Scattering of exchange spin waves from a helimagnetic layer sandwiched between two semi-infinite ferromagnetic media

V. D. Poimanov [1], A. N. Kuchko [2] and V. V. Kruglyak [3,*]
[1] Faculty of Physics and Technology, Donetsk National University, 24 Universitetskaya Street, Donetsk 83001, Ukraine
[2] Faculty of Physics and Mathematics, Igor Sikorsky Kyiv Polytechnic Institute, 37 Prospect Peremohy, Kyiv 03056, Ukraine
[3] Department of Physics and Astronomy, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom

(Received 8 November 2019; revised 1 June 2020; accepted 19 August 2020; published 10 September 2020)

We have calculated the scattering (reflection and transmission) coefficients of linear exchange spin waves normally incident upon a helimagnetic layer sandwiched between two semi-infinite ferromagnetic media. Our calculations show that, despite the helimagnetic order induced in the layer by the Dzyaloshinskii-Moriya interaction (DMI), the scattering is reciprocal and insensitive to the presence of the helimagnetic order in the layer. This comes as a result of the disappearance of the DMI from the boundary conditions in the considered geometry under the small-amplitude approximation and from the specific form of the nonreciprocity of the spin-wave dispersion relation in the helimagnetic material. We show that the helimagnetic layer's interfaces act as a system of two semicrossed polarizers for the circularly polarized spin waves incident from the ferromagnetic media. This results from the ellipticity of the magnetic precession induced by the easy-plane anisotropy in the helimagnetic layer. Our calculations also reveal the importance of evanescent solutions to correctly describe the spin-wave scattering in samples with elliptical precession. Our findings will aid development of magnonic devices containing helimagnetic constituents.

DOI: 10.1103/PhysRevB.102.104414

I. INTRODUCTION

It is widely recognized that noncentrosymmetric magnetic materials [1–6] bear great promise for magnonics [7,8]—the study of spin waves [9]—in terms of novel physical phenomena and device functionalities. The lack of inversion symmetry in such materials leads to an antisymmetric exchange coupling, known as Dzyaloshinskii-Moriya interaction (DMI), between spins [1,2]. This results in highly nonuniform static magnetic configurations [[3–6] and nonreciprocity of the spin-wave dispersion[3–6,10–12] and damping [13]. As with other areas of wave physics [14,15], the scattering of spin waves in nonuniform magnonic media and waveguides [16–32] is an essential aspect in magnonics. Of particular relevance to spin-wave devices [7,8] is scattering from local nonuniformities within otherwise homogeneous magnonic media or waveguides [16–27,29-32]. It is therefore tempting to realize (e.g., using the DMI) nonreciprocal scattering of spin waves from some sort of a magnetic nonuniformity, since this could lead, e.g., to the creation of spin-wave diodes [33]. However, the great majority of the magnonic devices studied so far scatter spin waves reciprocally. Exceptions include the Fanolike [34] system from Ref. [23], where the nonreciprocity is due to the chirality of the stray magnetic field from the precessing magnetization, and the topology-induced skew scattering of spin waves from magnetic skyrmions in thin films with DMI [25,26].

Here, we study theoretically the scattering of exchange spin waves from a thin helimagnetic layer [3–6] sandwiched between two semi-infinite ferromagnetic media, as a model system in which the effects of the DMI and associated spin-wave nonreciprocity could be observed. However, our analysis show that, in the considered geometry, the scattering of spin waves from the helimagnetic layer is reciprocal and does not appear to feel the presence of the helimagnetic ordering in the layer, in contrast, e.g., to Ref. [35]. This outcome results from the disappearance of the DMI from the boundary conditions in the considered geometry under the small-amplitude approximation and from the specific form of the nonreciprocity of the spin-wave dispersion relation. The ellipticity of precession in the helimagnet requires us to account for evanescent spin-wave modes, exponentially decaying from each interface into the adjacent materials. The helimagnetic layer's interfaces then act as a system of two polarizers for the incident spin waves, which have circular polarization.

The paper is organized as follows. In Sec. II, we introduce our theoretical model and the micromagnetic ground state in the helimagnetic layer. In Sec. III, we derive the spin-wave dispersion and general solutions for the constituent ferromagnetic and helimagnetic materials. Section IV presents our main analytical results for the complex reflection and transmission coefficients of spin waves scattered from a helimagnetic layer embedded within a ferromagnetic matrix. In Secs. V and VI, we discuss our results and present our conclusions, respectively.

