# **Multiple robust Dirac states in hexagonal lattice induced by** *p***-***d* **electron-counting rule and bilayer stacking**

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In this paper, we predict two-dimensional transition metal boride TiB<sub>2</sub>C<sub>2</sub> and HfB<sub>2</sub>C<sub>2</sub> nanosheets, which exhibit favorable mechanical and thermal properties. Young's moduli of  $\text{TiB}_2\text{C}_2$  and  $\text{HfB}_2\text{C}_2$  are 137.8 and 128.9 N/m, respectively. Both nanosheets could sustain up to 500 K based on our *ab initio* molecular dynamics results. In addition, both nanosheets are semimetallic, exhibiting 12 Dirac cones in the first Brillouin zone, six pairs of which form compensated electron-hole pockets. Under different types of external strains, both  $\text{TiB}_2\text{C}_2$ and  $\text{HfB}_2\text{C}_2$  are robust, and the Dirac cones could be tuned to be anisotropic. We propose that the multiple Dirac cones are induced by a combination of the *p*-*d* electron-counting rule and bilayer stacking; to verify our opinion, we also predicted and proved that  $TB_2Si_2$  (*T* = Ti, Zr, Hf) are mechanical stable two-dimensional nanosheets with multiple Dirac cones based on phonon and electronic band structure calculations.

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## **I. INTRODUCTION**

Since the discovery of graphene [\[1\]](#page-8-0), Dirac fermions have attracted great interest, especially in the two-dimensional (2D) hexagonal lattice. Dirac fermions, which emerge in the Brillouin zone (BZ) as a linear cross point between the valence band and conduction band, possess some extraordinary properties such as high electron mobility and low-power dissipation, and they are very promising for next-generation electronics. Experiments have confirmed some hexagonal Dirac materials in group IV 2D materials such as graphene [\[1\]](#page-8-0), silicene  $[2]$ , germanene  $[3]$ , stanene  $[4]$ , etc. The hexagonal lattice is prone to exhibit Dirac states for its symmetry associated with some covalent bond property.

Inspired by the discovery of graphene, which is stabilized by  $sp^2$  and three-coordinate bond character (each atom has  $s^2 p^2$  valence electrons), it is rational to speculate that a neighbor group such as boron could form stable 2D Dirac materials with one missing electron supplemented. Following this idea, one could stabilize the hexagonal lattice by the introduction of additional boron atoms into the honeycomb lattice to balance out the two- and multicenter bonds [\[5–7\]](#page-8-0). And this rule provides great opportunities in this field because additional boron atoms could be introduced in the honeycomb lattice with a lot of different arrangements [\[6–8\]](#page-8-0).

In addition, transition metals (TMs) with additional *d* electrons are also promising alternatives to form stable a TM boride hexagonal lattice. TiB<sub>2</sub> [\[9\]](#page-8-0) was the first predicted stable 2D Dirac TM boride hexagonal lattice, which extended

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the Dirac materials to metal diborides; each Ti atom stands above the center of the honeycomb lattice. This strategy was also extensively followed to predict stable 2D  $\text{ZrB}_2$  [\[10,11\]](#page-8-0), HfB<sub>2</sub> [\[12\]](#page-8-0), FeB<sub>2</sub> [\[13\]](#page-8-0), and FeB<sub>6</sub> [\[14\]](#page-8-0), all of which possess Dirac states. In addition, with both metal and boron atoms introduced in the honeycomb lattice,  $CuB<sub>3</sub>$ ,  $CuB<sub>6</sub>$ , and  $Cu<sub>7</sub>B<sub>15</sub>$  are also predicted to be stable nanosheets, and the  $Cu<sub>7</sub>B<sub>15</sub>$  nanosheet exhibits a Dirac nodal line in the BZ  $[15]$ .

