Backscattering off a driven Rashba impurity at the helical edge

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The spin degree of freedom is crucial for both understanding and exploiting the particular properties of the edges of two-dimensional topological insulators. In the absence of superconductivity and magnetism, Rashba coupling is the most relevant single-particle perturbation in this system. Since Rashba coupling does not break time reversal symmetry, its influence on transport properties is visible only if processes that do not conserve the single-particle energy are included. Paradigmatic examples of such processes are electron-electron interactions and time-dependent external drivings. We analyze the effects of a periodically driven Rashba impurity at the helical edge, in the presence of electron-electron interactions. Interactions are treated by means of bosonization, and the backscattering current is computed perturbatively up to second order in the impurity strength. We show that the backscattering current is nonmonotonic in the driving frequency. This property is a fingerprint of the Rashba impurity, being absent in the case of a magnetic impurity in the helical liquid. Moreover, the nonmonotonic behavior allows us to directly link the backscattering current to the Luttinger parameter K, encoding the strength of electron-electron interactions.

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I. INTRODUCTION

Since their theoretical prediction [1–3] and subsequent experimental discovery [4], quantum spin Hall (QSH) systems have attracted significant attention in view of their possible applications in spintronics [5–9] and topological quantum computation [10]. Accordingly, the number of host materials for QSH systems is increasing. In addition to semiconducting heterostructures, such as HgTe/CdTe- [4] and InAs/GaSb- [11] based systems, bismuthene on SiC [12], WTe2 [13], and jacutingaite [14] have been shown to be two-dimensional topological insulators. The bulk band gap, and hence the temperature range in which the QSH effect can be seen, is, in the latter materials, substantially enhanced. However, in order to employ the QSH effect in applications, it is crucial to be able to generate one-dimensional helical channels with long coherence length and to develop technologies enabling their manipulation. In this respect, HgTe-based heterostructures still present advantages. In such systems, in fact, the coherence length can reach several micrometers [15,16]. Moreover, the QSH effect can sustain strong magnetic fields [17–20], and the possibility to induce superconductivity has been demonstrated [21,22]. Recently, quantum constrictions between edges on different sides of the sample were also realized [23]. The combination of these particularities make certain functionalities viable [24–28].

The mechanisms allowing for the manipulation of helical edge states share the common trait of enabling the coupling among particles (electrons or holes) with different spins. This necessity essentially emerges due to spin-momentum locking. Several possibilities along these lines have been considered. Magnetic barriers [29–33], which could, in principle, allow for the implementation of many interesting proposals, have not been experimentally achieved in QSH systems yet. Alternatively, the possibility of locally manipulating Rashba coupling by means of external gates has been analyzed [34,35]. It opens the way to interesting electron quantum optics applications [36,37].

At the same time, Rashba coupling at the helical edge has been the object of intense research with the aim of understanding the possible mechanisms that generate backscattering, even in the absence of explicit time reversal symmetry breaking [38–41]. While the combination of strictly local spin-orbit coupling, interactions invariant under spin rotations, and an unbounded linear spectrum does not lead to backscattering [40], relaxing any of these conditions can, indeed, imply a backscattering current [41]. This effect manifests itself in two different configurations: A Rashba scatterer in an otherwise standard helical Luttinger liquid [38,39,41] and a normal scatterer in the so-called generic interacting helical liquid [42–45]. A generic helical liquid is a helical liquid in which the spin quantization axis defined by spin-momentum locking depends on the quasimomentum [42,46–48]. In both cases the combined effect of a spin-dependent term and an interaction term mixing the single-particle states matters.

The possibility of mixing single-particle states at the helical edge is not exclusive to electron-electron interactions. Time-dependent processes can result in similar effects. Recently, this principle was used to demonstrate that the effect of current noise in a generic helical liquid in the presence of spinindependent impurities can, indeed, result in a backscattering current [49].

In this paper, we take a complementary point of view. We consider a standard helical liquid in the presence of a periodically driven Rashba scatterer. Moreover, since interactions at the helical edge are material dependent [23,50–52] and potentially important [53,54], we include them by means of bosonization [55]. We perturbatively compute the backscattering current as a function of the frequency of the driving. We find that, even in the weak interaction regime, the dependence is nonmonotonic. This is in striking contrast to the behavior of the usual time-dependent impurities in single-channel Luttinger liquids [56–59] (we refer to this case as "magnetic impurity" [60] throughout the paper). Furthermore, we use such nonmonotonicity to define a quantity that depends on only the Luttinger parameter K, encoding the strength of electron-electron interactions. This dependence can, in principle, allow for a determination of K, a notoriously difficult task, without the need to consider power-law dependencies.

The rest of the paper is structured as follows. In Sec. II, we present the model and the formalism we use for the computation of the backscattering current. In Sec. III, we present the general result. In Sec. IV, we focus on the discussion of the nonmonotonic behavior and its implications. We then present the analysis of some relevant limiting cases in Sec. V. Finally, in Sec. VI, we draw our conclusions. Some details of the calculation are described in three Appendixes.

