Atomic-orbital-free intrinsic ferromagnetism in electrenes

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Atomic orbitals play fundamental roles in the modern theory of magnetism, not only providing local moments via their partial occupation, but also offering exchange interactions through their direct or indirect hybridization. Here, we report atomic-orbital-free intrinsic ferromagnetism in monolayer electrides or electrenes, in which excess electrons act as anions. Taking the honeycomb LaBr₂ (La³⁺Br₂⁻ · e^{-}) as an example, our first-principles calculations, in combination with a low-energy effective model in the basis of the Wannier function and Anderson's superexchange theory, reveal that the excess electron is localized at the center of the hexagon, which leads to the spontaneous formation of a local moment due to the strong Stoner instability of the associated state near the Fermi level (E_f). The large off-site Coulomb interaction and extended tails of both wave and Wannier functions indicate a significant spatial extension of the anionic electron state in LaBr₂. The overlap of the long tails mediates an extended ferromagnetic direct exchange to second-nearest neighbors (up to 7–8 Å), in sharp contrast to the conventional direct exchange which is short ranged due to the overlap of atomic orbitals with limited spatial extension. The dual nature, localization and extension, of the anionic electron state results in a unique magnetic mechanism in such atomic-orbital-free intrinsic two-dimensional ferromagnets.

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Introduction. Ferromagnetism (FM) in two-dimensional (2D) materials has been a hot research topic, because such materials offer special advantages of high data storage density and easy integration into semiconductor devices [1]. Recently, Gong et al. [2] and Huang et al. [3] first succeeded in fabricating intrinsic 2D ferromagnets, Cr₂Ge₂Te₆ and CrI₃ respectively, demonstrating the possibility of fabricating robust 2D intrinsic ferromagnets. Quickly, more intrinsic 2D ferromagnets, including Fe₃GeTe₂ [4], VSe₂ [5], CrOBr [6], CrSeBr [7], CrWGe₂Te₆ [8], MnPSe₃ [9], CoH₂ [10], and CrBr₃ [11] were reported. It is noted that the magnetism in these intrinsic 2D magnetic materials can be understood by the conventional theory of magnetism developed for 3D magnetic materials. For example, the formation of local magnetic moments in FM semiconductors Cr₂Ge₂Te₆ and CrI₃ is associated with the magnetic element Cr, which has partially filled 3d orbitals. The strongly localized nature of the 3d states under the crystal field favors spontaneous spin polarization and leads to the formation of local moments. These local moments are coupled via the 5p orbital of Te or I, by the superexchange within the Goodenough-Kanamori-Anderson (GKA) rules [12]. For ferromagnetic systems with free electrons, such as FM metals Fe₃GeTe₂ or VSe₂, the itinerary carriers can mediate a long-range indirect exchange between the local moments within the 3d orbitals via the Ruderman-Kittel-Kasuya-Yosida (RKKY) or Zener double exchange [13,14]. As can be seen, atomic orbitals play an essential role in conventional magnetic materials. Without them, there would be neither local moments of spin-polarized electrons nor exchange interactions through direct or indirect orbital hybridization.

In a family of intrinsic electron-rich materials, electrides or electrenes in 2D, excess electrons (named anionic electrons) are confined in geometrically interstitial sites and do not occupy any atomic orbital, acting as anions [15]. Such anionic electrons are loosely bounded and demonstrate some unique physical properties, such as superconductivity [16,17], topological matters [18–21], and Dirac plasmons [22]. Given the unique feature of the anionic electron state in electrenes and the enormous interest in 2D intrinsic ferromagnetic materials, it is natural to ask whether electrenes can be a new type of intrinsic 2D magnetic system. With such a motivation, we screened more than 6000 structures in the 2D materials database 2DMatPedia [23] for magnetic electrenes. Our search resulted in nine magnetic electrenes, and to our surprise, four of them (LaBr₂, La₂Br₅, Sc₇Cl₁₀, and Ba₂LiN) contain no magnetic elements [24]. A question then arises: What is the origin of magnetism in such electrenes without magnetic elements?

