Suppression of superfluidity by dissipation: An application to failed superconductors

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The ground states of bosons have been classified into superfluids, bosonic quantum Hall states, Mott insulators, and Bose glass. Recent experiments in two-dimensional clean superconductors under an external magnetic field B strongly suggest the existence of the fourth quantum state of Cooper pairs, i.e., Bose metal or quantum metal, where the resistivity remains constant at lowest temperature. However, its theoretical understanding remains unsettled. In this paper, we propose a model where the vortices behave as quantum bosons subject to dissipation at the normal core. We discuss that those bosons remain metallic even at zero temperature by generalizing the Feynman picture of superfluidity in the first quantization formulation and also by a perturbative calculation for a field theoretical model in the second quantization formalism. This result indicates that the resistivity ρ of Bose metal at zero temperature behaves as $\rho \sim \rho_n(B/B_{c2})$ with ρ_n being the residual resistivity in the normal state and B_{c2} the second upper critical field. It also predicts that the Bose metal is missing in superclean superconductors and at the dirty limit or in granular superconductors.

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I. INTRODUCTION

Recent experiments show the possible metallic state down to the lowest temperature with variable resistivity in clean two-dimensional superconductors under an external magnetic field, in sharp contrast to the conventional picture that the metallic state appears only at the quantum critical point between the insulator and superconductor [1]. Superconductors in this metallic phase are called "failed superconductors". Since we are interested in the temperature region much lower than the superconducting gap, the Cooper pairs can be regarded as charge $2e$ bosons, and hence the problem is regarded as that of the bosons. Under an external magnetic field B, the vortices are relevant to the transport properties. The vortex is accompanied with the winding of the phase θ of the Cooper pair, and the vortex motion results in the time dependence of θ and the voltage drop via the Josephson relation. Therefore, there is a relation between the conductivities σ_{Cooper} and σ_{vortex} of two models in two dimensions, i.e., $\sigma_{\text{Cooper}}\sigma_{\text{vortex}} = (\frac{(2e)^2}{h})^2$ with h being the Planck constant [2–4]. Therefore, one can discuss the conductivity of the system by analyzing the dynamics of the quantum vortices, which act as the repulsive bosons. Under an external magnetic field, the number of vortices is that of the magnetic flux measured in units of $\phi_0 = hc/(2e)$, and the many-body ground state of this vortex system is the keen issue to understand the Bose metal.

Although the vortices behave basically as bosons, there is an important difference. The vortex is a composite particle, where the order-parameter amplitude vanishes at the core. There are bound states at the core, the energy separation of which is $\delta \sim \Delta^2/\varepsilon_F$ with Δ being the superconducting gap and ε_F the Fermi energy [5]. The broadening $\hbar/\tau$ due to the finite lifetime by impurity scattering gives another energy scale. When $\hbar/\tau \ll \delta$, the system is called "superclean," but typically δ is extremely small for the low-temperature superconductors and this case is very rare. For $\delta \ll \hbar/\tau \ll \Delta$, the system is called "clean," while it is "dirty" for $\Delta \ll \hbar/\tau$. We are interested in the situation of clean two-dimensional superconductors, where the bound states at the core of each vortex constitute the continuous spectra with finite density of states at zero energy. The particle-hole excitation at the normal core [6,7] can be regarded as a heat bath with a continuum spectrum, which causes the dissipation associated with the motion of the vortex as Bardeen and Stephen discussed [8]. This dissipation is taken into account by the method introduced by Caldeira and Leggett [9], the coupling to harmonic oscillators.

There are many papers on the dissipative XY model [10–13], describing the dynamics of the resistively shunted Josephson-junction array [14,15]. However, the XY model is an effective model of bosons only at integer fillings [16,17]. Away from integer fillings, e.g., in the dilute limit, the action contains the first-order time derivative term [18], which is complex, and the Monte Carlo study in the phase representation is difficult because of the sign problem. This difference is important in the context of the positive magnetoresistance of failed superconductors, since in the dilute limit the number of vortices changes continuously as we increase the magnetic field. Also, the effect of dissipation on the dilute boson system has been studied in Ref. [19], but that study is only for the one-dimensional system. The analytical argument we will discuss here is different from that argument relying on bosonization, which is valid only in the one-dimensional system.

The bosonic system at zero temperature is known to be a perfect superfluid, i.e., $\rho_s = \rho$, where ρ_s is the superfluid density and ρ is the total particle density, if the system does not break the Galilean invariance [20,21] (for a similar discussion in the case of the superconductivity, see Refs. [22,23]). The point of the argument is that, if we write down the effective action for the phase variable ϕ [24], the Galilean invariance enforces the action to be the functional of only $\partial_t\phi - (\nabla\phi)^2/2m$. Since the coefficient of $\partial_t\phi$ in the effective action is the total particle number density, it forces the coefficient of the $(\nabla\phi)^2/2$ term to be the total density also, i.e., $\rho_s = n/m = \rho$. At finite temperature, since the imaginary time action at finite temperature is not Galilean invariant, the above argument does not apply. Therefore, at finite temperature $\rho_s \neq \rho$ [23,24], in accordance with Landau's famous expression of ρ_s in terms of the thermal distribution of the quasiparticle [25].