II. MODEL

A. Fermionic representation

The edge states of a QSH insulator are characterized by spin-momentum locking, meaning that right and left movers are uniquely associated with a specific spin polarization [53]. We define the fermionic operators $\psi_R(x) \equiv \psi_{R\uparrow}(x)$ and $\psi_L(x) \equiv \psi_{L\downarrow}(x)$. The edge Hamiltonian in the presence of an external bias V can be written as $\hat{H}_{\text{edge}} = \hat{H}_0 + \hat{H}_{\text{int}} + \hat{H}_V$, with

$$\hat{H}_0 = \int dx[-iv_F(\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L)], \tag{1}$$

$$\hat{H}_{\text{int}} = 2\pi v_F g_2 \int dx[\psi_R^\dagger(x)\psi_L^\dagger(x)\psi_L(x)\psi_R(x)], \tag{2}$$

$$\hat{H}_V = \int dx[\mu_L \psi_L^\dagger(x)\psi_L(x) - \mu_R \psi_R^\dagger(x)\psi_R(x)] . \tag{3}$$

The helical liquid is characterized by the first two terms: The noninteracting part $\hat{H}_0$ describes linear dispersing helical fermions with Fermi velocity v_F; $\hat{H}_{\text{int}}$ is the only relevant contact-interaction term allowed by spin-momentum locking if the system is not at half-filling [53]. The external bias enters our description through $\hat{H}_V$, with $eV = \mu_L - \mu_R$ [56], with e being the electron charge; for clarity, we take $\mu_R = 0$. We imagine that before the addition of any other perturbation, the system has reached a quasiequilibrium state with respect to the external leads. These leads are assumed to be space separated enough that it makes sense to assign a different chemical potential to each of the fermionic species, mimicking the effect of a finite voltage [61]. Additionally, we introduce the perturbation, a single time-dependent Rashba impurity. The associated Hamiltonian is [39] $\hat{H}_R = \int dx \mathcal{H}_R(x)$, with

$$\mathcal{H}_R(x) = \alpha(x,t)[\partial_x \psi_R^\dagger(x)\psi_L(x) - \psi_R^\dagger(x)\partial_x\psi_L(x) + \text{H.c.}], \tag{4}$$

where $\alpha(x,t)$, assumed to be real, is the Rashba matrix element. In an experimental setting, such a term can be induced with a local, time-dependent electric field, via gating or a scanning tunneling microscope tip. We consider the impurity to be turned on only after the system has reached quasiequilibrium with the leads. Moreover, it must be situated far away from both leads. In this case, the quasiequilibrium distribution functions of left and right movers are not substantially affected if the driving period is smaller than the electron injection time [62,63]. We assume zero temperature for simplicity.

B. Bosonic representation

In order to treat electron-electron interaction in a compact way, we employ bosonization [55,64,65]. Fermionic operators are then expressed as

$$\psi_{R/L}(x) = \frac{\kappa_{R/L}}{\sqrt{2\pi a}} e^{\pm i\sqrt{4\pi}\Phi_{R/L}(x)} e^{\pm ik_F x}. \tag{5}$$

In Eq. (5), $\Phi_{R/L}(x)$ are Hermitian bosonic fields, $\kappa_{R/L}$ are the Klein factors, k_F is the Fermi momentum, and a is a short-distance cutoff. The usual dual fields are defined as $\Phi(x) = \Phi_R(x) + \Phi_L(x)$ and $\Theta(x) = \Phi_R(x) - \Phi_L(x)$ and satisfy canonical commutation relations $[\Phi(x), \partial_y\Theta(y)] = -i\delta(x-y)$. The density of right movers is related to the bosonic fields by $\hat{\rho}_R(x) = \frac{1}{\sqrt{\pi}}\partial_x\Phi_R(x)$, with similar notation for left movers. We obtain $\hat{H}_{LL} = \hat{H}_0 + \hat{H}_{\text{int}}$ in the standard form:

$$\hat{H}_{LL} = \hbar\frac{v}{2}\int dx\left[\frac{1}{K}[\partial_x\Phi(x)]^2 + K[\partial_x\Theta(x)]^2\right]. \tag{6}$$

The parameter $K = \sqrt{(1-g_2)/(1+g_2)}$ encodes the interaction strength: $0 < K < 1$ means a repulsive interaction, with $K \to 1$ being the noninteracting limit. In what follows, we restrict ourselves to $K > 1/2$, as it is typical for QSH edge states. The Hamiltonian in Eq. (6), supplemented by the chirality-dependent chemical potential given in Eq. (3), can also be interpreted in terms of the inhomogeneous Luttinger liquid model [66–70] in the case in which the interacting helical liquid is coupled to noninteracting electron reservoirs. The inhomogeneous Luttinger liquid model describes one-dimensional fermions subject to a space-dependent interaction strength which is assumed to be slowly varying on the scale of the Fermi wavelength. Such variation is then modeled by position-dependent parameters K and v. In helical liquids, the bosonic sound velocity is not renormalized by interactions in the usual Luttinger liquid fashion, i.e., $vK \neq v_F$. This is due to broken Galilean invariance at low energies in a quantum spin Hall system at the edge, related to the linearity of the spectrum. In fact, the dependence of the velocity on the parameter K can be derived as [65,71]

$$v = v_F\left[1 - \left(\frac{1-K^2}{1+K^2}\right)^2\right]^{1/2} \equiv v_F\,\lambda(K). \tag{7}$$

