Rapid Communications

Exceptional points in Fermi liquids with quadrupolar interactions

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We show the existence of non-Hermitian degeneracies, known as *exceptional points*, in the collective mode spectrum of Fermi liquids with quadrupolar interactions. Through a careful analysis of the analytic properties of the dynamic quadrupolar susceptibility, we show that, in the weak attractive region, two stable collective modes coalesce to an exceptional point. We completely characterize this singularity, explicitly showing its topological properties. Experimental signatures are also discussed.

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Introduction. Open quantum systems play a central role in most applications of quantum mechanics [1]. An important theoretical tool to describe dissipative quantum systems is the modeling of locally nonconservative systems by effective non-Hermitian Hamiltonians [2–4]. These types of Hamiltonians have several counterintuitive properties. Perhaps one of the most striking ones is the appearance of non-Hermitian degeneracies [5] known as *exceptional points* (EPs) [6,7].

When a non-Hermitian Hamiltonian continuously depends on external parameters, it could happen that, for certain values of the parameters, two or more eigenvalues coalesce to an EP. However, this is not a usual degeneracy, as observed in Hermitian systems. In an EP, not only do the eigenvalues coincide but also the eigenvectors become linearly dependent [8], reducing in this way the dimension of the subspace associated with the degenerated eigenvalue. This singularity of the Hilbert space has remarkable topological consequences [9–13]. The relation between EPs and dynamical phase transitions was recognized early in theoretical as well as experimental works [14–17].

In recent years, exciting findings of EPs are showing up in very different contexts, strengthening the broad interest in this subject; from nuclear [18] and atomic physics [19–21] to Bose-Einstein condensates [22] and strongly correlated fermion systems [23], passing through microwave cavities [24] and SWAP gates in spin systems [16,25]. Moreover, the topological properties of EPs were experimentally studied in metamaterial setups [26,27].

In this Rapid Communication, we report the existence of exceptional points in the spectrum of collective excitations of Fermi liquids [28] with higher-order Landau parameter interactions. Fermi liquids with quadrupolar interactions caught the attention of the condensed matter community because it is the simplest model supporting an isotropic-nematic transition [29]. Nematic fluctuations play a crucial role in several strongly correlated systems, such as cuprates and Fe-based superconductors and a variety of quantum Hall effects [30].

Collective excitations of Fermi liquids with quadrupolar interactions have been studied in different regimes [29,31–35]. Here, we explicitly show the appearance of a non-Hermitian singularity for weak quadrupolar attraction. We completely characterize this exceptional point, by analyzing the Hilbert space structure and its topological properties. Finally, we discuss some possible experimental setups.

Model. We consider the simplest model of bidimensional spinless fermions with local quadrupolar interactions. The Hamiltonian is

$$H = \int d^2r \left\{ \psi^\dagger(\mathbf{r})\epsilon(\boldsymbol{\nabla})\psi(\mathbf{r}) + \frac{F_2}{4}\mathrm{Tr}[Q^2(\mathbf{r})] \right\}, \tag{1}$$

where $\psi(r)$ is a spinless fermionic field operator. The bare dispersion relation is given by $\epsilon(\boldsymbol{\nabla})$, where ∇ is the two-dimensional gradient operator. F_2 is the quadrupolar coupling constant. The quadrupolar fermionic density $Q_{ij} = \psi^\dagger(\mathbf{r})[\nabla_i\nabla_j - (\delta_{ij}/2)\nabla^2]\psi(\mathbf{r})$, with $i = 1, 2$, is a symmetric traceless tensor of rank 2, invariant under π rotations.