II. MODEL AND GROUND STATE

We consider a layer of a helimagnetic material "H" ($-d/2 < z < +d/2$, where d is the thickness of the layer)

*Corresponding author: V.V.Kruglyak@exeter.ac.uk

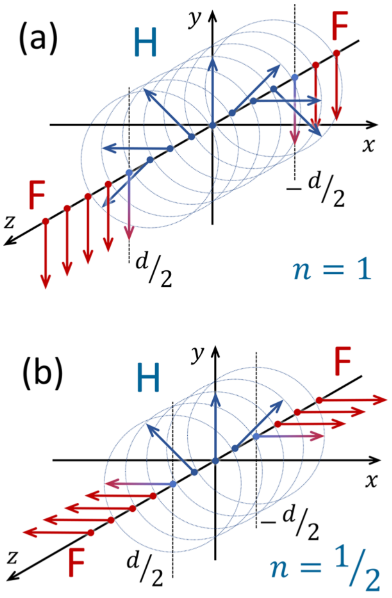

FIG. 1. The ground state magnetic configurations realized in samples with $n = 1$ and $n = \frac{1}{2}$ turns of the helix in the H layer are shown schematically in panels (a) and (b), respectively. The magnetization in the H layer rotates according to Eq. (4), while the F media are uniformly magnetized collinear to y and x axes when the H layer contains (a) integer (e.g., one) and (b) half-integer (e.g., a half) numbers of turns of the helicoid, respectively.

parallel to the x-y plane and sandwiched between two semi-infinite media made of a ferromagnetic material "F" ($z < -d/2$ and $z > +d/2$), as shown in Fig. 1. Throughout the paper, we use subscripts "H" and "F" for quantities characterizing materials H and F, respectively. We consider two special cases of the thickness of the helimagnetic layer containing either an integer number n, or a half-integer number $n + \frac{1}{2}$ of helix periods (turns).

The magnetization dynamics are described by the Landau-Lifshitz equation [9]

$$\frac{\partial \boldsymbol{M}_{\mathrm{H(F)}}}{\partial t} = -\gamma_{\mathrm{H(F)}}[\boldsymbol{M}_{\mathrm{H(F)}} \times \boldsymbol{H}_{\mathrm{eff,H(F)}}] + \frac{\alpha_{\mathrm{H(F)}}}{M_{\mathrm{H(F)}}}\left[\boldsymbol{M}_{\mathrm{H(F)}} \times \frac{\partial \boldsymbol{M}_{\mathrm{H(F)}}}{\partial t}\right], \quad (1)$$

where $\boldsymbol{M}_{\mathrm{H(F)}}$ is the magnetization, $\gamma_{\mathrm{H(F)}}$ is the gyromagnetic ratio, $\alpha_{\mathrm{H(F)}}$ is the dimensionless (Gilbert) damping constant, and t is the time. We do not account for the nonlocal damping [13], so as not to overcomplicate the calculations. The effective magnetic field is

$$\boldsymbol{H}_{\mathrm{eff,H(F)}} = -\frac{\delta w_{\mathrm{H(F)}}}{\delta \boldsymbol{M}_{\mathrm{H(F)}}}, \quad (2)$$

where $w_{\mathrm{H(F)}}$ is the volume magnetic energy density

$$w_{\mathrm{H(F)}} = \tfrac{1}{2}\lambda^2_{\mathrm{H(F)}}(\nabla \boldsymbol{M}_{\mathrm{H(F)}})^2 + \tfrac{1}{2}D_{\mathrm{H(F)}}\boldsymbol{M}_{\mathrm{H(F)}}[\nabla \times \boldsymbol{M}_{\mathrm{H(F)}}] \pm \tfrac{1}{2}\beta_{\mathrm{H(F)}}(\hat{\boldsymbol{n}}_{\mathrm{H(F)}}\boldsymbol{M}_{\mathrm{H(F)}})^2. \quad (3)$$

Here, $\lambda_{\mathrm{H(F)}} = \frac{\sqrt{2A_{\mathrm{H(F)}}}}{M_{\mathrm{H(F)}}}$ is the magnetic exchange length, where $A_{\mathrm{H(F)}}$ is the symmetric exchange constant. Both materials are uniaxial with anisotropy axes parallel to $\hat{\boldsymbol{n}}_{\mathrm{H(F)}}$. In the helimagnet, we have $\hat{\boldsymbol{n}}_{\mathrm{H}} \parallel \hat{z}$, while the direction of $\hat{\boldsymbol{n}}_{\mathrm{F}}$ (and so, of the anisotropy axis in the ferromagnet) will be defined later. The dimensionless anisotropy constant is $\beta_{\mathrm{H(F)}} > 0$. The $+$ and $-$ signs correspond to materials H (with "easy plane" anisotropy) and F (with "easy axis" anisotropy), respectively. $D_{\mathrm{H(F)}}$ is the strength of the DMI (of bulk origin [36,37]), such that $D_{\mathrm{H}} \equiv D$ and $D_{\mathrm{F}} \equiv 0$. The Dzyaloshinskii vector is assumed to be parallel to $\hat{z}$.

