Rapid Communications

Resistivity and its fluctuations in disordered many-body systems: From chains to planes

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We study a quantum particle coupled to hard-core bosons and propagating on disordered ladders with R legs, ranging from $R = 1$ (chains) to $R \gg 1$ (planes). The particle dynamics is studied within the framework of rate equations for the boson-assisted transitions between the Anderson states. We demonstrate that for finite $R < \infty$ and sufficiently strong disorder the dynamics is nondiffusive, while two-dimensional planar systems with $R \to \infty$ remain diffusive for arbitrarily strong disorder. The transition from diffusive to subdiffusive regimes may be identified via statistical fluctuations of resistivity. Close to the transition, the corresponding distribution function in the diffusive regime has fat tails which decrease much slower than $1/\sqrt{L}$, where L is the system size. Finally, we present evidence that similar non-Gaussian fluctuations arise also in standard models of many-body localization.

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Introduction. Many-body localization (MBL) emerged from the Anderson localization, by taking into account interactions between particles [1,2]. Later on, it was found that MBL is more general and also captures systems for which the noninteracting limit is delocalized [3–6]. The basic idea is that at large disorder, ergodicity is broken despite the presence of many-body interactions, as it is by now supported by numerous studies on quantum chains [7–24] and is consistent with several experimental studies of cold atoms in optical lattices [25–30]. Disordered, interacting systems exhibit very slow relaxation [3,6,29,31–48] that shows up also in systems which are not localized, e.g., due to too weak disorder or due to the SU(2) spin symmetry [49–53]. Then, the dynamics is typically subdiffusive [19,41,54–59], what is frequently considered as a precursor to localization [19,41,54–59] and has been mainly attributed to the presence of the so-called weak links [29,39,60,61].

The transport properties of the disordered systems with interactions in higher dimensions are much less explored. The standard numerical methods allow the study of too small systems or too short evolution times to judge on the long-time properties of macroscopic setups. Still, analytical arguments based on the presence of ergodic (delocalized) grains [62] suggest that MBL is stable only in one-dimensional (1D) systems provided that interactions decay exponentially with distance. On the other hand, the experiments show signatures of localization also in two-dimensional (2D) [27,29] and three-dimensional [25] systems. Thus, the dynamics of strongly disordered systems beyond 1D remains largely an open problem.

Here, we study a single quantum particle which is coupled to hard-core bosons. The particle propagates on a disordered ladder with R legs, ranging from $R = 1$ (chains) to $R \to \infty$ (planes). The system's dynamics is modeled via rate equations (REs) emerging from the Fermi golden rule (FGR) for transitions between the localized Anderson states [59,63]. We obtain unbiased numerical results for rather large systems with $N \sim 10^4$ sites and up to $R = 10^2$ legs. In other words, the approach is simple enough so that we are able to explore how the system's dynamics depends on its dimensionality. Previous studies of the same Hamiltonian on a single chain ($R = 1$) revealed that for strong disorder the particle dynamics is subdiffusive [64] and that such dynamics may be well described within the FGR approach [59,63].

Our results indicate that sufficiently strong disorder causes a transition between diffusive and subdiffusive regimes for arbitrary $R < \infty$. For weaker disorder, the diffusion constant $\mathcal{D}$ decreases almost exponentially with increasing disorder and is a self-averaging quantity with respect to various realizations of the disorder. Namely, the sample-to-sample fluctuations of $\mathcal{D}$ are Gaussian and its width decreases with system length L as $1/\sqrt{L}$. Upon approaching the transition to subdiffusion, we observe strong non-Gaussian fluctuations of the effective resistivity, defined here as the inverse diffusion constant $\rho = \mathcal{D}^{-1}$. The probability distribution of ρ reveals fat tails, $f(\rho) \propto \rho^{-(1+\alpha)}$, with $\alpha \to 1$ and the size dependence is much weaker than $1/\sqrt{L}$. In order to test whether such statistical fluctuations arise beyond the latter system and approach, we numerically calculate the distributions of $\mathcal{D}$ for three prototype quantum many-body models describing disordered spin chains. Our results suggest that the fat-tailed statistical fluctuations of resistivity are generic for all considered models.

Particle in a disordered potential. We study a quantum particle on a ladder containing R legs of length L coupled to itinerant hard-core bosons [63],

$$H = -t\sum_{\langle i,j\rangle} c_i^\dagger c_j + \sum_j \varepsilon_j n_j + g\sum_j n_j(a_j^\dagger + a_j)$$
$$+\omega_0 \sum_j a_j^\dagger a_j - t_b \sum_{\langle i,j\rangle} a_i^\dagger a_j, \qquad (1)$$

where ε_j are independent random potentials uniformly distributed in $[-W, W]$. Here, $c_j^\dagger$ and $a_j^\dagger$ are local fermion and hard-core boson operators ($a_j^\dagger a_j^\dagger = 0$), respectively. We set $t = 1$, $\omega_0 = g = 1$, and $t_b = 0.2$ and restrict our studies to the case of an infinite temperature, $\beta = 1/k_B T \to 0$.