The Galilean invariance and also the continuous translational symmetry at $T = 0$ are explicitly broken if we introduce the lattice potential or the disorder, leading to the Mott insulator [26] or the Bose glass [17,27,28]. Also, the application of the magnetic field breaks the Galilean invariance and leads to the quantum Hall state [29], where the longitudinal resistivity is zero. Another possible source of the loss of the Galilean invariance is the nonlocal interaction along the time direction which arises after we integrate out the gapless degrees of freedom. This depletion of the superfluid component due to retarded interaction has been studied in Ref. [30], where the gapless degree of freedom is the gauge field which mediates the interaction between vortices. Note that the translational symmetry is kept intact even in the presence of the dissipation, which nonetheless breaks the Galilean invariance, in the model we study below. The finite normal-state resistivity is assumed to be due to the short-range impurity potential and the translational symmetry is recovered by averaging, the contribution of which to the vortex pinning can be neglected. The pinning is mostly due to the inhomogeneity or defect of the size comparable to the coherence length of the superconductors.

In this paper, we will discuss the effect of the gapless degrees of freedom, i.e., the effect of the dissipation, on the bosonic many-body system as a model for quantum vortices in two-dimensional superconductors.

II. MODEL

The phenomenological action for the system of many bosons in the presence of the dissipation is

$$S = \int_0^\beta d\tau \left(\sum_i \frac{m}{2} \dot{\vec{r}}_i^2 + \sum_{i>j} V_{i,j} \right) + \frac{\eta}{4\pi} \sum_i \int_0^\beta d\tau \int_0^\beta d\tau' \frac{\pi^2}{\beta^2} \left(\frac{\vec{r}_i(\tau) - \vec{r}_i(\tau')}{\sin \frac{\pi}{\beta}(\tau - \tau')} \right)^2, \quad (1)$$

where i is the labeling of the bosons, $V_{i,j}$ is the repulsive interaction between bosons, m is the mass of the bosons, β is the inverse temperature, and the last term represents the effect of the Ohmic heat bath [9,31]. We note that the dissipation acting on the vortices is known to be Ohmic [5]. We neglected the effective interaction between the bosons induced by the coupling to the heat bath [32].

III. EXTENDED FEYNMAN ARGUMENT

Feynman developed a theory of superfluidity in ^{4}He in terms of the world-line path integral in the first quantization scheme [33,34]. He pointed out that for the bosonic system we should sum over all the boundary conditions such that the final coordinates $\vec{r}_i(\beta)$ are some permutation of the initial coordinates $\vec{r}_i(0)$. Then he assumed the initial coordinates $\vec{r}_i(0)$ to be on some specific lattice, and approximated the statistical weight of the exchange event to be the one of free particles with renormalized mass. Then the problem boils down to the summation over all directed polygons made of edges on the lattice, and at low enough temperature the typical size of the polygon diverges, and that leads to the superfluidity. Namely, the superfluidity is characterized by the presence of the macroscopically large exchange processes; it appears in the form of the large fluctuation of winding number [35,36]. In the absence of the dissipation, if we assume that the effect of the repulsive interaction is simply renormalizing the mass of the bosons, the action for the macroscopic exchange process can be obtained from the single-particle off-diagonal density matrix of the free particle, which is given by $y(|\boldsymbol{r} - \boldsymbol{r}'|) \propto \exp[-m(\boldsymbol{r} - \boldsymbol{r}')^2/(2\beta\hbar^2)]$, so the action for the exchange event is proportional to β^{-1}. Therefore, as $\beta \to \infty$, the entropy of the macroscopic exchange processes, which is constant as a function of temperature, overcomes the action for the exchange process, so the bosonic system shows superfluidity at finite temperature. More concretely, following Feynman, we approximate the partition function of the system by the one of the classical statistical problem of directed polygons on a lattice and write it as $Z = \sum_L y(d)^L g(L)$, where L is the number of edges of the polygon, d is the lattice constant, and $g(L)$ is the total number of the polygons with L links. Here we again note that y can be approximated by the *off-diagonal* single-particle density matrix, rather than the *diagonal* one as is used for the criterion of the superfluidity in previous literature [37], although the Lindemann-type criterion may be a good *necessary* condition for the superfluidity. In other words, what determines the action for the exchange is $\langle p^2 \rangle$, the second moment of the momentum, rather than $\langle r^2 \rangle$, the second moment of the position, since the off-diagonal density matrix represents the information of the momentum distribution through the Wigner transform as

$$y(|\boldsymbol{r} - \boldsymbol{r}'|) \propto \exp[-(\boldsymbol{r} - \boldsymbol{r}')^2 \langle p^2 \rangle / (2\hbar^2)]. \quad (2)$$

Here the function y is Gaussian since the Caldeira-Leggett action is quadratic, and we assume that this form remains valid even in the presence of the interaction between particles. $\langle p^2 \rangle^{-1}$ and $\langle r^2 \rangle$ show drastically different behavior in the presence of the dissipation: The former remains constant down to $\beta \to \infty$, while the latter diverges as $\log \beta$ [38]. The reason for finiteness of $\langle p^2 \rangle^{-1}$ was clearly explained by Caldeira and Leggett (see Refs. [9,38]).