Collective modes are encoded in the dynamic quadrupolar susceptibility (DQS) $\chi_{ijlm}(\omega, \mathbf{q}) = \langle Q_{ij}(-\omega, -\mathbf{q})Q_{lm}(\omega, \mathbf{q})\rangle$. DQS have been intensively studied [29,31,36,37] in the vicinity of a quantum critical point, where non-Fermi-liquid behavior is expected. Conversely, in this Rapid Communication we study the dynamic response in the Fermi-liquid regime. Since the quadrupolar moment has two degrees of freedom, the susceptibility has essentially two independent polarizations, the longitudinal $\chi_2^+(\omega, \mathbf{q})$ and the transversal polarization $\chi_2^-(\omega, \mathbf{q})$. These quantities have been computed using different approximation approaches [29,31–33,38]. In the limit of small momentum $q \ll k_F$, where k_F is the Fermi momentum, the result is [31] (please see the Supplemental Material [39] for a detailed description of the calculation)

$$\chi_2^\pm(\omega, \mathbf{q}) = \frac{\chi_0^0(s) \pm \chi_4^0(s)}{1 - F_2[\chi_0^0(s) \pm \chi_4^0(s)]}, \tag{2}$$

where

$$\chi_{2\ell}^0 = \left[-\delta_{\ell,0} + K_0(s)\left(\frac{1 - K_0(s)}{1 + K_0(s)}\right)^\ell \right], \tag{3}$$

with $K_0(s) = s/\sqrt{s^2 - 1}$. Equation (3) with $\ell = 0, 2$ are the bare density and quadrupolar susceptibilities, respectively.

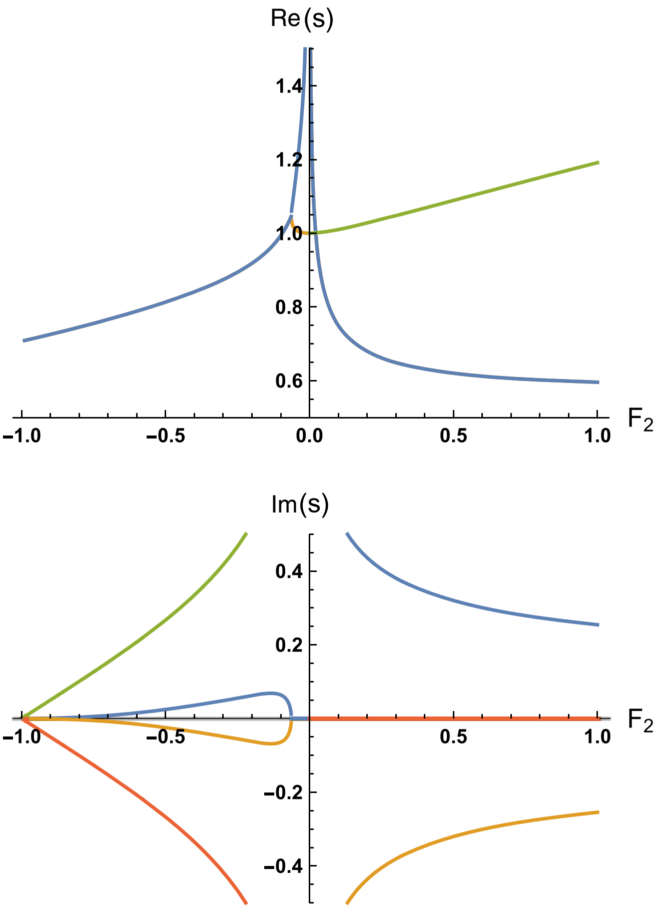

FIG. 1. Collective modes from the longitudinal polarized component of the DQS $\chi_2^+(s)$. In the upper panel we plot Re$[s(F_2)]$ while in the lower panel we depict Im$[s(F_2)]$.

Equation (2) has the usual structure of an effective interaction in the traditional random phase approximation (RPA). Due to the locality of the quadrupolar interaction (i.e., F_2 does not depend on $\mathbf{q}$), the DQS is not a function of ω and $\mathbf{q}$ independently. Instead, it depends on the dimensionless variable $s = \omega/qv_F$, where ω is the frequency and qv_F is the maximum energy of a particle-hole excitation with momentum $q = |\mathbf{q}|$ and Fermi velocity $v_F = |\mathbf{v_F}|$. It is worth mentioning that in the computation of Eq. (2), rotational invariance and particle-hole symmetry were imposed.

