Topological quantization of the classical stochastic transport of a magnetic skyrmion driven by a ratchetlike spin-polarized electric current at finite temperature

Shan-Chang Tang and Yu Shi *

Department of Physics & State Key Laboratory of Surface Physics, Fudan University, Shanghai 200433, China

(Received 13 June 2018; revised 31 August 2020; accepted 14 September 2020; published 23 September 2020)

We consider a magnetic skyrmion driven by a spin-polarized electrical current that is periodic in time, and is periodic and asymmetric in a direction different from that of the current itself. We study its classical stochastic transport in a finite temperature, by using the Fokker-Planck equation of the probability distribution, derived from the stochastic equation of motion, the Langevin equation. We also perform numerical simulation of the original Landau-Lifshitz-Gilbert equation describing the spins constituting the skrymion. The probabilistic average velocity of the skyrmion is along the direction of the periodicity. When the thermal energy is much lower than the potential energy, and their ratio is also much smaller than that between the time periodicity and the diffusion time, the time and probabilistic average velocity is the ratio between the spatial and temporal periodicities multiplied by topological integer called the Chern number. This result provides a practical way of realizing topological numbers in classical stochastic systems and suggests a convenient way of manipulating skyrmions at finite temperatures.

DOI: 10.1103/PhysRevB.102.104428

I. INTRODUCTION

Magnetic skyrmions have attracted a lot of attention both experimentally and theoretically [1], at least since its observation in the chiral magnetic material MnSi [2]. Many studies have been made on its visibility [3–5], its motion driven by a spin-polarized electric current [6–10], its transport in presence of a temperature gradient [11–13], the thermal effects [14–16], and the effect of Magnus force [17], among others.

In this paper, we consider a magnetic skyrmion moving on a two-dimensional space at a finite temperature, driven by a spin-polarized electric current that is periodic in time and is periodic and asymmetric in a certain direction. Moreover, there exists a random magnetic field with thermal fluctuations, representing the effect of a finite temperature. Therefore, its motion is described by a two-dimensional Langevin equation obtained from the stochastic Landau-Lifshitz-Gilbert (SLLG) equation for the constituent spins. We show that it turns out to be a two-dimensional generalization of thermal ratchet adiabatically driven by an asymmetric potential that is periodic in both space and time, in addition to the thermal fluctuations [18].

Consequently, when the temperature is low enough, the time and probabilistic average of the velocity of the magnetic skyrmion is equal to a topologically invariant integer called the Chern number multiplied by the ratio between the spatial and temporal periodicities. This is referred to as topological quantization. We have also performed numerical simulations based on the SLLG equation, which not only confirm the analytical result, but also demonstrate the breaking of the quantization when either the adiabatic condition is unsatisfied or when the temperature is so high that the spins fail to constituent a particlelike skyrmion even before the adiabatic condition is violated. Finally, we also propose an experimental setup for the demonstration of the topological quantization.

The rest of the paper is organized as the following. In Sec. II, we give the Fokker-Planck equation describing the stochastic motion of the skyrmion. In Sec. III, we obtain the formula for the average velocity. In Sec. IV, we show that the system is exactly a two-dimensional generalization of the adiabatic thermal ratchet. In Sec. V, the condition is discussed for the topological quantization of the time and probabilistic average velocity. In Sec. VI, we present the numerical simulations based on the SLLG equation for the constituent spins, which demonstrate the topological quantization as well as its breakdown for high enough temperature. An experimental setup is designed in Sec. VII. A summary is made in Sec. VIII.

II. FOKKER PLANCK EQUATION FOR A SKYRMION

The dynamics of the constituent spins of the skrymion on a two-dimensional space is described by the SLLG equation [10–16]

$$\frac{\partial \mathbf{n}}{\partial t} + (\mathbf{v}_s \cdot \nabla)\mathbf{n} = -\frac{1}{\hbar}\mathbf{n} \times (\mathbf{H}_{\text{eff}} + \mathbf{R}) + \alpha \mathbf{n} \times \frac{\partial \mathbf{n}}{\partial t} + \beta \mathbf{n} \times (\mathbf{v}_s \cdot \nabla)\mathbf{n}, \quad (1)$$

where $\mathbf{n} \equiv \mathbf{n}(x, y)$ represents the direction of the spin at (x, y), satisfying $\mathbf{n}^2 = 1$, α represents the damping effect, $\mathbf{v}_s = -\frac{a^3}{2e}\mathbf{j}$ is the spin velocity, where $\mathbf{j}$ is the spin-polarized electric current density multiplied by its spin polarization and divided by the magnetic saturation. a is the lattice constant of the local spins, β is the nonadiabatic coefficient, usually $\beta \ll \alpha$, $\mathbf{H}_{\text{eff}} \equiv -\frac{\delta \mathcal{H}_S}{\delta \mathbf{n}}$ is the effective magnetic field, $\mathcal{H}_S$ being the