In order to derive RE, we first diagonalize the single-particle part of the Anderson Hamiltonian [first two terms in Eq. (1)], $H_{SP} = \sum_l \epsilon_l \varphi_l^\dagger \varphi_l$, where $\varphi_l = \sum_i \phi_{li} c_i$ and ϕ_{li} are single-particle eigenfunctions. We then use the FGR to calculate the transition rates Γ_{lk} between different $l \neq k$ Anderson states $|l\rangle = \varphi_l^\dagger |0\rangle$. The emerging REs allow us to study large system sizes $N = LR \lessapprox 10^3$, whereas for $N \sim 10^4$ we use a simplified FGR (SFGR). In the latter approach, we neglect the momentum dependence of matrix elements for particle-boson interactions and assume a uniform bosonic density of states. In the case of a single chain, the explicit form of Γ_{lk} has been derived in Refs. [59,63] for FGR and SFGR, respectively. For convenience, we recall the main steps of the derivations in the Supplemental Material [65].

To directly address the transport, we consider an open system with the current source at the left rung and the current drain at the right rung of the ladder, i.e., we study a system with current flowing (on average) along the legs, as described by the RE,

$$\frac{dn_l}{dt} = I_l + \sum_{k\neq l}(\Gamma_{kl} n_k - \Gamma_{lk} n_l). \qquad (2)$$

Here, n_l is the occupation of the state $|l\rangle$ and $I_l = I_l^s + I_l^d$ accounts for the source and the drain, respectively,

$$I_l^s = \mathcal{I}_0 \sum_{i\in \mathrm{left}} |\phi_{li}|^2, \quad I_d^s = -\mathcal{I}_0 \sum_{i\in \mathrm{right}} |\phi_{li}|^2, \qquad (3)$$

where the summations are carried out over the left- and right-edge rungs. Since ϕ_{li} are normalized, the total injected current $\sum_l I_l^s = R\mathcal{I}_0$, hence, $\mathcal{I}_0$ is the current density. Then, the diffusion constant $\mathcal{D}$ is obtained from the relation between the current density and the gradient of the particle density, $\mathcal{D} = -\mathcal{I}_0/\nabla n_i$ with $n_i = \sum_l n_l |\phi_{li}|^2$ and n_l representing the stationary solution of RE (2). We refer to the Supplemental Material [65] for technical details on the stationary solution.

In Fig. 1 we depict one of main results of this Rapid Communication, i.e., the diffusion constant $\mathcal{D}$ as a function of the disorder strength W for various dimensionalities R, as calculated with FGR and SFGR. It is evident that $\mathcal{D}$ decays exponentially with increasing W [66–68], and that this dependence extends to very large W for the 2D system, being partially consistent with Ref. [69]. In the regime of finite $\mathcal{D}$, the spatial variations of n_i along the legs are linear, as shown in Figs. 2(a) and 2(c). However, for strong enough disorder $W \sim W_c$, the variation becomes inhomogeneous due to the formation of weak links, exemplified in Figs. 2(b) and 2(d). The threshold value W_c increases with R, but apparently

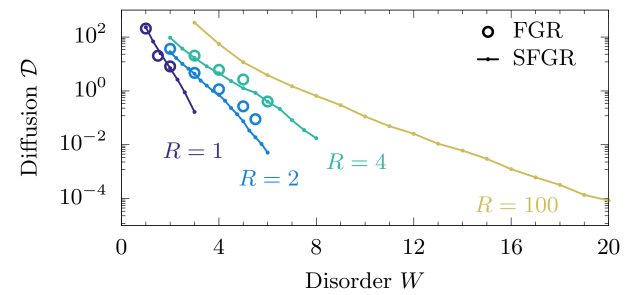

FIG. 1. Diffusion constant $\mathcal{D}$ obtained from the rate equations with the Fermi golden rule (FGR) for $L = 200$ and simplified FGR (SFGR, see text for details) for $N = LR = 10^4$ with various numbers of legs R up to a 2D system for $R \gg 1$.

remains finite provided that $R < \infty$. Weak links ultimately lead to vanishing of $\mathcal{D}$. Such behavior signals a transition (or a crossover) at $W \sim W_c$ between the diffusive and subdiffusive regimes. Results in Fig. 1 show data obtained for a single (typical) realization of disorder. In the remaining, we study sample-to-sample fluctuations which support the diffusive-subdiffusive transition.

Sample-to-sample fluctuations. In order to explain the statistical fluctuations of $\mathcal{D}$, we consider as a toy model a single chain ($R = 1$) where the FGR transitions are restricted to Anderson states on neighboring sites, $\Gamma_{kl} \sim \delta_{k,l+1}$, $I_l^s \simeq \mathcal{I}_0 \delta_{l1}$, and $I_l^d \simeq \mathcal{I}_0 \delta_{lL}$. Then, one derives from the stationary solution of Eq. (2) that $n_l - n_{l+1} = \mathcal{I}_0/\Gamma_{l,l+1}$, and consequently

$$\rho = \mathcal{D}^{-1} \simeq \frac{n_1 - n_L}{L\mathcal{I}_0} = \frac{1}{L}\sum_l \tau_l, \quad \tau_l = \frac{1}{\Gamma_{l,l+1}}. \qquad (4)$$

As previously demonstrated for the toy model [59,63], the transition times $\tau_l = \Gamma_{l,l+1}^{-1}$ can be well approximated via independent random variables with a power-law probability distribution function $f_\tau(\tau) \propto \tau^{-(\alpha+1)}$ for large enough τ. Within this simplification, ρ in Eq. (4) becomes an average of