It is interesting that the bilayer boron honeycomb lattice with an appropriate TM sandwiched between the layers could also be stable 2D Dirac materials, such as  $CrB<sub>4</sub>$  [\[16\]](#page-8-0),  $MoB<sub>4</sub>$  [\[17\]](#page-8-0),  $WB<sub>4</sub>$  [\[18\]](#page-8-0), and  $UB<sub>4</sub>$  [\[19\]](#page-8-0) nanosheets. The Dirac states and stability are predictable considering that Cr atoms could offer the  $3d^4$  electrons to the four boron atoms. Compared with  $TB_2$  nanosheets, the  $TB_4$  nanosheets could exhibit multiple Dirac states along the Γ -*M*-*K*-Γ high-symmetry line in the BZ.

In addition to theoretical investigations, 2D TM borides have also been investigated and successfully synthesized in experimental research. Cepek *et al.* [\[20\]](#page-8-0) synthesized MgB<sub>2</sub> thin films in ultrahigh-vacuum conditions via molecular beam epitaxy (MBE), and this research provided the possibility of controlled doping of  $MgB_2$ . Ti $B_2$  sheets synthesized at the interface of TiC and  $B_4C$  also have been reported [\[21\]](#page-8-0). Rich AlB<sub>2</sub>-type bulk structures  $(ScB_2, TiB_2, VB_2, ZrB_2, ZrB_2)$  $NbB<sub>2</sub>, MoB<sub>2</sub>, TaB<sub>2</sub>, HfB<sub>2</sub>, YB<sub>2</sub>, etc.)$  [\[22–27\]](#page-8-0) offer great opportunities to 2D TM borides. In addition, 2D MoB was also successfully synthesized by etching the Al atoms from MAB phase (where M, A, and B are metal, aluminum, and boron atoms, respectively)  $Mo<sub>2</sub>AlB<sub>2</sub> [28]$  $Mo<sub>2</sub>AlB<sub>2</sub> [28]$ ; this method is similar to that of synthesizing MXene from MAXenes [\[29,30\]](#page-8-0).

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FIG. 1. (a) The geometric structures, (b) and (c) phonon dispersion, (d) and (e) AIMD results at 500 K of bond-length variations, and (f) in-plane isotropic biaxial strain and energy curve of  $TIB_2C_2$  and  $Hfb_2C_2$  nanosheets, respectively.

To further explore and extend the 2D Dirac state TM boride nanosheet family, in this paper, we find that  $TiB_2C_2$  and  $HfB_2C_2$  nanosheets are also stable nanosheets, and both of them exhibit six pairs of Dirac cones which form compensated electron-hole pockets in the first BZ. The Dirac states are both robust with the absence of spin-orbit coupling (SOC) against different types of external strains. In combination with density function theory (DFT) and the tight-binding model, we propose a mechanism in which multiple robust Dirac states in a hexagonal lattice are induced by the combination of the *p*-*d* electron-counting rule and bilayer stacking. We further verify this mechanism with successful prediction of stable multiple Dirac state hexagonal lattice  $TiB_2Si_2$ ,  $ZrB_2Si_2$ , and  $HfB_2Si_2$  nanosheets. The mechanism is very promising for exploring 2D Dirac state materials.

## **II. COMPUTATIONAL DETAILS**

Our calculations are based on the DFT method to carry out the analysis of the structural stabilities and the electronic structures. Numerical calculations are performed by plane-wave-based DFT within the generalized gradient approximation [\[31\]](#page-8-0), and the exchange-correlation potential of the Perdew-Burke-Ernzerhof functional is used as implemented in the Vienna Ab initio Simulation Package (VASP) [\[32\]](#page-8-0). The projector augmented wave potentials explicitly include three valence electrons for B  $(2s^22p^1)$  and four for Ti  $(3d^24s^2)$  and Hf  $(5d^26s^2)$ . The phonon band structures of the corresponding crystal structures are computed by the PHONOPY package [\[33\]](#page-8-0), which is integrated with the density functional perturbation theory [\[34\]](#page-8-0). To validate the thermal stability of the nanosheets, *ab initio* molecular dynamics

<span id="page-2-0"></span>TABLE I. Structure parameters (Å) and mechanical properties  $(N/m)$  of TiB<sub>2</sub>C<sub>2</sub> and HfB<sub>2</sub>C<sub>2</sub> nanosheets.