We limit our consideration to the case of zero bias magnetic field. Then, the energy of the helimagnet is minimized when its magnetization forms a helix described by [3,4]

$$M_{\mathrm{H},0,x} = -M_{\mathrm{H}} \sin(K_{\mathrm{H}}z), \qquad M_{\mathrm{H},0,y} = M_{\mathrm{H}} \cos(K_{\mathrm{H}}z), \quad M_{\mathrm{H},0,z} = 0, \quad (4)$$

where the wave vector $\boldsymbol{K}_{\mathrm{H}}$ and period of the helix l are

$$\boldsymbol{K}_{\mathrm{H}} = \frac{D}{\lambda^2_{\mathrm{H}}}\hat{z}, \quad l = \frac{2\pi}{K_{\mathrm{H}}}. \quad (5)$$

In an infinite helimagnetic sample, the energy of the helix is invariant relative to its translations along the $\hat{z}$ axis. In our model, we would like to keep the position of the helix fixed by Eqs. (4). So, we adjust the orientation of the anisotropy axes in material F to ensure that in the ground magnetic state: (i) Eqs. (4) are satisfied in material H, (ii) the magnetization of the ferromagnetic media is uniform (and collinear to $\hat{\boldsymbol{n}}_{\mathrm{F}}$), and (iii) the boundary conditions at the interfaces between the H layer and the F media

$$\boldsymbol{M}_{\mathrm{H}} \times \boldsymbol{M}_{\mathrm{F}} = 0, \qquad \lambda^2_{\mathrm{F}}\boldsymbol{M}_{\mathrm{F}} \times \frac{\partial \boldsymbol{M}_{\mathrm{F}}}{\partial z} = \lambda^2_{\mathrm{H}}\boldsymbol{M}_{\mathrm{H}} \times \left(\frac{\partial \boldsymbol{M}_{\mathrm{H}}}{\partial z} - [\boldsymbol{K}_{\mathrm{H}} \times \boldsymbol{M}_{\mathrm{H}}]\right), \quad (6)$$

are satisfied. The boundary conditions represent a limiting case of those derived in Ref. [38], assuming a strong ferromagnetic coupling between the helimagnet and ferromagnet and neglecting any interface anisotropy. Our boundary conditions can also be obtained from those in Ref. [39] by setting the components of the DMI tensor (Eq. (7) in Ref. [39]) as $D_1 = D_2 = D_5 = 2D$, $D_3 = D_4 = 0$. The required ground state is realized if $\hat{\boldsymbol{n}}_{\mathrm{F}} \parallel \hat{\boldsymbol{x}}$ ($\hat{\boldsymbol{n}}_{\mathrm{F}} \parallel \hat{\boldsymbol{y}}$) when the helimagnetic layer fits a half-integer (integer) number of helix turns. Figures 1(a) and 1(b) illustrate the static magnetic configurations realized in samples with $n = 1$ and $n = \frac{1}{2}$ turns of the helix in the H layer, respectively.

In this paper, we only consider small-amplitude exchange spin waves. So, we linearize Eq. (1), to obtain the linearized Landau-Lifshitz equation

$$\frac{\partial \boldsymbol{m}_{\mathrm{H(F)}}}{\partial t} = -\gamma_{\mathrm{H(F)}}([\boldsymbol{M}_{\mathrm{H(F)},0} \times \boldsymbol{h}_{\mathrm{eff,H(F)}}] + [\boldsymbol{m}_{\mathrm{H(F)}} \times \boldsymbol{H}_{\mathrm{eff,H(F)},0}]) + \frac{\alpha_{\mathrm{H(F)}}}{M_{\mathrm{H(F)}}}\left[\boldsymbol{M}_{\mathrm{H(F)},0} \times \frac{\partial \boldsymbol{m}_{\mathrm{H(F)}}}{\partial t}\right], \quad (7)$$

where subscripts 0 denote static quantities, while $\boldsymbol{m}_{\mathrm{H(F)}}$ and $\boldsymbol{h}_{\mathrm{eff,H(F)}}$ are the small dynamic perturbations to the magnetization and effective magnetic field, respectively.

