# Electrical magnetochiral effect and kinetic magnetoelectric effect induced by chiral exchange field in helical magnetics

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The features of spin and charge transport in conductive helimagnets, which are due to the action of an inhomogeneous exchange magnetic field on the spin of conduction electrons, are theoretically studied. The interaction between the spin of moving particles and an inhomogeneous external magnetic field was first recorded in the famous Stern-Gerlach experiment that investigated the quantum nature of spin. In the present paper, we have demonstrated that two physical effects—the electrical magnetochiral effect (EMChE) and the kinetic magnetoelectric effect (KMEE)—can be explained through the interaction of the spins of itinerant electrons in chiral helimagnets with spatially inhomogeneous effective magnetic field of exchange origin. All parameters of the EMChE and KMEE are presented in terms of both the characteristic frequencies of spin relaxation of conduction electrons in a helimagnet and the frequencies of their Larmor precession in external and internal exchange fields. It has been shown that the effective frequency of conduction-electron spin relaxation in a helimagnet contains three components: (i) the rate of spin-lattice relaxation caused by spin-orbit scattering of conduction electrons by defects of the crystal lattice, (ii) the rate of change in the average spin of electrons due to their "diffusion" escape from a region with a given direction of the average spin to a region with a different direction of spin density, and (iii) the contribution of the Larmor precession of the spins of electrons moving in the helimagnet's exchange field that assigns the precession axis altering its direction in space. The peculiarities of the EMChE and KMEE that substantially depend on the ratio of the above-listed spin relaxation rates and the angular frequencies of electron precession are described. The numerical estimates performed show that the mechanism of generating EMChE provides the effect magnitude sufficient to be experimentally detected in metallic helimagnets. The frequency regions of spin relaxation and spin precession are determined to observe a giant electrical magnetochiral effect and resonant behavior of the chiral magnetoresistance. We have called the appropriate effect "magnetochiral kinetic resonance" (MChKR). The physical nature of MChKR is elucidated. The latter arises due to the coincidence of the Larmor precession frequency of an electron in the effective field and the phase change frequency of the helicoidal exchange field acting on the electron moving along the helicoid's axis with a speed equal to that of the electron flow. We have demonstrated how the experimental studies of the KMEE can be used to directly determine the chirality of helimagnets.

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## I. INTRODUCTION

Chirality (from the Greek "$\chi\varepsilon\iota\rho$" = hand)—is a fundamental concept used for describing a wide range of material objects for which no mirror symmetry exists. The chirality in solids can produce unique phenomena caused by the peculiarities of their chiral-dependent response to electric and magnetic fields. Magnetic crystals, the ordering of which is due to antisymmetric exchange interaction of relativistic origin, called the Dzyaloshinskii-Moriya interaction [1,2], can exhibit magnetic chirality. The latter emerge as various helicoidal magnetic structures. The lack of symmetry of spatial inversion in the corresponding crystal structures is the reason for the origin of the Dzyaloshinskii-Moriya interaction and magnetic chirality.

A distinctive feature of spin-transport phenomena in chiral helimagnets is the existence of specific nonreciprocity effects arising under passing both charge and spin fluxes through a magnetized helimagnet. Owing to the breakage of both the symmetry of spatial inversion and the symmetry of time reversal, chiral magnets as materials that demonstrate spin-dependent nonreciprocity effects are attracting ever-greater interest. Therefore, conductive chiral helimagnets are promising materials for creating on their basis a new class of spintronics devices.

The chirality effects on forming the magnetotransport properties of conductors with mirror isomer symmetry were first discovered in 1985 [3]. The authors of this work, based on symmetry considerations, predicted the kinetic magnetoelectric effect (KMEE). It manifests itself in the appearance of magnetization of charge carriers flowing through an isomer, with the magnetization being proportional to the electric field acting in the metal. According to [3], the proportionality coefficient between the magnetization vector and the electric

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field vector is a pseudoscalar for cubic crystals or isotropic stereoisometric materials, with the sign for two isomers with different chiralities being different. In [4], it is shown that the KMEE arises in metallic antiferromagnets with a helicoidal wave of spin density.

The influence of chirality on nonreciprocal magnetotransport was first reported in the paper [5] published in 2001. It was shown that the nonlinear electrical resistance of a chiral conductor should include a nonreciprocal chiral-dependent contribution proportional to both an external magnetic field and an electric current flowing through the conductor. The sign of this contribution is determined by the sign of chirality. The predicted phenomenon was called "electrical magnetochiral anisotropy" or EMChA. Two possible microscopic mechanisms responsible for the emergence of EMChA were identified: (i) chiral-dependent scattering of conduction electrons and (ii) a current-induced effective internal magnetic field generated by KMEE. Later, the name of the EMChA effect was substituted for the electrical magnetochiral effect (EMChE).

The paper [6] offers a theoretical nonlinear current-voltage characteristic found for a conductive helimagnet and predicts the probability of the existence of a diode effect in noncoplanar helimagnets. Here, special mention is made of the spin asymmetry of the energy spectrum of conduction electrons in spiral exchange fields as a mechanism causing this effect.

EMChE induced by chiral spin fluctuations has been discovered experimentally in a MnSi chiral helimagnet [7]. EMChE has been also experimentally appraised in a $CrNb_3S_6$ monoaxial chiral magnet [8] within a wide range of magnetic fields at temperatures below the temperature of magnetic ordering.

Based on a numerical calculation, the authors of [9] have shown the possibility of the existence of nonreciprocity effects of spin transport in helimagnets. For evaluating nonreciprocal spin transport in helimagnets, the authors employed the Landauer method based on Green's functions for a one-dimensional Kondo lattice model.

Recently, the authors of this paper proposed an approach for describing spin transport in conductive helimagnets [10]. Sequential quantum-mechanical accounting for forces acting on an electron spin in an inhomogeneous magnetic field underlies the concept. In helimagnets, the role of an inhomogeneous magnetic field is played by an effective field of exchange origin, exerted by the spins forming a chiral helicoidal magnetic order on the conduction electrons. The developed technique is based on the method of a quantum kinetic equation, which allows one to immediately combine the sequential quantum-mechanical description of the spin dynamics of conduction electrons and the semiclassical approach to the description of their orbital motion.

Manifestations of the action of an external inhomogeneous magnetic field on the motion of spin-possessing particles are well known. It would be appropriate here to recall the classical Stern-Gerlach experiments described in [11], which involve an inhomogeneous magnetic field to change the nature of the motion of free silver atoms. It is reported in [10] that making allowance for the impact of an inhomogeneous exchange field on the conduction-electron spins in a monoaxial chiral helimagnet of the "simple helix" type naturally explains the microscopic nature of KMEE in helimagnets. Moreover, the above interaction causes the appearance of an additional chiral spin-dependent contribution to the electrical resistance of helimagnets.

Based on the equations derived in [10], the present paper constructs a microscopic theory of electrical magnetochiral and kinetic magnetoelectric effects produced by the exchange interaction between itinerant electrons and localized electrons responsible for the formation of helimagnetism. The findings secured describe the nonreciprocity effects when passing both electric and spin currents through a helimagnet.

## II. BASIC EQUATIONS OF ELECTRON SPIN TRANSPORT IN SPATIALLY INHOMOGENEOUS CONDUCTIVE MAGNETICS

To describe the spin-transport properties of magnetically inhomogeneous conductors such as helimagnets, we will use the mathematical apparatus of the quantum kinetic equation as a base. By means of the quantum kinetic equation, it is possible to take into account the relationship of charge kinetics and spin dynamics of conduction electrons in a relatively simple way. The quantum kinetic equation has been formulated and successfully used to disclose spin-transport phenomena in paramagnetic metals in [12,13]. In this paper, we will resort to the results of [10] and apply the apparatus of the quantum kinetic equation to delineate the spin-transport properties of conduction electrons coupled by the exchange interaction with a system of electrons localized on the internal $d$ or $f$ shells of lattice atoms whose magnetic moments form a helicoidal magnetic order in a conductor. The general issues of composing quantum kinetic equations and their boundary conditions for describing the spin kinetics of conduction electrons and localized spins coupled by the exchange interaction were considered in [14].

Let us look into the spin state of conduction electrons through the electron spin operator $\hat{\mathbf{s}}$. The spin states of electrons localized in the $d$ or $f$ shells of atoms located at the sites of the crystal lattice with coordinates $\mathbf{r}_i$ can be characterized using the operators $\hat{\mathbf{S}}_i$ (index $i$ is the site number). The interaction of conduction electrons and localized spins can be expressed by the well-known Hamiltonian of the $s$-$d$ exchange model as follows: $\hat{\mathcal{H}}_{s-d} = -\sum_i \mathcal{J}(\mathbf{r}-\mathbf{r}_i)\hat{\mathbf{s}}\cdot\hat{\mathbf{S}}_i$, where $\mathcal{J}(\mathbf{r}-\mathbf{r}_i)$ is the exchange integral. The spin operator $\hat{\mathbf{s}}$ and the magnetic moment operator $\hat{\boldsymbol{\mu}}$ of conduction electrons are determined through the Pauli spin matrices $\hat{\boldsymbol{\sigma}}$ by the relations $\hat{\mathbf{s}} = (\hbar/2)\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\mu}} = -\mu\hat{\boldsymbol{\sigma}}$, where $\mu = g\mu_0/2$, $g$ is the $g$ factor of an electron, and $\mu_0$ is the Bohr magneton. The quantum-mechanical average of the sum of the magnetic moment operators $\hat{\boldsymbol{\mu}}_i$ of localized electrons governs macroscopic magnetization $\mathbf{M}$ of the helicoidal system. In the general case, the magnetization $\mathbf{M}$ is a function of the coordinate $\mathbf{r}$ and time $t$.