In this Rapid Communication, taking LaBr₂ as an example, we demonstrate the intrinsic atomic-orbital-free magnetism in this group of 2D materials. The results of our first-principles calculations using the density functional theory (DFT) indicate that the excess electron in LaBr₂ does not occupy any atomic orbital but localizes at the center of the hexagon of

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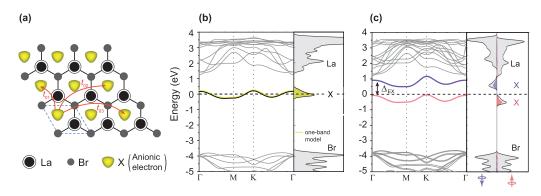


FIG. 1. (a) LaBr₂ (La³⁺Br₂⁻ · e^{-}) ML in the *H*-phase MoS₂ structure with ELF maps (isosurface value = 0.75) in yellow. The dashed diamond denotes the unit cell. Only the ELF on interstitial sites is shown for clarity. The anionic electron in the geometric space, labeled as X, is at the center of each hexagon. The red arrows are the schematic representation of the hopping paths between anionic electrons, and the calculated hopping parameters are $t_{01} = 12.50$ meV, $t_{02} = 53.18$ meV, and $t_{03} = -9.11$ meV, respectively. Band structure and density of states of ML LaBr₂ (b) without and (c) with spin polarization. The yellow band and DOS in (b) are from the one-band tight-binding model.

its honeycomb hexagonal crystal structure [see details and justification of the calculation methodologies based on VASP, Wannier function, constrained random phase approximation (cRPA), and Monte Carlo simulations in the Supplemental Material (SM) [25] (see also Refs. [26-42] therein)]. Each anionic electron carries a magnetic moment of $1\mu_B$, which corresponds to the s = 1/2 interstitial state characterized by a half-filled band near the Fermi level. The underlying physics of the magnetic exchange interaction is further understood by a low-energy effective model, in which the parameters are mapped from the Wannier functions by the cRPA. We show that while the localization of the anionic electron state induces the magnetic moment, the strong overlap of the spatially extended tails of the wave and Wannier functions results in a direct-exchange interaction over an extended range to the second-nearest neighbors (up to 7–8 Å). This is in sharp contrast to the *conventional* direct exchange in magnetic materials which is very short ranged due to the limited spatial extension of the atomic orbitals.

Structure of monolayer LaBr₂. Layered bulk LaBr₂ was synthesized experimentally in 1989 [43]. Its crystal structure resembles that of the 2*H* phase of MoS₂ (space group $P6_3/mmc$) as shown in Fig. 1(a). The excess electron in La³⁺Br₂⁻ · e⁻, as an ion, is confined in the center of the hexagon and is free from any atomic orbitals [Fig. 1(a)]. The calculated exfoliation energy of LaBr₂ is 0.27 J/m² which is lower than that of Ca₂N (1.13 J/m²) [44] and graphene (0.43 J/m²) [45]. This indicates that monolayer (ML) LaBr₂ can be exfoliated from bulk LaBr₂, similar to how electrene Ca₂N is isolated experimentally from its bulk counterpart by liquid exfoliation [46]. Furthermore, both the calculated phonon spectrum and *ab initio* molecular dynamics simulation confirm the thermodynamic structural stability of ML LaBr₂ [47,48].

Origin of local moments. In conventional magnetic materials, the formation of local moments is associated with partially filled atomic orbitals [3,4]. Their strongly localized nature favors spin polarization and leads to the formation of local moments. However, the magnetic moments in ML LaBr₂ are of a different nature. As shown in Fig. 1(b), there is a narrow band near the Fermi level, which is mainly from the anionic

electrons labeled as X in Fig. 1(a). According to the Stoner criterion, $g(E_f)I > 1$, where $g(E_f)$ is the density of states (DOS) at E_f and I denotes the Stoner parameter [49], this strongly localized state of the interstitial electron is unstable and would lead to spontaneous spin polarization. The electronic structure of ML LaBr₂ calculated with the spin polarization [Fig. 1(c)] shows a local moment of $1\mu_B$ per unit cell (u.c.). The spinexchange splitting energy (Δ_{EX}) is around 1 eV, comparable to that of the Mn_{3d} orbital (1.62 eV) for $M = 2\mu_B$ [50]. The calculated electron localization function (ELF), projected DOS, and spin density all confirm that the magnetic moment is mainly contributed by the atomic-orbital-free anionic electron localized at a geometrically interstitial region [25].