*yushi@fudan.edu.cn

Hamiltonian, which is

$$\mathcal{H}_\mathcal{S} = \iint dxdy\Big[\frac{J}{2}(\nabla \mathbf{n})^2 + \frac{D}{a}\mathbf{n}\cdot(\nabla\times\mathbf{n}) - \frac{1}{a^2}\mathbf{B}\cdot\mathbf{n} - \frac{K}{a^2}n_z^2\Big], \tag{2}$$

in the continuous case, and is

$$\mathcal{H}_\mathcal{S} = -J\sum_{\langle ij\rangle}\mathbf{n}_i\cdot\mathbf{n}_j - D\sum_{\langle ij\rangle}\hat{\mathbf{e}}_{ij}\cdot\mathbf{n}_i\times\mathbf{n}_j - \mathbf{B}\cdot\sum_i\mathbf{n}_i - K\sum_i n_{iz}^2, \tag{3}$$

in the discrete case [1], where J is the exchange interaction constant, D is the Dzyaloshinskii-Moriya interaction strength [19,20], $\mathbf{B}$ is the external magnetic field in the unit of energy, K is the anisotropic constant, and $\langle ij\rangle$ represents the nearest neighbors. In our present work, a crucial element is an additional random magnetic field $\mathbf{R}$, which characterizes the effect of the finite temperature T, with $\langle R_i(\mathbf{r},t)\rangle = 0$, $\langle R_i(\mathbf{r},t)R_j(\mathbf{r}',t')\rangle = 2\alpha\hbar k_BTa^2\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')$, where $i, j = x, y$, $\hbar$ is the Planck constant, and k_B is the Boltzmann constant.

A skyrmion is a topologically stable spin texture, characterized by a winding number, which is unchanged by continuous deformation. Hence it can move as a particlelike object, governed by the equation of motion, which can be derived from SLLG equation using Thiele's method [14,21]. Because of the randomness of $\mathbf{R}$, the equation of motion of the skyrmion is a Langevin equation

$$\alpha_d\Big[\dot{\mathbf{q}} - \frac{\beta}{\alpha}\mathbf{v}_s\Big] + \alpha_m\hat{\mathbf{z}}\times[\dot{\mathbf{q}} - \mathbf{v}_s] = \boldsymbol{\nu}(t), \tag{4}$$

where the stochastic variable $\mathbf{q} = (q_x, q_y)$ represents the position of the skyrmion as a whole, $\alpha_d \equiv \alpha\iint dxdy(\frac{\partial\mathbf{n}}{\partial x})^2$, and $\alpha_m \equiv \iint dxdy\mathbf{n}\cdot(\frac{\partial\mathbf{n}}{\partial x}\times\frac{\partial\mathbf{n}}{\partial y})$ describe the effects from the spin texture, namely, damping and magnus effects [17], $\boldsymbol{\nu} = (\nu_x, \nu_y)$ is the stochastic force due to the random magnetic field $\mathbf{R}$, satisfying

$$\langle\nu_i(t)\rangle = 0,\ \langle\nu_i(t)\nu_j(t')\rangle = 2\frac{\alpha_dk_BTa^2}{\hbar}\delta_{ij}\delta(t-t'). \tag{5}$$

The stochastic motion of the skrymion can also be described in terms of the probability density $\rho(\mathbf{r},t)$, satisfying the corresponding Fokker-Planck equation [22]

$$-\frac{\partial\rho(\mathbf{r},t)}{\partial t} = \mathcal{D}\mathcal{O}\rho(\mathbf{r},t), \tag{6}$$

where

$$\mathcal{D} \equiv \frac{\alpha_dk_BTa^2}{\hbar(\alpha_m^2+\alpha_d^2)},$$

$$\mathcal{O} = -\nabla^2 + \frac{\partial}{\partial x}(C_1v_{sx} + C_2v_{sy}) + \frac{\partial}{\partial y}(-C_2v_{sx} + C_1v_{sy}),$$

with

$$C_1 \equiv \hbar\frac{\frac{\beta}{\alpha}\alpha_d^2 + \alpha_m^2}{\alpha_dk_BTa^2},\quad C_2 \equiv \hbar\frac{(\frac{\beta}{\alpha}-1)\alpha_m}{k_BTa^2}.$$

The probability current density $\boldsymbol{\mathcal{J}} = (\mathcal{J}_x, \mathcal{J}_y)$ is given by

$$\mathcal{J}_x = \mathcal{D}\Big[(C_1v_{sx} + C_2v_{sy}) - \frac{\partial}{\partial x}\Big]\rho, \tag{7}$$

$$\mathcal{J}_y = \mathcal{D}\Big[(-C_2v_{sx} + C_1v_{sy}) - \frac{\partial}{\partial y}\Big]\rho. \tag{8}$$

One can find that the probability current $\iint\boldsymbol{\mathcal{J}}dxdy$ gives the instantaneous velocity $\dot{\mathbf{q}}$ averaged over probability distribution, that is,

$$\langle\dot{\mathbf{q}}\rangle = \mathbf{J} = \iint\boldsymbol{\mathcal{J}}dxdy. \tag{9}$$

III. PROBABILISTIC AVERAGE VELOCITY OF A SKYRMION

The Fokker-Planck equation can be solved by transforming the Fokker-Planck operator to an Hermitian operator, under the condition [22]

$$\frac{\partial}{\partial y}(C_1v_{sx} + C_2v_{sy}) = \frac{\partial}{\partial x}(-C_2v_{sx} + C_1v_{sy}). \tag{10}$$

Without loss of generality, we choose the direction of the driving electric current to be x direction, that is, $v_{sx} = v_s$, $v_{sy} = 0$. Hence the above condition is reduced to

$$\frac{\partial v_s}{\partial y} + \kappa\frac{\partial v_s}{\partial x} = 0, \tag{11}$$

where

$$\kappa \equiv \frac{C_2}{C_1} = \frac{(\frac{\beta}{\alpha}-1)\alpha_d\alpha_m}{\frac{\beta}{\alpha}\alpha_d^2 + \alpha_m^2}.$$

Therefore the most general form of v_{sx} is

$$v_s(x,y,t) = M(x-\kappa y) + N(t), \tag{12}$$

where $M(x-\kappa y)$ is a function of $x-\kappa y$, N is a function of t.

