Coulomb blockade oscillations of heat conductance in the charge Kondo regime

D. B. Karki
Division of Quantum State of Matter, Beijing Academy of Quantum Information Sciences, Beijing 100193, China

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We develop a method of theoretically investigating the charge, energy, and heat transport in the presence of the charge Kondo correlations. The Coulomb blockade oscillations of heat conductance in the single-electron transistor exhibiting charge Kondo effects are investigated. We explore the Wiedemann-Franz ratio in both charge-single-channel and charge-two-channel Kondo regimes. The close connections of our findings with the recent experiments on multichannel charge Kondo effects are discussed.

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I. INTRODUCTION

The rapid progress of quantum technologies has led to new experiments in addition to the development of new theoretical approaches to understand quantum transport phenomena at the nanoscale [1]. A prototypical nanoscale device of common interest is a single-electron transistor (SET), which contains a metallic grain, the quantum dot (QD), tunnel coupled to two electronic reservoirs [1,2]. The transport characterization of such a SET is mainly governed by the Coulomb blockade (CB) phenomena [3]. Consequently, the electronic conductance and the thermopower of a SET show the CB oscillations as a function of the gate voltage as reported in the seminal work [4]. The theory of CB oscillation based on the sequential tunneling approach originally developed in Ref. [3], however, does not consistently explain the low-temperature transport characteristics of a SET since the corresponding transport is dominated by the inelastic cotunneling mechanisms [5,6].

The complete theory of low-temperature thermoelectrics of a SET came after the seminal works [7–11]. These studies have investigated the CB oscillations of thermoelectric coefficients of a SET with the QD strongly coupled to one of the leads by a quantum point contact as in the experiment reported in Ref. [6]. Interestingly, the SET setup with the QD strongly coupled to one of the leads was shown to be associated with the single-channel Kondo (1CK) and two-channel Kondo (2CK) effects [7]. The Kondo effect involving the Coulomb blockade in a QD arises from the two possible charge states in the QD which are adjusted to the same energy by tuning the gate voltage to the critical point [8]. These two charge states then behave fundamentally the same as the effective spin eigenstates, and hence, the electron spin plays the role of two Kondo channels [12]. Therefore, the spin polarization by magnetic field results in the charge-1CK effects, and the electrons with an effective spin $1/2$ provide the realization of charge-2CK effects in the original formulation [7].

The Kondo paradigm provides a valuable tool for understanding the physics of strongly correlated systems both in Fermi-liquid (FL) and non-FL (NFL) regimes [13–22]. So far, much progress has been made toward understanding spin Kondo effects that stem from a localized spin at a discrete energy level in QD nanostructures [23]. Although both charge Kondo effects and spin Kondo effects can be observed in the QD, their behaviors and the corresponding fundamental mechanisms are strikingly different [12]. Consequently, one expects different scaling behaviors of the transport properties in the spin and charge versions of the Kondo effects.

The charge-1CK falls into the FL universality class; the description of charge-2CK is beyond the scope of FL theory [17,19]. Therefore, the theoretical advancements made in Refs. [7–11] also provide a viable way of experimentally accessing the NFL regime, as reported in recent experiments on the charge-2CK effect [24]. In addition, since the first proposal [7], various transport properties in charge-1CK and charge-2CK regimes have been intensively investigated by different theoretical methods [8–11,25–28]. In recent years, the interest in charge-2CK has been expanded to the study of energy and heat transport [29,30], which can be measured with existing experimental setups [24,31].

For a QD-based SET in the presence of the voltage bias and temperature gradient, various measures of thermoelectric response have been the subject of recent experimental and theoretical studies [1]. It has been almost two decades since the development of a full-fledged theory of thermopower in the charge-2CK regime of a QD [9]. The original theory has been extended to more complex geometries exhibiting charge-2CK effects [25,26]. However, a lot less attention has been paid to the investigation of heat response in the charge-2CK regime, although it is within experimental access [24,30]. In this work, we aim to fill this gap by developing a general theoretical framework dealing with the charge, energy, and heat transport in the charge Kondo regime based on the original proposal [9]. In particular, we focus mainly on the aspects of heat transport in the presence of the charge Kondo correlation because the corresponding charge and energy transport were thoroughly investigated in Refs. [9,10].

This paper is organized as follows. In Sec. II, we discuss briefly the common measures of thermoelectric transport, including the electronic conductance G, thermopower S (Seebeck coefficient), and electronic thermal conductance $\mathcal{K}$.

In addition, we present the connection between G and S as predicted by the semiclassical Cutler-Mott relation. We also discuss the impact of the Kondo correlation on the Wiedemann-Franz ratio connecting G, S, and $\mathcal{K}$. We present the model description and an outline of the calculation of the transmission coefficient, the fundamental quantity characterizing the transport of charge, energy, and heat, of a QD-based SET in Sec. III. Section IV is devoted to the investigation of the charge, energy, and heat transport in the case of spinless electrons exhibiting the charge-1CK effects. The transport in the charge-2CK regime is explored in Sec. V. Section VI contains the conclusion of our work together with possible future research plans based on the present work.