			Lattice TM-B TM-C B-C Thickness $Y_{2D}$	
$TiB_2C_2$		2.758 2.400 2.333 1.595	3.501	137.8
$HfB_2C_2$		2.823 2.542 2.472 1.633	3.809	128.9

(AIMD) simulations lasting for 5 ps with a time step of 1.0 fs are performed. The AIMD simulations were in the *NVT* ensemble, and the temperature was controlled by using the Nosé-Hoover method [\[35\]](#page-8-0).

In this paper, we take a large plane-wave cutoff of 600 eV and set the energy convergence criterion to be  $10^{-6}$  eV. A Γ-centered  $21 \times 21 \times 1$  Monkhorst-Pack *k*point mesh is used for the Brillouin-zone integrations in the nanosheets  $[36]$ . The total energies are calculated using the linear tetrahedron method with Bloch corrections [\[37\]](#page-8-0). During structural relaxation, atomic positions are relaxed until the Hellman-Feynman forces are less than  $10^{-3}$  eV/Å.

## **III. RESULT AND DISCUSSION**

# A. Geometric structure and stability of TiB<sub>2</sub>C<sub>2</sub> **and HfB2C2 nanosheets**

As shown in Fig.  $1(a)$ , the symmetry of the optimized geometric structure of the  $TiB_2C_2$  and  $HfB_2C_2$  nanosheets is  $D_{3d}$ , and the lattice parameters are 2.758 and 2.823 Å for TiB<sub>2</sub>C<sub>2</sub> and HfB<sub>2</sub>C<sub>2</sub>, respectively. They both consist of one layer of Ti or Hf atoms sandwiched by two layers of graphenelike hexagonal B-C atoms rather than C-C atoms. The thicknesses of  $TiB_2C_2$  and  $HfB_2C_2$  nanosheets are 3.501 and 3.809 Å, respectively. Detailed structure properties are shown in Table I.

We further confirmed the stability of the two nanosheets with the calculations of cohesive energy, AIMD, and phonon band structures. The cohesive energy per atom  $E_{coh} = (E_{TM} +$  $2E_B + 2E_C - E_{TMB_2C_2}$ )/5, where  $E_{TM}$ ,  $E_B$ ,  $E_C$ , and  $E_{TMB_2C_2}$ are the total energies per Ti or Hf atom, per B atom, per C atom, and per  $TB_2C_2$  formula unit, respectively. The cohesive energies are 6.33 and 6.76 eV per atom for the  $TiB_2C_2$  and  $HfB_2C_2$  nanosheets, respectively. The results are larger than those of the FeB<sub>2</sub> (4.87 eV) and FeB<sub>6</sub> (5.79 eV) nanosheets [\[13,14\]](#page-8-0).

We calculated the phonon dispersion relations of both nanosheets; the results are presented in Figs.  $1(b)$  and  $1(c)$ , which suggest that they are both mechanically stable in the absence of imaginary frequencies in the phonon band structures.

To further confirm the thermal stability of both nanosheets, we performed an AIMD simulation with a  $5 \times 5 \times 1$  supercell of 125 atoms. From the AIMD simulated structural patterns at a temperature of 500 K **(**see Figs. [1\(d\),](#page-1-0) [1\(e\)](#page-1-0) and S1 [\[38\]](#page-8-0)**)**, one sees that overall, the local bond structure remains intact. The result confirms the thermal stability of  $TiB_2C_2$  and  $HfB_2C_2$  at 500 K.