Consider a coordinate transformation

$$\begin{pmatrix}u\\v\end{pmatrix} = W\begin{pmatrix}x\\y\end{pmatrix}, \tag{13}$$

where

$$W \equiv \frac{1}{\sqrt{1+\kappa^2}}\begin{pmatrix}1 & -\kappa\\ \kappa & 1\end{pmatrix}. \tag{14}$$

The Fokker-Planck operator can be rewritten as

$$\mathcal{O} = -\nabla^2 + \zeta\sqrt{1+\kappa^2}\frac{\partial}{\partial u}[M(\sqrt{1+\kappa^2}u) + N(t)], \tag{15}$$

where

$$\zeta = \frac{1}{\mathcal{D}}\frac{\frac{\beta}{\alpha}\alpha_d^2 + \alpha_m^2}{\alpha_d^2 + \alpha_m^2} = \hbar\frac{\frac{\beta}{\alpha}\alpha_d^2 + \alpha_m^2}{\alpha_dk_BTa^2}.$$

The probability current density can be rewritten as $\boldsymbol{\mathcal{J}} = (\mathcal{J}_u, \mathcal{J}_v)$, with

$$\mathcal{J}_u = \mathcal{D}\Big\{\zeta\sqrt{1+\kappa^2}[M(\sqrt{1+\kappa^2}u) + N(t)] - \frac{\partial}{\partial u}\Big\}\rho, \tag{16}$$

$$\mathcal{J}_v = -\mathcal{D}\frac{\partial}{\partial v}\rho. \tag{17}$$

The stochastic variable $\mathbf{q}$ can be rewritten as $\mathbf{q} = (q_u, q_v)$, with

$$\begin{pmatrix} q_u \\ q_v \end{pmatrix} = W \begin{pmatrix} q_x \\ q_y \end{pmatrix}, \tag{18}$$

satisfying

$$\dot{q}_u = - D\partial_{q_u}\Phi(q_u) + \xi_u, \tag{19}$$

$$\dot{q}_v = \xi_v, \tag{20}$$

with

$$\begin{pmatrix} \xi_u \\ \xi_v \end{pmatrix} = W \begin{pmatrix} \xi_x \\ \xi_y \end{pmatrix}. \tag{21}$$

The components of the probabilistic average velocity are

$$\langle \dot{q_u} \rangle = \iint \mathcal{J}_u du dv, \tag{22}$$

$$\langle \dot{q_v} \rangle = \iint \mathcal{J}_v du dv. \tag{23}$$

Note that in terms of coordinates (u, v), the system is still two dimensional. Nevertheless, the motion on v direction is purely diffusion. It is straightforward to obtain $\langle \dot{q_v} \rangle = 0$. In the following, we set out to calculate $\langle \dot{q_u} \rangle$.

In terms of

$$\Phi(u, t) = -\zeta\sqrt{1+\kappa^2}\int_0^u du' M(\sqrt{1+\kappa^2}u') - \zeta\sqrt{1+\kappa^2}N(t)u, \tag{24}$$

the Fokker-Planck operator $\mathcal{O}$ can be transformed to a Hermitian operator

$$\mathcal{H} = e^{\Phi/2}\mathcal{O}e^{-\Phi/2} = -\nabla^2 + U, \tag{25}$$

where

$$U = -\frac{1}{2}\frac{\partial^2\Phi}{\partial u^2} + \frac{1}{4}\left(\frac{\partial\Phi}{\partial u}\right)^2. \tag{26}$$

Consequently, the Fokker-Planck equation is transformed to a Schrödinger-like equation

$$-\frac{\partial\psi(u, v, t)}{\partial t} = \left(\mathcal{D}\mathcal{H} - \frac{1}{2}\frac{\partial\Phi}{\partial t} - \frac{\partial \ln\sqrt{Z}}{\partial t}\right)\psi(u, v, t), \tag{27}$$

where

$$\psi(u, v, t) = \rho(u, v, t)e^{\Phi/2}\sqrt{Z} \tag{28}$$

with $Z \equiv \iint e^{-\Phi}dxdy$.

The probability current, i.e., the probabilistic average of the velocity, which is on u direction rather than x direction, is thus

$$J_u(t) = \iint \mathcal{J}_u(u, v, t)dudv = -2\mathcal{D}\iint \psi_0(u, v, t)\frac{\partial}{\partial u}\psi(u, v, t)dudv, \tag{29}$$

where $\psi_0 = \frac{1}{\sqrt{Z}}e^{-\Phi/2}$ is the "ground state wave function" corresponding to the equilibrium state $\rho_0 = \frac{1}{\sqrt{Z}}e^{-\Phi}$.