II. THERMOELECTRIC TRANSPORT COEFFICIENTS

The general setup for the thermoelectric transport contains the quantum impurity (QD) tunnel coupled to two external electron reservoirs. The left (L) and right (R) reservoirs are in equilibrium, separately, at temperatures T_γ ($\gamma = \mathrm{L, R}$) and chemical potentials μ_γ. Once the temperature gradient $\Delta T \equiv T_\mathrm{L} - T_\mathrm{R}$ and the voltage bias $e\Delta V \equiv \mu_\mathrm{L} - \mu_\mathrm{R}$ are established across the setup, the heat current I_h and charge current I_c start to flow. For simplicity of the presentation, henceforth, we use the system of atomic units $e = k_\mathrm{B} = \hbar = 1$. The charge and heat currents in the linear response theory are then connected by the Onsager relations [32,33],

$$\begin{pmatrix} I_\mathrm{c} \\ I_\mathrm{h} \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta V \\ \Delta T \end{pmatrix}. \quad (1)$$

The Onsager transport coefficients L_{ij} in Eq. (1) provide all the thermoelectric measurements of interest in the linear response regime [34]. To further deal with the coefficients L_ij, we set up the transport integrals in terms of the transmission coefficient $\mathcal{T}(\varepsilon, T)$ [35–38]:

$$\mathscr{L}_n \equiv \frac{1}{4T} \int_{-\infty}^{\infty} d\varepsilon \, \frac{\varepsilon^n}{\cosh^2\left(\frac{\varepsilon}{2T}\right)} \mathcal{T}(\varepsilon, T), \quad n = 0, 1, 2, \quad (2)$$

where T is the reference temperature.

The transport integrals $\mathscr{L}_n$ are then directly connected to the Onsager transport coefficients L_{ij}, namely, $L_{11} = \mathscr{L}_0$ and $L_{12} = \mathscr{L}_1/T$ [39]. In addition L_{12} and L_{21} are related by the Onsager reciprocity relation, and the coefficient L_{22} relates the electronic thermal conductance with L_{11} and L_{12} [1]. While the electronic conductance directly follows from $\mathscr{L}_0$, the thermopower or the Seebeck coefficient is usually obtained as $S = \mathscr{L}_1/\mathscr{L}_0 T$. In the case of noninteracting electrons in a metal, the Cutler-Mott relation [40] connects the thermopower S_CM with the logarithmic derivative of the energy-dependent electronic conductance $G(E)$ with respect to the energy E

$$S_\mathrm{CM} = \frac{\pi^2}{3} T \, \frac{d \ln G(E)}{dE}. \quad (3)$$

The deviation of S from S_CM amounts to strong electron correlation in the system [41]. In addition to the Seebeck effect, the Peltier effects are also of common interest in generic thermoelectric experiments [1]. The Peltier effect describes the generation of a heat current I_h due to the charge current I_c driven in a circuit under the isothermal condition $T_\mathrm{L} = T_\mathrm{R}$ by an applied voltage bias ΔV. The Peltier coefficient Π^γ associated with the γ reservoir is defined as $\Pi^\gamma = I_\mathrm{h}^\gamma / I_\mathrm{c}|_{T_\mathrm{L} = T_\mathrm{R}}$ [42]. This coefficient provides valuable information about the characterization of how good a material is for thermoelectric solid-state refrigeration or power generation [1]. In addition, the linear response (LR) Peltier coefficient Π_0 is related to the corresponding Seebeck coefficient $S = S^\mathrm{LR}$ via the Kelvin relation $\Pi_0 = TS$ [39].

Investigations beyond thermopower are usually done by electronic thermal conductance $\mathcal{K}$. The Wiedemann-Franz (WF) law connects the electronic thermal conductance $\mathcal{K}$ to the electrical conductance G in the low-temperature regime of a macroscopic sample by a universal constant, the Lorenz number L_0, defined as $L_0 \equiv \mathcal{K}/GT = \pi^2/3$ [1,35]. However, transport through nanodevices is generally expected to violate the WF law even in the FL regime [1]. Interestingly, the WF law was recently reported to be satisfied even in the NFL regime of Kondo effects [30,43]. The violation or the validation of the WF law is usually accounted for by studying the Lorenz ratio R, which is expressed in terms of the transport integrals [44],

$$R(T) \equiv \frac{L(T)}{L_0} = \frac{3}{(\pi T)^2} \left[\frac{\mathscr{L}_2}{\mathscr{L}_0} - \left(\frac{\mathscr{L}_1}{\mathscr{L}_0} \right)^2 \right]. \quad (4)$$

Any deviation of R from unity, therefore, amounts to the violation of the WF law.

From the preceding discussion, it is apparent that the fundamental quantities for characterizing the LR transport properties of a SET considered in this work are the transport integrals $\mathscr{L}_\mathrm{n}$. Furthermore, $\mathscr{L}_\mathrm{n}$ are the function of the transmission coefficient $\mathcal{T}(\varepsilon, T)$, which depends on the detail of the model. In the following section, we discuss in detail the calculation of the transmission coefficient applicable to both the charge-1CK and charge-2CK regimes of a QD-based SET.