A rigorous quantum-mechanical description of a system of exchange-coupled, itinerant $s$ and localized $d$ ($f$) electrons can be based on the system's statistical-operator equation that satisfies the well-known von Neumann equation. Given the interaction $\hat{\mathcal{H}}_{s-d}$ between the $s$ and $d$ subsystems, the description of the total system is oversimplified in the mean-field approximation. This approximation represents the exchange interaction $\hat{\mathcal{H}}_{s-d}$ as the action of the inhomogeneous exchange field $\tilde{\mathbf{B}} = \Lambda \mathbf{M}$ on the electron spin, where $\Lambda = \int d\mathbf{r} \mathcal{J}(\mathbf{r})/(g\mu_0)^2$. In this case, the von Neumann equation for the total statistical operator boils down to an equation for the one-electron operator $\hat{\rho}$ as follows: $\partial\hat{\rho}/\partial t + (i/\hbar)[\hat{\mathcal{H}}, \hat{\rho}] = 0$. The latter contains the effective one-electron Hamiltonian $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_e + \hat{\mathcal{H}}_\mu + \hat{V}$. Here $\hat{\mathcal{H}}_0 = (\mathbf{p} - e\mathbf{A}/c)^2/2m_e$ is the kinetic energy operator of an electron with a quasimomentum $\mathbf{p}$ and effective mass $m_e$, which moves in a constant homogeneous external magnetic field $\mathbf{B} = \text{rot}\mathbf{A}$, $\mathbf{A}$ being the vector potential; $\hat{\mathcal{H}}_e = -e\mathbf{E}\cdot\mathbf{r}$ is the operator of interaction between the electron charge $e$ and an electric field $\mathbf{E}$; $\hat{\mathcal{H}}_\mu = \mu\mathbf{B}^{(\text{eff})}\cdot\hat{\boldsymbol{\sigma}}$ is the energy operator of the interaction between the electron magnetic moment and the effective magnetic field $\mathbf{B}^{(\text{eff})} = \mathbf{B} + \tilde{\mathbf{B}}$. The operator $\hat{V} = U + (\hbar/4m_e^2c^2)\hat{\boldsymbol{\sigma}}\cdot[\partial U/\partial\mathbf{r}\times\mathbf{p}]$ is responsible for the spin-orbit interaction of an electron with scatterers—all kinds of crystal lattice defects that create a scattering potential $U$. The Hamiltonian $\hat{\mathcal{H}}_0$ controls the electron energy spectrum $\varepsilon_\mathbf{p}$ as a function of the quasi-momentum $\mathbf{p}$ that is assumed to be isotropic and quadratic for simplicity.

Let us enter the quantum distribution function $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ as the one-electron density matrix $\hat{\rho}$ written down in the Wigner representation and averaged over the configurations of the scattering potential $\hat{f}(\mathbf{r}, \mathbf{p}, t) = \sum_\mathbf{k} e^{i\mathbf{kr}/\hbar}\langle\hat{\rho}_{\mathbf{p}+\mathbf{k}/2 \quad \mathbf{p}-\mathbf{k}/2}\rangle$. Here $\hat{\rho}_{\mathbf{p}\,\mathbf{p}'}$ are the matrix elements of the operator $\hat{\rho}$ of the eigenfunctions of the operator $\hat{\mathcal{H}}_0$, with these functions matching the energy $\varepsilon_\mathbf{p}$. The brackets $\langle\cdots\rangle$ stands for the averaging operation over all possible configurations of the scattering potential $U$.

The quantum kinetic equation for $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ can be deduced from the von Neumann equation for $\hat{\rho}$. This can be done through the Wigner representation of the density matrix, followed by averaging over the configurations of the scattering potential and passing to the semiclassical limit in describing the orbital motion of the electron. The details of this rather cumbersome procedure can be found in the Supplemental Material [15], but here we give only the final equation for $\hat{f}(\mathbf{r}, \mathbf{p}, t)$:

$$\frac{\partial\hat{f}}{\partial t} + \mathbf{v}\cdot\frac{\partial\hat{f}}{\partial\mathbf{r}} + \left\{e\mathbf{E} + \frac{e}{c}[\mathbf{v}\times\mathbf{B}] - \mu\frac{\partial}{\partial\mathbf{r}}(\hat{\boldsymbol{\sigma}}\cdot\tilde{\mathbf{B}})\right\}\cdot\frac{\partial\hat{f}}{\partial\mathbf{p}} + \mu\frac{i}{\hbar}(\mathbf{B} + \tilde{\mathbf{B}})\cdot[\hat{\boldsymbol{\sigma}}, \hat{f}] + \hat{\mathcal{R}} = 0. \tag{1}$$

The second term of the left side of Eq. (1) keeps the electron velocity $\mathbf{v} = \partial\varepsilon_\mathbf{p}/\partial\mathbf{p}$. The first two summands of the third term in the curly braces are nothing but the Lorentz force exerted by the electric field $\mathbf{E}$ and the homogeneous magnetic field $\mathbf{B}$. The summand $-\mu\partial/\partial\mathbf{r}(\hat{\boldsymbol{\sigma}}\cdot\tilde{\mathbf{B}})$ can be treated as a quantum addition to the classical Lorentz force arising in an inhomogeneous field $\tilde{\mathbf{B}}$ due to the electron spin. It is this summand that determines all the features of electric and spin transport in conductive magnets having inhomogeneous magnetic structures. The last term $\hat{\mathcal{R}}$ on the left side of Eq. (1), called the collision integral, governs the relaxation of the quantum distribution function $\hat{f}$ to its instantaneous locally equilibrium value $\hat{f}_L$ and is expressed through the deviation $\delta\hat{f} = \hat{f} - \hat{f}_L$. The reader can find a specific form of the collision integral for the spin-orbit scattering mechanism in [10].

Being a quantum generalization of the well-known and widely used Boltzmann equation to the case of the presence of a spin moment of charge carriers, the quantum kinetic equation (1) for $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ is perhaps the simplest and most effective theoretical tool for studying charge and spin transport, conditional upon considering the orbital motion of electrons in the language of classical mechanics. Having solved the quantum kinetic equation for $\hat{f}(\mathbf{r}, \mathbf{p}, t)$, we can obtain material equations to relate the fluxes of electric charge and spin moment of electrons to external electric and magnetic fields that induce them.

The quantum distribution function $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ being the $2\times 2$ density matrix of a spin 1/2 can be represented as a linear combination of the unit matrix and Pauli matrices in the following way: $\hat{f}(\mathbf{r}, \mathbf{p}, t) = 1/2[n(\mathbf{r}, \mathbf{p}, t) + \mathbf{s}(\mathbf{r}, \mathbf{p}, t)\cdot\hat{\boldsymbol{\sigma}}]$. The newly introduced distribution functions $n(\mathbf{r}, \mathbf{p}, t)$ and $\mathbf{s}(\mathbf{r}, \mathbf{p}, t)$ are defined as $n(\mathbf{r}, \mathbf{p}, t) = \text{Tr}\hat{f}(\mathbf{r}, \mathbf{p}, t)$ and $\mathbf{s}(\mathbf{r}, \mathbf{p}, t) = \text{Tr}\hat{\boldsymbol{\sigma}}\hat{f}(\mathbf{r}, \mathbf{p}, t)$, respectively. From now on, $\text{Tr}\hat{M}$ implies the operation of taking the trace (spur) of the matrix $\hat{M}$. The function $n(\mathbf{r}, \mathbf{p}, t)$ has the meaning of the electron density distribution function. Having summed up $n(\mathbf{r}, \mathbf{p}, t)$ over all possible values of the momentum $\mathbf{p}$, we arrive at the density of the number of electrons at a given point in space at a given instant of time, $N(\mathbf{r}, t) = \sum_\mathbf{p} n(\mathbf{r}, \mathbf{p}, t)$. Summing up the product $\mathbf{v}n(\mathbf{r}, \mathbf{p}, t)$ over $\mathbf{p}$ yields the electron flux density $\mathbf{I}(\mathbf{r}, t) = \sum_\mathbf{p} \mathbf{v}n(\mathbf{r}, \mathbf{p}, t)$. The product $e\mathbf{I}(\mathbf{r}, t)$ is the electric current density $\mathbf{j}(\mathbf{r}, t)$. By analogy with $n(\mathbf{r}, \mathbf{p}, t)$, the function $\mathbf{s}(\mathbf{r}, \mathbf{p}, t)$ may be referred to as the spin density distribution function. Summing up $\mathbf{s}(\mathbf{r}, \mathbf{p}, t)$ over $\mathbf{p}$, we come up with the quantity $\mathbf{S}(\mathbf{r}, t) = \sum_\mathbf{p} \mathbf{s}(\mathbf{r}, \mathbf{p}, t)$. It may bear the name of the spin density. Summing up the tensor product of vectors $\mathbf{v}\otimes\mathbf{s}(\mathbf{r}, \mathbf{p}, t)$ over $\mathbf{p}$, we obtain the tensor quantity $\boldsymbol{J}(\mathbf{r}, t) = \sum_\mathbf{p} \mathbf{v}\otimes\mathbf{s}(\mathbf{r}, \mathbf{p}, t)$. It will be as the spin-current density tensor.