Mechanism of ferromagnetic exchange. It is known that the magnetic exchange between the local moments, either directly or indirectly, plays a key role in determining the magnetic properties, such as ferromagnetism or antiferromagnetism (AFM). The conventional direct exchange due to the direct overlap of atomic orbitals is usually short ranged. The range of interaction can be extended in *d-p-d* superexchange, *p-d* exchange, and *s-f* RKKY [13,14,51–53]. Nevertheless, all these interactions are based on atomic orbitals and none of these magnetic exchange models is applicable to the *atomicorbital-free* magnetism in LaBr₂.

In order to understand the coupling between the magnetic anionic electrons, we examine the nature of the anionic electron state. In Fig. 2(a), we compare the radial distributions of the wave functions of the anionic electron state and the Cr_{3d} and I_{5n} orbitals in CrI_3 which is a well-known 2D ferromagnetic material [3]. It is found that the state of the anionic electron is almost as localized as the atomic d state, and meanwhile is more extended than the atomic p state. Based on this duality (localization and extension) of the anionic electron state, we propose that the formation of the local moment is from the localized feature, while the ferromagnetic interaction between the atomic-orbital-free anionic electrons in ML LaBr₂ is mediated by an overlap of the extended anionic electron state, as illustrated in Fig. 2(b). Based on this mechanism, the weakening of either the localization or the extension of the anionic electron state is expected to reduce the magnetism. For example, applying an in-plane biaxial tensile strain of

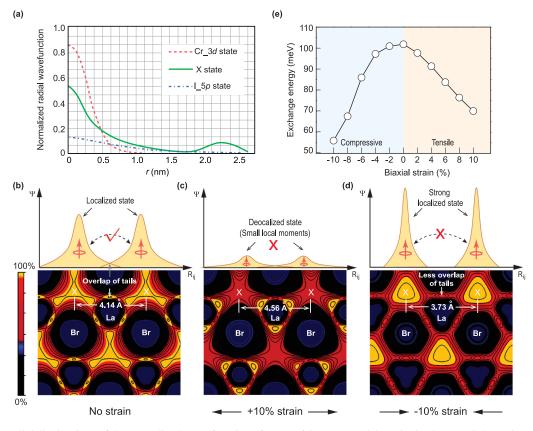


FIG. 2. (a) Radial distributions of the normalized wave function of states of Cr_{3d} , I_{5p} , and the anionic electron. Schematics of strain-induced changes in the localization of wave functions and their overlap (upper panel) as well as the calculated spin density (lower panel) of (b) strain-free ML LaBr₂, (c) ML LaBr₂ with 10% tensile strain, and (d) ML LaBr₂ with 10% compressive strain. The isovalue is $0.001e/Å^3$. (e) The strain dependence of the exchange energy, which is calculated by the total energy difference between the AFM and FM configurations.

10% leads to a decrease of local moments, while an in-plane biaxial compressive strain of 10% reduces the overlap between the tails of the wave functions, as shown in Figs. 2(c) and 2(d), respectively, both resulting in a weaker exchange interaction compared with the undeformed structure, as shown in Fig. 2(e).

Low-energy effective model. While the DFT results above have clearly shown the unique dual nature of the atomic-orbital-free magnetic state of the anionic electron in LaBr₂, we construct a low-energy effective model in terms of the Wannier function associated with the interstitial state of the anionic electron to further understand the underlying mechanism. It is because this model allows us to explicitly include the Coulomb interaction on the nonatomic interstitial site and nonlocal direct exchange interaction. We adopt the one-orbital extended Hubbard model in which the Hamiltonian of the system based on the second quantization is given by [54,55]