III. MODEL HAMILTONIAN

We consider a QD (metallic island) coupled to the two electronic reservoirs, the left (L) lead and the right (R) lead. While the coupling between the QD and left lead is provided by a tunneling junction, that with the right lead is achieved by a single-channel quantum point contact (QPC) with reflection amplitude $|r|$ (see Ref. [7] for details). In addition, the conductance of the tunneling junction connecting the QD to left lead G_L is assumed to be much smaller than that of the other junction G_R. This results in the thermal equilibrium between the QD and the right lead. For this case, namely, the QD coupled weakly to the left contact and strongly to the right contact, the explicit form of the low-energy Hamiltonian $\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_\mathrm{L} + \mathscr{H}_\mathrm{R} + \mathscr{H}_\mathrm{C}$ is well known in the language of bosonization [9] (for clarity of presentation, in the following we write the Hamiltonian from Ref. [9]). Here the noninteracting part of the Hamiltonian reads [9,10]

$$\mathscr{H}_0 = \sum_{k\sigma} \varepsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_\sigma \varepsilon_\sigma d_\sigma^\dagger d_\sigma + \frac{v_\mathrm{F}}{2\pi} \sum_\sigma \int_\infty^\infty \left\{ \pi^2 \Pi_\sigma^2(x) + [\partial_x \phi_\sigma(x)]^2 \right\} dx, \quad (5)$$

where the operator $c_{k\sigma}$ annihilates an electron in the momentum state k with spin $\sigma = \uparrow, \downarrow$ in the left lead, d_σ annihilates an electron of spin σ in the QD, and v_F stands for the Fermi velocity. The bosonized displacement operator ϕ_σ and corresponding conjugated momentum operator Π_σ satisfy the usual commutation relation connecting the δ function: $[\phi(x), \Pi(y)] = i\delta(x-y)$. The Hamiltonian $\mathscr{H}_L$ describing the tunneling from the left lead to the QD is given by

$$\mathscr{H}_L = \sum_{k\sigma} (t_k c^\dagger_{k\sigma} d_\sigma + \text{H.c.}), \tag{6}$$

with t_k being the tunneling amplitude. The backscattering in the QPC is accounted for by the Hamiltonian

$$\mathscr{H}_R = -\frac{D}{\pi}|r| \sum_\sigma \cos[2\phi_\sigma(0)], \tag{7}$$

where D is the bandwidth. In addition, the Coulomb interaction in the QD is described by the Hamiltonian

$$\mathscr{H}_C = E_C\left[\hat{n} + \frac{1}{\pi}\sum_\sigma \phi(0) - N\right]^2. \tag{8}$$

Here $\hat{n}$ is the integer-valued operator that commutes with the electron annihilation operator of the left contact ψ_L, N is a dimensionless parameter proportional to the gate voltage, and E_C is the charging energy of the island (see Ref. [10] for details).

IV. TRANSMISSION COEFFICIENT IN THE CHARGE KONDO REGIME

In the following we describe the tunneling through the left contact within the second order of perturbation in the corresponding tunneling matrix element t_k. Under this assumption the electron transport processes are explained in terms of the tunneling density of states (DOS) of the left lead $\nu_L(\varepsilon)$ and that of the QD $\nu_D(\varepsilon)$ [9]. In addition, due to the weak energy dependence, henceforth, we consider the case of $\nu_L(\varepsilon) = \nu_L$. We then define the finite-temperature T transmission coefficient $\mathcal{T}(\varepsilon, T)$ characterizing the charge, energy, and heat transport through the left contact by the following relation [7,8]:

$$\mathcal{T}(\varepsilon, T) = -\frac{G_L}{\nu_0}\,\nu_D(\varepsilon, T). \tag{9}$$

In Eq. (9), $G_L \equiv 2\pi\nu_L\nu_0\langle|t_{kk'}|^2\rangle$ is the conductance of the left barrier for noninteracting electrons in the QD, and ν_0 stands for the DOS in the QD which is no longer renormalized by the electron interactions. The tunneling DOS of the QD is then given in terms of the electron Green's function (GF) [7,8],

$$\nu_D(\varepsilon, T) = -\frac{1}{\pi}\cosh\left(\frac{\varepsilon}{2T}\right)\int_{-\infty}^{\infty}\mathcal{G}\left(\frac{1}{2T}+it\right)e^{i\varepsilon t}\,dt, \tag{10}$$

where $\mathcal{G}(\tau) = -\langle T_\tau \psi_L(\tau)\psi_L^\dagger(0)\rangle$ is the Matsubara GF defined in terms of the operator ψ_L, which annihilates an electron in the QD at the position of the left contact [8].

Rescaling the operator $\psi_L \equiv \psi_L F$ in terms of another operator F which lowers $\hat{n}$ by unity (which means the commutation relation $[F, \hat{n}] = F$ has to be satisfied), we define the GF $\mathcal{G}(\tau)$ given in Eq. (10) as the product of the noninteracting part $\mathcal{G}_0(\tau)$ and the correlator $K(\tau)$ accounting for the electron interactions in the system [7–11],

$$\mathcal{G}(\tau) = \mathcal{G}_0(\tau)\,K(\tau) = -\frac{\pi\nu_0 T}{\sin(\pi T\tau)}\,K(\tau). \tag{11}$$

Here the time order correlator $K(\tau)$ is defined in terms of $F(\tau)$ such that $K(\tau) \equiv \langle T_\tau F(\tau)F^\dagger(\tau)\rangle$, with T_τ being the corresponding time ordering operator.