To investigate the in-plane stiffness of both nanosheets, we calculated the energies with different isotropic surface strains



FIG. 2. The electronic band structures of (a) and (b)  $TiB_2C_2$  and (c) and (d)  $HfB_2C_2$  nanosheets.

along the *a* and *b* axes. Figure  $1(f)$  shows the relative energy  $dE$  ( $dE = E - E_0$ ) per unit cell as a function of isotropic surface strain  $e = (A - A_0)/A_0$ . We could get the surface Young's modulus according to  $Y_{2D} = A_0^{-1} (\partial^2 E / \partial e^2)_{e=0}$ , where  $A_0$  is equilibrium surface area of the unit cell.  $Y_{2D} = 137.8$  N/m for the TiB<sub>2</sub>C<sub>2</sub> nanosheet, and  $Y_{2D} = 128.9$  N/m for the HfB<sub>2</sub>C<sub>2</sub> nanosheet. The in-plane Young's moduli are about 40% and 37% of that of graphene, and they exhibit high mechanical strength.

Considering the experimental feasibility of our proposed structures, they could be synthesized by controlled doping from TM borides using the molecular beam epitaxy technique as we mentioned above [\[20\]](#page-8-0). In addition, chemical vapor deposition (CVD) growth also offers opportunities to obtain high-quality and large-size alloyed single- or multilayer transition metal 2D nanosheets. TiB<sub>2</sub>C<sub>2</sub> and HfB<sub>2</sub>C<sub>2</sub> can be grown well on specific substrates (such as Ag, Cu, Si,  $SiO<sub>2</sub>$ , and so on) under extreme conditions (e.g., high temperature, strong oxidizing agents, and so on). For example, Liu *et al.* reported the synthesis of  $Mo_{1-x}W_{x}S_2$  alloys by sulfurizing as-prepared Mo1−*<sup>x</sup>*W*<sup>x</sup>* thin films through the CVD process [\[39\]](#page-8-0), and Xu *et al.* also reported the synthesis of largearea high-quality 2D ultrathin  $Mo<sub>2</sub>C$  crystals using the CVD method  $[40]$ .

#### **B.** The Dirac states in  $TIB_2C_2$  and  $HFB_2C_2$  nanosheets

We plotted the electronic band structure along highsymmetry points  $\Gamma$ -*M*-*K*- $\Gamma$  as well as the three-dimensional (3D) valence and conduction band surface to analyze the

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FIG. 3. The electronic band projection structures of  $TiB_2C_2$  nanosheets.

electronic characters of  $TiB_2C_2$  $TiB_2C_2$  $TiB_2C_2$  and  $HfB_2C_2$  (see Figs. 2 and S2 [\[38\]](#page-8-0)). There are two Dirac cones around the *K* point; one is at the *K* point, and the other is at the offsymmetry point between Γ and *K*, which we denote as *D*. Two Dirac cones are characterized with compensated electron-hole pockets.

To get further understanding of the contribution of the two Dirac cones, we plot the projected band structures of  $TiB_2C_2$  and  $HfB_2C_2$  nanosheets in Figs. 3 and [4,](#page-4-0) respectively. Considering the *D3d* point group symmetry, the five *d* orbitals of TiB<sub>2</sub>C<sub>2</sub> will split into three groups:  $e_1^*$  ( $d_{xz}$  degenerate with  $d_{yz}$ ,  $e_1$  ( $d_{xy}$  degenerate with  $d_{x^2-y^2}$ ), and  $a_1^*$  ( $d_{z^2}$ ). In Fig. 3, for *p* and *d* orbitals, the radii of the different colored lines are proportional to the corresponding orbital contribution. The result suggests that the first Dirac point at *K* is mainly from degenerate  $d_{yz}$  and  $d_{xz}$  as well as degenerate  $d_{xy}$  and  $d_{x^2-y^2}$  of Ti atoms and *pz* orbitals of boron and carbon atoms. The result implies that the four orbitals of Ti atoms mainly hybridize with  $p<sub>z</sub>$  orbitals of boron and carbon atoms. The second Dirac

point at *D*, which locates along  $\Gamma$  to *K*, is mainly from the three *d* orbitals ( $d_{xy}$ ,  $d_{x^2-y^2}$ , and  $d_{z^2}$ ) of Ti atoms. We also made a comparison with the  $\dot{C}$  *p* orbital and B *p* orbital of TiB<sub>2</sub>C<sub>2</sub> (see Fig. S3 [\[38\]](#page-8-0)); the result suggests that the *p* orbitals of B and C exhibit similar band dispersion, and B *p* orbitals contribute more states than those of C *p* orbitals at the *K* point. The electronic band character of the  $HfB_2C_2$  nanosheet follows the same rule as that of  $TiB_2C_2$ .

