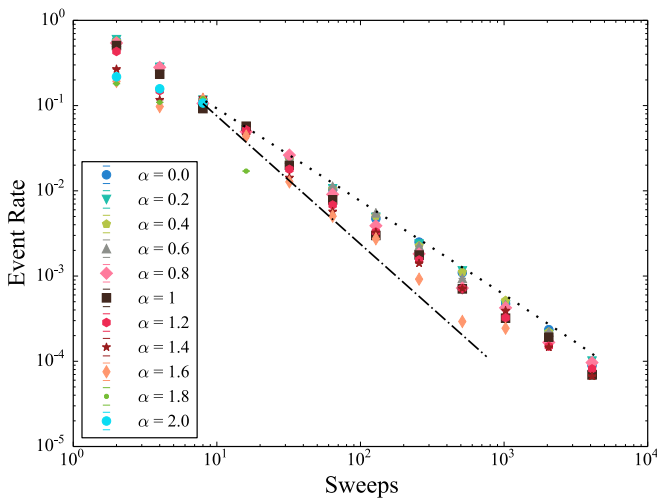


FIG. 10. Instantaneous rates for the number of record barrier crossings as a function of time, for every α value in SK. The instantaneous rate decreases as a power law for all but the highest admixture values. In the glassy regime, the deceleration is essentially hyperbolic (dotted line), while the rate drops more sharply for $\alpha > 0.6$, up to roughly $t^{-1.5}$ at $\alpha = 1.6$ (dash-dotted line), beyond which further record events become immeasurably rare.

significant that new valleys are not encountered beyond the first ~ 100 sweeps.

V. CONCLUSION

Our study explores the distinction between glassy relaxation and ordinary coarsening, which is often ignored in the description and analysis of aging systems. Focusing on families of models that interpolate between either extreme, we not only apply measures [30,31] that clearly indicate the difference but also show a rather sharp transition in the nonequilibrium behavior between those extremes that, for the Edwards-Anderson model on a cubic lattice, appears to coincide with the (equilibrium) zero-temperature transition between spin glass and ferromagnet [51]. The corresponding transition that we find at a subextensive scale in SK seems to have been unnoticed.

While the distinction we are making between a coarsening (ferromagnetic) and an aging (glassy) regime can be seen as semantic, considering that both, algebraic as well as logarithmic growing domains, are commonly portrayed as coarsening [45], the difference in dynamic behavior after a quench is profound. The picture that emerges is one of a largely convex landscape on one side with invariant energetic barriers in the case of coarsening, a system that despite its often complex network of fractal interfaces locally homogenizes rather quickly. On the other side, we find a very hierarchical landscape [37–41] with energetic (and potentially entropic) barriers that grow with deeper entrenchment within the landscape, rendering all but record fluctuations ineffective for relaxation.

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