The Rashba Hamiltonian instead becomes [38,71–73]

$$\hat{H}_R = \int dx\Bigg[\alpha(x,t) i\frac{\kappa_L\kappa_R}{\sqrt{\pi}a}\sum_{m=\pm} e^{2imk_F x} \times\left(\partial_x\Theta(x)e^{im\sqrt{4\pi}\Phi(x)} + m\frac{e^{im\sqrt{4\pi}\Phi(x)}}{\sqrt{\pi}a}\right)\Bigg]. \quad (8)$$

Moreover, the bias term can be written as $\hat{H}_V = \int dx\, \frac{\mu_L}{2\sqrt{\pi}}[\partial_x\Phi(x) - \partial_x\Theta(x)]$. It can be shown that the presence of this term is equivalent to the following shift in bosonic fields [61,71]:

$$\Phi \to \Phi + \frac{\omega_0 t}{\sqrt{4\pi}}, \qquad \partial_x\Theta \to \partial_x\Theta - \frac{eV}{\sqrt{4\pi}v_F}, \quad (9)$$

with $\omega_0 = eV/\hbar$. We remark that, following the approach of Ref. [61], we consider the electron reservoirs to be spatially extended and noninteracting. Consequently, we consider the zero modes of the system to be noninteracting as well, given the influence of the leads [71]. For this reason, v_F, rather than vK, appears in Eq. (9).

C. Form of the backscattering current

In the absence of impurities, the current passing through a helical liquid is e^2V/h, meaning that the system shows perfect conductance quantization [66–68]. Here, we want to consider the impact of a periodically driven Rashba impurity to lowest order in the impurity strength. For simplicity, we consider a perfectly localized impurity, parametrized by $\alpha(x,t) = \alpha_0 a\sin(\omega t)\delta(x - x_0)$. The variation of the current is associated with the rate of variation in the number of right and left movers. Therefore, we need to calculate the expectation value of the operator,

$$\hat{I}_{BS}(t) \equiv e(\dot{\hat{n}}_R - \dot{\hat{n}}_L) = 2e\dot{\hat{n}}_R = -2\frac{i}{\hbar}e[\hat{n}_R, \hat{H}_R], \quad (10)$$

where the (normal ordered) number operator is defined as $\hat{n}_R = \int dx\hat{\rho}_R(x) = \int dx : \psi_R^\dagger(x)\psi_R(x) :$. As we show in Appendix A, the bosonized version of the backscattering currents reads

$$\hat{I}_{BS}(t) = -\int dx\, \alpha(x,t) 2e\frac{\kappa_L\kappa_R}{\sqrt{\pi}\hbar a}\sum_{m=\pm} e^{2imk_F x} \times\left(m\,\partial_x\Theta(x)e^{im\sqrt{4\pi}\Phi(x)} + \frac{e^{im\sqrt{4\pi}\Phi(x)}}{\sqrt{\pi}a}\right). \quad (11)$$

We remark that the shifts in Eq. (9) have to be implemented in the current operator as well. In order to calculate its expectation value at a generic time t, we treat the impurity as a time-dependent perturbation and use a Kubo-like approach, which has been used in similar impurity problems (see, for example, Refs. [49,56,57,59,74]),

$$\langle\hat{I}_{BS}(t)\rangle = \frac{i}{\hbar}\int_{-\infty}^{t} dt'\langle 0|[\hat{H}_R(t'), \hat{I}_{BS}(t)]|0\rangle. \quad (12)$$

In Eq. (12), $|0\rangle$ is the ground state of $\hat{H}_{LL}$ in Eq. (6), which generates also time evolution for the operators. If we did not perform any shift in the bosonic fields, $|0\rangle$ should be the ground state of $\hat{H}_{LL} + \hat{H}_V$.