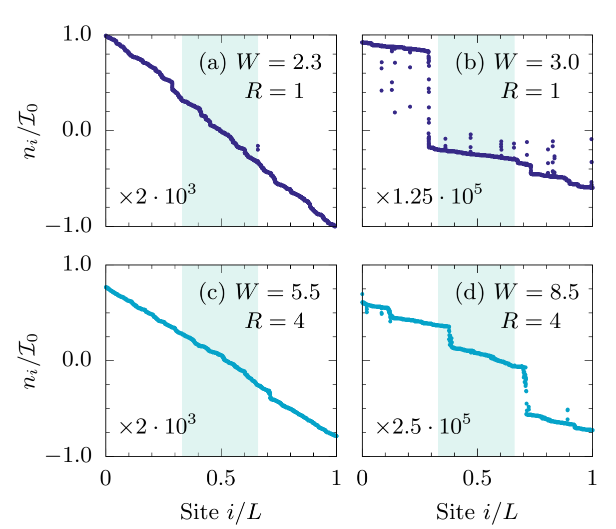

FIG. 2. Spatial profiles of n_i for $N = 10^4$. (a), (b) and (c), (d) show results for $R = 1$ and $R = 4$, respectively. Shaded regions represent sections of the system where the diffusion constant is calculated. Results for $R > 1$ are averaged over the rungs.

L-independent random variables with the transition to subdiffusion at $\alpha = 1$ [70].

For the toy model, we focus on the diffusive regime, $1 < \alpha < 2$, where the average transition time is finite $\langle \tau \rangle = \int_0^\infty d\tau\, f_\tau(\tau)\tau < \infty$, but $\langle \tau^2 \rangle$ diverges, thus the fluctuations of ρ are non-Gaussian. It is well established [70] that for the fat-tailed (the so-called α-stable) distributions, the random variable

$$u = \frac{1}{L^{1/\alpha}}\left(\sum_{l=1}^{L} \tau_l - L\langle \tau \rangle\right) = L^{(\alpha-1)/\alpha}(\rho - \langle \tau \rangle) \qquad (5)$$

has a limit distribution $f_u(u)$ for $L \to \infty$ and asymptotically $f_u(u) \propto u^{-(\alpha+1)}$. Clearly, the latter determines the tails as well as the L dependence of the resistivity distribution $f_\rho(\rho)$. In particular, close to the transition to the subdiffusive regime, $\alpha \to 1$, the exponent in the right-hand side of Eq. (5) vanishes, $(\alpha - 1)/\alpha \to 0$. As a consequence, one obtains a weak, at most logarithmic, L dependence of $f_\rho(\rho)$. The fat tails can be observed from the cumulative and the complementary cumulative distribution functions of $\mathcal{D}$ and ρ, respectively,

$$F_{\mathcal{D}}(\mathcal{D}) = \int_0^{\mathcal{D}} d\mathcal{D}' f_{\mathcal{D}'}(\mathcal{D}') \simeq \frac{\mathcal{D}^\alpha}{\alpha L^{\alpha-1}}, \quad \mathcal{D} \ll \langle \tau \rangle^{-1}, \qquad (6)$$

$$F^c_\rho(\rho) = \int_\rho^\infty d\rho' f_\rho(\rho') \simeq \frac{1}{\alpha L^{\alpha-1}\rho^\alpha}, \quad \rho \gg \langle \tau \rangle. \qquad (7)$$

A similar (albeit not identical) toy model in Ref. [39] also leads to a power-law dependence of $f_\rho(\rho)$, however, with a size dependence governed always by the prefactor $\sqrt{L}$ in Eq. (5). Hence, the L dependence is essential for testing the present scenario.

It is by far not evident whether the same properties survive when the transition rates are not independent random variables connecting only neighboring sites, but instead are obtained fully from FGR or SFGR. In Fig. 3 we present $F_{\mathcal{D}}(\mathcal{D})$ and $F^c_\rho(\rho)$ calculated for a two-leg ladder ($R = 2$) directly from the stationary solution of Eqs. (2) and with SFGR transition rates. In the same figure, we also display (insets) results, obtained from the toy model, Eq. (4), with $f_\tau(\tau) = \alpha/\tau^{\alpha+1}$ for $\tau \geqslant 1$, where we used $\alpha = 1.05$. For modest disorder shown in Fig. 3(a) we confirm that $F_{\mathcal{D}}(\mathcal{D})$ represents an error function, in agreement with the Gaussian fluctuations of $\mathcal{D}$ and its width decreasing approximately as $1/\sqrt{L}$ (not shown). However, upon approaching the subdiffusive regime, as in Fig. 3(b), the $F_{\mathcal{D}}(\mathcal{D})$ clearly differs from the Gaussian case. Results for $F^c_\rho(\rho)$ and $F_{\mathcal{D}}(\mathcal{D})$ now agree with the analytical predictions, Eqs. (6) and (7) for $\alpha \to 1$, as well as with numerical results for a finite toy model, shown in the insets of Figs. 3(b) and 3(c). In particular, the statistical fluctuations of $\rho \gg \langle \tau \rangle$ (or $\mathcal{D} \to 0$) only weakly depend on L. Since the toy model shows a transition between the diffusive and subdiffusive regimes exactly at $\alpha = 1$ [70], we expect a similar transition also for the studied ladders with hard-core bosons. To be specific, here we understand the transition as a sudden change of the transport properties. Additional numerical data supporting such a scenario are discussed in the Supplemental Material [65]. However due to numerical limitations, we are not in a position to exclude a scenario of (quite sharp) diffusion-subdiffusion crossover within our approach and beyond the FRG approximation.