Plugging the transmission coefficient given by Eq. (9) into the transport integrals (2), followed by using the exact integrals calculated by the method of contour integration

$$\mathcal{I}_n \equiv \int_{-\infty}^{\infty}\frac{x^n\,e^{i2Ttx}}{\cosh x}\,dx, \quad \mathcal{I}_0 = \frac{\pi}{\cosh(\pi Tt)},$$
$$\mathcal{I}_1 = i\frac{\pi^2}{2}\frac{\sinh(\pi Tt)}{\cosh^2(\pi Tt)}, \quad \mathcal{I}_2 = \frac{\pi^2}{4}\left[\frac{2\pi}{\cosh^3(\pi Tt)} - \mathcal{I}_0\right], \tag{12}$$

we obtain the most general form of the transport coefficients in terms of the electron correlator $K(\tau)$:

$$\mathscr{L}_0 = -\pi T\frac{G_L}{2}\int_{-\infty}^{\infty}dt\;K\left(\frac{1}{2T}+it\right)\frac{1}{\cosh^2(\pi Tt)},$$
$$\mathscr{L}_1 = -i(\pi T)^2\frac{G_L}{2}\int_{-\infty}^{\infty}dt\;K\left(\frac{1}{2T}+it\right)\frac{\sinh(\pi Tt)}{\cosh^3(\pi Tt)},$$
$$\mathscr{L}_2 = -(\pi T)^3\frac{G_L}{2}\int_{-\infty}^{\infty}dt\,K\left(\frac{1}{2T}+it\right)\frac{2}{\cosh^4(\pi Tt)} - (\pi T)^2\mathscr{L}_0. \tag{13}$$

Computation of thermoelectric transport coefficients in Eq. (13) essentially needs the explicit form of the electron correlator $K(\tau)$. In addition, $K(\tau)$ also depends on the number of conduction channels in the contact which connects the left lead and the QD. For our purpose of investigating the single-channel and two-channel charge Kondo effects, it is, however, sufficient to consider a single-channel contact connecting the left lead and the QD. Furthermore, since the electron spin behaves fundamentally the same as the Kondo channels, the transport mechanism for spinless electrons and that for the spin-1/2 electrons are strikingly different. In the following we discuss the case of spinless electrons and that with spin 1/2 separately.

V. THE CHARGE-1CK REGIME: SPINLESS ELECTRONS

In the case of spinless electrons, which is generally achieved by applying the magnetic field, the SET setup exhibits the charge-1CK effect. The time order correlator in the charge-1CK regime has been shown to possess a compact analytical expression [10],

$$K\left(\frac{1}{2T}+it\right) = \left(\frac{\pi^2 T}{\gamma E_C}\right)^2\frac{1}{\cosh^2(\pi Tt)}\Big[1 - 2\gamma\xi|r|\cos(2\pi N) - i4\pi^2\xi\gamma|r|\frac{T}{E_C}\sin(2\pi N)\tanh(\pi Tt)\Big], \tag{14}$$

where $\xi \simeq 1.59$ is a constant and the symbol γ is related to the Euler constant C such that $\ln\gamma = C$. In addition, we explicitly assumed the low-temperature regime satisfying the condition $T \ll E_C$. Substitution of the correlator (14) into Eq. (13) results in the following expressions for the transport integrals:

$$\mathscr{L}_0 = \frac{2}{3}G_L\left(\frac{\pi^2 T}{\gamma E_C}\right)^2[1-2\gamma\xi|r|\cos(2\pi N)],$$
$$\mathscr{L}_1 = -\frac{8\pi^7\xi G_L}{15\gamma}T\left(\frac{T}{E_C}\right)^3|r|\sin(2\pi N),$$
$$\mathscr{L}_2 = \frac{2}{5}G_L(\pi T)^2\left(\frac{\pi^2 T}{\gamma E_C}\right)^2[1-2\gamma\xi|r|\cos(2\pi N)]. \quad (15)$$

To obtain the compact form of the transport integrals (15), we integrated out the t variable by the method of contour integration. Namely, we used integrals of the form

$$\mathscr{O}_n \equiv \int_{-\infty}^{\infty}dt\,\frac{e^{iat}}{\cosh^n(\pi T t)}; \quad (16)$$

for $n = 4$ and $n = 6$

$$\mathscr{O}_4 = -\frac{a\pi}{6(\pi T)^4}\frac{(a^2+4\pi^2T^2)}{\sinh(a/2T)},$$
$$\mathscr{O}_6 = -\frac{a\pi}{120(\pi T)^6}\frac{(a^4+20\pi^2a^2T^2+64\pi^4T^4)}{\sinh(a/2T)}. \quad (17)$$

The transport coefficients given by Eq. (15) result in the expressions of the thermopower and the Lorenz ratio

$$S = -\frac{4\pi^3\xi\gamma}{5E_C}|r|\sin(2\pi N)\,T, \quad (18)$$
$$R = \frac{9}{5} - \frac{3}{\pi^2}S^2. \quad (19)$$

To obtain the thermopower expression (18), we used only the leading term in the conductance. We note that the expression of thermopower presented in Eq. (15) was already established in Ref. [9]. The quadratic temperature scaling of the electronic conductance $\mathscr{L}_0$ and the linear suppression of thermopower with temperature as well as the reflection coefficient as seen from Eqs. (14) and (18) signify the FL behavior is similar to the single-channel spin Kondo effects. Moreover, at the particle-hole (PH) symmetric point ($S = 0$) the Lorenz ratio attains a universal constant of $9/5$ even at finite temperature [45].

In addition, from the Lorenz ratio given in Eq. (18), one can analyze the low-temperature behavior of the electronic thermal conductance $\mathcal{K} \propto \mathscr{L}_0 RT$. Since $G \propto T^2$ and $S \propto T$, the leading temperature scaling of the conductance is $\mathcal{K} \propto T^3$. This cubic temperature scaling of $\mathcal{K}$ is in contrast to the single-channel spin Kondo effects where the leading temperature scaling behavior of $\mathcal{K}$ is linear due to the presence of unitary conductance.