Collective modes. The DQS is an analytic function of s, having poles and cuts. It has branch points at $s = \pm 1$; the threshold of Landau damping $\omega = \pm v_F q$. We will focus on the longitudinal polarization $\chi_2^+(s)$ since, as we will show, this component displays an EP. Collective modes are computed by solving the algebraic equation $F_2[\chi_0^0(s) + \chi_4^0(s)] = 1$. We have numerically solved it for F_2 running from the strong attractive ($F_2 = -1$) to the strong repulsive regime ($F_2 > 1$). We display the result in Fig. 1. In the upper panel, we show the real part of the collective modes as function of F_2, while in lower panel we show the imaginary part. In the repulsive region ($F_2 > 0$), we observe a stable (real) mode that tends to $s = 1$ when $F_2 \to 0$. This is the quadrupolar equivalent of the Landau zero sound. In addition, a damped mode also appears in the same region. The stable mode is continuously extended to the weak attractive region $F_2 \lesssim 0$. However, in this regime, there is another stable mode with a divergent behavior, $s \to +\infty$ when $F_2 \to 0^-$. The existence of such a mode was reported in Ref. [33]. Interestingly, there is a special point, F_2^c, where both stable modes meet together. For $F_2 < F_2^c$, these modes become damped, as can be clearly seen in the lower panel of Fig. 1. We can also observe an overdamped mode (purely imaginary) in all of the attractive region. This mode is the precursor of the isotropic-nematic phase transition that occurs at $F_2 = -1$ and has already been extensively studied [29,31].

Exceptional point. In order to analytically characterize the singularity at $F_2 = F_2^c$, we first observe that $s(F_2^c) \gtrsim 1$, being well separated from the cut $s^2 < 1$. On the other hand, the singularity is sufficiently close to $s = 1$, allowing us to try a series expansion of $\chi_2^+(s)$ in the neighborhood of $s = 1$. For simplicity, let us work with the inverse of the DQS, $\mathcal{L}^+(s) = [\chi_2^+(s)]^{-1}$. Expanding this quantity in terms of the variable $\sqrt{(s-1)/2}$, we find the following expansion (please see Supplemental Material [39] for details of the calculation),

$$\mathcal{L}^+(s) = -F_2 + \sqrt{\frac{s-1}{2}} + 5\left(\frac{s-1}{2}\right) + O[(s-1)^{3/2}]. \quad (4)$$

Longitudinal quadrupolar fluctuations $\delta Q^+(s, \mathbf{q})$ are governed by the effective action

$$S_{\text{eff}} = \int \frac{d\omega d^2q}{(2\pi)^3} \mathcal{L}^+(s)|\delta Q^+(s, \mathbf{q})|^2. \quad (5)$$

The collective modes are given by the roots of $\mathcal{L}^+(s) = 0$. Using Eq. (4), we obtain

$$s_\pm = \frac{1}{25}\{(26 + 10F_2) \pm \sqrt{20F_2 + 1}\}. \quad (6)$$

$s_\pm(F_2)$ has a square-root singularity (branch point) at $F_2^c = -1/20$. At this point, both zeros are degenerated, $s_\pm(F_2^c) = 51/50$. We depict the real and imaginary part of $s_\pm(F_2)$ in Fig. 2. Thus, the approximation made in Eq. (4) for $|s-1| \ll 1$ correctly captures the presence of the degeneracy point observed in the numerical computation of Fig. 1. The square-root singularity is a typical signature of an exceptional point [40].