III. ANALYTIC RESULT OF THE BACKSCATTERING CURRENT

We focus on the dc component of the current, e.g., $\langle\hat{I}_{BS}^{\rm dc}\rangle = \frac{1}{\tau}\int_{t_1}^{t_1+\tau} dt\langle\hat{I}_{BS}(t)\rangle$, where t_1 is a generic time after the impurity has been completely switched on and τ is the driving period (so there is no dependence on the initial phase of the impurity). As a result of a tedious but straightforward calculation (reported in Appendix B), we obtain

$$\begin{aligned}\langle\hat{I}_{BS}^{\rm dc}\rangle =\; & e\frac{\alpha_0^2\, a^{2K}}{2\pi\hbar^2 v^{2(K+1)}}\frac{2K+1}{K\,\Gamma(2+2K)} \times\sum_{r=\pm} r|\omega + r\omega_0|^{2K+1}{\rm sgn}(\omega + r\omega_0)\\ & - e\frac{\alpha_0^2 a^{2K}}{\pi\hbar^2 v^{2K+1}}\frac{\omega_0}{v_F}\frac{1}{\Gamma(2K+1)}\sum_{r=\pm}|\omega + r\omega_0|^{2K}\\ & + e\frac{\alpha_0^2 a^{2K}}{2\pi\hbar^2 v^{2K}}\frac{\omega_0^2}{2v_F^2}\frac{1}{\Gamma(2K)} \times\sum_{r=\pm} r|\omega + r\omega_0|^{2K-1}{\rm sgn}(\omega + r\omega_0).\end{aligned} \quad (13)$$

In Eq. (13), $t_a = a/v$ is a short-time cutoff, and $\Gamma(x)$ is the Euler gamma function. For the sake of simplicity, we neglected exponential factors of the kind $\exp -(t_a|\omega \pm \omega_0|)$ because we are considering $\omega, \omega_0 \ll E_g$, with E_g being the bulk gap. In our model, the bulk gap E_g can, indeed, be identified with $\hbar v/a$ (the high-energy cutoff). It is convenient to introduce the dimensionless parameter

$$\gamma(K) \equiv \frac{\alpha_{\rm ad}^2 (ak_F)^{2K}}{4\pi K^2\,\Gamma(2K)\,\lambda^{2(K+1)}(K)k_F^2}, \quad (14)$$

which contains a dimensionless version of the impurity matrix element $\alpha_{\rm ad} = \alpha_0 ak_F/\hbar v_F$, while $\lambda(K)$ is defined in Eq. (7). In this way, the backscattering current can be written as

$$\langle\hat{I}_{BS}^{\rm dc}\rangle = e\left(\frac{\hbar}{E_F}\right)^{2K}\gamma(K)\mathfrak{I}(K,\omega,\omega_0), \quad (15)$$

where $E_F = \hbar v_F k_F$. The function $\mathfrak{I}$ in Eq. (15), which has the dimensions $1/s^{2(K+1)}$, is instead given by the sum of three contributions, each one containing different K-dependent powers of $\omega \pm \omega_0$:

$$\mathfrak{I}(K,\omega,\omega_0) = f_{2K+1} + f_{2K} + f_{2K-1}, \quad (16)$$

with

$$f_{2K+1}(\omega,\omega_0) = \sum_{r=\pm} r|\omega + r\omega_0|^{2K+1}{\rm sgn}(\omega + r\omega_0), \quad (17a)$$

$$f_{2K}(\omega,\omega_0) = -2\lambda K\omega_0\sum_{r=\pm}|\omega + r\omega_0|^{2K}, \quad (17b)$$

$$f_{2K-1}(\omega,\omega_0) = (\lambda K\omega_0)^2\sum_{r=\pm} r|\omega + r\omega_0|^{2K-1}{\rm sgn}(\omega + r\omega_0). \quad (17c)$$

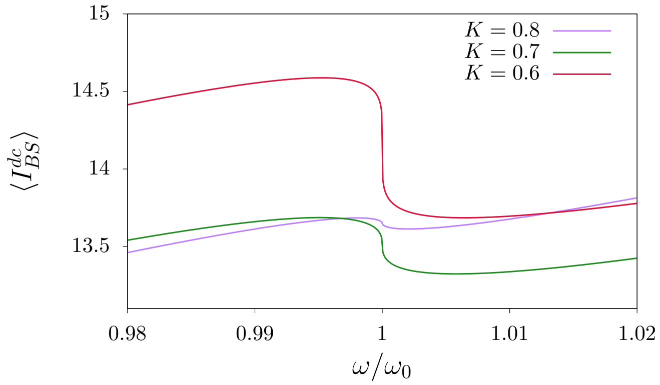

FIG. 1. Zoom around $\omega = \omega_0$ of $\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle$, measured in units of $[e\alpha_0^2 k_F^2 (k_F a)^{(2K)}/(\hbar^2\omega_0)]$ for $K = 0.8$ (violet), $K = 0.7$ (green), and $K = 0.6$ (red). We further set $\hbar = 1$ and $E_F = 0.1\hbar\omega_0$.

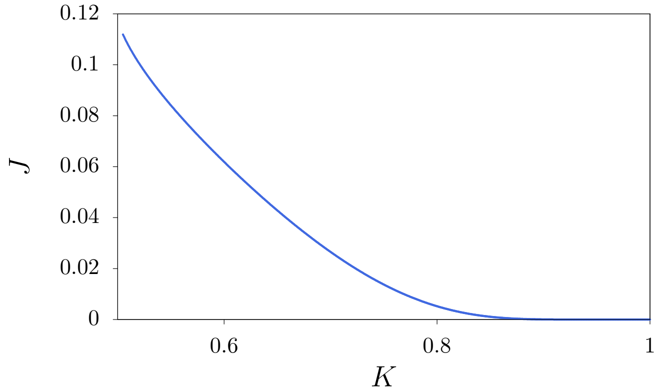

FIG. 2. The relative jump defined in the text as a function of the interaction parameter K.