If we assume that the effect of the interaction can be renormalized to the effective mass of the particle, from the well-known result of the quantum Brownian motion [38],

$$\langle p^2 \rangle = \frac{M}{\beta} + 2M \frac{\mu_1\mu_2}{\mu_1 - \mu_2} [\psi(1 + \mu_1\beta) - \psi(1 + \mu_2\beta)], \quad (3)$$

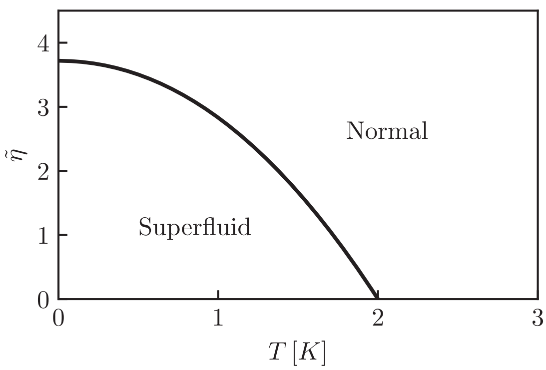

FIG. 1. Schematic phase diagram obtained from Feynman's argument combined with the expression for the off-diagonal density matrix in the presence of the Ohmic dissipation, Eq. (3). $\tilde{\eta} = \eta d^2/\hbar$, where d is the interparticle distance. We set $\lambda = \hbar^2/(2M) = 6.0596\,\text{Å}^2\,\text{K}$, $d = 3.570\,\text{Å}$, and $\hbar\omega_D = 10\,\text{K}$. The phase boundary is calculated from the condition $\langle p^2\rangle\,(T, \tilde{\eta}) = \langle p^2\rangle\,(T = 2\,\text{K}, \tilde{\eta} = 0)$, i.e., we assumed that the transition temperature for the dissipationless system is $T = 2\,\text{K}$.

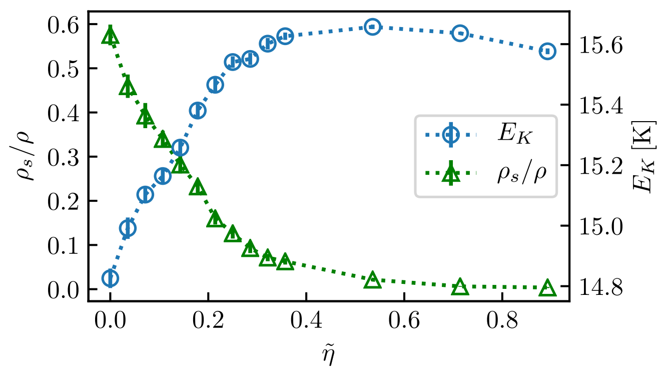

FIG. 2. The superfluid fraction ρ_s/ρ and the kinetic energy E_K for the three-dimensional system as a function of $\tilde{\eta} = \eta d^2/\hbar$, where $d = 3.570\,\text{Å}$ is the interparticle distance. The number of particles is $N = 64$, the particle density is $\rho = 0.02198\,\text{Å}^{-3}$, the temperature is $T = 2\,\text{K}$, the imaginary time step is $5 \times 10^{-3}\,\text{K}^{-1}$, and the cutoff of the bath is set to be $\tau_c = 0.2$. The blue circles represent the kinetic energy, while the green triangles represent the superfluid fraction.

where M is the effective mass of bosons, $\psi(x)$ is the digamma function, $\mu_{1/2} = \hbar(\omega_D \pm \sqrt{\omega_D^2 - 4\gamma\omega_D})/(4\pi)$, $\gamma = \eta/M$, and ω_D is the cutoff of the spectrum of the bath. Then, since $\langle p^2\rangle$ decreases as we lower the temperature and saturates at finite value, we expect that the transition temperature, which is the temperature where the entropy of macroscopic exchange $g(L)$ and the action for the exchange y^L compete, monotonically decreases and reaches zero as we increases the coupling η. This behavior is schematically shown in Fig. 1. We note that the superfluid state and the normal state of vortices correspond to the insulating and the failed superconductor phase of electrons, respectively. The critical η at $T = 0$ can be estimated from $d^2\,\langle p^2\rangle_{T=0}\,/\hbar^2 = 2Md^2\mu_1\mu_2[\ln{(\mu_1/\mu_2)}]/[\hbar^2(\mu_1 - \mu_2)] \sim 1$. If we further assume $\omega_D \gg \gamma$, the above condition simplifies to $\tilde{\eta}[\ln(\omega_D/\gamma)]/(\hbar\pi) \sim 1$, where $\tilde{\eta} = d^2\eta/\hbar$.