The dynamics described by Eq. (4) is nonlocal in time. However, since the degeneracy is separated from the cut, we can further expand $\mathcal{L}^+(s)$ in the neighborhood of $s = s_\pm$. In addition, we observe that the local character of the interaction imposes that $\mathcal{L}^+(s)$ only depends on the dimensionless variable s. Thus, we can consider quadrupolar fluctuations $\delta Q^+(s)$, ignoring any momentum dependence not scaling with s. The consequence is that all collective modes in this approximation have a linear dispersion relation $\omega \sim v_F q$. This is a good approximation for weak interactions. However, it breaks down in the strongly attractive regime ($F_2 \sim -1$), where nonlocal interactions $F_2(q)$ are essential [31]. With these considerations, we arrive at the effective action

$$S_{\text{eff}} = \int ds\{(s-\epsilon_1)(s-\epsilon_2) + w^2\}|\delta Q^+(s)|^2, \quad (7)$$

where $\epsilon_1 = (1/25)(27 + 10F_2)$, $\epsilon_2 = (1/25)(25 + 10F_2)$, and $w = (1/25)\sqrt{20|F_2|}$ are real positive numbers in the vicinity of the EP. The zeros of the Lagrangian are given of course by Eq. (6).

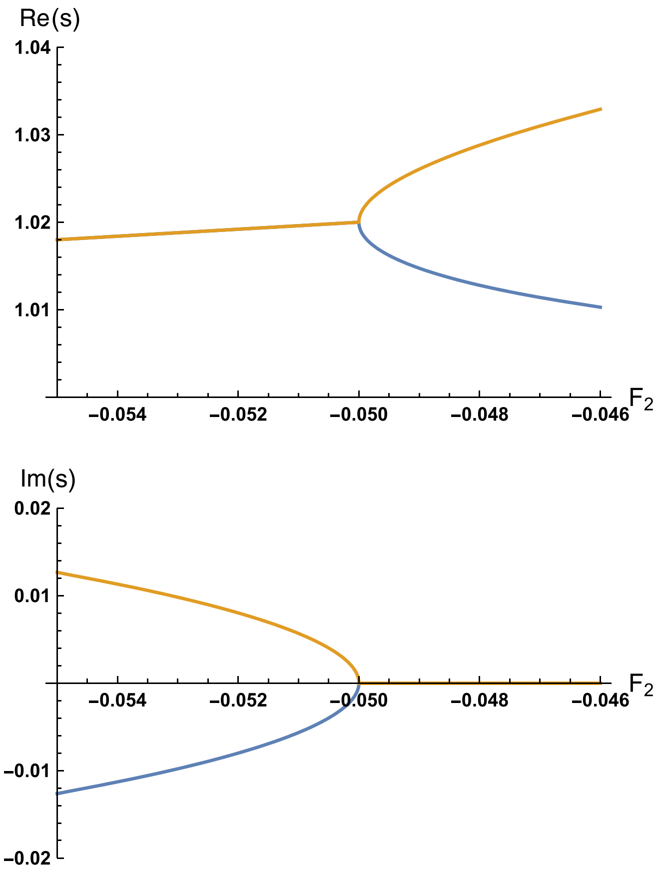

FIG. 2. Solutions of $\mathcal{L}^{+}(s_{\pm}) = 0$, given by Eq. (6) as a function of the parameter F_2. The upper panel shows the real part of $s_{\pm}$, while lower one depicts the imaginary part. The point $F_2 = -1/20$, where both eigenvalues coalesce and the imaginary part emerges, is the exceptional point.

In order to rewrite the effective action in the Hamiltonian formalism (first order in time), we introduce a two-component vector field $\delta Q^{+} = (\delta Q_1, \delta Q_2)$. In terms of this field, the effective action reads (please see Supplemental Material [39] for details)

$$S_{\text{eff}} = \int ds (\delta Q^{+})^{\dagger} (sI - H_{\text{eff}}) \delta Q^{+}, \tag{8}$$

where I is the 2×2 identity matrix and the effective Hamiltonian is

$$H_{\text{eff}} = \begin{pmatrix} \epsilon_1 & iw \\ iw & \epsilon_2 \end{pmatrix}. \tag{9}$$

It is straightforward to verify that, integrating out the vector component δQ_2, we obtain the effective action of Eq. (7) for the field δQ_1. Therefore, the dynamics near the singularity is driven by a 2×2 symmetric effective Hamiltonian (non-Hermitian), which determines the properties of the EP [41].