VI. THE CHARGE-2CK REGIME: ELECTRONS WITH SPIN

The SET setup with electrons having an effective spin 1/2 exhibits the charge-2CK effect in the low-temperature regime $T \ll E_C$. The time order correlator accounting for the interactions in the case of spin-1/2 electrons as computed in Ref. [9] is given by

$$K\left(\frac{1}{2T}+it\right) = \frac{\pi\Gamma T}{2\gamma E_C}\mathcal{I}_0 - \frac{2T}{E_C}|r|^2\sin(2\pi N)\ln\left(\frac{E_C}{\Gamma+T}\right)\mathcal{I}_1,$$
$$\mathcal{I}_n = \frac{1}{\cosh(\pi T t)}\int_{-\infty}^{\infty}dx\,\frac{x^n}{x^2+\Gamma^2}\frac{e^{itx}}{\cosh(x/2T)}, \quad (20)$$

where we defined the gate voltage dependent parameter $\Gamma \equiv 8\gamma E_C|r|^2\cos^2(\pi N)/\pi^2$. By substituting Eq. (20) into Eq. (13), we obtained the following expressions of the transport integrals:

$$\mathscr{L}_0 = \frac{\Gamma G_L}{8\gamma E_C}\mathcal{P}_0, \quad \mathscr{L}_2 = \frac{G_L\Gamma}{48\gamma E_C}T^2\mathcal{J},$$
$$\mathscr{L}_1 = -\frac{G_L}{6\pi}\frac{T^2}{E_C}|r|^2\sin(2\pi N)\ln\left(\frac{E_C}{\Gamma+T}\right)\mathcal{P}_1. \quad (21)$$

The coefficients appearing in Eq. (21) are expressed in the form of integrals,

$$\mathcal{P}_n = \int_{-\infty}^{\infty}dx\frac{x^{2n}(\pi^2+x^2)}{x^2+(\Gamma/T)^2}\frac{1}{\cosh^2(x/2)}, \quad n = 0, 1,$$
$$\mathcal{J} = \int_{-\infty}^{\infty}dx\,\frac{x^4+4\pi^2x^2+3\pi^4}{x^2+(\Gamma/T)^2}\,\frac{1}{\cosh^2(x/2)}. \quad (22)$$

To arrive at Eq. (21), we integrated out the t variable exactly by using the method of contour integration on the following integrals:

$$\int_{-\infty}^{\infty}dt\,\frac{e^{ixt}}{\cosh^5(\pi T t)} = -\pi\frac{x^4+10(\pi T)^2x^2+9(\pi T)^4}{24(\pi T)^5\,\cosh(x/2T)},$$
$$\int_{-\infty}^{\infty}dt\,e^{ixt}\frac{\sinh(\pi T t)}{\cosh^4(\pi T t)} = -\frac{i\pi x}{6(\pi T)^4}\frac{x^2+(\pi T)^2}{\cosh(x/2T)},$$
$$\int_{-\infty}^{\infty}dt\,\frac{e^{itx}}{\cosh^3(\pi T t)} = -\frac{\pi}{2(\pi T)^3}\frac{x^2+(\pi T)^2}{\cosh(x/2T)}. \quad (23)$$

The general behavior of the transport integrals in Eq. (21) can be studied by numerical integration of Eq. (22). Nonetheless, the most interesting behavior of the transport integrals is obtainable by an exact analytical procedure. Since the integrals in Eq. (22) possess two parameters, T and Γ, we define two asymptotic regimes, $T \gg \Gamma$ and $T \ll \Gamma$. To calculate analytically the asymptotes of $\mathcal{P}_0$ and $\mathcal{J}$ in the regime of $T \gg \Gamma$, we use the Lorentzian approximation of the δ distribution, $\delta(x) = \lim_{a\to 0}\frac{a}{\pi}\frac{1}{x^2+a^2}$, to obtain

$$\frac{\Gamma}{T}\mathcal{P}_0\bigg|_{T\gg\Gamma} = \pi^3, \quad \frac{\Gamma}{T}\mathcal{J}\bigg|_{T\gg\Gamma} = 3\pi^5. \quad (24)$$

The similar result for $\mathcal{P}_1$ is obtained, however, by expanding the corresponding integrand in the Taylor series with respect to Γ/T and using the Sommerfeld integrals

$$\mathcal{P}_1|_{T\gg\Gamma} = \int_{-\infty}^{\infty}dx\frac{x^2+\pi^2}{\cosh^2(x/2)} + \mathcal{O}\left(\frac{\Gamma}{T}\right)^2 = \frac{16}{3}\pi^2. \quad (25)$$

In addition for the calculation of $\mathcal{P}_1$ in the regime $T \gg \Gamma$, we also expanded the logarithmic factor in Eq. (22). Exploiting these procedures, we obtain the following analytical asymptotes of the transport integrals:

$$\mathscr{L}_1|_{T\gg\Gamma} = -\frac{8\pi G_L}{9}\frac{T^2}{E_C}\ln\left(\frac{E_C}{T}\right)|r|^2\sin(2\pi N),$$
$$\mathscr{L}_0|_{T\gg\Gamma} = \frac{\pi^3 G_L T}{8\gamma E_C},\quad \mathscr{L}_2|_{T\gg\Gamma} = \frac{\pi^5 G_L T^3}{16\gamma E_C}. \tag{26}$$