III. SPIN-WAVE DISPERSION RELATIONS AND GENERAL SOLUTIONS

In this section, we derive expressions for the spin-wave dispersion in the constituent materials. The dynamic magnetization is assumed to have the same time dependence, i.e., $\boldsymbol{m}_{\mathrm{H(F)}} \propto \exp(-i\omega t)$. Furthermore, we neglect any magnetic nonuniformity in the x-y plane, i.e., we only consider normally incident spin waves (propagating along the z axis).

The static effective magnetic field in the helimagnet with magnetization described by Eq. (4) is equal to zero, i.e., $\boldsymbol{H}_{\mathrm{eff,H,0}} = 0$. For the components of the dynamic effective field, we obtain from Eqs. (2) and (3)

$$\begin{aligned} h_{\mathrm{eff,H},x} &= \lambda_H^2 \frac{\partial^2 m_{\mathrm{H},x}}{\partial z^2} + D\frac{\partial m_{\mathrm{H},y}}{\partial z}, \\ h_{\mathrm{eff,H},y} &= \lambda_H^2 \frac{\partial^2 m_{\mathrm{H},y}}{\partial z^2} - D\frac{\partial m_{\mathrm{H},x}}{\partial z}, \\ h_{\mathrm{eff,H},z} &= \lambda_H^2 \frac{\partial^2 m_{\mathrm{H},z}}{\partial z^2} - \beta_{\mathrm{H}} m_{\mathrm{H},z}. \end{aligned} \tag{8}$$

Furthermore, we introduce circular variables as follows [3,4]:

$$\begin{aligned} m_{\mathrm{H},\pm} &= (m_{\mathrm{H},x} \pm i m_{\mathrm{H},y}) e^{\mp iK_{\mathrm{H}} z}, \\ M_{\mathrm{H},0,\pm} &= \pm i M_{\mathrm{H}} e^{\pm iK_{\mathrm{H}} z}, \\ h_{\mathrm{eff,H},\pm} &= \lambda_H^2 \frac{\partial^2 m_{\mathrm{H},\pm}}{\partial z^2} \mp iD\frac{\partial m_{\mathrm{H},\pm}}{\partial z}. \end{aligned} \tag{9}$$

Then, Eq. (7) is reduced to the following system of equations:

$$\begin{aligned} \left(-\lambda_{\mathrm{H}}^2 \frac{\partial^2}{\partial z^2} + \beta_{\mathrm{H}} - i\alpha_{\mathrm{H}}\frac{\omega}{\omega_{\mathrm{H}}}\right) m_{\mathrm{H},z} + i\frac{\omega}{\omega_{\mathrm{H}}} m_{\mathrm{H},\pm} &= 0, \\ \left(-\lambda_{\mathrm{H}}^2 \frac{\partial^2}{\partial z^2} - i\alpha_{\mathrm{H}}\frac{\omega}{\omega_{\mathrm{H}}}\right)(m_{\mathrm{H},+} + m_{\mathrm{H},-}) - 2i\frac{\omega}{\omega_{\mathrm{H}}} m_{\mathrm{H},z} &= 0, \end{aligned} \tag{10}$$

where $\omega_{\mathrm{H}} = \gamma M_{\mathrm{H}}$. Noting that Eq. (7) must preserve the length of the magnetization vector and that the ground magnetic state is described by Eq. (4), we obtain $m_{\mathrm{H},+} - m_{\mathrm{H},-} = 0$. This allows us to exclude one equation from system (10), obtaining in a matrix form

$$\begin{pmatrix} -\lambda_{\mathrm{H}}^2 \frac{\partial^2}{\partial z^2} - i\alpha_{\mathrm{H}}\frac{\omega}{\omega_{\mathrm{H}}} & -i\frac{\omega}{\omega_{\mathrm{H}}} \\ i\frac{\omega}{\omega_{\mathrm{H}}} & -\lambda_{\mathrm{H}}^2 \frac{\partial^2}{\partial z^2} + \beta_{\mathrm{H}} - i\alpha_{\mathrm{H}}\frac{\omega}{\omega_{\mathrm{H}}} \end{pmatrix} \begin{pmatrix} m_{\mathrm{H},+} \\ m_{\mathrm{H},z} \end{pmatrix} = 0. \tag{11}$$