Hilbert space and topology. The Hilbert space spanned by the basis $\psi_{\pm}$ and its dual, spanned by $\phi_{\pm}$, are in general different in non-Hermitian Hamiltonian systems. They are defined by

$$H_{\text{eff}} \psi_{\pm} = s_{\pm} \psi_{\pm}, \tag{10}$$

$$H_{\text{eff}}^{\dagger} \phi_{\pm} = s_{\pm}^{*} \phi_{\pm}. \tag{11}$$

Biorthogonality requires $\langle \phi_i | \psi_j \rangle = \delta_{ij}$ with $i, j = \pm$. Since the effective Hamiltonian is symmetric, the dual space is spanned by $\phi_{\pm} = \psi_{\pm}^{*}$. Solving Eq. (10), we find

$$\psi_{\pm} = c^{\pm} \begin{pmatrix} 1 \\ \frac{-i}{\sqrt{1-z}} [1 \mp z^{1/2}] \end{pmatrix}, \tag{12}$$

where $c^{\pm}$ are complex normalization constants. We have introduced the variable $z = 1 + 20F_2$, in order to have the EP at $z = 0$. As anticipated, not only does $s_{+} = s_{-}$ at the EP, but the eigenvectors collapse to $\psi_{\pm}^{\text{EP}} = c^{\pm}(1, -i)$. This fact produces that $\langle \phi^{\text{EP}} | \psi^{\text{EP}} \rangle = 0$, which is evidently in conflict with biorthogonality. In this way, the EP is a singularity in the structure of the Hilbert space [8]. This singularity induces remarkable topological properties. To show this, let us compute the geometric phase that the wave function picks up when the EP is winded in parameter space. For this, we analytically continue z to the complex plane and define the Berry phase as $\gamma = i \oint_C d\ell \cdot \mathbf{A}$, where the one-form $\mathbf{A} = \langle \phi_{+} | \nabla \psi_{+} \rangle / \langle \phi_{+} | \psi_{+} \rangle$ [42], C is a closed path, and ∇ is the gradient in parameter space z. The equivalent definition with ϕ_{-} and ψ_{-} eigenvectors provides the same result. Notice that $\mathbf{A}$ is ill defined at the EP since, at this point, the denominator is zero. The particular structure of the Hilbert space and its dual allows us to rewrite the vector form as a total derivative (locally a pure gauge), $\mathbf{A} = (1/2) \nabla \ln \langle \phi_{+} | \psi_{+} \rangle$. Thus, the EP is a branch point of the logarithm. Each time the phase of $\langle \phi_{+} | \psi_{+} \rangle$ winds the branch point, the logarithm picks up a $2\pi i$ term. This property does not depend on the specific path, provided the path encircles the EP. Thus, we can compute γ considering a very small circumference around the EP. Using Eq. (12), we find for $|z| \ll 1$, $\langle \phi_{+} | \psi_{+} \rangle \sim z^{1/2}$. Due to the square-root singularity, the phase of $\langle \phi_{+} | \psi_{+} \rangle$ is half the phase of z. Therefore, taking the path C winding *two times* the EP, the Berry phase $\gamma = \pi$, in agreement with results obtained for general symmetric non-Hermitian Hamiltonians [43]. In this way, in encircling the EP, it is necessary to wind four times the singularity to return to the original state [44]. Recently, this unique topology of EPs was experimentally confirmed in metamaterial setups [26,27].