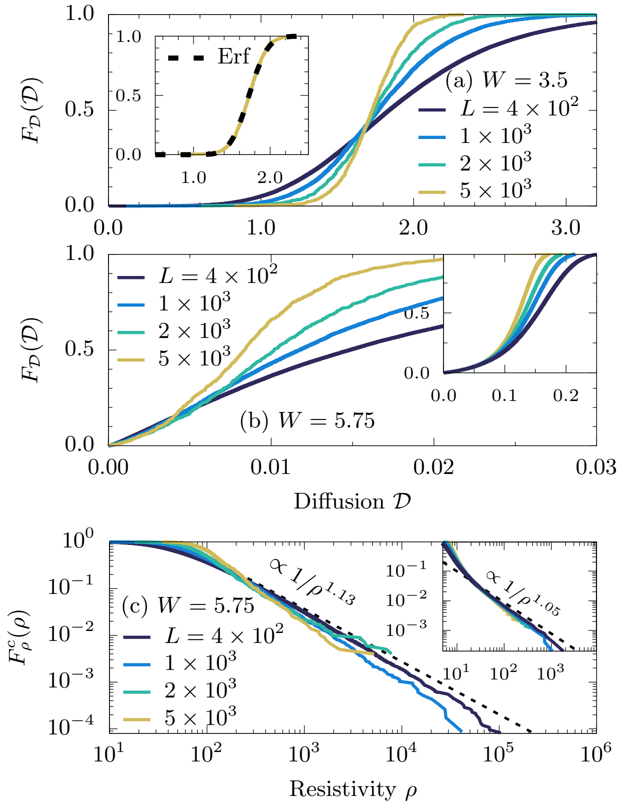

FIG. 3. Results obtained via the rate equations with SFGR for $R = 2$. (a), (b) Cumulative distribution functions of the diffusion constant $F_{\mathcal{D}}(\mathcal{D})$ and (c) complementary distribution functions of the resistivity $F^c_\rho(\rho)$. Inset (a): $F_{\mathcal{D}}(\mathcal{D})$ for $L = 5 \times 10^3$ fitted with the error function. Insets (b) and (c): The same as in the main plots but for a chain with independent random transition times with a fat-tailed distribution, Eq. (4).

Diffusivity of the planar system. The toy model also offers a simple explanation of why the 2D system remains diffusive for arbitrary W, as shown in Fig. 1. To this end, we construct the lower bound on $\mathcal{D}$ and demonstrate that it is nonzero. Consider a network with only nearest-neighbor transitions. We set a threshold transition time $\tau_{\text{th}} < \infty$ and check τ_l on each link in the network. For links with $\tau_l < \tau_{\text{th}}$ we replace τ_l with τ_{th} and remove links which do not satisfy the latter inequality. As a consequence, the values of all τ_l increase, hence we end up with a percolation problem for a system with obviously smaller $\mathcal{D}$ than the original system. The density of the removed links $\int_{\tau_{\text{th}}}^\infty d\tau\, f_\tau(\tau) = 1/(\alpha\tau_{\text{th}}^\alpha)$ may be tuned to an arbitrarily small number via increasing τ_{th}, thus the system may be tuned above the percolation threshold for arbitrary $\alpha > 0$. Consequently, the transport is always diffusive. Percolation threshold determines the density of missing links which may block the transport in 2D, hence it cannot be blocked by rare weak links, as noted previously in Ref. [40].

Fluctuations in disordered spin chains. Finally, we check whether such anomalous fluctuations may arise beyond the semiclassical RE approach in fully many-body models. To