IV. STOCHASTIC MOTION OF THE SKYRMION AS A TWO-DIMENSIONAL THERMAL RATCHET

We first recall the thermal ratchet. Consider an overdamped particle moving along one-dimensional space, governed by the Langevin equation

$$\eta\dot{x} = -V'(x, t) + \xi(t), \tag{30}$$

where V is the total potential periodic in both the one-dimensional coordinate x and in time t, and is asymmetric in x, $\xi(t)$ is a random force satisfying $\langle\xi(t)\rangle = 0$ and $\langle\xi(t)\xi(s)\rangle = 2\eta k_B T\delta(t-s)$, η is the proportional constant between the frictional or damping force and the velocity. Because of the existence of the time-dependent force, a nonzero unidirectional current develops [23], which would be absent if there were no time-dependent force even though the potential is asymmetric.

For model (30), it has been shown that when the temperature is low enough, the average velocity of the particle is a basic unit multiplied by a Chern number, a topologically invariant integer [18]. The quantitative criterion for the lowness of the temperature is

$$\frac{k_B T}{V_0} \ll \min\left(1, \frac{\mathcal{T}}{\tau_D}\right), \tag{31}$$

where V_0 is the amplitude of the potential energy V, $\mathcal{T}$ is the time periodicity of the potential, $\tau_D = L^2/\mathcal{D}$ is the diffusion time over the spatial period L of V. $\frac{k_B T}{V_0} \ll 1$ represents the dominance of the potential energy over the thermal fluctuation, while $\frac{k_B T}{V_0} \ll \frac{\mathcal{T}}{\tau_D}$ is the further requirement of adiabaticity in this case [18].

We now show that the stochastic motion of the skyrmion is exactly a two-dimensional generalization of this adiabatic thermal rachet. For this purpose, we rewrite the Langevin equation of the skyrmion (4) as

$$\dot{\boldsymbol{q}} = -\mathcal{D}\nabla\Phi(\mathbf{r}, t) + \boldsymbol{\xi}(t), \tag{32}$$

where $\boldsymbol{\xi}(t) = (\xi_x(t), \xi_y(t))$, with $\xi_x = \frac{\alpha_d v_x + \alpha_m v_y}{\alpha_d^2 + \alpha_m^2}$, $\xi_y = \frac{-\alpha_m v_x + \alpha_d v_y}{\alpha_d^2 + \alpha_m^2}$, Φ is exactly as given in (24).

It can be seen that the skyrmion becomes a two-dimensional generalization of the thermal rachet if $M(x - \kappa y)$ is inversion-symmetric and periodic in $x - \kappa y$, in other words, $M = M(\sqrt{1+\kappa^2}u)$ is periodic in u, with periodicity denoted as L, while $N(t)$ is periodic in time t, with the temporal periodicity denoted as $\mathcal{T}$. Note that the direction of the electric current, i.e., x direction, is not the direction of u, along which M is periodic. Hence our model is truly two dimensional. Below we will envisage an experimental setup realizing such a situation.

The inversion-asymmetry of M can be maintained even though its gradient is inversion-symmetric, for example, if M is a sine function u, which is inversion-asymmetric, its gradient is a cosine function, which is inversion-symmetric.

V. TIME AVERAGE OF THE PROBABILITY CURRENT

The probability current is just the probabilistic average of the velocity. Now we calculate its time average, which is called the time and probabilistic average velocity.

We consider the case that the potential energy dominates the thermal energy, that is,

$$\Phi_0 \gg 1, \tag{33}$$

where Φ_0 is the amplitude of Φ [18]. In this case, the Bloch bands of $\mathcal{H}$ is derived from the low levels in the deep potential wells of U, which typically contains a double-well structure in each period of the ratchet, even if Φ has one well in each period, as the dominant term $(\partial_u \Phi)^2$ in U has only half the period of Φ, and thus must contain two wells in each period of Φ. These two wells are made inequivalent by the weaker term, $\partial_u^2 \Phi$, which has the full periodicity of V. The band gap ΔE can thus be estimated to be $\sim \partial_u^2 \Phi$ [18]. Thus

$$\Delta E \sim \frac{\Phi_0}{L^2}. \tag{34}$$

The adiabatic condition is

$$\mathcal{T} \gg \frac{1}{\mathcal{D}\Delta E}, \tag{35}$$

where ΔE is the gap between the lowest and the second-lowest eigenvalues of $\mathcal{H}$. Hence in the case that the potential energy dominates the thermal energy, by substituting Eq. (34) to the adiabatic condition (35), one obtains

$$\mathcal{T} \gg \frac{L^2}{\mathcal{D}\Phi_0}. \tag{36}$$

We can define $\Phi_0 \equiv V_0/k_B T$, then the condition (31) is reproduced.