The analytic expression for the backscattering current represents the main result of the paper. It is hence important to comment on its validity. As in the case of the driven magnetic impurity [56], the divergence of the backscattering current for $K \leqslant 1/2$ at $\omega = \omega_0$ makes our perturbative result, in the proximity of that point and for strong interactions, unreliable. Moreover, $\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle$ should be a small correction to e^2V/h. This is true for a certain range of ω. As ω grows, at some point, we leave the validity regime of the perturbation theory. A reasonable estimate, with $\alpha = 0.16\,\mathrm{nm\,eV}$, $k_F = 0.05\,\mathrm{nm}^{-1}$, $v_F = 5.5 \times 10^{-5}\mathrm{m/s}$, $k_F a \approx 1$, gives, for the case in Fig. 1, a ratio $\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle/(e^2V/h) \approx 15\%$, confirming the consistency of our perturbative scheme.

IV. NONMONOTONICITY OF $\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle$

For a moderately interacting helical liquid with $K > 1/2$, $\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle$ is a continuous function of ω: f_{2K-1} presents a divergence only in its derivatives. Far away from $\omega = \omega_0$, f_{2K-1} is negligible, and $\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle$, as a function of ω, always has a positive derivative. Close to ω_0 this is not true because the term proportional to $|\omega - \omega_0|^{2K-1}$ in f_{2K-1} contributes with a negative diverging derivative. Therefore, $\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle$ is not monotonously growing, but rather exhibits a kink across ω_0, as shown in Fig. 1. To quantify this kink, we call $\omega_{\max}$ the local maximum to the left of ω_0 and $\omega_{\min}$ the local minimum to the right of ω_0 and define the relative jump as

$$J \equiv \frac{\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle(\omega_{\max}) - \langle \hat{I}_{BS}^{\mathrm{dc}}\rangle(\omega_{\min})}{\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle(\omega_{\max})}. \tag{18}$$

The dependence of J on K is shown in Fig. 2. This definition is convenient since all the prefactors to $\mathfrak{I}$ in Eq. (15), containing system-specific parameters such as the Fermi velocity, cancel out. Moreover, J does not depend on the external bias eV since we can write $\mathfrak{I}(K, \omega, \omega_0) = \omega_0^{2K+1}\mathfrak{I}'(K, \omega/\omega_0)$. Therefore, J depends only on the interaction strength K. Thus, J can serve as an alternative way to extract K from transport measurements without the need to measure a power-law behavior. We remark that in the case of a magnetic impurity these considerations do not apply, as the related backscattering current decreases monotonously as ω increases [56].

V. LIMITS

A. Static limit

As a first sanity check, we calculate the static impurity limit, $\omega \to 0$, of the backscattering current in Eq. (13) and compare it with Ref. [71]. By using Eq. (7), we obtain

$$\lim_{\omega\to 0}\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle = 2e\gamma(K)\left(\frac{\hbar}{E_F}\right)^{2K}(vK - v_F)^2|\omega_0|^{2K+1}\mathrm{sgn}(\omega_0). \tag{19}$$

As realized in Ref. [71], the possibility that $vK \neq v_F$ when interactions are present implies a single particle contribution to the backscattering current, which scales as $\sim V^{2K+1}$.

B. Noninteracting limit

In the noninteracting limit $K = 1$, $v = v_F$, the dc component of the current of Eq. (13) becomes

$$\begin{aligned}\lim_{K\to 1}\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle &= +e\frac{\alpha_0^2 a^2}{2\pi\hbar^2 v^4}\left(\omega_0^3 + 3\omega^2\omega_0\right)\\ &\quad - e\frac{\alpha_0^2 a^2}{2\pi\hbar^2 v^4}2\omega_0\left(\omega^2 + \omega_0^2\right)\\ &\quad + e\frac{\alpha_0^2 a^2}{2\pi\hbar^2 v^4}\omega_0^3\\ \Rightarrow\quad \lim_{K\to 1}\langle \hat{I}_{BS}^{\mathrm{dc}}\rangle &= \frac{\alpha_0^2 a^2}{2\pi\hbar^3 v_F^4}\omega^2 e^2 V.\end{aligned} \tag{20}$$

We have used $\Gamma(n+1) = n!$. The fermionic calculation in Appendix C confirms the formula above and elucidates its interpretation, which is the following: The backscattering current at second order originates from one-photon resonant processes that transfer electrons from one branch of the linear spectrum to the opposite one, together with a change in energy of $\pm\hbar\omega$. This process is, in principle, equally probable for both fermionic species. However, for an energy window set by the external bias (of size eV) the electrons from the branch with lower chemical potential energy cannot backscatter into the opposite one because of Pauli principle. Therefore, the net effect is a decrease of the current originally set by the external bias. This result is in agreement with the results of Ref. [49]

for noninteracting QSH edge electrons with a generic spin texture in momentum space. The ω^2 factor in Eq. (20) ensures the absence of backscattering current in the static limit $\omega \to 0$, as expected [71].