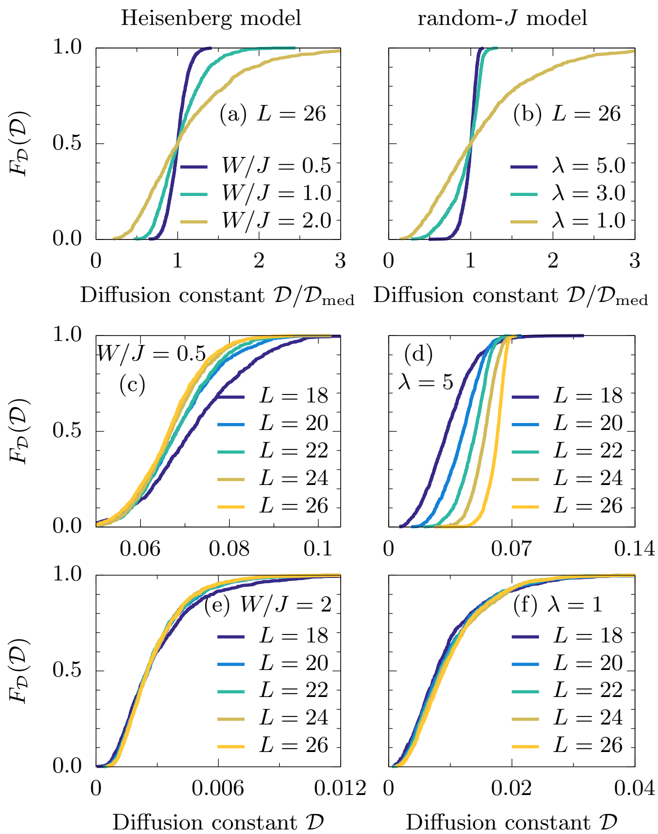

FIG. 4. Cumulative distribution functions $F_\mathcal{D}(\mathcal{D})$ for various disorder strengths (W) and system sizes (L) calculated in the ergodic regimes of the random-field Heisenberg model (left column) and the random-J Heisenberg model with $f_J(J) = \lambda J^{\lambda-1}$ distribution (right column). $\mathcal{D}$ in (a) and (b) are normalized by the median $\mathcal{D}_{\rm med}$. Results are obtained from $N_r = 10^3$ disordered samples.

this end, we consider the standard model of MBL, i.e., the Heisenberg chain with quenched disorder introduced via random, on-site magnetic fields [1,2]. It is expected that the transition from ergodic to nonergodic regimes takes place at $W_c/J \sim 3.7$, where J is the antiferromagnetic exchange coupling [8], but larger values of W_c have also been reported recently [71–75]. Furthermore, it has been argued that the MBL phase in this model is preceded by the subdiffusive regime [29,39,60,61], although the subdiffusion may be a transient phenomenon that does not occur in the asymptotic limit $L \to \infty$, $t \gg 1$, as suggested in Refs. [74,76,77]. For this reason, we study also the random-J Heisenberg chain with a singular distribution of J, $f_J(J) = \lambda J^{\lambda-1}$ for $0 \leqslant J \leqslant 1$. It emerges as an effective model for spin dynamics in the Hubbard chain with a random charge potential [56,78–80]. Numerical results for this random-J Heisenberg chain [56,78,79] indicate that the spin dynamics is subdiffusive for $\lambda < 1$. Moreover, one may show [65] that the diffusion constant $\langle \mathcal{D} \rangle = 0$ for $\lambda < 0.5$, while spins remain delocalized due to the SU(2) symmetry [49,50,52,53].

In order to extract the diffusion constant, we calculate the low-frequency regular part of the spin conductivity $C(\omega)$, i.e., $\mathcal{D} = C(\omega \to 0)$. We refer to the Supplemental Material [65] and Ref. [81] for technical details. In Fig. 4, we present the cumulative distribution functions $F_\mathcal{D}(\mathcal{D})$ obtained for the disordered quantum spin chains. For small disorder, $F_\mathcal{D}(\mathcal{D})$ may be well fitted by the error function, reflecting the Gaussian distribution of $\mathcal{D}$. Despite a limited range of accessible sizes, we clearly see that the width of the Gaussian decreases with L [see Figs. 4(c) and 4(d)]. On the other hand, increasing the disorder strength changes the functional form of $F_\mathcal{D}(\mathcal{D})$. It is evident from the results presented in Figs. 4(e) and 4(f) that the distribution becomes non-Gaussian and weakly L dependent. Similar conclusions can also be reached for the diffusive regime in the random-transverse-field Ising chain [22,82], as shown in the Supplemental Material [65]. Such behavior, i.e., a non-Gaussian, L-independent $F_\mathcal{D}(\mathcal{D})$ distribution function, closely resembles the results presented in Fig. 3(b) for the RE approach. The latter similarity suggests that the fat-tailed fluctuations of resistivity at the diffusion-subdiffusion transition are generic for all discussed spin chains. Here, due to the even more severe limitations of numerical methods the latter claim should be considered as a well-justified conjecture.

Conclusions. We have studied the transport of a quantum particle coupled to hard-core bosons in a disordered potential. The geometry of the R-leg ladders allowed tuning the system between one-dimensional ($R = 1$) and two-dimensional ($R \to \infty$) cases. We have demonstrated that sufficiently strong disorder prevents diffusion and causes subdiffusive transport for any finite R, which implies that the weak-link scenario survives also for $R > 1$. On the other hand, planar systems ($R \to \infty$) appear to be always diffusive, albeit the diffusion constant decreases exponentially with the disorder strength and may eventually become undetectably small. We have shown that the vicinity of the subdiffusion regime may be identified via fat-tailed statistical fluctuations of resistivity between different realizations of disorder. A similar universality with a simple toy model suggests a well-defined diffusion-subdiffusion transition (at least within present numerical limitations). Numerical results obtained for various models of disordered spin chains suggest that the latter fluctuations occur also for other 1D disordered many-body quantum systems.

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[1] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Metal-insulator transition in a weakly interacting many-electron system with localized single-particle states, Ann. Phys. **321**, 1126 (2006).

[2] V. Oganesyan and D. A. Huse, Localization of interacting fermions at high temperature, Phys. Rev. B **75**, 155111 (2007).

[3] P. Sierant, D. Delande, and J. Zakrzewski, Many-body localization due to random interactions, Phys. Rev. A **95**, 021601(R) (2017).

[4] Y. Bar Lev, D. R. Reichman, and Y. Sagi, Many-body localization in system with a completely delocalized single-particle spectrum, Phys. Rev. B **94**, 201116(R) (2016).