From the transport integrals presented in Eq. (26), we obtain the expressions for the thermopower and Lorenz ratio:

$$S|_{T\gg\Gamma} = -\frac{64\gamma}{9\pi^2}\ln\left(\frac{E_C}{T}\right)|r|^2\sin(2\pi N), \tag{27}$$
$$R|_{T\gg\Gamma} = \frac{3}{2} - \frac{3}{\pi^2}(S|_{T\gg\Gamma})^2. \tag{28}$$

We note that the thermopower equation (27) (originally uncovered in Ref. [9]) diverges for vanishingly small temperature due to the assumed asymptotic behavior $T \gg \Gamma$. However, the thermopower vanishes in the middle of the Coulomb blockade valley $N = 1/2$, which corresponds to the exact PH symmetric point. At this PH symmetric point, the Lorenz ratio becomes a universal number $R = 3/2$, and hence, the electronic thermal conductance acquires the quadratic temperature scaling behavior $\mathcal{K}|_{T\gg\Gamma,N=1/2} \propto T^2$.

To obtain the correct behavior of thermopower in the very low temperature regime, in the following we consider the opposite limit $T \ll \Gamma$. We use a technique similar to that exploited for the discussion of the $T \gg \Gamma$ regime; namely, we Taylor expand the integrand of the integrals $\mathcal{P}_{0,1}$ and $\mathcal{J}$ given in Eq. (21) with respect to the small parameter T/Γ and retain the lowest-order term. Then, using the Sommerfeld integrals, we obtain the asymptotic forms of the transport integrals in the low-temperature regime satisfying $T \ll \Gamma$,

$$\mathscr{L}_1|_{T\ll\Gamma} = -\frac{\pi^7 G_L}{60\gamma^2}T\left(\frac{T}{E_C}\right)^3\frac{1}{|r|^2}\frac{\sin(\pi N)}{\cos^3(\pi N)}\ln\left(\frac{E_C}{\Gamma}\right),$$
$$\mathscr{L}_0|_{T\ll\Gamma} = \frac{2G_L\pi^2}{3\gamma E_C\Gamma}T^2,\quad \mathscr{L}_2|_{T\ll\Gamma} = \frac{2G_L\pi^4}{5\gamma E_C\Gamma}T^4. \tag{29}$$

Therefore, the thermopower and the Lorenz ratio in the limit of $T \ll \Gamma$ read [46]

$$S|_{T\ll\Gamma} = -\frac{\pi^3}{5}\frac{T}{E_C}\tan(\pi N)\ln\left[\frac{\pi^2/8\gamma}{|r|^2\cos^2(\pi N)}\right], \tag{30}$$
$$R|_{T\ll\Gamma} = \frac{9}{5} - \frac{3}{\pi^2}(S|_{T\ll\Gamma})^2. \tag{31}$$

From the last equation it is seen that the Lorenz ratio also attains a universal number at the PH symmetric point similar to the single-channel situation (for the corresponding discussion of spin-2CK effects, see Ref. [43]). The beyond-asymptotic behavior of the Lorenz ratio is obtained by numerically solving Eq. (22) for R, which is presented in Fig. 1, showing the interplay of R between the regimes $T \ll \Gamma$ and $\Gamma \ll T$. The T^2 scaling of the conductance and the linear temperature scaling of the thermopower in the low-temperature regime $T \ll \Gamma$ result in the cubic temperature scaling behavior of the corresponding electronic thermal conductance $\mathcal{K}|_{T\ll\Gamma} \propto T^3$.

The temperature scaling behavior and complete interplay of other thermoelectric measures such as the power factor $Q \equiv S^2G$ and the dimensionless figure of merit $ZT \equiv S^2GT/\mathcal{K}$ (neglecting the phonon contribution to the thermal conductance) can be easily obtained from Eqs. (26) and (29). Since the power factor Q depends linearly on the conductance of the left contact $G_L \ll G_R$, the model of the charge-2CK regime suggested in Ref. [9], however, suffers from the smallness of the power factor from an application point of view. Given that, however, the figure of merit to the lowest order of approximation does not depend on G_L, thus providing the appreciable value of ZT.

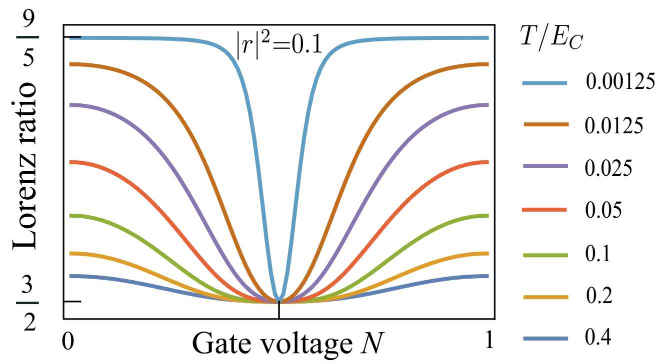

FIG. 1. Variation of the Lorenz ratio with gate voltage at the strong-coupling regime $T \ll \Gamma$ of the charge-2CK effect for fixed temperatures.