Under this condition, one can use the adiabatic perturbation theory to obtain time and probabilistic average velocity [18],

$$\begin{aligned} \overline{\langle \dot{u} \rangle} &= \frac{L}{\mathcal{T}}\mathcal{C}, \\ \overline{\langle \dot{v} \rangle} &= 0, \end{aligned} \tag{37}$$

where $\mathcal{C}$ is an integer called Chern number. In terms of the original coordinates (x, y),

$$\begin{aligned} \overline{\langle \dot{x} \rangle} &= \frac{1}{\sqrt{1+\kappa^2}} \frac{L}{\mathcal{T}}\mathcal{C}, \\ \overline{\langle \dot{y} \rangle} &= -\frac{\kappa}{\sqrt{1+\kappa^2}} \frac{L}{\mathcal{T}}\mathcal{C}, \end{aligned} \tag{38}$$

which indicates that the average velocity of the skrymion is the basic unit multiplied by an integer. It is "quantized" in the sense that is an integer multiply of a basic unit, though it is a classical system.

VI. NUMERICAL SIMULATION

We have also performed numerical simulations of the SLLG equation, by using the Runge-Kutta method on a 100×864 lattice with the periodic boundary condition. The size is so chosen as $100/864 \approx \kappa$. Inspired by a numerical work on a thermal ratchet [24], we assume the polarized electric current to be

$$\begin{aligned} j_x = &-j_c\left[\cos k_c(x-\kappa y) + \frac{1}{2}\cos 2k_c(x-\kappa y)\right] \\ &- A\cos\left(\frac{2\pi}{\mathcal{T}}t\right), \end{aligned} \tag{39}$$

which is in the unit of $\frac{2e}{a^2\tau}$, where $\tau \equiv \frac{\hbar}{J}$ is the time unit. Hence $v_{sx} = \frac{a}{\tau} j_x$.

We first examine the parameter regimes for the dominance of the potential energy and the adiabatic condition. Substituting Eq. (39) into Eq. (24), we obtain

$$\begin{aligned} \Phi(u,t) = &-\frac{\frac{\beta}{\alpha}\alpha_d^2 + \alpha_m^2}{\alpha_d \frac{k_B T}{J} a} j_c\Big[\sin(\sqrt{1+\kappa^2}k_c u) \\ &+ \frac{1}{4}\sin(2\sqrt{1+\kappa^2}k_c u)\Big] \\ &- \frac{\frac{\beta}{\alpha}\alpha_d^2 + \alpha_m^2}{\alpha_d \frac{k_B T}{J} a}\sqrt{1+\kappa^2} A\cos\left(\frac{2\pi}{\mathcal{T}}t\right)u, \end{aligned} \tag{40}$$

which is then estimated by using the following magnitude: $\sin(\sqrt{1+\kappa^2}u) + \frac{1}{4}\sin 2k(\sqrt{1+\kappa^2}u) \sim 1$, $u \sim L$, $A\cos(\frac{2\pi}{\mathcal{T}}t) \sim \frac{A}{2}$. Thus the potential energy dominates the thermal energy when

$$\Phi_0 \sim \frac{\frac{\beta}{\alpha}\alpha_d^2 + \alpha_m^2}{\alpha_d \frac{k_B T}{J}} \frac{L}{a}\left(\frac{j_c}{2\pi} + \frac{A}{2}\right) \gg 1, \tag{41}$$

which is substituted into the gap formula (34), reducing the adiabatic condition (35) to

$$\mathcal{T} \gg \frac{L}{a}\frac{\tau}{\frac{j_c}{2\pi} + \frac{A}{2}}. \tag{42}$$

Supposing the parameter values to be $\alpha_d \sim 1$, $\alpha_m \sim 10$, $j_c \sim 0.1$, $A \sim 0.1$, $L/a \sim 100$, as will be used in our simulation, we have

$$\Phi_0 \sim 10^3\left(\frac{J}{k_B T}\right) \gg 1 \tag{43}$$

as the condition for the dominance of potential energy, and

$$\mathcal{T} \gg 10^3\tau \tag{44}$$

as the adiabatic condition in this case. It can be found that $\tau_D = \frac{L^2}{D} = \frac{L^2\hbar(\alpha_m^2+\alpha_d^2)}{\alpha_d k_B T a^2} = \tau\frac{\alpha_m^2+\alpha_d^2}{\alpha_d}\frac{L^2}{a^2}\frac{1}{\frac{k_B T}{J}} \approx 10^2 \times 10^4 \times 10\tau = 10^7\tau$.

Now we perform simulation in the case of potential energy dominance $\Phi_0 \gg 1$ and under the further adiabatic condition $\mathcal{T} \gg 10^3\tau$. In this limit, for various values of $k_B T/J$, we obtain the average velocity as a function of j_c, as shown in Fig. 1.

Exactly as our theory above has predicted, the simulation result clearly indicates that for $k_B T/J < 0.1$, the average velocity of the skyrmion is indeed an integer multiply of $\frac{1}{\sqrt{1+\kappa^2}}\frac{L}{\mathcal{T}}$, which is about 0.02 in the unit of a/τ,

At the highest value of the temperature used in our simulation, $k_B T = 0.1$, there is a small deviation from the quantized value. Note that this is not because the adiabatic condition is violated, but is due to the fact that the magnetic skyrmion becomes unstable at such a temperature. At this temperature, for the constituent spins, it becomes inappropriate to use the equation of motion (4) for the whole particlelike skyrmion, which is the basis of the above theory of topological quantization.