# **C. Robust Dirac states in the**  $TIB_2C_2$  **and**  $HFB_2C_2$  **nanosheets against various external strains**

As we mentioned in the Introduction, the  $\text{ZrB}_2$  nanosheet exhibits multiple Dirac states. While the one along  $\Gamma$  to *M* is sensitive to external isotropic biaxial strain, the other one along Γ to *K* is robust against isotropic biaxial strain. For TiB<sub>2</sub>C<sub>2</sub> and HfB<sub>2</sub>C<sub>2</sub> nanosheets, both Dirac cones are robust against isotropic biaxial strain as well as anisotropic strains. As shown in Fig. [5,](#page-4-0) as for the electronic structures

<span id="page-4-0"></span>





FIG. 5. The electronic band structures of TiB<sub>2</sub>C<sub>2</sub> under isotropic strains of (a) −3% and (b) 3% and anisotropic strains of (c) −3% and (d) 3%.



FIG. 6. The electronic band structures of  $HfB_2C_2$  under isotropic strains of (a)  $-4.5\%$  and (b) 4% and anisotropic strains of (c)  $-3\%$ and (d) and (e) 3%; 3D band structure around the *K* point is shown in (e) to confirm the Dirac cone shift.

under isotropic strains of  $-3\%$  [see Fig. [5\(a\)\]](#page-4-0) and 3% [see Fig.  $5(b)$ ], the result suggests that both Dirac cones at the *K* and *D* points are robust, with the one at *K* shifting down a bit, and the electron-hole pockets could survive the tensile strains. To further check the robustness of the Dirac cones, we further calculated anisotropic strains of −3% [see Fig. [5\(c\)\]](#page-4-0) and 3% [see Fig.  $5(d)$ ] along the (1 –1 0) direction. The result also verified the robustness even in the absence of the  $D_{3d}$ point group symmetry, which was broken by the anisotropic strains. To investigate the mechanism of Dirac evolution under anisotropic strains, we further plotted the band structure along Γ to *S* (0.5 0.5 0.0); the result implies that under anisotropic strains, *K* is not degenerate any longer, while both of the Dirac states still remain in line with  $R_a = R_b$ , where  $R_a$  and  $R_b$ are reciprocal lattice vectors (see Fig. S4 [\[38\]](#page-8-0)). The  $\text{HfB}_2\text{C}_2$ nanosheet exhibits a similar robustness character against these external strains. Both Dirac cones also survive under isotropic

strains of  $-4.5\%$  [see Fig. 6(a)] and 4% [see Fig. 6(b)] as well as anisotropic strains of  $\pm 3\%$  [see Figs. 6(c) and 6(d)]. To manipulate the energy level of the Dirac cones, we found that both Dirac cones could be shifted to Fermi energy level under isotropic strains of −4.5%. In addition, the anisotropy of the Dirac cones could be obviously enhanced under external strains, which is desirable for manipulating the direction of electron propagation [\[41,42\]](#page-8-0).

As mentioned above, the Dirac states are from the *d* orbitals of the TM and *p* orbitals of B and C atoms; the Dirac cone contribution is different from that of  $TiB<sub>2</sub>$ , in which the Dirac cone is mainly from *d* orbitals of Ti, and the Dirac cone could survive the SOC effect. However, for both the  $TiB_2C_2$ and  $HfB_2C_2$  nanosheets, the Dirac cones are broken under the SOC effect, especially for the 5*d* transition metal Hf. As shown in Fig. [7,](#page-6-0) the gaps are 6.6 (18.5) and 177 (30.1) meV for the Dirac cones at *K* (*D*) for the TiB<sub>2</sub>C<sub>2</sub> (HfB<sub>2</sub>C<sub>2</sub>) nanosheet, respectively. The result suggests that when strain is −4.5%, the  $HfB_2C_2$  nanosheet could be successfully tuned to exhibit a metal-insulator transition, which is desirable for electronic devices.