[5] X. Li, D.-L. Deng, Y.-L. Wu, and S. Das Sarma, Statistical bubble localization with random interactions, Phys. Rev. B **95**, 020201(R) (2017).

[6] M. Mierzejewski, J. Herbrych, and P. Prelovšek, Universal dynamics of density correlations at the transition to the many-body localized state, Phys. Rev. B **94**, 224207 (2016).

[7] C. Monthus and T. Garel, Many-body localization transition in a lattice model of interacting fermions: Statistics of renormalized hoppings in configuration space, Phys. Rev. B **81**, 134202 (2010).

[8] D. J. Luitz, N. Laflorencie, and F. Alet, Many-body localization edge in the random-field Heisenberg chain, Phys. Rev. B **91**, 081103(R) (2015).

[9] F. Andraschko, T. Enss, and J. Sirker, Purification and Many-Body Localization in Cold Atomic Gases, Phys. Rev. Lett. **113**, 217201 (2014).

[10] P. Ponte, Z. Papić, F. Huveneers, and D. A. Abanin, Many-Body Localization in Periodically Driven Systems, Phys. Rev. Lett. **114**, 140401 (2015).

[11] A. Lazarides, A. Das, and R. Moessner, Fate of Many-Body Localization under Periodic Driving, Phys. Rev. Lett. **115**, 030402 (2015).

[12] R. Vasseur, S. A. Parameswaran, and J. E. Moore, Quantum revivals and many-body localization, Phys. Rev. B **91**, 140202(R) (2015).

[13] M. Serbyn, Z. Papić, and D. A. Abanin, Quantum quenches in the many-body localized phase, Phys. Rev. B **90**, 174302 (2014).

[14] D. Pekker, G. Refael, E. Altman, E. Demler, and V. Oganesyan, Hilbert-Glass Transition: New Universality of Temperature-Tuned Many-Body Dynamical Quantum Criticality, Phys. Rev. X **4**, 011052 (2014).

[15] E. J. Torres-Herrera and L. F. Santos, Dynamics at the many-body localization transition, Phys. Rev. B **92**, 014208(R) (2015).

[16] M. Távora, E. J. Torres-Herrera, and L. F. Santos, Inevitable power-law behavior of isolated many-body quantum systems and how it anticipates thermalization, Phys. Rev. A **94**, 041603(R) (2016).

[17] C. R. Laumann, A. Pal, and A. Scardicchio, Many-Body Mobility Edge in a Mean-Field Quantum Spin Glass, Phys. Rev. Lett. **113**, 200405 (2014).

[18] D. A. Huse, R. Nandkishore, and V. Oganesyan, Phenomenology of fully many-body-localized systems, Phys. Rev. B **90**, 174202 (2014).

[19] S. Gopalakrishnan, K. R. Islam, and M. Knap, Noise-Induced Subdiffusion in Strongly Localized Quantum Systems, Phys. Rev. Lett. **119**, 046601 (2017).

[20] J. Hauschild, F. Heidrich-Meisner, and F. Pollmann, Domain-wall melting as a probe of many-body localization, Phys. Rev. B **94**, 161109(R) (2016).

[21] J. Herbrych, J. Kokalj, and P. Prelovšek, Local Spin Relaxation within the Random Heisenberg Chain, Phys. Rev. Lett. **111**, 147203 (2013).

[22] J. Z. Imbrie, Diagonalization and Many-Body Localization for a Disordered Quantum Spin Chain, Phys. Rev. Lett. **117**, 027201 (2016).

[23] R. Steinigeweg, J. Herbrych, F. Pollmann, and W. Brenig, Typicality approach to the optical conductivity in thermal and many-body localized phases, Phys. Rev. B **94**, 180401(R) (2016).

[24] J. Herbrych and J. Kokalj, Effective realization of random magnetic fields in compounds with large single-ion anisotropy, Phys. Rev. B **95**, 125129 (2017).

[25] S. S. Kondov, W. R. McGehee, W. Xu, and B. DeMarco, Disorder-Induced Localization in a Strongly Correlated Atomic Hubbard Gas, Phys. Rev. Lett. **114**, 083002 (2015).

[26] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, Observation of many-body localization of interacting fermions in a quasi-random optical lattice, Science **349**, 842 (2015).

[27] J.-Y. Choi, S. Hild, J. Zeiher, P. Schauß, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, I. Bloch, and C. Gross, Exploring the many-body localization transition in two dimensions, Science **352**, 1547 (2016).

[28] P. Bordia, H. P. Lüschen, S. S. Hodgman, M. Schreiber, I. Bloch, and U. Schneider, Coupling Identical 1D Many-Body Localized Systems, Phys. Rev. Lett. **116**, 140401 (2016).

[29] P. Bordia, H. Lüschen, S. Scherg, S. Gopalakrishnan, M. Knap, U. Schneider, and I. Bloch, Probing Slow Relaxation and Many-Body Localization in Two-Dimensional Quasiperiodic Systems, Phys. Rev. X **7**, 041047 (2017).

[30] J. Smith, A. Lee, P. Richerme, B. Neyenhuis, P. W. Hess, P. Hauke, M. Heyl, D. A. Huse, and C. Monroe, Many-body localization in a quantum simulator with programmable random disorder, Nat. Phys. **12**, 907 (2016).