Experimental signatures. Information about the collective excitations of strongly correlated systems can be obtained by measuring momentum-resolved dynamic susceptibility in the meV scale [45]. The detection of a stable mode near the usual zero sound could be an indication of the presence of an EP. Moreover, pump-probe spectroscopy [46–48] yields important information on the dynamic response in the time domain. An experimental signature can be obtained from $\chi_2^{+}(\mathbf{q}, t)$, by Fourier transforming the DQS in the neighborhood of the EP. For $F_2 > F_2^c$, the retarded susceptibility is

$$\text{Re}[\chi_2^{+}(\mathbf{q}, t)] = 2 v_F q \left[\frac{\sin(\omega_{-} t)}{\omega_{-}} \right] \cos(\omega_{+} t) \Theta(t), \tag{13}$$

where $\omega_{\pm} = (s_{+} \pm s_{-}) v_F q / 2$ and $\Theta(t)$ is the Heaviside distribution. We clearly observe two well-separated timescales since $\omega_{+}/\omega_{-} \gg 1$. At the EP, $\omega_{-} = 0$ and $\sin(\omega_{-} t)/\omega_{-} \to t$. Thus, the signature of the EP is a growing linear modulat-

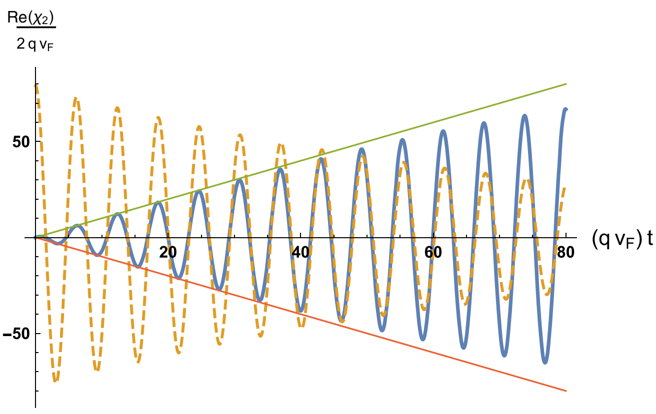

FIG. 3. $\text{Re}[\chi_2(\mathbf{q},t)]/2v_Fq$ as a function of $(v_Fq)t$. The continuous line is plotted with Eq. (13) by fixing $F_2 = F_2^c + 0.005$. The dashed line is the damped mode for $F_2 = F_2^c - 0.005$. The linear functions are the exact modulating function at the EP, $F_2 = F_2^c = -1/20$.

ing function of time, $\chi_2^+(\mathbf{q},t) \sim t\cos(\omega_+ t)$. An approximate linear modulation can be observed on a huge range of intermediate times, even when the coupling is not fine tuned at $F_2 = F_2^c$. On the other hand, for $F_2 < F_2^c$, the dynamic response dramatically changes since the modulation is exponentially damped $\chi_2^+(\mathbf{q},t) \sim \exp\{-|\omega_-|t\}\cos(\omega_+ t)$. We depict these different regimes in Fig. 3. The abrupt change in the dynamical response at the EP should also be captured in quantum quench setups [49]. Another interesting possibility is to look for signatures on the AC electrical conductivity [50].

Summary and discussion. We have shown the existence of an EP in the collective mode spectrum of a Fermi liquid with weak attractive quadrupolar interactions. We completely characterize this singularity in terms of the Hilbert space structure as well as through its topological properties. We have also provided experimental signatures in the dynamical response. More complex models of Fermi liquids could lead to higher-dimensional singularities, such as exceptional lines or surfaces [51,52]. For instance, if we consider isotropic density interactions (F_0) in addition to the quadrupolar ones [33], we still find square-root singularities which, in the limit of small F_0, take the form $s_+ - s_- = \sqrt{1 + 20F_2 + 4F_0}$. In this way, the spectrum has an *exceptional line* parametrized by $F_2 + F_0/5 = -1/20$.

Concluding, non-Hermitian singularities appear in the spectrum of collective modes of Fermi liquids with higher angular momentum attractive interactions. Specific properties, such us the singularity location and dimensionality, are model dependent. However, its existence, its topological properties, and experimental signatures are robust results. It could be important to investigate the influence of these singularities in the single quasiparticle spectrum and its effect on charge transport and other out-of-equilibrium properties.

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