VI. CONCLUSIONS

We have analytically computed the backscattering current due to a δ-like harmonically driven Rashba scatterer in a helical Luttinger liquid, up to second order in the impurity strength. The result is qualitatively different from the case of a time-dependent magnetic barrier. It can hence help elucidate the role of impurities on the transport properties of quantum spin Hall systems. Interestingly, our results allow for the definition of an experimentally accessible quantity that depends only on the strength of electron-electron interactions, through the Luttinger parameter K. Such a quantity could allow for a measurement of K without the need of measuring power-law dependencies.

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APPENDIX A: BACKSCATTERING CURRENT OPERATOR

In this Appendix, we calculate explicitly the commutator in Eq. (10) that gives the expression for the backscattering current operator, making use of the bosonized version for the density operator $\hat{\rho}_R(x) = \frac{1}{\sqrt{\pi}}\partial_x\Phi_R(x)$ and the Rashba Hamiltonian, Eq. (8). With the help of $[A, e^B] = Ce^B$, if $C = [A, B]$ and $[A, C] = [B, C] = 0$, we can compute the first commutator

$$[\partial_x\Phi_R(x), e^{i\sqrt{4\pi}\Phi(y)}] = -\sqrt{\pi}\delta(x-y)e^{i\sqrt{4\pi}\Phi(y)},$$

where we used standard identities from bosonization [65]. Moreover, we obtain for the other commutator $[\partial_x\Phi_R(x), \partial_y\Theta(y)] = \frac{i}{2}\partial_y[\delta(x-y)]$ directly from the canonical commutation relation of the dual fields. The associated term, when integrated over space, vanishes because we are considering that $\lim_{x\to\pm\infty}\alpha(x) = 0$. Putting everything together, we obtain the expression for the backscattering current operator in Eq. (11).

APPENDIX B: DETAILS ON THE BOSONIZATION CALCULATION

We show here the details of the calculation of Eq. (12) in the bosonic language. Upon substituting Eq. (8) [with $\alpha(x,t) = \alpha_0(t)a\delta(x-x_0)$] and (11) and using the fact that $\langle\kappa_L\kappa_R\kappa_L\kappa_R\rangle = -1$, we obtain

$$\langle\hat{I}_{BS}(t)\rangle = \frac{i}{\hbar}\int_{-\infty}^{t}dt'\Bigg\{\sum_{m_1,m_2=\pm} ie\frac{2\alpha_0(t)\alpha_0(t')}{\pi\hbar}e^{2i(m_1+m_2)k_Fx_0}e^{i\omega_0(m_1t'+m_2t)}\Bigg[m_2\Bigg(\sum_{j=\mathrm{I}}^{\mathrm{IV}}C_R^{\mathrm{j}}(x_0,t;x_0,t';m_1,m_2)\Bigg)$$
$$-\frac{eV}{2\hbar v_F}a(1+m_1m_2)C_R^{\mathrm{I}}(\cdots) + \frac{e^2V^2}{4\hbar^2v_F^2}a^2m_1C_R^{\mathrm{I}}(\cdots) - \frac{eV}{2\hbar v_F}a\big(C_R^{\mathrm{IV}}(\cdots) + m_1m_2C_R^{\mathrm{III}}(\cdots)\big)\Bigg] - \text{c.c.}\Bigg\}. \tag{B1}$$

In Eq. (B1), $(\cdots)$ stands for $(x_0, t; x_0, t'; m_1, m_2)$. The correlation functions are defined as

$$C_R^{\mathrm{I}}(x,t;x',t';m_1,m_2) = \frac{m_1m_2}{\pi a^2}\langle 0|e^{i\sqrt{4\pi}m_1\Phi(x',t')}e^{im_2\sqrt{4\pi}\Phi(x,t)}|0\rangle, \tag{B2a}$$

$$C_R^{\mathrm{II}}(x,t;x',t';m_1,m_2) = \langle 0|\partial_{x'}\Theta(x',t')e^{im_1\sqrt{4\pi}\Phi(x',t')}\partial_x\Theta(x,t)e^{im_2\sqrt{4\pi}\Phi(x,t)}|0\rangle, \tag{B2b}$$

$$C_R^{\mathrm{III}}(x,t;x',t';m_1,m_2) = \frac{m_1}{\sqrt{\pi}a}\langle 0|e^{im_1\sqrt{4\pi}\Phi(x',t')}\partial_x\Theta(x,t)e^{im_2\sqrt{4\pi}\Phi(x,t)}|0\rangle, \tag{B2c}$$

$$C_R^{\mathrm{IV}}(x,t;x',t';m_1,m_2) = \frac{m_2}{\sqrt{\pi}a}\langle 0|\partial_x\Theta(x',t')e^{im_1\sqrt{4\pi}\Phi(x',t')}e^{im_2\sqrt{4\pi}\Phi(x,t)}|0\rangle. \tag{B2d}$$