To find the spin-wave dispersion, we seek solutions of system (11) in the form of planes waves

$$m_{\mathrm{H},+} = iC_{\mathrm{H}} e^{ik_{\mathrm{H}} z}, \quad m_{\mathrm{H},z} = D_{\mathrm{H}} e^{ik_{\mathrm{H}} z}, \tag{12}$$

where k_{H} is the "reduced" wave number of spin waves in the helimagnet, i.e., the wave number in a system rotating with the static magnetization as defined by Eq. (9) [3]. This substitution converts Eq. (11) into a system of algebraic equations for amplitudes C_{H} and D_{H}. By equating to zero the determinant of this algebraic system, we obtain the complex dispersion relation as

$$\left(\frac{\omega}{\omega_{\mathrm{H}}}\right)^2 = \left(\lambda_{\mathrm{H}}^2 k_{\mathrm{H}}^2 + \beta_{\mathrm{H}} - i\alpha_{\mathrm{H}}\frac{\omega}{\omega_{\mathrm{H}}}\right)\left(\lambda_{\mathrm{H}}^2 k_{\mathrm{H}}^2 - i\alpha_{\mathrm{H}}\frac{\omega}{\omega_{\mathrm{H}}}\right). \tag{13}$$

Equation (13) predicts that for each value of ω, there are four roots for k_{H}: two ($\pm k_{\mathrm{H,p}}$) corresponding to propagating waves and two ($\pm ik_{\mathrm{H,e}}$) corresponding to evanescent waves, where

$$k_{\mathrm{H},{\mathrm{p} \atop \mathrm{e}}} = \frac{1}{\lambda_{\mathrm{H}}}\sqrt{\sqrt{\left(\frac{\omega}{\omega_{\mathrm{H}}}\right)^2 + \left(\frac{\beta_{\mathrm{H}}}{2}\right)^2} \mp \frac{\beta_{\mathrm{H}}}{2} \pm i\alpha_{\mathrm{H}}\frac{\omega}{\omega_{\mathrm{H}}}}. \tag{14}$$

These waves have elliptical precession with ellipticities $\eta_{\mathrm{H}} \equiv D_{\mathrm{H}}/C_{\mathrm{H}}$ given by

$$\begin{aligned} \eta_{\mathrm{H},{\mathrm{p} \atop \mathrm{e}}} &= \frac{\omega_{\mathrm{H}}}{\omega}\left(\lambda_{\mathrm{H}}^2 k_{\mathrm{H}}^2 - i\alpha_{\mathrm{H}}\frac{\omega}{\omega_{\mathrm{H}}}\right) \\ &= \pm\frac{\omega_{\mathrm{H}}}{\omega}\left(\sqrt{\left(\frac{\omega}{\omega_{\mathrm{H}}}\right)^2 + \left(\frac{\beta_{\mathrm{H}}}{2}\right)^2} \mp \frac{\beta_{\mathrm{H}}}{2}\right). \end{aligned} \tag{15}$$

The ellipticities of the propagating and evanescent waves are related via

$$\eta_{\mathrm{H,p}}\eta_{\mathrm{H,e}} = -1. \tag{16}$$

For the spin-wave dispersion in the ferromagnetic material, similar calculations yield

$$\left(\frac{\omega}{\omega_{\mathrm{F}}}\right)^2 = \left(\beta_{\mathrm{F}} + \lambda_{\mathrm{F}}^2 k_{\mathrm{F}}^2 - i\alpha_{\mathrm{F}}\frac{\omega}{\omega_{\mathrm{F}}}\right)^2, \tag{17}$$

where $\omega_{\mathrm{F}} = \gamma M_{\mathrm{F}}$. Like for the helimagnetic material, for each value of $\omega > \beta_{\mathrm{F}}$, we have four roots for k_{F}: two ($\pm k_{\mathrm{F,p}}$) corresponding to propagating waves and two ($\pm ik_{\mathrm{F,e}}$) corresponding to evanescent waves, where

$$k_{\mathrm{F},{\mathrm{p} \atop \mathrm{e}}} = \frac{1}{\lambda_{\mathrm{F}}}\sqrt{\frac{\omega}{\omega_{\mathrm{F}}} \mp \beta_{\mathrm{F}} \pm i\alpha_{\mathrm{F}}\frac{\omega}{\omega_{\mathrm{F}}}}. \tag{18}$$

These spin waves are circularly polarized at all frequencies, with ellipticities given by $\eta_{\mathrm{F,p}} = 1$ and $\eta_{\mathrm{F,e}} = -1$. The dispersion of propagating spin waves described by Eq. (18) is plotted in Fig. 2(a). In contrast to the helimagnetic material, the ferromagnetic dispersion has a frequency gap proportional to the strength of the uniaxial anisotropy β_{F}.