The spin-transport properties of systems in which the mean free path of conduction electrons is comparable to or exceeds the characteristic scale of the change in the fields $\tilde{\mathbf{B}}$ and $\mathbf{E}$, as well as the characteristic linear dimensions of the sample, should be described by the quantum kinetic equation (1). In the opposite limiting case when the mean free path of electrons is the smallest parameter of the length dimension, this problem can be significantly simplified by describing the above system through densities and flows instead of distribution functions. Then, Eq. (1) for the quantum distribution function produces a closed set of equations for the densities $N(\mathbf{r}, t)$, $\mathbf{S}(\mathbf{r}, t)$ and flows $\mathbf{I}(\mathbf{r}, t)$, $\boldsymbol{J}(\mathbf{r}, t)$. Methodological particulars for this set of equations to be derived from the quantum kinetic equation for $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ are given in the Supplemental Material [15]. Here, we present the resulting set of equations for $N(\mathbf{r}, t)$, $\mathbf{S}(\mathbf{r}, t)$, $\mathbf{I}(\mathbf{r}, t)$, $\boldsymbol{J}(\mathbf{r}, t)$:

$$\frac{\partial}{\partial t}N + \frac{\partial}{\partial\mathbf{r}}\cdot\mathbf{I} = 0, \tag{2}$$

$$\frac{\partial}{\partial t}\mathbf{S} + \frac{\partial}{\partial\mathbf{r}}\cdot\boldsymbol{J} + [\mathbf{S}\times(\boldsymbol{\Omega} + \tilde{\boldsymbol{\Omega}})] + \frac{1}{\tau_S}\delta\mathbf{S} = 0, \tag{3}$$

$$\frac{\partial}{\partial t}\mathbf{I} + \frac{v_F^2}{3}\frac{\partial}{\partial\mathbf{r}}\delta N - \frac{e}{m_e}\mathbf{E}N - [\boldsymbol{\Omega}_C\times\mathbf{I}] + \frac{\hbar}{2m_e}\left(\frac{\partial}{\partial\mathbf{r}}\otimes\tilde{\boldsymbol{\Omega}}\right)\cdot\mathbf{S} + \frac{1}{\tau_O}\mathbf{I} + \frac{1}{\tau_{SO}}{}^3\boldsymbol{\epsilon}\cdot\cdot\boldsymbol{J} = 0, \tag{4}$$

$$\frac{\partial}{\partial t}\boldsymbol{J} + \frac{v_F^2}{3}\frac{\partial}{\partial \mathbf{r}} \otimes \delta\mathbf{S} - \frac{e}{m_e}\mathbf{E}\otimes\mathbf{S} - [\boldsymbol{\Omega}_C \times \boldsymbol{J}] + [\boldsymbol{J}\times(\boldsymbol{\Omega}+\tilde{\boldsymbol{\Omega}})] + \frac{\hbar}{2m_e}\left(\frac{\partial}{\partial \mathbf{r}}\otimes\tilde{\boldsymbol{\Omega}}\right)\delta N + \frac{1}{\tau_O}\boldsymbol{J} + \frac{1}{\tau_{\rm SO}}{}^3\boldsymbol{\epsilon}\cdot\mathbf{I} = 0, \tag{5}$$

The quantity $\delta\mathbf{S} = \mathbf{S} - \mathbf{S}_L$ appearing in Eqs. (3)–(5) is the deviation of the spin density $\mathbf{S}$ from its local equilibrium value $\mathbf{S}_L = -\chi(\mathbf{B}+\tilde{\mathbf{B}})/\mu$, where $\chi$ is the Pauli susceptibility of the electron gas. Analogously, the quantity $\delta N = N - N_0$ is the deviation of the electron number density $N$ from its equilibrium value $N_0$. Equations (4) and (5) include the quantity ${}^3\boldsymbol{\epsilon}$ as an absolutely antisymmetric unit tensor of the third rank. The sign "⊗" denotes the tensor product of vectors, and the sign "··" stands for the double scalar product of tensors. Equations (3)–(5) introduce the notations $\boldsymbol{\Omega} = \gamma\mathbf{B}$, $\tilde{\boldsymbol{\Omega}} = \gamma\tilde{\mathbf{B}}$, $\boldsymbol{\Omega}_C = (|e|/m_e c)\mathbf{B}$, where $\gamma = 2\mu/\hbar$ is the gyromagnetic ratio. The frequencies $\Omega$ and $\tilde{\Omega}$ characterize the angular velocity of the electron spin precession in fields $\mathbf{B}$ and $\tilde{\mathbf{B}}$, respectively. The quantity $\Omega_C$ determines the angular velocity of the orbital motion of electrons along cyclotron orbits under the action of the Lorentz force in field $\mathbf{B}$. Expressions for the electron relaxation time $\tau_O$ during its orbital motion, spin relaxation time $\tau_S$, and relaxation time $\tau_{\rm SO}$ caused by asymmetric spin-orbit electron scattering, as well as for the quantity $v_F$ that has the meaning of the average electron velocity near the Fermi surface, can be found in [10,15].

The system at hand can be compiled more simply by disregarding the effects of time dispersion in solving Eqs. (4) and (5). In the process, the characteristic frequency of change in the internal and external fields is assumed to be small compared to the quasimomentum relaxation rate $v_O = 1/\tau_O$. Suppose that the cyclotron frequency $\Omega_C$; the Larmor frequencies $\Omega$, $\tilde{\Omega}$; and the asymmetric spin-orbit scattering velocity $v_{\rm SO} = 1/\tau_{\rm SO}$ are also small compared to $v_O$. In addition, we restrict ourselves to considering systems whose deviation from the state of electroneutrality can be ignored—$\delta N$ is thought to be negligible as compared to $N_0$. For conductors with a metallic nature of conductivity, this condition is almost exactly met. Under the above assumptions made, we can omit the first, second, fourth, and seventh terms on the left side of Eq. (4) and omit the first, fourth, fifth, sixth, and eighth terms in Eq. (5).

Let us introduce the magnetization $\mathbf{m} = -\mu\mathbf{S}$ of conduction electrons and the deviation of the electron magnetization from the local equilibrium state, $\delta\mathbf{m} = \mathbf{m} - \chi(\mathbf{B}+\tilde{\mathbf{B}})$. Given the approximations accepted, using Eq. (4), we come to an equation that determines the magnitude of the field $\mathbf{E}$ acting in a metal at a given electric current density $\mathbf{j}$:

$$\mathbf{E} = \rho_0\mathbf{j} + \mathbf{E}_{\rm HM}, \quad \mathbf{E}_{\rm HM} = -\frac{1}{eN_0}\left(\frac{\partial}{\partial \mathbf{r}}\otimes\tilde{\mathbf{B}}\right)\cdot\mathbf{m}, \tag{6}$$

where $\rho_0 = m_e/N_0 e^2\tau_O$ is a component of the specific electrical resistivity of the system under consideration, not related to helicoidal ordering. If the condition $E_{\rm HM}/\rho_0 j \ll 1$ holds true, the approximate equality $\mathbf{E} \approx \rho_0\mathbf{j}$ can be used for calculating the spin current from Eq. (5). As a result, we arrive at an expression for the spin-current tensor:

$$\boldsymbol{J} = \frac{D}{\mu}\frac{\partial}{\partial \mathbf{r}}\otimes\delta\mathbf{m} - \frac{1}{e\mu N_0}\mathbf{j}\otimes\mathbf{m}, \tag{7}$$

where $D = v_F^2\tau_O/3$ is the electron diffusion coefficient. It can be shown that taking the field component $\mathbf{E}_{\rm HM}$ into account reduces the diffusion coefficient along the spiral axis by $\delta D$, the relative value of which is $\delta D/D = 3\chi\tilde{B}^2/N_0 m_e v_F^2$. If one claims that the condition $\delta D/D \ll 1$ is always satisfied, the assumptions made above are justified. Plugging (7) into (3) yields the desired closed equation for $\delta\mathbf{m}$:

$$\frac{\partial}{\partial t}\mathbf{m} - D\left(\frac{\partial}{\partial \mathbf{r}}\right)^2\delta\mathbf{m} + \frac{1}{eN_0}\frac{\partial}{\partial \mathbf{r}}\cdot(\mathbf{j}\otimes\delta\mathbf{m}) + [\delta\mathbf{m}\times(\boldsymbol{\Omega}+\tilde{\boldsymbol{\Omega}})] + \frac{1}{\tau_S}\delta\mathbf{m} = -\frac{\chi}{eN_0}\frac{\partial}{\partial \mathbf{r}}\cdot(\mathbf{j}\otimes\tilde{\mathbf{B}}). \tag{8}$$

Equation (8) is valid for any configurations of the vectors of the inhomogeneous magnetization $\mathbf{M}$ and the field $\tilde{\mathbf{B}}$, altering their direction in space and time.

## III. SPIN DYNAMICS OF CONDUCTION ELECTRONS IN CHIRAL HELIMAGNETS

The present section applies the equations obtained above to describe the transport properties of conductive chiral helimagnets. Let us examine the following configuration of the system and external fields. Suppose that the wave vector $\mathbf{q}$ of the magnetic helix of a helimagnet, the vector of the external constant magnetic field $\mathbf{B}$, the vector of the electric field $\mathbf{E}$, and the vector of the density of the current $\mathbf{j}$ passing through the helimagnet are spatially oriented along the OZ axis, the direction of which is defined by the unit vector $\mathbf{e}_z$. Apart from the current density vector $\mathbf{j}$, the vector of the drift velocity $\mathbf{w} = \mathbf{j}/eN_0$ of electrons should be introduced. For characterizing the direction of the drift velocity vector $\mathbf{w}$, the unit vector $\mathbf{i} = \mathbf{w}/w$ is worth employing.

Consider a monoaxial helicoidal magnet. Suppose that its local magnetization $\mathbf{M}$ depends only on the $z$ coordinate, and the OZ axis rests on the screw axis of symmetry. We assume that the length of the magnetic helicoid vector $M = |\mathbf{M}|$ is external field independent and is a characteristic of the helicoidal state. Vector $\mathbf{M}$ can be represented as the sum $\mathbf{M} = \mathbf{M}_t + \mathbf{M}_\ell$, where $\mathbf{M}_t$ and $\mathbf{M}_\ell$ are the transverse (relative to the axis of the helicoid) and longitudinal components of the helicoid magnetization. Taking magnetic helicoids with the $z$-independent longitudinal magnetization $\mathbf{M}_\ell$ as an example, we characterize the direction of $\mathbf{M}_\ell$ by the unit vector $\mathbf{b}$. Then, $\mathbf{M}_\ell = \mathbf{b}M\sin\Theta$. A magnetic helicoid with a nonzero value of $M_\ell$ is referred to as a conical magnetic helix (CMH). There exists a configuration when $M_\ell \equiv 0$ and the magnetization $\mathbf{M}$ lies in the $XY$ plane, which corresponds to a "simple spiral" spin ordering.