Below, we will show a strong support for this physical argument by the numerical Monte Carlo calculation of the superfluid density. This calculation confirms that the interaction between particles does not drastically affect the picture of superfluidity by Feynman even in the presence of dissipation.

IV. RESULT OF THE NUMERICAL CALCULATION

We calculated the superfluid density for the boson system characterized by the action (1) with the worm algorithm in continuous space [39,40] using the winding number formula [35,36]. We implemented the canonical version [41,42] where the Monte Carlo moves do not change the number of particles and employed the Aziz potential [43] for the interaction. We performed the numerical calculation for three-dimensional and two-dimensional systems. We set the particle number N, the particle density ρ, and the temperature T to be $N = 64$, $\rho = 0.02198\,\text{Å}^{-3}$, and $T = 2\,\text{K}$ for the three-dimensional system and $N = 25$, $\rho = 0.0432\,\text{Å}^{-2}$, and $T = 0.5\,\text{K}$ for the two-dimensional system. We note that the numerical calculation performed in the clean system with the Aziz potential and same ρ gave the transition temperature $T_c = 2.193 \pm 0.006\,\text{K}$ for the three-dimensional system and $T_c = 0.653 \pm 0.010\,\text{K}$ for the two-dimensional system [39,40]. Note that the transition in two dimensions corresponds to the Kosterlitz-Thouless transition. The convergence was checked by binning analysis [44]. Following Ref. [40], we employed the Chin approximation [45] for the interaction term. The dissipative term was discretized as [46,47]

$$\begin{aligned}\frac{\eta}{2\pi}\sum_i\sum_{k>k'}\frac{\pi^2}{N_\tau^2}\frac{[\vec{r}_i(k)-\vec{r}_i(k')]^2}{\sin^2[\frac{\pi}{N_\tau}(k-k')]}\\ =: \sum_i\sum_{k>k'}K(k-k')[\vec{r}_i(k)-\vec{r}_i(k')]^2,\end{aligned}\tag{4}$$

where N_τ is the number of the Trotter step, and k and k' label the time slice. To avoid the divergence associated with the discontinuity at $k = 0$ and N_τ, we introduce the UV cutoff for K as $K(k - k') = K[(1 - \tau_c)N_\tau]$ for $(1 - \tau_c)N_\tau \leqslant k - k' \leqslant N_\tau - 1$; this form of cutoff is naturally realized if we introduce the ultraviolet cutoff for the spectrum of the heat bath. We set $\tau_c = 0.2$ for the three-dimensional system and $\tau_c = 0.05$ for the two-dimensional system.

The result of the calculation for the three-dimensional system is shown in Fig. 2 (green triangle). We can clearly see that ρ_s monotonically decreases as a function of $\tilde{\eta}$. We also calculated the kinetic energy (blue circles), which characterizes how strong the bosons fluctuate in imaginary time. We can see the increase of the kinetic energy as a function of $\tilde{\eta}$, which comes both from the suppression of fluctuation of each boson and from the suppression of the exchange event by dissipation. We note that the fluctuation and the exchange event are known to lower the kinetic energy [36].

Another important quantity is the off-diagonal density matrix, which can be easily calculated in the worm algorithm. We numerically estimated the off-diagonal density matrix $n(r)$,