Performing standard bosonization calculations [64,65], we obtain

$$\sum_{j=\mathrm{I}}^{\mathrm{IV}}C_R^{\mathrm{j}}(x_0,t;x_0,t';m_1,m_2) = \delta_{m_1,-m_2}\frac{1}{a^2}\frac{2K+1}{2\pi K}\left(\frac{t_a}{t_a+i(t'-t)}\right)^{2(K+1)},$$

where $t_a = a/v$, with v being the bosonic excitation velocity. In general, every correlation function gives a $\delta_{m_1,-m_2}$ factor, so that the first term in the second line vanishes. We also derive

$$C_R^{\mathrm{I}}(\cdots) = -\delta_{m_1,-m_2}\frac{1}{\pi a^2}\left(\frac{t_a}{t_a+i(t'-t)}\right)^{2K},$$
$$C_R^{\mathrm{IV}}(\cdots) + m_1m_2C_R^{\mathrm{III}}(\cdots) = -\delta_{m_1,-m_2}\frac{2}{\pi a}\left(\frac{t_a}{t_a+i(t'-t)}\right)^{2K+1}, \tag{B3}$$

and with that

$$\langle \hat{I}_{BS}(t)\rangle = -\frac{2}{\hbar}\text{Im}\int_{-\infty}^{t} dt' ie\frac{2\alpha_0(t)\alpha_0(t')}{\pi\hbar}\left\{(e^{-i\omega_0(t'-t)} - e^{i\omega_0(t'-t)})\left[\frac{2K+1}{2\pi K a^2}\left(\frac{t_a}{t_a+i(t'-t)}\right)^{2(K+1)} + \frac{e^2V^2}{4\hbar^2 v_F^2}\left(\frac{t_a}{t_a+i(t'-t)}\right)^{2K}\right] + \frac{eV}{2\hbar v_F}(e^{i\omega_0(t'-t)} + e^{-i\omega_0(t'-t)})\frac{2}{\pi a}\left(\frac{t_a}{t_a+i(t'-t)}\right)^{2K+1}\right\}. \tag{B4}$$

We now consider a time-periodic impurity $\alpha_0(t) = \alpha_0 \sin(\omega t)$ as in the main text and switch to the integration variable $\tau = t' - t$ in such a way that the dc component of the current is

$$\langle \hat{I}_{BS}^{\text{dc}}\rangle = -e\frac{\alpha_0^2}{2\pi\hbar^2}\int_{-\infty}^{\infty} d\tau e^{i\omega\tau}(e^{-i\omega_0\tau} - e^{i\omega_0\tau})\left\{\frac{2K+1}{2\pi K a^2}\left[\left(\frac{t_a}{t_a+i\tau}\right)^{2(K+1)} - \text{c.c.}\right] + \frac{e^2V^2}{4\hbar^2 v_F^2}\left[\left(\frac{t_a}{t_a+i\tau}\right)^{2K} - \text{c.c.}\right]\right\} - \frac{\alpha_0^2}{\pi^2\hbar^2 a^3}\frac{eV}{\hbar v_F}\int_{-\infty}^{\infty} d\tau e^{i\omega\tau}(e^{i\omega_0\tau} + e^{-i\omega_0\tau})\left[\left(\frac{t_a}{t_a+i\tau}\right)^{2K+1} + \text{c.c.}\right]. \tag{B5}$$

If we make use of the integrals

$$\begin{aligned}
\int_{-\infty}^{\infty} dx\, e^{i\Omega x}\left[\left(\frac{t_a}{t_a+ix}\right)^{2K+2} - \left(\frac{t_a}{t_a-ix}\right)^{2K+2}\right] &= \frac{2\pi e^{-t_a|\Omega|}t_a^{2K+2}}{\Gamma(2K+2)}|\Omega|^{2K+1}\text{sgn}(\Omega),\\
\int_{-\infty}^{\infty} dx\, e^{i\Omega x}\left[\left(\frac{t_a}{t_a+ix}\right)^{2K} - \left(\frac{t_a}{t_a-ix}\right)^{2K}\right] &= \frac{2\pi e^{-t_a|\Omega|}t_a^{2K}}{\Gamma(2K)}|\Omega|^{2K-1}\text{sgn}(\Omega),\\
\int_{-\infty}^{\infty} dx\, e^{i\Omega x}\left[\left(\frac{t_a}{t_a+ix}\right)^{2K+1} + \left(\frac{t_a}{t_a-ix}\right)^{2K+1}\right] &= \frac{2\pi e^{-t_a|\Omega|}t_a^{2K+1}}{\Gamma(2K+1)}|\Omega|^{2K},
\end{aligned} \tag{B6}$$