When exposed to the magnetic field $\mathbf{B}$ directed along the axis of the helicoid, a simple spiral can transform into a conical one. Upon switching-on the field $\mathbf{B}$, the magnetization $\mathbf{M}$ deviates from the $XY$ plane by a finite angle $\Theta = \Theta(B)$. The latter's magnitude depends on the magnitude of the magnetic field $B = |\mathbf{B}|$, with the direction of the helicoid's longitudinal

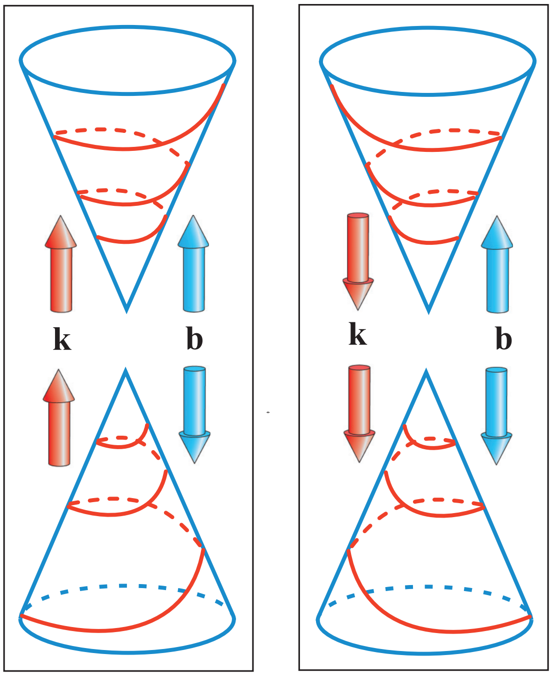


FIG. 1. Schematic representation of possible configurations of a conical magnetic helix, defined by the tensor $\boldsymbol{c} = \mathbf{k} \otimes \mathbf{b}$ and the $\mathbf{k}$ and $\mathbf{b}$ vectors. The left and right patterns reflect enantiomorphic states with symmetry of the group of right and left screws, respectively.

magnetization vector $\mathbf{M}_\ell$ coinciding with the direction of the field $\mathbf{B}$ assigned by the unit vector $\mathbf{b} = \mathbf{B}/B$.

The direction of the helicoid's transverse magnetization vector $\mathbf{M}_t$ is given by the unit vector $\mathbf{h} = \mathbf{h}(z)$. Then, the transverse magnetization is $\mathbf{M}_t = \mathbf{h}M\cos\Theta$. The dependence of the vector $\mathbf{h}$ on $z$ is assumed to be harmonic. For definiteness, let $\mathbf{h} = \mathbf{e}_x \cos qz \pm \mathbf{e}_y \sin qz$, where the signs "$\pm$" belong to the "right" and "left" helices, respectively.

To characterize the direction of the "twisting" of the magnetic helicoid, we introduce the unit vector $\mathbf{h}'$ that sets out the direction of the $\partial/\partial z\, \mathbf{h}(z)$ derivative vector through the relation $\partial/\partial z\, \mathbf{h}(z) = q\mathbf{h}'$. The direction of the twisting of the magnetic helix is unambiguously determined by the vector $\mathbf{k} = [\mathbf{h} \times \mathbf{h}']$ called the helimagnet chirality vector. Also, for convenience, the vector $\mathbf{q} = q\mathbf{k}$ and the scalar characteristic of the helix, $K = (\mathbf{k} \cdot \mathbf{e}_z)$, should be entered. They bear the name of the wave vector of the magnetic helix and the chirality of the helimagnet, respectively. A positive chirality value $K = +1$ corresponds to a right-handed helix, whereas a negative chirality $K = -1$ refers to a left-handed helix.

Let us look into the symmetry properties of a conical magnetic helix. The symmetry of CMH is defined by two unit vectors: the chirality vector $\mathbf{k}$ and the vector $\mathbf{b}$ that determines the direction of the axis of the magnetic cone. The common characteristic of CMH is the tensor dyadic product $\boldsymbol{c} = \mathbf{k} \otimes \mathbf{b}$. Figure 1 schematically shows four possible configurations of the $\mathbf{k}$ and $\mathbf{b}$ vectors together with a symbolic image of the cone of the corresponding magnetic spiral.

The continuum model of the chiral magnet offers the helicoid axis as the screw axis of infinite-order symmetry. Symmetry elements are rotations around the helical axis with simultaneous translation along it. The symmetry group is polar and enantiomorphic. In Fig. 1, the left pattern represents states with right screw symmetry. The right pattern involves the enantiomers of these states. The second-rank symmetric tensor $\boldsymbol{c} = \mathbf{k} \otimes \mathbf{b}$ is characterized by its three main invariants, with only the main invariant $\boldsymbol{e} \cdot\cdot\, \boldsymbol{c}$ being nonzero. Here, $\boldsymbol{e}$ is the second-rank unit tensor. It is easy to see that $\boldsymbol{e} \cdot\cdot\, \boldsymbol{c} = \mathbf{k} \cdot \mathbf{b}$. The vector $\mathbf{c}_z = (\mathbf{k} \cdot \mathbf{b})\mathbf{e}_z$ that defines two oppositely directed polar axes governs the scalar product of the tensor $\boldsymbol{c}$ and the unit vector $\mathbf{e}_z$. Hence, it follows that each of the symmetry groups of the right and left screws is naturally divided into subgroups. For one of these groups, the $\mathbf{k}$ and $\mathbf{b}$ vectors are codirectional ($\mathbf{k} \uparrow\uparrow \mathbf{b}$) and $\boldsymbol{e} \cdot\cdot\, \boldsymbol{c} = +1$, and for the others they are oppositely directed ($\mathbf{k} \uparrow\downarrow \mathbf{b}$) and $\boldsymbol{e} \cdot\cdot\, \boldsymbol{c} = -1$.

Under the above approximations, Eq. (8) for the magnetization $\delta\mathbf{m}$ appears as

$$\frac{\partial}{\partial t}\mathbf{m} - D\frac{\partial^2}{\partial z^2}\delta\mathbf{m} + (\mathbf{w}\cdot\mathbf{e}_z)\frac{\partial}{\partial z}\delta\mathbf{m} + [\delta\mathbf{m} \times (\boldsymbol{\Omega} + \tilde{\boldsymbol{\Omega}})] + \frac{1}{\tau_S}\delta\mathbf{m} = -\chi(\mathbf{w}\cdot\mathbf{e}_z)\frac{\partial}{\partial z}\tilde{\mathbf{B}}. \tag{9}$$

The solution to Eq. (9) for $\delta\mathbf{m}$ needs to be sought as the sum $\delta\mathbf{m} = \delta\mathbf{m}_\ell + \delta\mathbf{m}_t$. The first ($\delta\mathbf{m}_\ell$) and the second ($\delta\mathbf{m}_t$) summands are responsible for the longitudinal and transverse components, respectively. The former of the desired solution does not depend on coordinates and time. The latter should be striven for in the form of a helicoid with the wave vector $\mathbf{q}$. From (9), we obtain a set of coupled equations for $\delta\mathbf{m}_\ell$ and $\delta\mathbf{m}_t$. The $\delta\mathbf{m}_\ell$ component parallel to the helicoid axis relates the transverse magnetization $\delta\mathbf{m}_t$ directly by the equation

$$\delta\mathbf{m}_\ell = [\mathbf{h} \times \delta\mathbf{m}_t]\tau_S\Omega_{\mathrm{HM}}\cos\Theta, \tag{10}$$

where $\Omega_{\mathrm{HM}} = \gamma\Lambda M$. Plugging the resulting relationship (10) into the second equation of this set leads to a closed equation for $\delta\mathbf{m}_t$:

$$\frac{\partial}{\partial t}\mathbf{m}_t + [\delta\mathbf{m}_t \times (\Omega_B\mathbf{b} - Kqw\mathbf{i})] + (\nu_S + \nu_D)\delta\mathbf{m}_t + [[\mathbf{h} \times \delta\mathbf{m}_t] \times \mathbf{h}]\nu_L\cos^2\Theta = [\mathbf{h} \times \mathbf{i}]\chi\Lambda KqwM\cos\Theta, \tag{11}$$

where $\nu_S = 1/\tau_S$, $\nu_D = q^2D$, and $\nu_L = \Omega_{\mathrm{HM}}^2\tau_S$ are the characteristic frequencies of various processes of spin relaxation of conduction electrons in a helimagnet, $\Omega_B = \Omega + \Omega_{\mathrm{HM}}\sin\Theta$.