The frequency dependence of the real part of the wave number $k_{\mathrm{H,p}}$ defined by Eq. (14) (i.e., the dispersion relation) is shown in Fig. 2(a). The characteristic feature of this dependence is the absence of the frequency gap at $\mathrm{Re}(k_{\mathrm{H,p}}) = 0$. The value of the uniaxial anisotropy constant β_{H} controls the curvature of the dispersion curve at low frequencies. Figure 2(b) shows the ellipticity of the propagating spin wave mode as a function of frequency [Eq. (15)], while we keep in mind that the ellipticity of the evanescent mode has a reciprocal dependence, Eq. (16). The ellipticity of the propagating (evanescent) wave tends to zero (diverges) at zero frequency and asymptotically approach 1 (–1) at high frequencies. This means that the precession becomes circular. The precession's chirality is opposite for the propagating and evanescent waves, i.e., the magnetization undergoes Larmor and anti-Larmor precession [40–42], respectively.

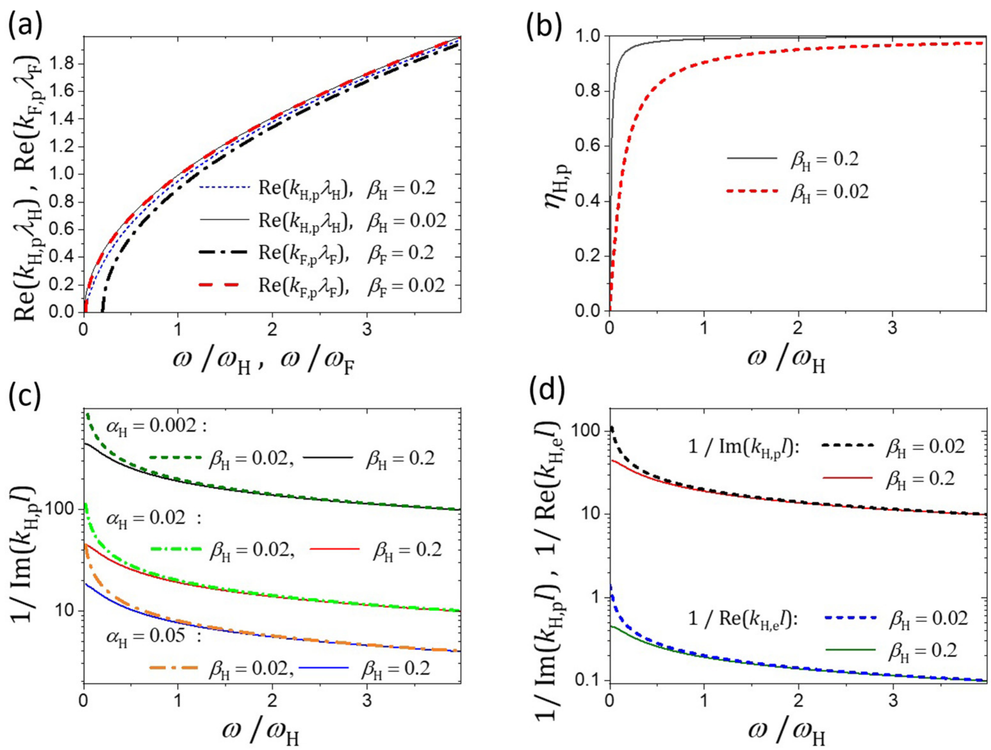

FIG. 2. (a) The dispersion relations of the propagating exchange spin waves in the helimagnetic, $\mathrm{Re}(k_{\mathrm{H,p}}\lambda_{\mathrm{H}})$, and ferromagnetic, $\mathrm{Re}(k_{\mathrm{F,p}}\lambda_{\mathrm{F}})$, media are plotted for characteristic values of the strength of the uniaxial anisotropy $\beta_{\mathrm{H(F)}}$. (b) The frequency dependence of the ellipticity of propagating spin waves is shown for the same values of β_{H} as in panel (a). (c) The penetration depth of the propagating spin waves into the helimagnetic layer is shown for characteristic values of $\beta_{\mathrm{H(F)}}$ and of the damping constant α_{H}. (d) The penetration depths of the propagating and evanescent spin waves into the helimagnetic layer are compared for the same values of $\beta_{\mathrm{H(F)}}$ as in panel (c) while keeping the damping constant value fixed at $\alpha_{\mathrm{H}} = 0.02$. In all panels, we assume $l = 5\lambda_{\mathrm{H}}$.

Figures 2(c) and 2(d) compare the depth of penetration of the propagating and evanescent spin waves into the helimagnetic layer. The penetration depth of the propagating modes $(\mathrm{Im}(k_{\mathrm{H,p}}))^{-1}$ is determined primarily by the value of the damping constant α_{H}, which has almost no effect on the penetration depth of the evanescent waves $(\mathrm{Re}(k_{\mathrm{H,e}}))^{-1}$. At low frequencies, the penetration depths of both propagating and evanescent waves are affected by the value of β_{H}: both depths decrease as β_{H} increases, making the precession more elliptical. At all frequencies, the penetration depth of evanescent modes is consistently smaller than that of propagating spin waves.