To study the deviation of thermopower from that predicted by the Cutler-Mott relation given in Eq. (3), first, we express the conductance Eq. (21) in terms of the polygamma function. In particular, the integral $\mathcal{P}_0$ in Eq. (21) is expressed as

$$\frac{\mathcal{P}_0}{4} = \left(\frac{\pi T}{\Gamma}\right)^2 + \left[1 - \frac{\Gamma}{2\pi T}\psi^{(1)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right]\left[1-\left(\frac{\pi T}{\Gamma}\right)^2\right],$$

where the trigamma function is defined as $\psi^{(1)}(y) = \sum_{n=0}^{\infty}(y+n)^{-2}$. For the SET considered in this work, we have the gate voltage $E \equiv 2E_C N$, which results in the Cutler-Mott thermopower S_{CM} [47]:

$$S_{CM} = -\frac{\pi^2}{6}\frac{T}{E_C}\frac{d\ln G}{dN} = \left[\frac{2\gamma|r|^2\sin(2\pi N)}{3}\right.$$
$$\left.\times\frac{4 - \frac{4\Gamma}{\pi T}\psi^{(1)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right) + \left[1-\left(\frac{\Gamma}{\pi T}\right)^2\right]\psi^{(2)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)}{\frac{2\Gamma}{\pi T} + \left[1-\left(\frac{\Gamma}{\pi T}\right)^2\right]\psi^{(1)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)}\right], \tag{32}$$

with $\psi^{(2)}(y) \equiv \frac{\partial}{\partial y}\psi^{(1)}(y)$ being the tetragamma function. The discrepancy between the thermopower calculated by the formula $S = \mathscr{L}_1/T\mathscr{L}_0$ from Eq. (21) and that calculated from the Cutler-Mott formula (32) is shown in Fig. 2. The significant deviation of thermopower at low temperature from that provided by the Cutler-Mott relation signifies the prominent role of strong electron correlation in the system. Similar behavior of the thermopower was reported in an experimental study of the single-channel spin Kondo effect [41].

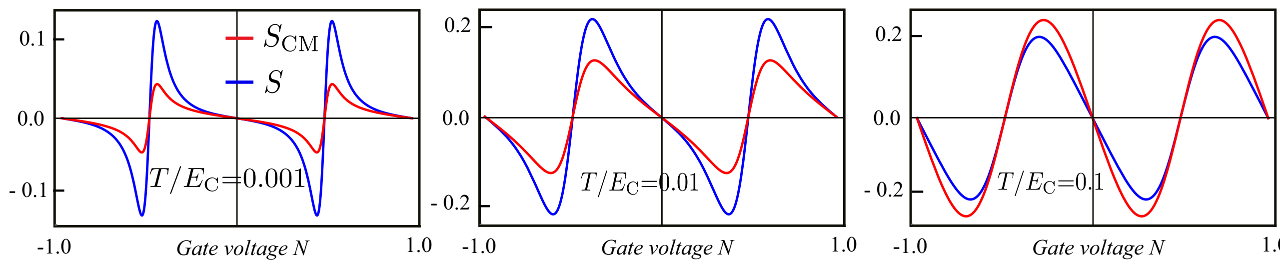

FIG. 2. Gate voltage variation of thermopower for the charge-2CK effect obtained from the Cutler-Mott relation S_{CM} and by the explicit calculation of thermopower S at different temperatures as indicated in the corresponding plots. For each plot we chose the reflection amplitude to be $|r|^2 = 0.1$.

VII. CONCLUSIONS

We investigated the heat transport through a SET in the charge-1CK (FL) and charge-2CK (NFL) regimes by extending the original theory of thermopower developed in Ref. [9]. In addition to the electronic conductance and the thermopower, we obtained compact analytical expressions for the Cutler-Mott thermopower and the WF ratio for both the charge-1CK and charge-2CK effects. We characterized the different transport regimes of a SET exhibiting charge Kondo correlations and explored the universal value of the WF ratio at the corresponding particle-hole symmetric point. The exact asymptotes and the temperature scaling behavior of electronic thermal conductance of a SET in charge-1CK and charge-2CK regimes were explored. Significant deviation of thermopower from the semiclassical Cutler-Mott relation was investigated and explained in terms of the charge Kondo correlations. An interesting direction for future work would be to investigate the heat transport in the presence of the charge Kondo correlation considering more complex setups [28] and to extend the presented calculations to the beyond-linear response regime. Investigations of the impact of channel asymmetry on the heat transport in the charge Kondo regime also appears to be a valid avenue for future research.