[31] M. Žnidarič, T. Prosen, and P. Prelovšek, Many-body localization in the Heisenberg *XXZ* magnet in a random field, Phys. Rev. B **77**, 064426 (2008).

[32] J. H. Bardarson, F. Pollmann, and J. E. Moore, Unbounded Growth of Entanglement in Models of Many-Body Localization, Phys. Rev. Lett. **109**, 017202 (2012).

[33] J. A. Kjäll, J. H. Bardarson, and F. Pollmann, Many-Body Localization in a Disordered Quantum Ising Chain, Phys. Rev. Lett. **113**, 107204 (2014).

[34] M. Serbyn, Z. Papić, and D. A. Abanin, Criterion for Many-Body Localization-Delocalization Phase Transition, Phys. Rev. X **5**, 041047 (2015).

[35] D. J. Luitz, N. Laflorencie, and F. Alet, Extended slow dynamical regime prefiguring the many-body localization transition, Phys. Rev. B **93**, 060201(R) (2016).

[36] M. Serbyn, Z. Papić, and D. A. Abanin, Universal Slow Growth of Entanglement in Interacting Strongly Disordered Systems, Phys. Rev. Lett. **110**, 260601 (2013).

[37] S. Bera, H. Schomerus, F. Heidrich-Meisner, and J. H. Bardarson, Many-Body Localization Characterized from a One-Particle Perspective, Phys. Rev. Lett. **115**, 046603 (2015).

[38] E. Altman and R. Vosk, Universal dynamics and renormalization in many-body-localized systems, Annu. Rev. Condens. Matter Phys. **6**, 383 (2015).

[39] K. Agarwal, S. Gopalakrishnan, M. Knap, M. Müller, and E. Demler, Anomalous Diffusion and Griffiths Effects Near the Many-Body Localization Transition, Phys. Rev. Lett. **114**, 160401 (2015).

[40] S. Gopalakrishnan, M. Müller, V. Khemani, M. Knap, E. Demler, and D. A. Huse, Low-frequency conductivity in many-body localized systems, Phys. Rev. B **92**, 104202 (2015).

[41] M. Žnidarič, A. Scardicchio, and V. K. Varma, Diffusive and Subdiffusive Spin Transport in the Ergodic Phase of a Many-Body Localizable System, Phys. Rev. Lett. **117**, 040601 (2016).

[42] Y. Bar Lev and D. R. Reichman, Dynamics of many-body localization, Phys. Rev. B **89**, 220201(R) (2014).

[43] Y. Bar Lev, G. Cohen, and D. R. Reichman, Absence of Diffusion in an Interacting System of Spinless Fermions on a One-Dimensional Disordered Lattice, Phys. Rev. Lett. **114**, 100601 (2015).

[44] O. S. Barišić, J. Kokalj, I. Balog, and P. Prelovšek, Dynamical conductivity and its fluctuations along the crossover to many-body localization, Phys. Rev. B **94**, 045126 (2016).

[45] J. Bonča and M. Mierzejewski, Delocalized carriers in the t-J model with strong charge disorder, Phys. Rev. B **95**, 214201 (2017).

[46] I. V. Protopopov and D. A. Abanin, Spin-mediated particle transport in the disordered Hubbard model, Phys. Rev. B **99**, 115111 (2019).

[47] M. Schecter, T. Iadecola, and S. Das Sarma, Configuration-controlled many-body localization and the mobility emulsion, Phys. Rev. B **98**, 174201 (2018).

[48] J. Zakrzewski and D. Delande, Spin-charge separation and many-body localization, Phys. Rev. B **98**, 014203 (2018).

[49] A. Chandran, V. Khemani, C. R. Laumann, and S. L. Sondhi, Many-body localization and symmetry-protected topological order, Phys. Rev. B **89**, 144201 (2014).

[50] A. C. Potter and R. Vasseur, Symmetry constraints on many-body localization, Phys. Rev. B **94**, 224206 (2016).

[51] P. Prelovšek, O. S. Barišić, and M. Žnidarič, Absence of full many-body localization in the disordered Hubbard chain, Phys. Rev. B **94**, 241104(R) (2016).

[52] I. V. Protopopov, W. W. Ho, and D. A. Abanin, Effect of SU(2) symmetry on many-body localization and thermalization, Phys. Rev. B **96**, 041122(R) (2017).

[53] A. J. Friedman, R. Vasseur, A. C. Potter, and S. A. Parameswaran, Localization-protected order in spin chains with non-Abelian discrete symmetries, Phys. Rev. B **98**, 064203 (2018).

[54] D. J. Luitz and Y. Bar Lev, Anomalous Thermalization in Ergodic Systems, Phys. Rev. Lett. **117**, 170404 (2016).

[55] D. J. Luitz and Y. Bar Lev, The ergodic side of the manybody localization transition, Ann. Phys. **529**, 1600350 (2016).

[56] M. Kozarzewski, P. Prelovšek, and M. Mierzejewski, Spin Subdiffusion in the Disordered Hubbard Chain, Phys. Rev. Lett. **120**, 246602 (2018).

[57] P. Prelovšek and J. Herbrych, Self-consistent approach to many-body localization and subdiffusion, Phys. Rev. B **96**, 035130 (2017).