In infinite homogeneous media, the evanescent solutions are neglected as unphysical, since they would grow indefinitely at one of the infinity limits otherwise. However, their account is needed when spin-wave scattering from a localized nonuniformity is studied [43–45]. Specifically, we will use the full general solution of the homogeneous system corresponding to Eq. (11)

$$m_{\mathrm{H},x} = i\cos(K_{\mathrm{H}}z)\big(C^{(+)}_{\mathrm{H,p}}e^{ik_{\mathrm{H,p}}z} + C^{(-)}_{\mathrm{H,p}}e^{-ik_{\mathrm{H,p}}z} + C^{(+)}_{\mathrm{H,e}}e^{-k_{\mathrm{H,e}}z} + C^{(-)}_{\mathrm{H,e}}e^{k_{\mathrm{H,e}}z}\big),$$

$$m_{\mathrm{H},y} = i\sin(K_{\mathrm{H}}z)\big(C^{(+)}_{\mathrm{H,p}}e^{ik_{\mathrm{H,p}}z} + C^{(-)}_{\mathrm{H,p}}e^{-ik_{\mathrm{H,p}}z} + C^{(+)}_{\mathrm{H,e}}e^{-k_{\mathrm{H,e}}z} + C^{(-)}_{\mathrm{H,e}}e^{k_{\mathrm{H,e}}z}\big),$$

$$m_{\mathrm{H},z} = \eta_{\mathrm{H,p}}\big(C^{(+)}_{\mathrm{H,p}}e^{ik_{\mathrm{H,p}}z} + C^{(-)}_{\mathrm{H,p}}e^{-ik_{\mathrm{H,p}}z}\big) + \eta_{\mathrm{H,e}}\big(C^{(+)}_{\mathrm{H,e}}e^{-k_{\mathrm{H,e}}z} + C^{(-)}_{\mathrm{H,e}}e^{k_{\mathrm{H,e}}z}\big). \tag{19}$$

The dependence on $K_{\mathrm{H}}z$ enters Eq. (19) because, due to the substitution of variables described by Eqs.(9), wave numbers $k_{\mathrm{H,p}}$ and decay rates $k_{\mathrm{H,e}}$ are defined in the coordinate frame that is “rotating” with the static magnetization [3,4]. This rotation of the static magnetization must be accounted for when calculating the overall orientation of the magnetization, e.g., when matching the magnetizations at interfaces with the ferromagnetic media using boundary conditions (6).

When the H layer fits an integer number n of helicoid turns (and therefore, $\boldsymbol{n}_{\mathrm{F}} \parallel \hat{\boldsymbol{y}}$ and $m_{\mathrm{F},y} = 0$), the full general solution of the homogeneous problem is

$$m_{\mathrm{F},x} = i(-1)^n\big(C^{(+)}_{\mathrm{F,p}}e^{ik_{\mathrm{F,p}}z} + C^{(-)}_{\mathrm{F,p}}e^{-ik_{\mathrm{F,p}}z} + C^{(+)}_{\mathrm{F,e}}e^{-k_{\mathrm{F,e}}z} + C^{(-)}_{\mathrm{F,e}}e^{k_{\mathrm{F,e}}z}\big),$$

$$m_{\mathrm{F},z} = C^{(+)}_{\mathrm{F,p}}e^{ik_{\mathrm{F,p}}z} + C^{(-)}_{\mathrm{F,p}}e^{-ik_{\mathrm{F,p}}z} - C^{(+)}_{\mathrm{F,e}}e^{-k_{\mathrm{F,e}}z} - C^{(-)}_{\mathrm{F,e}}e^{k_{\mathrm{F,e}}z}. \tag{20}$$

When the H layer fits $n + \frac{1}{2}$ helicoid turns (and therefore, $\boldsymbol{n}_{\mathrm{F}} \parallel \hat{\boldsymbol{x}}$ and $m_{\mathrm{F},x} = 0$), the static magnetizations on the opposite boundaries of the H layer are antiparallel, and we have for the general solution