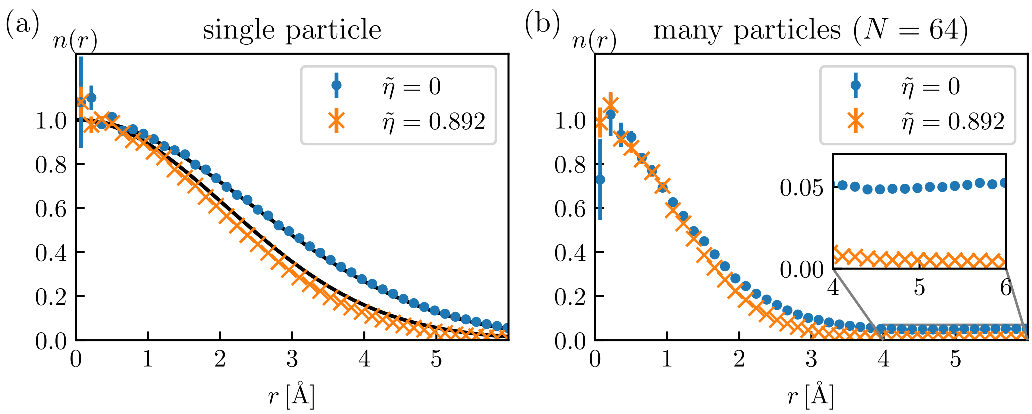

FIG. 3. The off-diagonal density matrix in three dimensions for (a) the system with a single particle and (b) the system with many particles ($N = 64$). The particle density is $\rho = 0.02198$ Å^{-3}, the temperature is $T = 2$ K, the imaginary time step is 5×10^{-3} K^{-1}, and the cutoff of the bath is set to be $\tau_c = 0.2$. $\tilde{\eta} = \eta d^2/\hbar$, where $d = 3.570$ Å. The solid lines for the single-particle case denote an analytical result based on Eqs. (2) and (3).

defined as [36]

$$n(|\vec{r}-\tilde{\vec{r}}|) = \frac{V}{Z}\int d\vec{r}_2^0 \dots d\vec{r}_N^0 \int_{\{\vec{r}_1(0),\vec{r}_2(0),\dots,\vec{r}_N(0)\}=\{\vec{r},\vec{r}_2^0,\dots,\vec{r}_N^0\}}^{\{\vec{r}_1(\beta),\vec{r}_2(\beta),\dots,\vec{r}_N(\beta)\}=\{\tilde{\vec{r}},\vec{r}_2^0,\dots,\vec{r}_N^0\}} D\vec{r}_1(\tau)\dots D\vec{r}_N(\tau)e^{-S}, \quad (5)$$

where V is the volume of the system and Z is the partition function defined as

$$Z = \int d\vec{r}_1^0 d\vec{r}_2^0 \dots d\vec{r}_N^0 \int_{\{\vec{r}_1(0),\vec{r}_2(0),\dots,\vec{r}_N(0)\}=\{\vec{r}_1^0,\vec{r}_2^0,\dots,\vec{r}_N^0\}}^{\{\vec{r}_1(\beta),\vec{r}_2(\beta),\dots,\vec{r}_N(\beta)\}=\{\vec{r}_1^0,\vec{r}_2^0,\dots,\vec{r}_N^0\}} D\vec{r}_1(\tau)\dots D\vec{r}_N(\tau)e^{-S}. \quad (6)$$

The numerically estimated $n(r)$ in three dimensions for the single-particle system and the many-particle ($N = 64$) system are shown in Figs. 3(a) and 3(b). For the single-particle case, Fig. 3(a), we showed the off-diagonal density matrix obtained both from the numerical calculation (blue circles and orange crosses) and from the analytical expression for the single-particle off-diagonal density matrix, Eq. (2), where $\langle p^2\rangle$ is given by Eq. (3) (black curves). We can see that the numerically obtained off-diagonal density matrix agrees well with the analytic expression. As we can see, the effect of dissipation appears as the decrease of the width of the Gaussian distribution. For the many-particle case, Fig. 3(b), we can see that the dissipation does not change the width very much, but it leads to the decrease of the asymptotic value of the off-diagonal density matrix. The asymptotic value of the off-diagonal density matrix gives the condensate fraction $\tilde{n}_0$, and the numerically estimated $\tilde{n}_0$ is shown in Fig. 4. We can see the monotonic decrease of $\tilde{n}_0$ as a function of $\tilde{\eta}$, associated with the suppression of the exchange event.

We also show the result of the numerical calculation in two dimensions to show that the suppression of superfluidity by dissipation occurs independently of the dimensionality of the system. The superfluid fraction ρ_s/ρ, the kinetic energy E_K, and the off-diagonal density matrix are shown in Figs. 5(a) and 5(b). As we can see from Fig. 5(a), the superfluid fraction decreases as a function of $\tilde{\eta}$. Also, from Fig. 5(b), the tail of the off-diagonal density matrix is suppressed in the presence of dissipation. Therefore, the numerical results in the two-dimensional system also support the suppression of superfluidity by dissipation.

V. FIELD THEORETICAL MODEL

Here, we discuss the effect of dissipation on the superfluidity in the following field theoretical model:

$$S = \sum_{\omega_n,\boldsymbol{k}}\left(-i\omega_n + \frac{\boldsymbol{k}^2}{2m} - \mu\right)\bar{\psi}_{n,\boldsymbol{k}}\psi_{n,\boldsymbol{k}} + \frac{g}{2}\int d\tau d\boldsymbol{r}\bar{\psi}\bar{\psi}\psi\psi + \alpha\sum_{\omega_n,\boldsymbol{k}}|\omega_n|\rho_{\boldsymbol{k}}^n\rho_{-\boldsymbol{k}}^{-n}, \quad \left(\rho_{\boldsymbol{k}}^n = \sum_{\omega_m,\boldsymbol{q}}\bar{\psi}_{n+m,\boldsymbol{k}+\boldsymbol{q}}\psi_{m,\boldsymbol{q}}\right), \quad (7)$$

where ψ and $\bar{\psi}$ are the bosonic annihilation and creation operator, ω_n is the Matsubara frequency for bosons, g is the interaction strength, and α is the strength of the dissipation. This model obviously breaks the Galilean invariance because of the last term.