[58] Y. B. Lev, D. M. Kennes, C. Klöckner, D. R. Reichman, and C. Karrasch, Transport in quasiperiodic interacting systems: From superdiffusion to subdiffusion, Europhys. Lett. **119**, 37003 (2017).

[59] P. Prelovšek, J. Bonča, and M. Mierzejewski, Transient and persistent particle subdiffusion in a disordered chain coupled to bosons, Phys. Rev. B **98**, 125119 (2018).

[60] K. Agarwal, E. Altman, E. Demler, S. Gopalakrishnan, D. A. Huse, and M. Knap, Rareregion effects and dynamics near the manybody localization transition, Ann. Phys. **529**, 1600326 (2016).

[61] H. P. Lüschen, P. Bordia, S. Scherg, F. Alet, E. Altman, U. Schneider, and I. Bloch, Observation of Slow Dynamics Near the Many-Body Localization Transition in One-Dimensional Quasiperiodic Systems, Phys. Rev. Lett. **119**, 260401 (2017).

[62] W. De Roeck and F. Huveneers, Stability and instability towards delocalization in many-body localization systems, Phys. Rev. B **95**, 155129 (2017).

[63] M. Mierzejewski, P. Prelovšek, and J. Bonča, Einstein Relation for a Driven Disordered Quantum Chain in the Subdiffusive Regime, Phys. Rev. Lett. **122**, 206601 (2019).

[64] J. Bonča, S. A. Trugman, and M. Mierzejewski, Dynamics of the one-dimensional Anderson insulator coupled to various bosonic baths, Phys. Rev. B **97**, 174202 (2018).

[65] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.102.161111 for derivation of the transition rates, additional results supporting the diffussion-subdiffusion transition, as well as for more details of calculations for disordered quantum spin systems.

[66] O. S. Barišić and P. Prelovšek, Conductivity in a disordered one-dimensional system of interacting fermions, Phys. Rev. B **82**, 161106(R) (2010).

[67] P. Prelovšek, M. Mierzejewski, O. S. Barišić, and J. Herbrych, Density correlations and transport in models of many-body localization, Ann. Phys. **529**, 1600362 (2017).

[68] J. Šuntajs, J. Bonča, T. Prosen, and L. Vidmar, Quantum chaos challenges many-body localization, arXiv:1905.06345 .

[69] M. P. A. Fisher, G. Grinstein, and S. M. Girvin, Presence of Quantum Diffusion in Two Dimensions: Universal Resistance at the Superconductor-Insulator Transition, Phys. Rev. Lett. **64**, 587 (1990).

[70] J. P. Bouchaud and A. Georges, Anomalous diffusion in disordered media: Statistical mechanisms, models and physical applications, Phys. Rep. **195**, 127 (1990).

[71] T. Chanda, P. Sierant, and J. Zakrzewski, Time dynamics with matrix product states: Many-body localization transition of large systems revisited, Phys. Rev. B **101**, 035148 (2020).

[72] T. Devakul and R. R. P. Singh, Early Breakdown of Area-Law Entanglement at the Many-Body Delocalization Transition, Phys. Rev. Lett. **115**, 187201 (2015).

[73] E. V. H. Doggen, F. Schindler, K. S. Tikhonov, A. D. Mirlin, T. Neupert, D. G. Polyakov, and I. V. Gornyi, Many-body localization and delocalization in large quantum chains, Phys. Rev. B **98**, 174202 (2018).

[74] F. Weiner, F. Evers, and S. Bera, Slow dynamics and strong finite-size effects in many-body localization with random and quasiperiodic potentials, Phys. Rev. B **100**, 104204 (2019).

[75] R. K. Panda, A. Scardicchio, M. Schulz, S. R. Taylor, and M. Žnidarič, Can we study the many-body localisation transition?, Europhys. Lett. **128**, 67003 (2020).

[76] S. Bera, G. De Tomasi, F. Weiner, and F. Evers, Density Propagator for Many-Body Localization: Finite-Size Effects, Transient Subdiffusion, and Exponential Decay, Phys. Rev. Lett. **118**, 196801 (2017).

[77] P. Sierant, D. Delande, and J. Zakrzewski, Thouless Time Analysis of Anderson and Many-Body Localization Transitions, Phys. Rev. Lett. **124**, 186601 (2020).

[78] M. Środa, P. Prelovšek, and M. Mierzejewski, Instability of subdiffusive spin dynamics in strongly disordered Hubbard chain, Phys. Rev. B **99**, 121110(R) (2019).

[79] M. Kozarzewski, M. Mierzejewski, and P. Prelovšek, Suppressed energy transport in the strongly disordered Hubbard chain, Phys. Rev. B **99**, 241113(R) (2019).

[80] I. V. Protopopov, R. K. Panda, T. Parolini, A. Scardicchio, E. Demler, and D. A. Abanin, Non-Abelian Symmetries and Disorder: A Broad Nonergodic Regime and Anomalous Thermalization, Phys. Rev. X **10**, 011025 (2020).

[81] P. Prelovšek and J. Bonča, Ground state and finite temperature Lanczos methods, in *Strongly Correlated Systems - Numerical Methods*, edited by A. Avella and F. Mancini (Springer, Berlin, 2013).

[82] J. Z. Imbrie, On many-body localization for quantum spin chains, J. Stat. Phys. **163**, 998 (2016).