defining $t_a = a/v$, we arrive at

$$\begin{aligned}
\langle \hat{I}_{BS}^{\text{dc}}\rangle = {}& e\frac{\alpha_0^2 a^{2K}}{2\pi\hbar^2 v^{2(K+1)}}\frac{2K+1}{K\,\Gamma(2+2K)}\sum_{r=\pm} r e^{-t_a|\omega+r\omega_0|}|\omega+r\omega_0|^{2K+1}\text{sgn}(\omega+r\omega_0)\\
&- e\frac{\alpha_0^2 a^{2K}}{\pi\hbar^2 v^{2K+1}}\frac{\omega_0}{v_F}\frac{1}{\Gamma(2K+1)}\sum_{r=\pm} e^{-t_a|\omega+r\omega_0|}|\omega+r\omega_0|^{2K}\\
&+ e\frac{\alpha_0^2 a^{2K}}{2\pi\hbar^2 v^{2K}}\frac{\omega_0^2}{2v_F^2}\frac{1}{\Gamma(2K)}\sum_{r=\pm} r e^{-t_a|\omega+r\omega_0|}|\omega+r\omega_0|^{2K-1}\text{sgn}(\omega+r\omega_0).
\end{aligned} \tag{B7}$$

If we restrict ourselves to a regime where $t_a|\omega \pm \omega_0| \ll 1$, we obtain Eq. (13).

APPENDIX C: FREE-FERMION CALCULATION

The Rashba Hamiltonian in Eq. (4) with $\alpha(x,t) = \alpha_0(t)a\delta(x - x_0)$ has the momentum space representation

$$\hat{H}_R = -\frac{a\alpha_0(t)}{L}\sum_{k_1,k_2}[i(k_1+k_2)\hat{c}_{k_1R}^{\dagger}\hat{c}_{k_2L} + \text{H.c.}]. \tag{C1}$$

Writing the total number of right movers as $\hat{n}_R = \sum_k \hat{n}_{\text{R}k} = \sum_k \hat{c}_{kR}^{\dagger}\hat{c}_{kR}$, we can derive the expression for the backscattering current operator again via Eq. (10). The result is

$$\hat{I}_{BS} = -\frac{2ea\alpha_0(t)}{\hbar L}\sum_{k_1,k_2}(k_1+k_2)\big(\hat{c}_{k_1R}^{\dagger}\hat{c}_{k_2L} + \text{H.c.}\big). \tag{C2}$$

The Heisenberg evolution of the fermionic operators according to the clean edge Hamiltonian $\hat{H}_0 = \hbar v_F \sum_k k(\hat{n}_R - \hat{n}_L)$ is given by

$$\begin{aligned}
\hat{c}_{kR}(t) &= e^{-iv_F kt}\hat{c}_{kR},\\
\hat{c}_{kL}(t) &= e^{iv_F kt}\hat{c}_{kL}.
\end{aligned}$$

Using these expressions, we can calculate the current with Eq. (12). By repeatedly using Wick's theorem, we obtain

$$\langle \hat{I}_{BS}(t)\rangle = -\frac{2ea^2\alpha_0(t)}{L^2\hbar^2}\int_{-\infty}^{0} d\tau\alpha_0(t+\tau)\sum_{k_1',k_2}(k_1+k_2)^2 \times (e^{iv_F(k_1+k_2)(\tau)}\langle 0|\hat{n}_{k_1R} - \hat{n}_{k_2L}|0\rangle + \text{c.c.}),$$

where $\tau = t' - t$. Writing the impurity matrix element as $\alpha_0(\tau + t) = \alpha_0 \sin(\omega t)\cos(\omega\tau) + \alpha_0\cos(\omega t)\sin(\omega\tau)$, we see that only the first term contributes to the dc response. Therefore, after averaging over one driving period and performing the integration over τ, we obtain

$$\langle \hat{I}_{BS}^{\text{dc}}\rangle = -\frac{ea^2\alpha_0^2}{L^2\hbar^2}\sum_{k_1',k_2}(k_1+k_2)^2\langle 0|\hat{n}_{k_1R} - \hat{n}_{k_2L}|0\rangle\pi \times \{\delta[v_F(k_1+k_2)+\omega] + \delta[v_F(k_1+k_2)-\omega]\}. \tag{C3}$$

The Dirac δ functions ensure the conservation of energy in the one-photon processes that bring one electron into the opposite branch of the linear spectrum. The expectation value with respect to the ground state of the occupation numbers reads $\langle \hat{n}_{Rk} \rangle = \theta(-k + k_F^R)$ and $\langle \hat{n}_{Lk} \rangle = \theta(k - k_F^L)$, respectively. With our choice of chemical potentials, we have $k_F^R = 0$ and $k_F^L = -eV/\hbar v_F$. Upon substituting these expressions into the previous equation, forgetting about the high-energy cutoff in the spectrum, we obtain

$$\langle \hat{I}_{BS}^{\rm dc} \rangle = +\frac{\alpha_0^2 a^2 \omega^2 e^2 V}{2\pi \hbar^3 v_F^4}, \tag{C4}$$

which corresponds to Eq. (20) of the main text.

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