The $\nu_{\mathrm{HM}}$ sum of the three frequencies $\nu_S$, $\nu_D$, and $\nu_L$ controls the effective spin relaxation rate in the helimagnet: $\nu_{\mathrm{HM}} = \nu_S + \nu_D + \nu_L$. The contribution $\nu_S = 1/\tau_S$ is the spin-lattice relaxation rate caused by the dissipation of the nonequilibrium spin of conduction electrons by lattice defects. The contribution $\nu_D = q^2D$ describes the rate of change in the spin density of conduction electrons at a given point in space due to the "diffusion" escape of electron spins from this point. It should be underscored that spin diffusion in a conductive helimagnet is not a process of "flowing" of electrons with a given spin projection from one region to another due to the difference in electron concentration. In a helimagnet, the values of the nonequilibrium spin density vector at neighboring points of the helicoid's axis differ only in direction. Therefore, only the electron spin relaxation "in direction" obeys the diffusion. The $\nu_L = \Omega_{\mathrm{HM}}^2\tau_S$ component of the effective relaxation rate is due to the specific features of the precession motion of the spins of moving electrons in the effective exchange field of the helimagnet. The physical reason for the appearance of such a contribution lies in the Larmor spin precession of an electron traveling along the axis of the helicoid. In this case, the axis of the precession motion alters its direction. It is natural to call this mechanism of spin relaxation of conduction electrons in helimagnets as precessional.

The stationary solution to Eq. (11) has the form

$$\delta\mathbf{m} = \chi\Lambda M\cos(\Theta)\frac{(\nu_S+\nu_D)(\mathbf{q}\cdot\mathbf{w})[\mathbf{h}\times\mathbf{e}_z] + \varepsilon qw(\Omega_B-\varepsilon qw)\mathbf{h} - (1+\nu_D/\nu_S)\Omega_{\mathrm{HM}}\cos(\Theta)(\mathbf{q}\cdot\mathbf{w})\mathbf{e}_z}{\nu_B^2+(\Omega_B-\varepsilon qw)^2}, \tag{12}$$

where $\nu_B^2 = (\nu_S+\nu_D)(\nu_S+\nu_D+\nu_L\cos^2\Theta)$, $\varepsilon = {}^3\boldsymbol{e}\cdots[\mathbf{k}\otimes\mathbf{b}\otimes\mathbf{i}]$.

The newly introduced quantity $\varepsilon$ is defined as the triple scalar product of the third-rank unit tensor ${}^3\boldsymbol{e}$ and the triad $\mathbf{k}\otimes\mathbf{b}\otimes\mathbf{i}$ composed as the tensor product of three unit vectors $\mathbf{k}$, $\mathbf{b}$, and $\mathbf{i}$. Obviously, the quantity $\varepsilon$ is the only nonzero main invariant of the third-rank tensor $\mathbf{k}\otimes\mathbf{b}\otimes\mathbf{i}$ that characterizes the space-time symmetry of the properties of the helimagnet. The second-rank tensor $\boldsymbol{c} = \mathbf{k}\otimes\mathbf{b}$ introduced above characterizes the spatial symmetry of the conical magnetic helix, and, therefore, the quantity $\varepsilon$ can obviously be expressed through it: $\varepsilon = {}^3\boldsymbol{e}\cdots[\boldsymbol{c}\otimes\mathbf{i}]$. Having performed the triple scalar product operation, we can represent $\varepsilon$ in the following form:

$$\varepsilon = K(\mathbf{b}\cdot\mathbf{i}), \tag{13}$$

where $K = (\mathbf{e}_z\cdot\mathbf{k}) = \pm 1$ is the chirality of the helimagnet. This definition immediately implies that the chirality $K$ is a pseudoscalar. The $\mathbf{b}$ and $\mathbf{i}$ vectors are axial vectors. Their scalar product $(\mathbf{b}\cdot\mathbf{i})$ is a true scalar and, therefore, the quantity $\varepsilon$ defined in (13) is a pseudoscalar. The quantity $\varepsilon$ can take only two values: $\varepsilon = +1$ and $\varepsilon = -1$, depending on the mutual direction of the vectors $\mathbf{k}$, $\mathbf{b}$, and $\mathbf{i}$.

## IV. STERN-GERLACH ELECTRICAL MAGNETOCHIRAL EFFECT

Further discussion within the present paper includes, by default, the magnetic and transport properties of helimagnets that generate a conical magnetic helix under an applied external magnetic field. Based on the results obtained previously, it is not hard to examine the case of a conical spiral formed by internal fields of exchange origin. Substituting (12) into (6), we can find the relationship between the electric field and current density in the form $\mathbf{E} = \rho\mathbf{j}$, where the electrical resistivity $\rho$ is represented as the sum $\rho = \rho_0 + \delta\rho_B^{(\varepsilon)}$. The $\delta\rho_B^{(\varepsilon)}$ summand due to helimagnetism has the form

$$\delta\rho_B^{(\varepsilon)} = \rho_{\mathrm{HM}}R_B^{(\varepsilon)}, \quad \rho_{\mathrm{HM}} = \frac{\chi}{\nu_{\mathrm{HM}}}\left(\frac{q\Lambda M}{eN_0}\right)^2,$$
$$R_B^{(\varepsilon)} = \frac{\nu^2\cos^2\Theta}{\nu_B^2+(\Omega_B-\varepsilon qw)^2}. \tag{14}$$

Here $\nu^2 = (\nu_S+\nu_D)(\nu_S+\nu_D+\nu_L)$.

The $\delta\rho_B^{(\varepsilon)}$ quantity as a function of the magnetic field $B$ is evaluated within the range $0 \leqslant B \leqslant B_{\mathrm{FM}}$. The $B_{\mathrm{FM}}$ upper boundary of this range is the field of the phase transition from the system's helimagnetic state to the ferromagnetic one. At the phase transition point $B = B_{\mathrm{FM}}$, the quantity $R_B^{(\varepsilon)}$ vanishes and the total electrical resistance $\rho$ takes the value of $\rho_0$. Thus, the magnitude of $\rho_0$ determines the value of the electrical resistance $\rho$ at the point of the "helimagnet-ferromagnet" phase transition. This value is convenient to use as a reference level in experimentally studying the dependencies of the helimagnet's electrical resistance $\rho$ on a magnetic field. Therefore, theoretically, we define the specific magnetoresistance of the helimagnet as the difference $\rho - \rho_0$. Obviously, the $\delta\rho_B^{(\varepsilon)}$ quantity defined by relations (14) is the magnetoresistance of the helimagnet. In what follows, we will use the term "chiral magnetoresistance" (ChMR) to designate $\delta\rho_B^{(\varepsilon)}$ in the text.

The $\rho_{\mathrm{HM}}$ quantity appearing in relations (14) is the value of the ChMR for $B = 0$ in the limit of small measuring currents. This parameter is expressed only through the characteristics of the helimagnet. Consequently, the dimensionless quantity $R_B^{(\varepsilon)}$ can be ascribed to relative chiral magnetoresistance (RChMR). Since $\varepsilon$ can take only two values ($\varepsilon = \pm 1$), Eq. (14) constructs two possible magnetoresistance curves for the helimagnet: $R_B^{(+)}$ and $R_B^{(-)}$. The difference between $R_B^{(+)}$ and $R_B^{(-)}$ is a manifestation of electric magnetochiral anisotropy. For convenience, the quantity $R_B = (R_B^{(+)} + R_B^{(-)})/2$ and the magnetochiral anisotropy coefficient $A_B = (R_B^{(+)} - R_B^{(-)})/2R_B$ as a characteristic of the EMChA magnitude should be entered. Then,

$$R_B^{(\varepsilon)} = R_B(1+\varepsilon A_B), \tag{15}$$

where

$$A_B = \frac{2qw\Omega_B}{\nu_B^2+\Omega_B^2+(qw)^2}, \tag{16}$$

$$R_B = \frac{\nu^2\left[\nu_B^2+\Omega_B^2+(qw)^2\right]\cos^2\Theta}{\left[\nu_B^2+\Omega_B^2+(qw)^2\right]^2-(2qw\Omega_B)^2}. \tag{17}$$

Relying on (13)–(15), we arrive at the desired representation for the electrical resistance $\rho$:

$$\rho = \rho_0 + \rho_{\mathrm{HM}}R_B[1+KA_B(\mathbf{b}\cdot\mathbf{i})]. \tag{18}$$

Equation (18) itself is the most general formula of the electrical magnetochiral effect. Regarding $\rho$ as a function of three vectors $\mathbf{k}$, $\mathbf{b}$, and $\mathbf{i}$ at fixed values of the external magnetic field $B$ and the current density $j$ passing through the helimagnet, from (18) we obtain the following symmetry properties of the electrical resistance: $\rho(\mathbf{k},-\mathbf{b},-\mathbf{i}) = \rho(-\mathbf{k},\mathbf{b},-\mathbf{i}) = \rho(-\mathbf{k},-\mathbf{b},\mathbf{i}) = \rho(\mathbf{k},\mathbf{b},\mathbf{i})$.

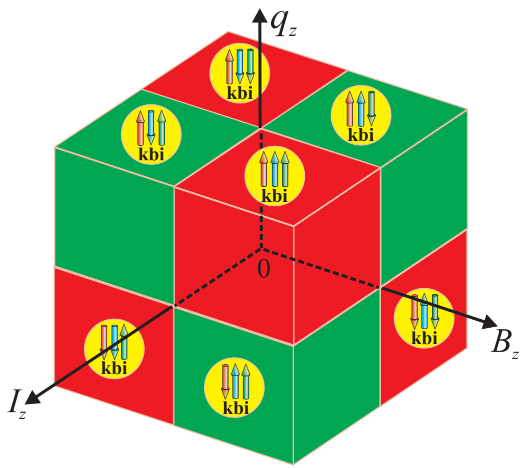


FIG. 2. A state diagram for a helimagnet in the variables $B_z$, $I_z$, $q_z$. Regions for $\varepsilon = +1$ are shown in red; regions for $\varepsilon = -1$ are shown in green. The colored arrows indicate the directions of the unit vectors **k**, **b**, and **i**.