# **D. Multiple Dirac cones induced by the** *p***-***d* **electron-counting rule and bilayer stacking**

As mentioned in the Introduction, there are many TM borides proposed to be Dirac materials. The result of Dirac characters in different TM borides listed in Table II suggests that, in the hexagonal boride lattice, boron atoms with three coordinates need four valence electrons to stabilize the lattice. For example, for  $B_2S$  [\[43\]](#page-8-0) and  $B_2Se$  [\[44\]](#page-8-0), each S or Se atom provides two  $p$  electrons to  $B_2$  components with a covalent bond property. For the TM borides, a similar rule dominates the lattice stability, which we named the *p*-*d* electron-counting rule for the hexagonal TM boride lattice. For TiB<sub>2</sub>, each Ti  $(4s^23d^2)$  contributes two electrons to the  $B_2$  honeycomb to stabilize the lattice. This rule is obviously consistent with other TM borides in Table II, including our previously proposed  $ZrB_2C_2$  [\[45\]](#page-8-0).

Besides the mechanism of lattice stability, another phenomenon implies that the multiple Dirac cone states (at *K* and off-symmetry *D*) are prone to appear in bilayer nanosheets. To investigate the evolution of the band dispersion in  $TB_2C_2$ , we calculated the electronic band of eight structures: only the Ti atom in the triangular unit cell with the lattice parameter of the  $TiB_2C_2$  nanosheet, only the B-C bilayer in the lattice (in the absence of Ti in  $TiB_2C_2$ ), and the  $TiB_2C_2$  multiple Dirac cone nanosheet with the evolution of distance between the TM and BC honeycomb (see Fig. [8\)](#page-6-0). The thickness *c* (perpendicular to the plane of the nanosheet) increasing from 1.3 $c_0$  to 2.5 $c_0$ , where  $c_0$  is the thickness of the ground state of  $TiB<sub>2</sub>C<sub>2</sub>$  nanosheet, the result shown in Fig. [8](#page-6-0) suggests that,

TABLE II. Dirac character in different 2D borides nanosheets: Dirac point position (DP) associated with symmetry (SYM).

	$B_2S$ , $B_2Se$	TiB <sub>2</sub>	ZrB <sub>2</sub>	FeB <sub>2</sub>	TMB <sub>4</sub> $(T = Cr, Mo, W)$	$TB_2C_2/TB_2Si_2$ $(T = Ti, Zr, Hf)$
SYM DP.	$D_{2h}$ $\Gamma$ -Y	$\mathcal{L}_{6v}$ $\Gamma$ -K	$C_{6v}$ $\Gamma$ -K, $\Gamma$ -M	$\mathcal{L}_{6v}$	$D_{6d}$ $\Gamma$ -K, K	$D_{3d}$ $\Gamma$ -K, K

<span id="page-6-0"></span>

FIG. 7. The electronic band structures of (a) and (b)  $TiB_2C_2$  and (c)  $HfB_2C_2$  with the SOC effect.



FIG. 8. Electronic band structures: (a) only Ti in the hexagonal lattice, (b) only the B-C bilayer in the hexagonal lattice, and (c)–(h)  $TiB_2C_2$ nanosheet with the lattice perpendicular to the plane increased by 30%, 50%, 70%, 100%, 120%, 150%, respectively. The states in cyan and gray are from Ti *d* and BC *p* orbitals, and the two bands which form Dirac cones we marked by a red line to follow the band evolution.

TABLE III. On-site energy (eV) evolution for *d* orbitals from the triangular Ti lattice to  $TiB<sub>2</sub>$  and  $TiB<sub>2</sub>C<sub>2</sub>$  nanosheets.