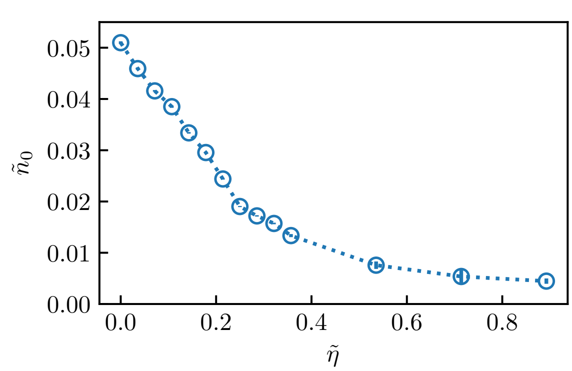

FIG. 4. The condensate fraction at zero momentum, $\tilde{n}_0$, estimated from the off-diagonal density matrix, for the three-dimensional system. The number of particles is $N = 64$, the particle density is $\rho = 0.02198$ Å^{-3}, the temperature is $T = 2$ K, the imaginary time step is 5×10^{-3} K^{-1}, and the cutoff of the bath is set to be $\tau_c = 0.2$. $\tilde{\eta} = \eta d^2/\hbar$, where $d = 3.570$ Å.

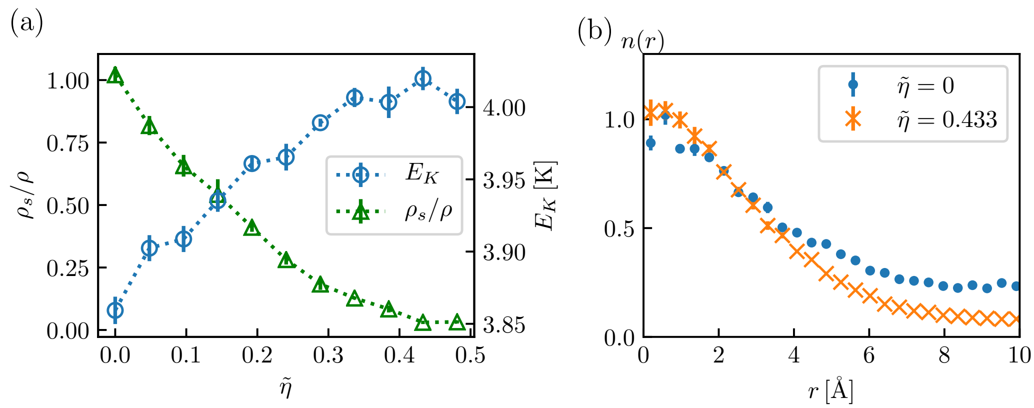

FIG. 5. (a) The kinetic energy E_K and the superfluid fraction ρ_s/ρ and (b) the off-diagonal density matrix for the two-dimensional system. The number of particles is $N = 25$, the particle density ρ is $\rho = 0.0432\,\text{Å}^{-2}$, the temperature is $T = 0.5\,\text{K}$, the imaginary time step is $5 \times 10^{-3}\,\text{K}^{-1}$, and the cutoff of the bath is set to be $\tau_c = 0.05$. $\tilde{\eta} = \eta d^2/\hbar$, where $d = 4.811\,\text{Å}$.

The idea behind the model Eq. (7) is the following. In the first quantized model above, each particle is subject to dissipation. This can be mapped to the finite diffusion constant or the conductivity of the many-particle system, and hence the dissipation enters as the many-body interaction. More explicitly, the propagator of the density $\rho(q,\omega)$ is expressed by

$$\Pi(q,\omega) = \langle \rho(q,\omega)\rho(-q,-\omega)\rangle = \frac{N(0)Dq^2}{Dq^2 + |\omega|} \tag{8}$$

where $N(0)$ is the density of electronic states at the Fermi energy, and D is the diffusion constant related to the conductivity $\sigma = e^2N(0)D$. In the action, the inverse of $\Pi(q,\omega)$ appears in front of $\rho(q,\omega)\rho(-q,-\omega)$, which is $\frac{1}{N(0)} + \frac{|\omega|}{N(0)Dq^2}$. For simplicity, we replace Dq^2 in the denominator by a constant. Then we obtain the last term of Eq. (7).