Change in the sign of $\varepsilon$ as one of the vectors **k**, **b**, and **i** changes its direction results in the following effects:

(i) Nonreciprocity effects. They make themselves felt in changing the value of the electrical resistance of the helimagnet upon varying either the direction of current flowing through the sample relative to the direction of the external magnetic field or the direction of the external magnetic field relative to the direction of the current: $\rho(\mathbf{k}, \mathbf{b}, -\mathbf{i}) \neq \rho(\mathbf{k}, \mathbf{b}, \mathbf{i})$, $\rho(\mathbf{k}, -\mathbf{b}, \mathbf{i}) \neq \rho(\mathbf{k}, \mathbf{b}, \mathbf{i})$;

(ii) The effect of electric magnetochiral anisotropy (EMChAE). It exhibits itself in the difference in the electrical resistance for two helimagnet samples with different chirality. In this case, the chirality vector **k** is differently oriented relative to the fixed directions of the magnetic field and current: $\rho(-\mathbf{k}, \mathbf{b}, \mathbf{i}) \neq \rho(\mathbf{k}, \mathbf{b}, \mathbf{i})$.

Since $\rho$ is indeed a function of one variable $\varepsilon$ that consists of a combination of the **k**, **b**, and **i** vectors, the above nonreciprocity effects have the same physical nature. Therefore, they can be mathematically described by the same relation, $\rho(-\varepsilon) \neq \rho(\varepsilon)$.

It is worth recalling that the physical cause of the appearance of the chiral magnetoresistance $\delta\rho_B^{(\varepsilon)}$ is to impact the spatially inhomogeneous exchange magnetic field on the conduction electrons carrying spin. Interaction between the spin of moving particles and an inhomogeneous magnetic field was first experimentally recorded in the famous Stern-Gerlach experiment [11]. In order to emphasize the physical nature of *the mechanism* of the electric magnetochiral effect described by Eqs. (16)–(18), we propose introducing the special term "Stern-Gerlach electrical magnetochiral effect" (SGEMChE) for its designation.

For visualizing the conditions for observing SGEMChE, Fig. 2 illustrates schematically the regions of the state diagram of a helimagnet in the variables $B_z$, $I_z$, $q_z$, which correspond to two different values of the parameter $\varepsilon$. Regions for $\varepsilon = +1$ are shown in red; regions for $\varepsilon = -1$ are shown in green.

Thus, the mechanism of interaction between the spin of conduction electrons and the inhomogeneous internal exchange field of the helimagnet naturally explains the fact of the existence of the magnetochiral anisotropy effect.

The issue of the magnitude of magnetochiral anisotropy and its dependence on the magnetic field deserves a separate discussion. Let $\Omega_{\rm FM} = \gamma B_{\rm FM}$ be the value of the frequency $\Omega$ of the Larmor precession of conduction electrons in the "helimagnet-ferromagnet" transition field $B = B_{\rm FM}$. Consequently, the value of the frequency $\Omega_B$ at the point $B = B_{\rm FM}$ is equal to $\Omega_{B_{\rm FM}} = \Omega_{\rm FM} + \Omega_{\rm HM}$. In this field, the spin relaxation frequency $\nu_B$ is $\nu_{B_{\rm FM}} \equiv \nu_S + \nu_D$ and the magnetochiral anisotropy coefficient $A_B$ takes the value of $A_{B_{\rm FM}} = 2qw\Omega_{B_{\rm FM}}/[\nu_{B_{\rm FM}}^2 + \Omega_{B_{\rm FM}}^2 + (qw)^2]$. In the event of the sufficiently large frequencies $\nu_S$, $\nu_D$, and $\nu_L$ and fulfilling the conditions $\Omega_{B_{\rm FM}}$, $qw \ll \nu_S$, $\nu_D$, and $\nu_L$, the magnetochiral anisotropy coefficient $A_B$ is small compared to unity for any magnetic field values: $A_{B_{\rm FM}} \approx 2qw\Omega_{B_{\rm FM}}/\nu_{B_{\rm FM}}^2$. If the spin relaxation rates $\nu_S$, $\nu_D$, and $\nu_L$ are small compared to $\Omega_{B_{\rm FM}}$ and $qw$, the asymptotic equality $A_{B_{\rm FM}} \approx 2qw\Omega_{B_{\rm FM}}/[\Omega_{B_{\rm FM}}^2 + (qw)^2]$ holds true for $A_{B_{\rm FM}}$. It is easy to see that the quantity $A_{B_{\rm FM}}$ turns out to be close to unity under the condition $qw = \Omega_{B_{\rm FM}}$. Thus, the magnetochiral anisotropy coefficient $A_B$ reaches its maximum possible value equal to unity near the phase transition point $B = B_{\rm FM}$ when the conditions $\nu_S, \nu_D, \nu_L \ll \Omega_{B_{\rm FM}}, qw$ are satisfied. In other words, in this case, the theory predicts the possibility of the existence of a giant electrical magnetochiral effect (GEMChE). A similar effect of abnormal nonreciprocal electric transport is observed experimentally in a chiral magnet, $CrNb_3S_6$ [8].

Under GEMChE conditions, very unusual behavior of the magnetochiral magnetoresistance $R_B^{(\varepsilon)}$ will be observed. More precisely, this concerns only the magnetoresistance branch $R_B^{(+)}$. It is easy to see from Eq. (14) that the value of the magnetoresistance $R_B^{(+)}$ is expected to rise in a resonant manner. The resonant growth will take place in helimagnets with small values of the spin relaxation frequency $\nu_B$ once the precession frequency $\Omega_B$ approaches the value of $qw$. On the contrary, the $R_B^{(-)}$ branch exhibits no resonance. Recall that the magnetoresistance $R_B^{(+)}$ describes such field configurations in the helimagnet that the quantity $\varepsilon = {}^3\boldsymbol{e} \cdots [\mathbf{k} \otimes \mathbf{b} \otimes \mathbf{i}] = +1$. This equality holds for all triples of collinear vectors **k**, **b**, and **i**, with one of the vectors being codirectional to the vector $\mathbf{e}_z$ and the directions of two others coinciding. In particular, $\varepsilon = +1$ provided that all four vectors **k**, **b**, **i**, and $\mathbf{e}_z$ are codirected.

The physics of the effect of a resonant increase in magnetoresistance for configurations with $\varepsilon = +1$ is absolutely clear. Let all vectors **k**, **b**, **i**, and $\mathbf{e}_z$ be codirectional. The magnetic moment of an electron moving with an average speed $w$ along the helicoid's axis precesses with the frequency $\Omega_B$. In this case, the transverse component of the magnetic moment rotates clockwise. In a coordinate system moving together with electrons at a speed $w$, the helicoid's transverse magnetization vector with a wave vector $q$ rotates clockwise with an angular velocity $\omega_w = qw$. Once the frequencies $\Omega_B$ and $\omega_w$ coincide, the precession of the magnetic moments of the conduction electrons moving at a speed $w$ becomes in phase with the rotation of the magnetization of the helicoid, followed by a resonant increase in the magnetoresistance.

Here, a direct analogy may be drawn with the conduction-electron spin resonance (CESR) phenomenon. The coincidence of the Larmor electron-precession frequency and frequency $\omega$ gives rise to a resonant absorption of the energy of an alternating external magnetic field of frequency $\omega$ by conduction electrons. In the case of a helimagnet, electrons move with a speed $w$ and along the wave vector $\mathbf{q}$; therefore, the frequency $\omega$ is replaced by the frequency $\omega_w = qw$. As far as the resonant behavior of the chiral magnetoresistance of a helimagnet is concerned, the analogy specified above makes it possible to give the name "magnetochiral kinetic resonance" to the effect detected.

For real helimagnets, the relaxation frequencies $\nu_S$, $\nu_D$, and $\nu_L$ can differ quite significantly both relative to each other and relative to the precession frequencies $\Omega_{\rm FM}$ and $\Omega_{\rm HM}$. Therefore, generally speaking, the behavior of magnetoresistance cannot be boiled down to the two cases described above. Next, some simplifying assumptions of a nonessential nature need to be made to illustrate the possible variability of the behavior of the magnetochiral anisotropy coefficient $A_B$ and the helimagnet's chiral magnetoresistance branch $R_B^{(\pm)}$. Under the assumptions made, the explicit dependencies of $A_B$ and $R_B^{(\pm)}$ on a magnetic field can be analytically constructed, without changing the qualitative picture of their behavior.

Let us further examine the dependence of the angle $\Theta$ characterizing the conical magnetic helix on the magnetic field $B$. The shape of the curve $\Theta(B)$ depends on the parameters of the exchange interaction of the helimagnet's localized spin moments, including both the symmetric Heisenberg exchange and the antisymmetric Dzyaloshinskii-Moriya exchange. Additionally, the curve $\Theta(B)$ is also affected by the type of magnetic crystalline anisotropy in a helimagnet. The helimagnet model that offers the Dzyaloshinskii-Moriya exchange interactions in the nearest-neighbor approximation and the second-order magnetic monoaxial anisotropy produces the simplest shape of $\Theta(B)$ as $\Theta(B) = \arcsin(B/B_{\rm FM})$. Such an approximation for $\Theta(B)$ yields a simple linear dependence of the precession frequency $\Omega_B$ on the magnetic field: $\Omega_B = \Omega_{B_{\rm FM}}(B/B_{\rm FM})$. As a result, the behavior of $A_B$ and $R_B^{(\pm)}$ as functions of the relative magnetic field $\beta = B/B_{\rm FM}$ is described by the following simple formulas:

$$A_B = \frac{2\Psi\beta}{\Upsilon^2 + \Gamma^2(1-\beta^2) + \Psi^2 + \beta^2}, \tag{19}$$

$$R_B^{(\pm)} = \frac{(\Upsilon^2 + \Gamma^2)(1-\beta^2)}{\Upsilon^2 + \Gamma^2(1-\beta^2) + (\beta \mp \Psi)^2}. \tag{20}$$

Here, we have entered the dimensionless parameters $\Upsilon = (\nu_S + \nu_D)/\Omega_{B_{\rm FM}}$, $\Gamma = \sqrt{\nu_L(\nu_S + \nu_D)}/\Omega_{B_{\rm FM}}$, and $\Psi = qw/\Omega_{B_{\rm FM}}$. Parameters $\Upsilon$ and $\Gamma$ control the relative intensity of various spin relaxation processes in a helimagnet, whereas the parameter $\Psi$ reflects the relative magnitude of the electron drift velocity.