$$\hat{H} = \sum_{ij,\sigma\sigma'} t_{ij}^{\sigma\sigma'} \hat{\alpha}_{i\sigma}^{\dagger} \hat{\alpha}_{j\sigma'} + \frac{1}{2} \sum_{i,\sigma\sigma'} U_{00} \hat{\alpha}_{i\sigma}^{\dagger} \hat{\alpha}_{i\sigma'}^{\dagger} \hat{\alpha}_{i\sigma'} \hat{\alpha}_{i\sigma} + \frac{1}{2} \sum_{ij,\sigma\sigma'} U_{ij} \hat{\alpha}_{i\sigma}^{\dagger} \hat{\alpha}_{j\sigma'}^{\dagger} \hat{\alpha}_{j\sigma'} \hat{\alpha}_{i\sigma} + \frac{1}{2} \sum_{ij,\sigma\sigma'} J_{ij}^{D} \hat{\alpha}_{i\sigma}^{\dagger} \hat{\alpha}_{j\sigma'}^{\dagger} \hat{\alpha}_{i\sigma'} \hat{\alpha}_{j\sigma},$$

$$(1)$$

where i(j) and $\sigma(\sigma')$ are site and spin indices; $\hat{\alpha}_{i\sigma}^{\dagger}(\hat{\alpha}_{i\sigma})$ are the creation (annihilation) operators and $t_{i\sigma'}^{\sigma\sigma'}$, U_{00} ,

 U_{ij} , and J_{ij}^D are the hopping parameters, on-site Coulomb, off-site Coulomb, and off-site direct-exchange interactions, respectively.

To map the parameters in this model, we construct a maximally localized Wannier function (MLWF) for the band of the anionic electron near E_f using the WANNIER90 package [56]. The hopping parameters $t_{ij}^{\sigma\sigma'}$ were obtained via the Wannier parametrization [56,57]. Based on the constructed MLWF, we then extract the partially screened U_{00} , U_{ij} , and J_{ij}^D using cRPA as implemented in VASP, excluding the "self-screening" transition for the band of the anionic electron (see details in SM [25]). Different MLWFs are constructed and the interstitial-centered MLWF is found to best describe the anionic electron state [25], which captures the extended tails around the well-localized *s*-symmetric body, as shown in the colored contour plot in the plane passing through the La atoms [Figs. 3(a) and 3(b)].

The cRPA based on this MLWF yields $t_{01} = 12.50$ meV, $U_{00} = 2.32$ eV, $U_{01} = 1.36$ eV, $J_{01}^D = 21.9$ meV, and $J_{02}^D =$ 1.9 meV. It is surprising that direct-exchange coupling remains significant between the second-nearest neighbors over 7.17 Å in LaBr₂, which is much longer than any atomic orbital. Furthermore, the strength of the off-site Coulomb interaction U_{01} is quite large (~60% of U_{00}), which indicates a significant spatial charge fluctuation in LaBr₂, in line with the extended nature of the anionic electron. The large U_{00} and U_{01} are the results of the weak screening from the atomic states

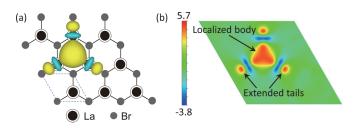


FIG. 3. The maximally localized Wannier function (a) and its colored contour plot in the plane passing through the La atoms (b) of the anionic electron state of LaBr₂. The isovalue of MLWF is set to 2.0. The localized *s*-symmetric body and its extended tails of Wannier function are indicated by black arrows.

as the anionic electron state near E_f is well isolated from other bands. It is remarkable that the anionic electron state in LaBr₂ has a significant spatial extension and overlaps in a long distance, leading to an extended direct-exchange interaction J_{ij}^D . This atomic-orbital-free direct exchange goes beyond the spatial limit of atomic-orbital overlap in other direct-exchange systems. It is noted that the effective Coulomb interaction $\tilde{U} = U_{00} - U_{01}$ of LaBr₂ is much larger than the hopping integral ($\tilde{U} \sim 77t_{01}$), which is within the range of a magnetic insulating phase on the triangular lattice [58,59], consistent with the localized nature of the anionic electron.