We calculated the superfluid density by the Bogoliubov approximation, i.e., substituted $\psi = \sqrt{\rho_0} + \phi$ and $\bar{\psi} = \sqrt{\rho_0} + \bar{\phi}$ and retained the terms up to quadratic order in $\phi, \bar{\phi}$. From the general argument [48,49], the normal component $\rho_n = \rho - \rho_s$ can be obtained from the transverse current-current response function $\chi^t(\omega, \boldsymbol{q})$ as $\rho_n/m = \lim_{\boldsymbol{q}\to 0} \chi^t(0,\boldsymbol{q})$. At one-loop order, it is given as [50] xs

$$\frac{\rho_n}{m} = \lim_{\boldsymbol{q}\to 0} \int \frac{d\epsilon_{\boldsymbol{k}}}{2\pi}\frac{d\omega}{2\pi}\frac{\epsilon_{\boldsymbol{k}}}{4i} \mathrm{tr}\big[\sigma_3 G^K_{\omega,\boldsymbol{k}}\sigma_3\big(G^R_{\omega,\boldsymbol{k}+\boldsymbol{q}} + G^A_{\omega,\boldsymbol{k}-\boldsymbol{q}}\big)\big], \tag{9}$$

where we assumed the two-dimensional system. The Green's functions are given as

$$G^{R/A}_{\omega,\boldsymbol{k}} = \frac{1}{\omega^2 - \omega_{\boldsymbol{k}}^2 \pm 2i\eta\omega\epsilon_{\boldsymbol{k}}} \times \begin{pmatrix} \omega + \epsilon_{\boldsymbol{k}} + g\rho_0 \mp i\eta\omega & -g\rho_0 \pm i\eta\omega \\ -g\rho_0 \pm i\eta\omega & -\omega + \epsilon_{\boldsymbol{k}} + g\rho_0 \mp i\eta\omega \end{pmatrix},$$

and $G^K_{\omega,\boldsymbol{k}} = \coth(\beta\omega/2)[G^R_{\omega,\boldsymbol{k}} - G^A_{\omega,\boldsymbol{k}}]$, where $\eta = 2\rho_0\alpha$. The number density can be calculated as

$$\rho = \rho_0 + \frac{1}{V}\sum_{\boldsymbol{k}}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} n_B(\omega) i\big[\big(G^R_{\omega,\boldsymbol{k}}\big)_{11} - \big(G^A_{\omega,\boldsymbol{k}}\big)_{11}\big]. \tag{10}$$

From now on, we consider the zero-temperature case, where the destruction of the superfluidity comes purely from the dissipation. The finite temperature case can be treated in a similar manner. We introduced the cutoff for η as $\eta\,\Theta(\omega_c^2 - \omega^2)$, where $\Theta(x)$ is the step function, and the hard cutoff for the energy $\epsilon_{\boldsymbol{k}}$ at ϵ_c. We chose $\epsilon_c < \omega_c$, so that the whole energy spectrum of the system is coupled to the heat bath. To calculate ρ_s, we regard η as a control parameter, calculate ρ_0 as a function of η, and then calculate $\rho_s(\eta, \rho_0(\eta))$. We used the parameters $\epsilon_c/(\rho g) = 900$, $\omega_c/(\rho g) = 1000$, $mg = 1$. The result of the calculation is shown in Figs. 6(a) and 6(b). We can see that ρ_s rapidly decreases and vanishes so the superfluidity is destroyed by the dissipation. This kind of behavior is also shown in Ref. [27], where the authors discussed the destruction of the superfluid by the static impurity potential, which is in contrast to our system where the translational symmetry is preserved but the time nonlocal action breaks the Galilean invariance.

Also, from Figs. 6(a) and 6(b), we can see that, at the critical η where $\rho_s = 0$, ρ_0 remains finite. This behavior is similar to the system with disorder [27,28], but the depletion of ρ_0 is large in this parameter region, so our one-loop calculation cannot determine whether or not ρ_0 is finite at the critical point. In fact, assuming the smooth behavior of the single-particle Green's function at the critical point, the Josephson relation [51,52] requires that both ρ_0 and ρ_s become zero. In spite of this uncertainty, we believe that the transition to the phase with $\rho_s = 0$ in this model remains intact, as is supported by our numerical calculation in a model with the different source of

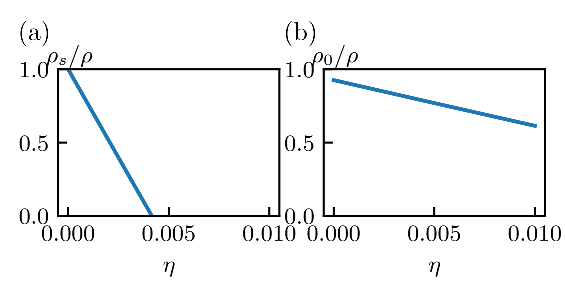

FIG. 6. (a) ρ_s/ρ and (b) ρ_0/ρ obtained from Eqs. (9) and (10). The parameters are $\epsilon_c/(\rho g) = 900$, $\omega_c/(\rho g) = 1000$, $mg = 1$.