	$d_{72}$	$d_{xz}$ , $d_{yz}$	$d_{xy}, d_{x^2-y^2}$
Ti	1.24	1.22	1.10
TiB <sub>2</sub>	0.99	0.67	0.88
$TiB_2C_2$	0.72	1.90	1.11

with the Ti and BC layer interaction evolution from  $2.5c_0$ to  $c_0$  (see Figs. [8](#page-6-0) and [2\)](#page-2-0), the ninth and tenth bands (red line) form two Dirac cones with the one at *D* shift upward. Compared with  $TiB<sub>2</sub>$ , whose states shift upward away from the Fermi energy at  $K$  and end up with one Dirac cone at the off-symmetry point, the bilayer BC interacts with Ti in inversion symmetry, and it could obviously change the on-site energy of Ti *d* orbitals with a crystal split sequence different from that of  $TiB<sub>2</sub>$ .

To investigate the origin of multiple Dirac states in these hexagonal TM borides, we further fitted the tight-binding Hamiltonian with maximally localized Wannier functions [\[46\]](#page-8-0) to the electronic band structure calculated by DFT. Table III lists the on-site energy of five *d* orbitals of Ti for the Ti triangular lattice,  $TiB_2$  nanosheet, and  $TiB_2C_2$  nanosheet. The schematic diagram of the *d* orbital energy split evolution is shown in Fig. 9: The crystal field split in the Ti triangular lattice is in the  $D_{6h}$  symmetry, and  $e_1$  is below the other two groups, between which the split is only 0.02 eV. The  $d$  orbital split for TiB<sub>2</sub> increased, and the energy sequence between  $e_1$  and  $e_1^*$  reversed under  $C_{6v}$  symmetry of  $C_{6v}$ . For TiB<sub>2</sub>C<sub>2</sub>, the symmetry is  $D_{3d}$ , and the crystal splitting increases more strongly than that of TiB<sub>2</sub>, and the energy split increases by 0.37 eV compared with that of the Ti triangular lattice. As shown in Fig. [3,](#page-3-0) the Dirac point at *D* is mainly from  $e_1$  and  $a_1^*$ , the *D* point is obviously shifted up compared with that of  $TiB<sub>2</sub>$ , and this shift dominates the second Dirac cone. The result suggests that for 2D TM borides, the *p*-*d* electron-counting rule and crystal field split induced by inversion-symmetric bilayer stacking are crucial for the emergence of multiple Dirac cones.

To verify our opinion, we further calculated the phonon and electronic band structures of  $TB_2Si_2$  ( $T = Ti$ ,  $Zr$  and Hf), all of which have been proved to be stable bilayer nanosheets



FIG. 9. Crystal field split for *d* orbitals in different lattices.



FIG. 10. Phonon and electronic band structures of (a) and (d)  $TiB_2Si_2$ , (b) and (e)  $ZrB_2Si_2$ , and (c) and (f)  $HfB_2Si_2$ , respectively.

exhibiting multiple Dirac cones (see Fig. 10); the structure and structure parameters are shown in Fig. S5 and Table S1 [\[38\]](#page-8-0).

#### **IV. CONCLUSION**

In summary, we predicted two different 2D  $TiB_2C_2$  and  $HfB_2C_2$  nanosheets, whose stabilities are verified by total structural atomic relaxations, the phonon dispersion calculations, and the AIMD calculations, which reveal that both nanosheets could hold up to the temperature of 500 K. The planar Young's moduli are 137.8 N/m for  $TiB_2C_2$  and 128.9 N/m for  $HfB_2C_2$ , which are about 40% and 37% of that of graphene, respectively. Both nanosheets exhibit 12 Dirac cones in their first BZ. All of the Dirac cones are robust under different types of external strains; the robustness of both Dirac cones is a promising property for electronic devices for its high electron-mobility and low-power dissipation. And the Dirac cones could be tuned by external strain to be anisotropic, which is desirable for manipulating the direction of electron propagation. We also proposed that for transition metal borides, multiple Dirac cones could be induced by the combination of the *p*-*d* electron-counting rule and bilayer stacking. We further proved that  $TB_2Si_2$  ( $T = Ti$ ,  $Zr$ , Hf) nanosheets are mechanically stable using this mechanism, which could also be used to predict other multiple Dirac state 2D materials.

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