In the limit of $U \gg t_{01}$, the overall isotropic exchange interaction of LaBr₂ can be described by the magnetic Anderson's model as follows [60],

$$J_{ij} = \frac{1}{\tilde{U}} T r_{\sigma} \{ \hat{t}_{ji} \hat{t}_{ij} \} - J_{ij}^D, \qquad (2)$$

where the first term is the antiferromagnetic superexchange from direct hopping between magnetic centers while the second term is the ferromagnetic direct exchange from the overlap of the wave functions of magnetic moments. Using the parameters obtained from Eq. (1), the nearest-neighboring kinetic AFM superexchange and direct FM exchange is 0.2 and 21.9 meV, respectively, and the overall isotropic exchange energy is -21.7 meV for LaBr₂. This isotropic exchange is stronger than the DFT result (-6.5 meV) because the latter is known for its inadequacy in describing the nonlocal directexchange interaction [61], justifying the rational choice of the low-energy effective model in unveiling the underlying exchange mechanism of atomic-orbital-free intrinsic magnetism in electrenes.

Possible magnetic enhancement. The trend of magnetic stability of LaBr₂ and its possible enhancement are studied by Monte Carlo (MC) simulations [62,63]. The calculated Curie temperature (T_c) and coercive field (H_c) are 235 K and 0.53 T, respectively, for pristine $LaBr_2$ (see Fig. 4). Since many experiments have demonstrated an increase of the Curie temperature of 2D FM semiconductors by charge doping [4,64,65], we perform calculations at different charge doping levels. The inset in Fig. 4(a) shows the dependence of the exchange energy on the doping concentration which is varied from -0.02e (electron doping) to +0.1e (hole doping) per unit cell. Results of our MC simulation indicate that 0.1e/u.c. hole doping increased both the T_c (341 K) and the $H_{\rm c}$ (0.71 T). Charge doping can be easily achieved experimentally by applying an external electric field [4,64,65]. This trend is in qualitative agreement with a recent experimental observation that hole doping increases both T_c and H_c of CrI₃ [64].

Conclusion. In conclusion, another type of 2D intrinsic ferromagnetic material, electrenes, with *atomic-orbital-free* magnetism is proposed. The physical origin of the local moments and magnetic exchange interactions in monolayer LaBr₂ is thoroughly investigated by the first-principles calculations, low-energy effective model, and magnetic Anderson's model. The anionic electron is localized but has significantly extended tails. The localized nature results in spin polarization and the formation of local moments. The overlap of its extended tails, on the other hand, promotes a direct-exchange interaction over an extended distance beyond the conventional direct exchange.

Finally, we wish to point out that we have taken $LaBr_2$, a layered electride, as an example to demonstrate the concept of anionic electron-induced ferromagnetic order. However,

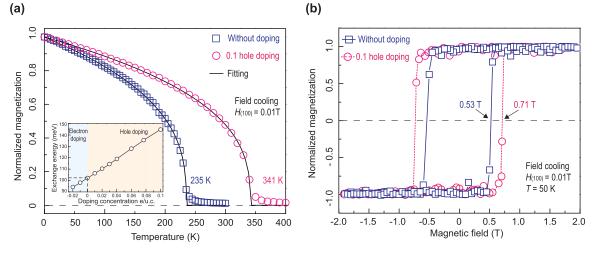


FIG. 4. (a) The temperature dependence of magnetization without (square) and with (circle) 0.1e/u.c. hole doping. A cooling field of 0.01 T is applied along the (100) direction. The inset shows the doping concentration dependence of the exchange energy. (b) The simulated magnetic field dependence of magnetization at the temperature of 50 K.

electrides are a big family [18,19,24,66,67]. The physical mechanism proposed in this Rapid Communication can be applied to ferromagnetism in other FM electrides. As reminded by Eick 30 years ago [68], they are "still a veritable gold mine" that are worthy of further studies on these intrinsic electron-rich materials which may have intriguing properties and unique technological applications. For example, very recently, Liu *et al.* reported an intrinsic quantum anomalous Hall effect induced by in-plane magnetization in LaCl [20,21]. Hirayama *et al.* proposed a class of electrides with exotic topological properties [19]. We hope this work will stimulate further experimental and theoretical studies to discover more magnetic electrenes and explore the spin physics in these types of 2D intrinsic magnetic systems, which may lead to practical spintronic applications.

Note added in proof. Recently, we became aware of a similar work by Badrtdinov and Nikolaev [69]. They investigated

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two of the four intrinsic ferromagnetic electrenes proposed in our work by similar theoretical approaches, uncovering the dual localized and extended nature of anionic electrons in magnetic electrenes.

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