We have also made simulations for various values of the driving period $\mathcal{T}$. As shown in Fig. 2, the average velocity of

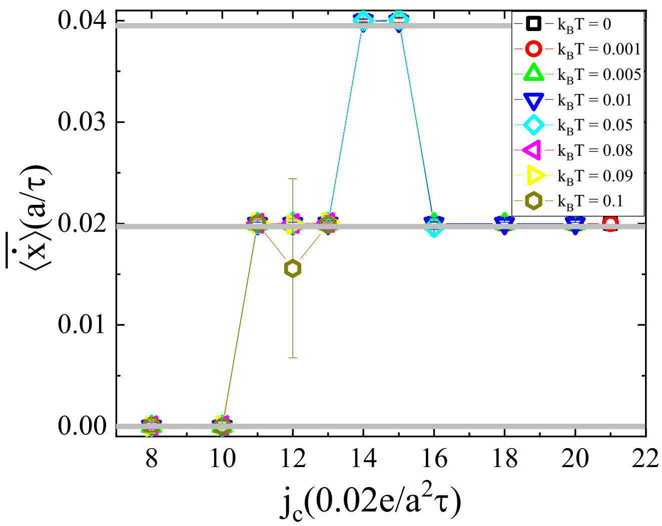

FIG. 1. The skyrmion's average velocity along x direction as a function of the amplitude j_c of the polarized electric current. The unit of the velocity is $\frac{a}{\tau}$, while the unit of j_c is $0.01\frac{2e}{a^2\tau}$. Different symbols and colors represent different values of k_BT in unit of J, i.e., k_BT/J. The grey line represents the analytically predicted value of the velocity. The parameter values used in the simulation are $k = \frac{2\pi}{100}$, $\kappa = \frac{100}{864}$, $\mathcal{T} = 5000\tau$. The amplitude A in time oscillation is fixed to be 0.2. The damping parameter α is 0.1, the magnetic field along the z direction B_z is $0.015J$, the Dzyaloshinskii-Moriya interaction constant D is $0.12J$; the anisotropic energy constant K is $0.01J$.

the skyrmion is obtained, in terms of the basic unit $\frac{1}{\sqrt{1+\kappa^2}}\frac{L}{\mathcal{T}}$. For $\mathcal{T} = 2000\tau$, the average velocity is quantized very well. However, the quantization is gradually lost with the decrease of the driving period, and is lost when $\mathcal{T} = 1700\tau$, which violates the adiabatic condition, in consistency with the above analysis.

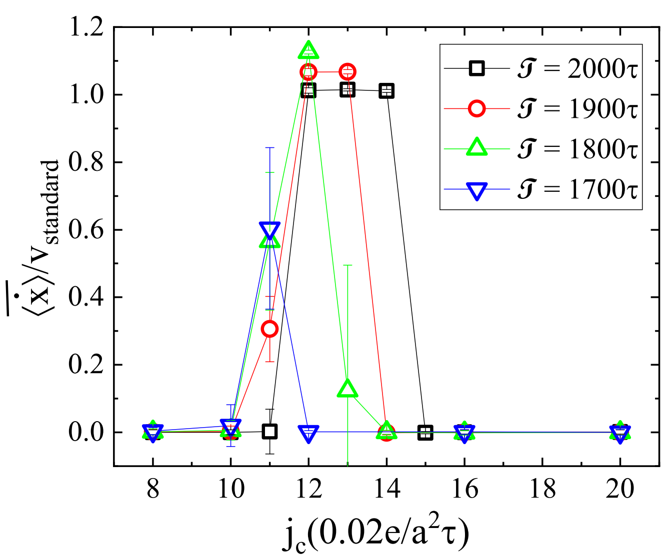

FIG. 2. The average velocity of the skyrmion divided by the basic unit $\frac{1}{\sqrt{1+\kappa^2}}\frac{L}{\mathcal{T}}$, which is denoted as v_{standard} here, as a function of j_c, for various values of the driving period $\mathcal{T}$. The temperature is fixed as $k_BT = 0.01J$.

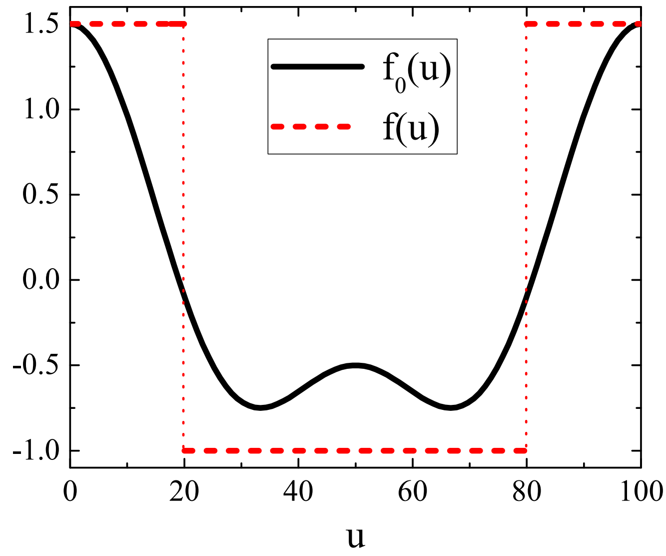

FIG. 3. Comparison between functions $f_0(u)$ and $f(u)$.