[1] G. Benenti, G. Casati, K. Saito, and R. Whitney, Phys. Rep. **694**, 1 (2017).
[2] M. S. Dresselhaus, G. Dresselhaus, X. Sun, Z. Zhang, S. B. Cronin, and T. Koga, Phys. Solid State **41**, 679 (1999).
[3] C. W. J. Beenakker and A. A. M. Staring, Phys. Rev. B **46**, 9667 (1992).
[4] A. A. M. Staring, L. W. Molenkamp, B. W. Alphenaar, H. van Houten, O. J. A. Buyk, M. A. A. Mabesoone, C. W. J. Beenakker, and C. T. Foxon, Europhys. Lett. **22**, 57 (1993).
[5] A. S. Dzurak, C. G. Smith, C. H. W. Barnes, M. Pepper, L. Martín-Moreno, C. T. Liang, D. A. Ritchie, and G. A. C. Jones, Phys. Rev. B **55**, R10197 (1997).
[6] S. Möller, H. Buhmann, S. F. Godijn, and L. W. Molenkamp, Phys. Rev. Lett. **81**, 5197 (1998).
[7] K. A. Matveev, Phys. Rev. B **51**, 1743 (1995).
[8] A. Furusaki and K. A. Matveev, Phys. Rev. B **52**, 16676 (1995).
[9] A. V. Andreev and K. A. Matveev, Phys. Rev. Lett. **86**, 280 (2001).
[10] K. A. Matveev and A. V. Andreev, Phys. Rev. B **66**, 045301 (2002).
[11] M. Turek and K. A. Matveev, Phys. Rev. B **65**, 115332 (2002).
[12] I. Affleck, J. Phys. Soc. Jpn. **74**, 59 (2005).
[13] J. Kondo, Prog. Theor. Phys. **32**, 37 (1964).
[14] P. W. Anderson, J. Phys. C **3**, 2436 (1970).
[15] G. Yuval and P. W. Anderson, Phys. Rev. B **1**, 1522 (1970).
[16] P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B **1**, 4464 (1970).
[17] P. Nozières, J. Low Temp. Phys. **17**, 31 (1974).
[18] K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975).
[19] P. Nozieres and A. Blandin, J. Phys. **41**, 193 (1980).
[20] A. M. Tsvelik and P. B. Wiegmann, Adv. Phys. **32**, 453 (1983).
[21] I. Affleck, Nucl. Phys. B **336**, 517 (1990).
[22] I. Affleck and A. W. W. Ludwig, Phys. Rev. B **48**, 7297 (1993).
[23] L. Kouwenhoven and L. Glazman, Phys. World **14**, 33 (2001).
[24] Z. Iftikhar, S. Jezouin, A. Anthore, U. Gennser, F. D. Parmentier, A. Cavanna, and F. Pierre, Nature (London) **526**, 233 (2015).
[25] T. K. T. Nguyen, M. N. Kiselev, and V. E. Kravtsov, Phys. Rev. B **82**, 113306 (2010).
[26] T. K. T. Nguyen and M. N. Kiselev, Phys. Rev. B **92**, 045125 (2015).
[27] L. A. Landau, E. Cornfeld, and E. Sela, Phys. Rev. Lett. **120**, 186801 (2018).
[28] T. K. T. Nguyen and M. N. Kiselev, Phys. Rev. B **97**, 085403 (2018).
[29] T. K. T. Nguyen and M. N. Kiselev, Phys. Rev. Lett. **125**, 026801 (2020).
[30] G. A. R. van Dalum, A. K. Mitchell, and L. Fritz, Phys. Rev. B **102**, 041111(R) (2020).
[31] Z. Iftikhar, A. Anthore, A. K. Mitchell, F. D. Parmentier, U. Gennser, A. Ouerghi, A. Cavanna, C. Mora, P. Simon, and F. Pierre, Science **360**, 1315 (2018).
[32] L. Onsager, Phys. Rev. **37**, 405 (1931).
[33] L. Onsager, Phys. Rev. **38**, 2265 (1931).
[34] T. A. Costi and V. Zlatić, Phys. Rev. B **81**, 235127 (2010).
[35] V. Zlatic and R. Monnier, *Modern Theory of Thermoelectricity* (Oxford University Press, Oxford, 2014).
[36] D. B. Karki and M. N. Kiselev, Phys. Rev. B **96**, 121403(R) (2017).
[37] D. B. Karki and M. N. Kiselev, Phys. Rev. B **100**, 125426 (2019).
[38] D. B. Karki and M. N. Kiselev, Phys. Rev. B **100**, 195425 (2019).
[39] T.-S. Kim and S. Hershfield, Phys. Rev. B **67**, 165313 (2003).

[40] M. Cutler and N. F. Mott, Phys. Rev. **181**, 1336 (1969).

[41] R. Scheibner, H. Buhmann, D. Reuter, M. N. Kiselev, and L. W. Molenkamp, Phys. Rev. Lett. **95**, 176602 (2005).

[42] E. N. Bogachek, A. G. Scherbakov, and U. Landman, Phys. Rev. B **60**, 11678 (1999).

[43] D. B. Karki and M. N. Kiselev, Phys. Rev. B **102**, 241402(R) (2020).

[44] D. B. Karki, Phys. Rev. B **102**, 115423 (2020).

[45] Note that the original proposal in Ref. [9] is limited to the temperature, which should be larger than the tunnel coupling strength to the left lead. Therefore, the exactly zero temperature limit cannot be reached in our work.

[46] This expression of thermopower was predicted in Ref. [9]. The contribution of our work is therefore the analysis of heat transport, namely, the behavior of the Lorenz ratio in the presence of the charge Kondo correlation.

[47] To obtain the analytical form of the Cutler-Mott thermopower, we first defined the integrals $\mathscr{M}_n = \frac{1}{4}\int s_{-\infty}^{\infty} dx\, \frac{x^{n+2}}{x^2+a^2}\frac{1}{\cosh^2(x/2)} = \frac{1}{a}\mathrm{Re}[Z_{n+2}(-ia)]$, with the function Z satisfying $Z_n(x) = ib_{n-1} + xZ_{n-1}(x)$ for $n \geqslant 1$. The constant factors b_n are obtained as $b_n \equiv \frac{1}{4}\int_{-\infty}^{\infty} dx\, \frac{x^n}{\cosh^2(x/2)}$, $b_0 = 1$, $b_2 = \frac{\pi^2}{3}$, $b_4 = \frac{7\pi^4}{15}, \ldots$. Then we used $Z_0(x) = \frac{1}{2\pi}\psi^{(1)}(\frac{1}{2} + \frac{ix}{2\pi})$, $\psi^{(1)}(y) = \sum_{n=0}^{\infty}\frac{1}{(y+n)^2}$.