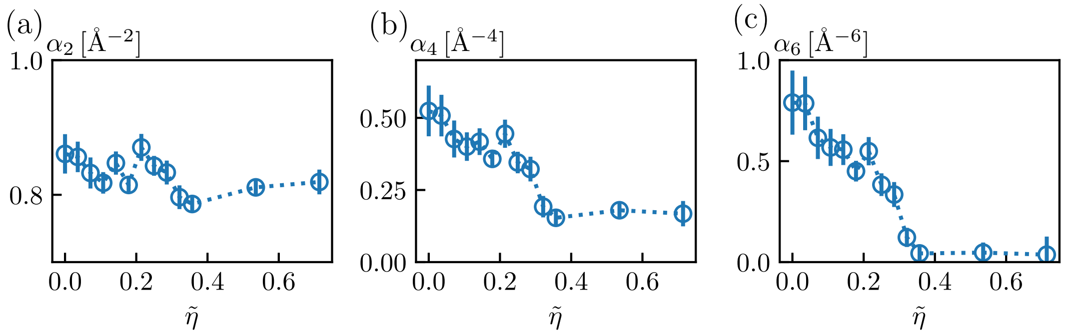

FIG. 7. The fitting parameters (a) α_2, (b) α_4, and (c) α_6, as a function of dissipation strength for the of-diagonal density matrix $n(r)$ in three dimensions. The fitting function is $\tilde{n}_0 + (1-\tilde{n}_0)f(r)$, where $f(x) = \exp(-\alpha_2 x^2/2! + \alpha_4 x^4/4! - \alpha_6 x^6/6!)$.

the Galilean symmetry breaking, i.e., Eq. (1), as we can see from Figs. 2 and 5(a).

VI. DISCUSSION

Our scenario predicts several experimental consequences. First, the Bose metal appears only in the clean superconductors where $\hbar/\tau \gg \delta \sim \Delta^2/\epsilon_F$, while it does not appear in the superclean case $\hbar/\tau \ll \Delta^2/\epsilon_F$. The latter can be realized for the superconductors with the large superconducting gap Δ and short coherence length. Once the disorder is stronger, it will pin the vortices and again the metallic state of vortices is rather difficult. The Anderson localization [53] becomes also relevant, and the superconductivity based on these localized states belongs to a different class [54]. The earlier studies on granular superconductors or dirty limit thin films correspond to this case, where the superconductor-insulator transition occurs without the metallic region between them. However, our scenario applies also to the dirty superconductors with $h/(e^2 k_F \xi) \ll \rho_n \ll h/e^2$ (k_F, Fermi wave number; ξ, coherence length of the superconductor; ρ_n, residual resistivity in the normal state). The expected behavior of the resistivity ρ of Bose metal at zero temperature is $\rho \sim \rho_n(B/B_{c2})$ with B_{c2} being the second upper critical field, since the motion of the vortices remains classical due to the suppressed quantum coherence, i.e., we expect a giant magnetoresistance, as is observed experimentally [1,55]. For a clean superconductor with $\rho_n \ll h/e^2$, the resistivity can be much smaller than the quantum resistance h/e^2. We note that the long-range interaction between the vortices does not spoil our scenario if we include the effect of the screening [30,56–58]. Also, the Berry phase of vortices [59], which is absent in the present paper since we assume the integer filling of the electrons, leads to the interference of the exchange events so that the superfluidity is further suppressed if we include the Berry phase term.

We also speculate that the effect of the normal core or dissipation discussed above affects the phase transition associated with the proliferation of the vortices, i.e., the transition not associated with the magnetic field. The point is that, if we extend the above dissipative action to the closed loop in the space-time, in the parameter region where the typical size of the vortex ring in the space-time is macroscopic, the exchange process between the rings is still suppressed for the same reason as above. Therefore, we expect a different phase compared to the usual proliferation of vortices in the bosonic superfluid.

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APPENDIX: THE DETAILS OF THE CALCULATION OF $\tilde{n}_0$

We estimated the condensate fraction $\tilde{n}_0$ in Fig. 4 by fitting the off-diagonal density matrix $n(r)$ with the function $\tilde{n}_0 + (1-\tilde{n}_0)f(r)$, where $f(x) = \exp(-\alpha_2 x^2/2! + \alpha_4 x^4/4! - \alpha_6 x^6/6!)$. This form of the fitting function is motivated by the one used in the absence of dissipation [60]. Here we ignored the contribution from the coupling term [60], since the form of the coupling term seems to be inapplicable in the presence of the dissipation. The ignorance of this term leads to an overestimation of n_0, but we believe that the qualitative trend as a function of $\tilde{\eta}$ can be captured by this simple fitting. We show the estimated value of α_2, α_4, and α_6 as a function of $\tilde{\eta}$ in Fig. 7. Although α_2, which represents the second cumulant of the distribution, does not change drastically as a function of $\tilde{\eta}$, α_4 and α_6 decrease, which indicates that the distribution becomes more and more Gaussian-like.

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