VII. EXPERIMENTAL REALIZATION

In our simulation above, the electric current is assumed to be given in Eq. (39), which can be written as

$$j_x = -j_c f_0(x - \kappa y) - A\cos\left(\frac{2\pi}{\mathcal{T}}t\right), \tag{45}$$

with

$$f_0(u) \equiv \cos(k_c u) + \tfrac{1}{2}\cos(2k_c u), \tag{46}$$

where $k_c = 2\pi/L$. In the simulation, we have chosen $L = 100a$.

Since the trigonometric functions are not easy to realize in the experiments, we replace $f_0(u)$ as

$$f(u) = \begin{cases} 1.5, & 0 \leqslant u < 20a, \\ -1 & 20a \leqslant u < 80a, \\ 1.5, & 80a \leqslant u < 100a, \end{cases} \tag{47}$$

moreover,

$$f(u + L) = f(u), \tag{48}$$

where $L = 100a$ in this example. As indicated in Fig. 3, $f_0(u)$ and $f(u)$ are close to each other. $f(u)$ can be rewritten as

$$f(u) = \begin{cases} 1.5, & -20a \leqslant u < 20a, \\ -1 & 20a \leqslant u < 80a. \end{cases} \tag{49}$$

Therefore, we propose a method of experimental realization of the polarized electric current

$$j_x = -j_c f(u) - A\cos\left(\frac{2\pi}{\mathcal{T}}t\right), \tag{50}$$

with $u = (x - \kappa y)/\sqrt{1+\kappa^2}$, as shown in Fig. 4. For easy realization, the polarized electric current is constant locally, as produced by local electrodes, but globally it satisfies Eq. (50), implementing the ratchetlike polarized electric current.

On the sample are planted many small electrodes, of which there are three kinds depicted as blue, red, and green. On each line with the slope $\frac{1}{\kappa}$, are electrodes with a same color. The

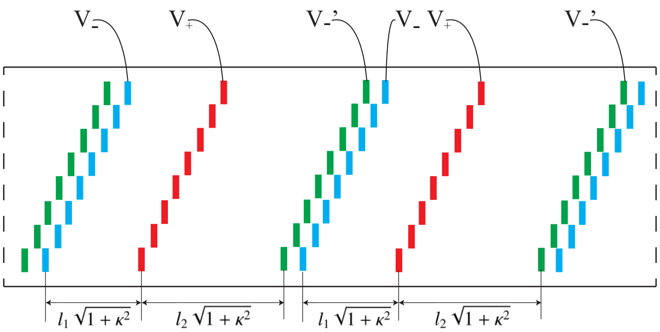

FIG. 4. A device that realizes the ratchetlike polarized electric current.

lines with the three colors alternate. The distance between the neighboring blue and red electrodes lines is $\sqrt{1+\kappa^2}l_1$, while that between the neighboring red and green lines is $\sqrt{1+\kappa^2}l_2$. Thus the spatial period along x direction is $\sqrt{1+\kappa^2}L$. $L = l_1 + l_2$ is the period in u. According to the values in the simulation, one can use $l_1 = 40a$, $l_2 = 60a$. But this is not necessary.

The distance between the neighboring green and blue lines is as small as possible. The red electrodes are all grounded, with

$$V_+ = 0. \tag{51}$$

The voltage of each blue electrode is

$$V_- = -\left(1.5j_c + A\cos\frac{2\pi}{\mathcal{T}}\right)\frac{2e}{a^2\tau}\frac{l_1\sqrt{1+\kappa^2}}{\sigma}, \tag{52}$$

where σ is the electrical conductivity of the material. The voltage of each green electrode is

$$V'_- = -\left(1.0j_c - A\cos\frac{2\pi}{\mathcal{T}}\right)\frac{2e}{a^2\tau}\frac{l_2\sqrt{1+\kappa^2}}{\sigma}. \tag{53}$$

Thus the electric current in the range of $l_1\sqrt{1+\kappa^2}$ is $\frac{\sigma(V_+-V_-)}{(-2e/a^2\tau)l_1\sqrt{1+\kappa^2}} = -1.5j_c - A\cos(\frac{2\pi}{\mathcal{T}}t)$, the electric current in the range of $l_2\sqrt{1+\kappa^2}$ is $\frac{\sigma(V'_- - V_+)}{(-2e/a^2\tau)l_2\sqrt{1+\kappa^2}} = 1.0j_c - A\cos(\frac{2\pi}{\mathcal{T}}t)$.

In the experiment, we should first generate a single skyrmion on the sample. Then apply the above voltages on the electrodes and measure the position of the skyrmion as a function of time, from which velocity of the skyrmion is obtained. The time and probabilistic average can be obtained by the averaging over the process of the transport.

VIII. SUMMARY

In summary, we have studied a magnetic skyrmion adiabatically driven by a ratchetlike polarized electric current and subject to thermal fluctuations of magnetic field. We show that the model exactly implements a two-dimensional generalization of an adiabatic thermal rachet, consequently, when the temperature is so low that the potential energy dominates the thermal energy while the adiabatic condition is also satisfied, the time and probabilistic average of its velocity is equal to the ration between the spatial and temporal periodicities multiplied by an integer called the topological Chern number. In our model, the direction of the spatial periodicity is not the direction of the electric current. This topological quantization is confirmed by our numerical simulation directly dealing with the constituent spins. We also design an experimental setup to produce the ratchetlike electric current. The topological quantization proposed here provides an interesting way of robust control of the skyrmion transport at low temperatures, which could be useful for magnetic storage and communication.