Further consideration requires numerical estimates of the $\Gamma$, $\Upsilon$, and $\Psi$ parameters typical of metallic helimagnets. Dysprosium can serve as an example of such materials. Taking the exchange-integral value equal to $\mathcal{J} \sim 0.1$ eV, we obtain an estimate of the frequency $\Omega_{\rm HM} \sim 10^{14}\ {\rm s}^{-1}$. For dysprosium, $M \sim 0.3$ T, which corresponds to the exchange interaction parameter $\Lambda \sim 10^4$. Suppose that the characteristic field of the phase transition from the system's helimagnetic state to the ferromagnetic one is $B_{\rm FM} \sim 1$ T. Then, an estimate of the frequency $\Omega_{\rm FM} \sim 10^{10}\ {\rm s}^{-1}$. Hence, it follows that $\Omega_{\rm FM} \ll \Omega_{\rm HM}$ and $\Omega_{B_{\rm FM}} \approx \Omega_{\rm HM}$. In the literature, there is no information on the spin-lattice relaxation rates $\nu_S$ in helimagnets. A lower order-of-magnitude estimate for $\nu_S$ can be obtained using experimental data on measurements of the spin resonance of conduction electrons in noble metals, for which the spin-lattice relaxation rate is $\sim 10^{11}\ {\rm s}^{-1}$. As an upper order-of-magnitude estimate for $\nu_S$, the momentum relaxation rate $\nu_O = 1/\tau_O$ equal to $\nu_O \sim 10^{14}\ {\rm s}^{-1}$ can be utilized. Let $\nu_S$ lie in the interval of $10^{12} - 10^{13}\ {\rm s}^{-1}$. Then, we have $\nu_S/\Omega_{\rm HM} \ll 1$. The diffusion relaxation rate $\nu_D = q^2 D$ is difficult to unambiguously estimate. This is because both the wave number $q$ of the magnetic helicoid and the diffusion coefficient $D$ may be temperature dependent. Therefore, for dysprosium, the temperature change from $T_C = 85$ K (the Curie temperature) to $T_N = 179$ K (the Néel temperature) varies the period $\lambda$ of the helicoid from 40 to 25 Å. As a consequence, the wave number $q$ of the magnetic helicoid grows. The quantity $D = v_F^2 \tau_O/3$ behaves similarly to electrical conductivity—it increases with dropping the temperature due to an increment in the momentum relaxation time $\tau_O$. For estimation, we use the average values $q$ and $\tau_O$. For this, we set $q \sim 10^7\ {\rm cm}^{-1}$, $\tau_O \sim 10^{-14}$ s. Finally, the relaxation frequency $\nu_D$ is evaluated as $\nu_D \sim 10^{16}\ {\rm s}^{-1}$. Hence, it immediately follows that $\nu_D/\nu_S \gg 1$ and $\nu_D/\Omega_{\rm HM} \gg 1$.

The parameter $\Psi$ can be rewritten as the ratio between the velocity $w$ and the value $\Omega_{\rm HM}/q$ (numerically, $\Omega_{\rm HM}/q \sim 10^7$ cm/s). The average electron velocity $w$ at the achievable current density is always much less than this value and, therefore, $\Psi \ll 1$. As a result, the theory parameters for metallic helimagnets can be estimated as $\Upsilon \sim \nu_D/\Omega_{\rm HM} \gg 1$, $\Gamma \sim \sqrt{\nu_D/\nu_S} \gg 1$, $\Psi \sim qw/\Omega_{\rm HM} \ll 1$.

Now, we may estimate the parameter $\rho_{\rm HM}$ relative to $\rho_0$ to further calculate the numerical value of the ChMR $\delta\rho_B^{(\varepsilon)}$. Given that $\nu_S \ll \nu_D \ll \nu_L$, the definition (14) implies that $\rho_{\rm HM}/\rho_0 \sim (\nu_S/\nu_O)(q/k_F)^2$, where $k_F \sim 10^8\ {\rm cm}^{-1}$ is the wave number of electrons on the Fermi surface. For the adopted values of the parameters $\nu_S$, $\nu_O$, and $q$, the estimate appears as the following: $\rho_{\rm HM}/\rho_0 \sim 10^{-3} \div 10^{-2}$. Thus, the chiral magnetoresistance $\rho_{\rm HM}$ in metallic helimagnets is two to three orders of magnitude less than $\rho_0$. However, this difference is quite enough to observe SGEMChE experimentally.

Under the conditions $\Upsilon \gg 1$, $\Gamma \gg 1$, and $\Psi \ll 1$, the behavior of $R_B^{(\pm)}$ will be described by the following formulas:

$$R_B^{(\pm)} = R_B(1 \pm A_B), \tag{21}$$

where

$$A_B \approx A\frac{\beta}{1 + \alpha(1-\beta^2)}, \tag{22}$$

$$R_B \approx 1 - \frac{\beta^2}{1 + \alpha(1-\beta^2)}. \tag{23}$$

Formulas (22) and (23) contain the parameters $A = 2qw(\Omega_{\rm FM} + \Omega_{\rm HM})/(\nu_S + \nu_D)^2$ and $\alpha = (\Omega_{\rm FM} + \Omega_{\rm HM})^2/\{\nu_S(\nu_S + \nu_D)\}$ are dependent on the

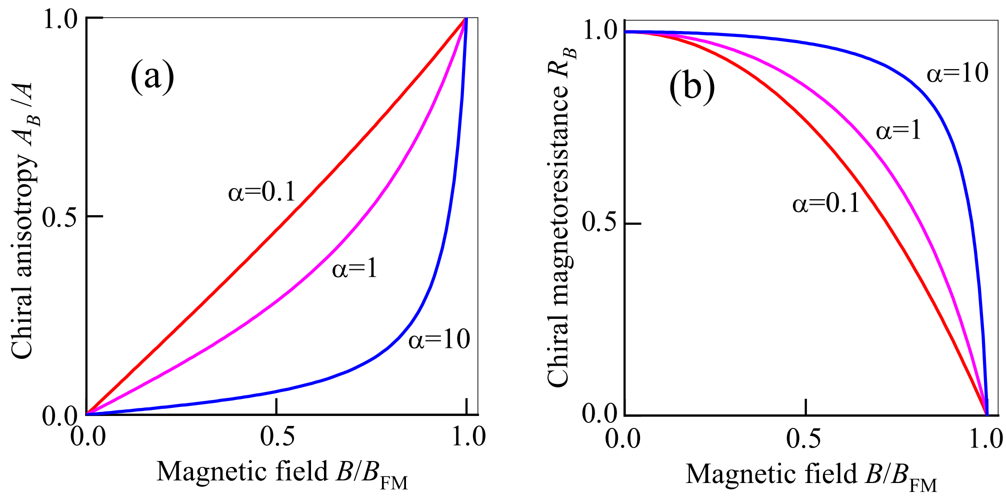


FIG. 3. (a) The relative magnetochiral anisotropy $A_B/A$ and (b) the relative magnetoresistance $R_B$ as a function of the reduced magnetic field $\beta = B/B_{FM}$ for the values of the parameters $\alpha = 0.1$, 1.0, and 10.

characteristics of the helimagnet. For metal helimagnets, the magnitude of the magnetochiral anisotropy coefficient $A_B$ is always small compared to unity due to the smallness of the parameter $A$; it monotonically increases with increasing the magnetic field strength until reaching its maximum possible value for $\beta = 1$. The magnetoresistance $R_B$ is a steadily decreasing function of the magnetic field and vanishes in the field of the helimagnet transition to the ferromagnetic state. The shape of the field $B$ dependencies of the quantities $A_B$ and $R_B$ is determined only by the value of the parameter $\alpha$. The estimates specified above show that the parameter $\alpha$ can be either greater or less than unity. For various values of the parameter $\alpha$, Fig. 3 sketches the dependencies of the relative quantities $A_B/A$ and $R_B$ on the reduced magnetic field $\beta = B/B_{FM}$.

## V. STERN-GERLACH KINETIC MAGNETOELECTRIC EFFECT

The issue of the longitudinal conduction-electron magnetization induced in a helimagnet by an electric current flowing along the axis of the helicoid (KMEE) deserves a separate discussion. From expression (12), the longitudinal magnetization induced by the electric current can be written as $\delta\mathbf{m}_\ell = -m_B(K\mathbf{i} + A_B\mathbf{b})$, where $m_B = \chi(qw/\gamma)(\Omega^2_{HM}/\nu_S\nu_{HM})R_B$, and $R_B$ and $A_B$ are the solutions to Eqs. (16) and (17).

The expression obtained for $\delta\mathbf{m}_\ell$ yields an important finding—the longitudinal electron magnetization is nonzero even in the absence of a magnetic field, when $A_B = 0$. The current flowing through the helimagnet samples induces longitudinal magnetization, with its vector having a different direction relative to the current flow direction for different chiralities. This result is in full agreement with that predicted in [3]. In a right-handed magnetic helix ($K = +1$), the flow of electrons moving in the $\mathbf{i}$ direction induces the longitudinal magnetization $\delta\mathbf{m}_\ell$. Its vector and the electron flow vector are pointing in opposite directions: $\delta\mathbf{m}_\ell \uparrow\downarrow \mathbf{i}$, whatever the flow direction $\mathbf{i}$ is. In the left-handed helix ($K = -1$), the vectors $\delta\mathbf{m}_\ell$ and $\mathbf{i}$ turn out to be codirectional: $\delta\mathbf{m}_\ell \uparrow\uparrow \mathbf{i}$, regardless of the flow direction $\mathbf{i}$.

The total magnetization of the helimagnet in an arbitrary magnetic field is composed of the magnetization $\mathbf{M}$ of localized electrons and the conduction-electron magnetization $\mathbf{m}{=}\chi(\mathbf{B} + \tilde{\mathbf{B}}) + \delta\mathbf{m}$. The longitudinal component $\boldsymbol{\mathcal{M}}$ of the total magnetization is given by

$$\boldsymbol{\mathcal{M}} = -m_B K\mathbf{i} + M_B\mathbf{b}, \tag{24}$$

where $M_B = M\sin\Theta + \chi(B + \Lambda M\sin\Theta) - A_B m_B$. Consequently, Eq. (24) provides the following symmetry properties of the total longitudinal magnetization $\boldsymbol{\mathcal{M}}$ as a function of the three unit vectors $\mathbf{k}$, $\mathbf{b}$, and $\mathbf{i}$:

$$\begin{aligned}&\boldsymbol{\mathcal{M}}(-\mathbf{k}, \mathbf{b}, -\mathbf{i}) = \boldsymbol{\mathcal{M}}(\mathbf{k}, \mathbf{b}, \mathbf{i}),\\ &\boldsymbol{\mathcal{M}}(-\mathbf{k}, -\mathbf{b}, \mathbf{i}) = \boldsymbol{\mathcal{M}}(\mathbf{k}, -\mathbf{b}, -\mathbf{i}) = -\boldsymbol{\mathcal{M}}(\mathbf{k}, \mathbf{b}, \mathbf{i}).\end{aligned} \tag{25}$$

Equation (24) describes the nonreciprocity effect that manifests itself in changing the value of the helimagnet's longitudinal magnetization with changing the direction of current passing through the sample: $\boldsymbol{\mathcal{M}}(\mathbf{k}, \mathbf{b}, -\mathbf{i}) \neq \boldsymbol{\mathcal{M}}(\mathbf{k}, \mathbf{b}, \mathbf{i})$. Besides, this equation governs EMChAE. The latter indicates that, for two helimagnets with different chiralities, the longitudinal magnetization values are different, all other things being equal: $\boldsymbol{\mathcal{M}}(-\mathbf{k}, \mathbf{b}, \mathbf{i}) \neq \boldsymbol{\mathcal{M}}(\mathbf{k}, \mathbf{b}, \mathbf{i})$.

The effect of magnetochiral anisotropy of longitudinal magnetization can be used for experimentally determining the chirality of helimagnet samples. For this purpose, the total longitudinal magnetization in two different directions of the magnetic field needs to be measured. As the vector characteristic of the magnetochiral anisotropy of longitudinal magnetization, it is convenient to choose the sum of the magnetizations $\bar{\boldsymbol{\mathcal{M}}} = \boldsymbol{\mathcal{M}}(\mathbf{k}, \mathbf{b}, \mathbf{i}) + \boldsymbol{\mathcal{M}}(\mathbf{k}, -\mathbf{b}, \mathbf{i})$. From (24), it is easy to see that $\bar{\boldsymbol{\mathcal{M}}} = -2m_B K\mathbf{i}$. When measuring the $\boldsymbol{\mathcal{M}}(\mathbf{k}, \mathbf{b}, \mathbf{i})$ and $\boldsymbol{\mathcal{M}}(\mathbf{k}, -\mathbf{b}, \mathbf{i})$ vectors, the direction of the vector $\bar{\boldsymbol{\mathcal{M}}}$ turns out to be independent of the magnetic field direction. The direction of the total magnetization $\bar{\boldsymbol{\mathcal{M}}}$ experimentally determined as codirectional with the direction of the electric current $\mathbf{j}$ stands for a helimagnet's positive chirality ($K = +1$)

corresponding to a right-handed magnetic helix. Conversely, if $\bar{\boldsymbol{\mathcal{M}}} \uparrow\downarrow \mathbf{j}$, the chirality of such a helimagnet is $K = -1$. For a helimagnet of any chirality, the absolute value of $\bar{\boldsymbol{\mathcal{M}}}$ of the magnetization $\bar{\mathcal{M}}$ is proportional to the current flowing through the helimagnet. Recently, experimental evidence for the existence of a kinetic magnetoelectric effect has been afforded in the paper [16] on investigating nuclear magnetic resonance in tellurium as a nonmagnetic semiconductor with strong spin-orbit interaction and having a chiral atomic ordering.

As follows from Eq. (24), the magnitude of the longitudinal magnetization is $\mathcal{M} = M_B - \varepsilon m_B$. At fixed values of the magnetic field and current, $\mathcal{M}$ takes two values: $\mathcal{M}_B^{(+)}$ and $\mathcal{M}_B^{(-)}$. The magnitude of the magnetochiral anisotropy in the kinetic magnetoelectric effect can be directly characterized through the difference between the magnetizations $\mathcal{M}_B^{(-)}$ and $\mathcal{M}_B^{(+)}$, equal to $\Delta\mathcal{M}_B = \mathcal{M}_B^{(-)} - \mathcal{M}_B^{(+)} = 2m_B$. When $B = 0$, the value of $\Delta\mathcal{M}_B$ reaches its maximum and amounts to $\Delta\mathcal{M}_0 = 2m_0$. For metallic helimagnets $\Delta\mathcal{M}_0 \approx 2\chi q w/\gamma$. The magnetizations $\mathcal{M}_B^{(\pm)}$ being linearly related to the chiral magnetoresistance $R_B^{(\pm)}$, all appropriate conclusions about the possibility of observing MChKR when studying the KMEE can be immediately ascribed to $\mathcal{M}_B^{(\pm)}$ as well (see previous section dedicated to EMChE).

## VI. CONCLUSIONS

In this paper, we have demonstrated that the interaction between the spins of conduction electrons and spatially inhomogeneous effective magnetic fields of exchange origin in chiral helimagnets provides a natural explanation for two physical effects: the electrical magnetochiral effect and the kinetic magnetoelectric effect. The manifestation of the interaction of the spin of moving particles with an inhomogeneous magnetic field was first experimentally recorded in the famous Stern-Gerlach experiment [11]. We have shown that both of these effects, EMChE and KMEE, have the same physical origin. The manifestation of these effects is characterized by the symmetry properties of the third-rank tensor $\mathbf{k} \otimes \mathbf{b} \otimes \mathbf{i}$. The latter involves a tensor product of three vectors: the helimagnet chirality vector $\mathbf{k}$, the vector $\mathbf{b}$ of the direction of the external magnetic field, and the vector $\mathbf{i}$ of the electron flow direction. All nonreciprocity effects in chiral helimagnets are "controlled" by the behavior of the pseudoscalar quantity $\varepsilon = {}^3\boldsymbol{e} \cdots [\mathbf{k} \otimes \mathbf{b} \otimes \mathbf{i}]$. It is defined as the triple scalar product of the third-rank unit tensor ${}^3\boldsymbol{e}$ and the tensor $\mathbf{k} \otimes \mathbf{b} \otimes \mathbf{i}$. This peculiarity of magnetochiral effects in helimagnets is explicitly expressed in terms of the chirality parameter $K = \pm 1$ by the relation $\varepsilon = K(\mathbf{b} \cdot \mathbf{i})$.

All parameters of the EMChE and KMEE are presented through the characteristic frequencies of spin relaxation of conduction electrons and the frequencies of their Larmor precession. It has been shown that the effective frequency $\nu_{\rm HM}$ of electron spin relaxation in the helimagnet includes three components: $\nu_S$, $\nu_D$, and $\nu_L$. The contribution of $\nu_S$ is the rate of spin-lattice relaxation caused by the scattering of conduction electrons by defects in the crystal lattice due to the presence of spin-orbit interaction. The $\nu_D$ summand describes the rate of change in the average electron spin upon "diffusively" escaping the electrons from a region with a given direction of the average spin into a region with a different direction of spin density. The $\nu_L$ component appears due to the specific features of the Larmor precession motion of the spins of moving electrons in the helimagnet's effective exchange field when the axis of the precession motion changes direction as the electron travels along the helicoid's axis. The peculiarities of the EMChE and KMEE depend on the ratio of the above spin relaxation frequencies and the angular frequencies of electron precession in a constant external magnetic field and in an effective internal exchange field.

Our numerical estimates show that the mechanism of generating EMChE provides the effect magnitude sufficient to be experimentally detected in metallic helimagnets. The frequency regions of spin relaxation and spin precession are determined to observe a giant electrical magnetochiral effect. Under GEMChE conditions, the magnetoresistance of a helimagnet exhibits resonant behavior. We have called the appropriate effect "magnetochiral kinetic resonance". The physical nature of MChKR has been revealed. It happens once the Larmor electron-precession frequency in the effective field coincides with the phase change frequency of the helicoidal exchange field. In the process, an electron undergoing the rotating exchange field with the above-mentioned frequency moves along the helicoid's axis with a speed equal to that of the electron flow.

We have described the manifestations of electric magnetochiral anisotropy in the kinetic magnetoelectric effect. It has been shown that experimental data on the KMEE can be applied to directly determine the chirality of helimagnets tested.

The effects mentioned in this work are caused by the formation of a long-period magnetic structure of the "conical spiral" type in a helimagnet subjected to an external magnetic field. There exist helimagnets that display magnetic ordering as a conical spiral in the absence of an external magnetic field. Examples of such "conical" helimagnets include holmium and erbium. All the effects predicted in the present paper for systems with a conical spiral formed under the action of an external magnetic field will be observed in conical helimagnets in the absence of an external magnetic field as well.

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