ACKNOWLEDGMENTS

This work was supported by National Natural Science Foundation of China (Grants No. 11374060 and No. 11574054).

[1] N. Nagaosa and Y. Tokura, Topological properties and dynamics of magnetic skyrmions, Nat. Nanotech. **8**, 899 (2013).
[2] S. Mühlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and P. Böni, Skyrmion lattice in a chiral magnet, Science **323**, 915 (2009).
[3] X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, Real-space observation of a two-dimensional skyrmion crystal, Nature (London) **465**, 901 (2010).
[4] X. Z. Yu, N. Kanazawa, Y. Onose, K. Kimoto, W. Z. Zhang, S. Ishiwata, Y. Matsui, and Y. Tokura, Near room-temperature formation of a skyrmion crystal in thin-films of the helimagnet FeGe, Nat. Mater. **10**, 106 (2011).
[5] S. Heinze, K. von Bergmann, M. Menzel, J. Brede, A. Kubetzka, R. Wiesendanger, G. Bihlmayer, and S. Blüegel, Spontaneous atomic-scale magnetic skyrmion lattice in two dimensions, Nat. Phys. **7**, 713 (2011).
[6] F. Jonietz, S. Mühlbauer, C. Pfleiderer, A. Neubauer, W. Münzer, A. Bauer, T. Adams, R. Georgii, P. Böni, R. A. Duine, K. Everschor, M. Garst, and A. Rosch, Spin transfer torques in MnSi at ultralow current densities, Science **330**, 1648 (2010).
[7] X. Z. Yu, N. Kanazawa, W. Z. Zhang, T. Nagai, T. Hara, K. Kimoto, Y. Matsui, Y. Onose, and Y. Tokura, Skyrmion flow near room temperature in an ultralow current density, Nat. Commun. **3**, 988 (2012).
[8] T. Schulz, R. Ritz, A. Bauer, M. Halder, M. Wagner, C. Franz, C. Pfleiderer, K. Everschor, M. Garst, and A. Rosch, Emergent electrodynamics of skyrmions in a chiral magnet, Nat. Phys. **8**, 301 (2012).
[9] K. Everschor, M. Garst, B. Binz, F. Jonietz, S. Müehlbauer, C. Pfleiderer, and A. Rosch, Rotating skyrmion lattices by spin torques and field or temperature gradients, Phys. Rev. B **86**, 054432 (2012).

[10] J. Iwasaki, M. Mochizuki, and N. Nagaosa, Universal current-velocity relation of skyrmion motion in chiral magnets, Nat. Commun. **4**, 1463 (2013).
[11] L. Kong and J. Zang, Dynamics of an Insulating Skyrmion Under a Temperature Gradient, Phys. Rev. Lett. **111**, 067203 (2013).
[12] S. Lin, C. D. Batista, C. Reichhardt, and A. Saxena, AC Current Generation in Chiral Magnetic iInsulators and Skyrmion Motion Induced by the Spin Seebeck Effect, Phys. Rev. Lett. **112**, 187203 (2014).
[13] M. Mochizuki, X. Z. Yu, S. Seki, N. Kanazawa, W. Koshibae, J. Zang, M. Mostovoy, Y. Tokura, and N. Nagaosa, Thermally driven ratchet motion of a skyrmion microcrystal and topological magnon Hall effect, Nat. Mater. **13**, 241 (2014).
[14] R. E. Troncoso and Á. S. Núñez, Thermally assisted current-driven skyrmion motion, Phys. Rev. B **89**, 224403 (2014).
[15] R. E. Troncoso and Á. S. Núñez, Brownian motion of massive skyrmions in magnetic thin films, Ann. Phys. **351**, 850 (2014).
[16] J. L. García-Palacios and F. J. Lázaro, Langevin-dynamics study of the dynamical properties of small magnetic particles, Phys. Rev. B **58**, 14937 (1998).
[17] C. Reichhardt, D. Ray, and C. J. Olson Reichhardt, Magnus-induced ratchet effects for skyrmions interacting with asymmetric substrates, New J. Phys. **17**, 073034 (2015).
[18] Y. Shi and Q. Niu, Quantization and corrections of adiabatic particle transport in a periodic ratchet potential, Europhys. Lett. **59**, 324 (2002).
[19] I. Dzyaloshinsky, A thermodynamic theory of "weak" ferromagnetism of antiferromagnetics, J. Phys. Chem. Solids **4**, 241 (1958).
[20] T. Moriya, Anisotropic superexchange interaction and weak ferromagnetism, Phys. Rev. **120**, 91 (1960).
[21] A. A. Thiele, Steady-State Motion of Magnetic Domains, Phys. Rev. Lett. **30**, 230 (1973).
[22] H. Risken, *The Fokker-Planck Equation* (Springer, Berlin, 1989).
[23] P. Reimann, Brownian motors: Noisy transport far from equilibrium, Phys. Rep. **361**, 57 (2002).
[24] R. Bartussek, P. Hänggi, and J. G. Kissner, Periodically rocked thermal ratchets, Europhys. Lett. **28**, 459 (1994).