$$\begin{aligned} m_{\mathrm{F},y} &= \mp i(-1)^n \big(C^{(+)}_{\mathrm{F,p}} e^{ik_{\mathrm{F,p}}z} + C^{(-)}_{\mathrm{F,p}} e^{-ik_{\mathrm{F,p}}z} \\ &\quad + C^{(+)}_{\mathrm{F,e}} e^{-k_{\mathrm{F,e}}z} + C^{(-)}_{\mathrm{F,e}} e^{k_{\mathrm{F,e}}z}\big), \\ m_{\mathrm{F},z} &= C^{(+)}_{\mathrm{F,p}} e^{ik_{\mathrm{F,p}}z} + C^{(-)}_{\mathrm{F,p}} e^{-ik_{\mathrm{F,p}}z} - C^{(+)}_{\mathrm{F,e}} e^{-k_{\mathrm{F,e}}z} - C^{(-)}_{\mathrm{F,e}} e^{k_{\mathrm{F,e}}z}, \end{aligned} \tag{21}$$

where the minus and plus signs in the expression for $m_{\mathrm{F},y}$ correspond to the ferromagnet on the left-hand side from the H layer (where $\boldsymbol{M}_{\mathrm{F},0} \parallel \hat{\boldsymbol{x}}$ for even values of n) and on the right-hand side from the H layer (where $\boldsymbol{M}_{\mathrm{F},0} \parallel \hat{\boldsymbol{x}}$ for odd values of n), respectively.

IV. SPIN-WAVE SCATTERING FROM THE HELIMAGNETIC LAYER

Let us consider a spin wave of unit amplitude incident on the helimagnetic layer from the left. The scattering coefficients are obtained by matching the general solutions of the homogeneous Eqs. (19), (20), and (21) at interfaces, i.e., for $z = \pm d/2$, using the linearized boundary conditions (6)

$$\begin{aligned} &[\boldsymbol{M}_{0,\mathrm{H}} \times \boldsymbol{m}_\mathrm{F}] - [\boldsymbol{M}_{0,\mathrm{F}} \times \boldsymbol{m}_\mathrm{H}] = 0, \\ &\left[\lambda_\mathrm{F}^2 \boldsymbol{M}_{0,\mathrm{F}} \times \frac{\partial \boldsymbol{m}_\mathrm{F}}{\partial z}\right] - \left[\lambda_\mathrm{H}^2 \boldsymbol{M}_{0,\mathrm{H}} \times \frac{\partial \boldsymbol{m}_\mathrm{H}}{\partial z}\right] = 0, \end{aligned} \tag{22}$$

where we have taken into account that, in the ground state described by Eq. (4),

$$\frac{\partial \boldsymbol{M}_{\mathrm{H},0}}{\partial z} - [\boldsymbol{K}_\mathrm{H} \times \boldsymbol{M}_{\mathrm{H},0}] = 0, \tag{23}$$

and moreover that, in the linear approximation,

$$\big[\lambda_\mathrm{H}^2 \boldsymbol{M}_{0,\mathrm{H}} \times [\boldsymbol{K}_\mathrm{H} \times \boldsymbol{m}_\mathrm{H}]\big] = 0. \tag{24}$$

Notably, the wave vector of the helix $\boldsymbol{K}_\mathrm{H}$ is absent from the linearized boundary conditions (22).

We take into account that, in the left-hand side ferromagnetic medium, there are three waves: two propagating (one incident and one reflected, with amplitudes of 1 and r_p, respectively) and one evanescent (decaying exponentially from the interface to the left), with amplitude of r_e. In the right-hand side ferromagnetic medium, there are two waves: one propagating (transmitted) and one evanescent (decaying exponentially from the interface to the right), with amplitudes of t_p and t_e, respectively. In the helimagnetic layer, there are four waves: two counterpropagating, with amplitudes $c_\mathrm{p}^{(\pm)}$, and two evanescent (one each decaying exponentially from each interface into the interior of the layer), with amplitudes $c_\mathrm{e}^{(\pm)}$.

The subsequent derivations depend on whether the helimagnetic layer contains an integer or half-integer number of turns of the helicoid. For an integer number of turns of the helicoid in the H layer, we have $K_\mathrm{H} d = 2\pi n$, i.e., $d = nl$. So, from Cartesian components of Eqs. (22) at the two interfaces, we obtain eight equations

$$\begin{aligned} &(\mu_{\mathrm{H},x(z)} - \mu_{\mathrm{FL},x(z)})|_{z=-\frac{d}{2}} = 0, \quad (\mu_{\mathrm{H},x(z)} - \mu_{\mathrm{FR},x(z)})|_{z=\frac{d}{2}} = 0, \\ &\left(A_\mathrm{H} \frac{\partial \mu_{\mathrm{H},x(z)}}{\partial z} - A_\mathrm{F} \frac{\partial \mu_{\mathrm{FL},x(z)}}{\partial z}\right)\bigg|_{z=-\frac{d}{2}} = 0, \quad \left(A_\mathrm{H} \frac{\partial \mu_{\mathrm{H},x(z)}}{\partial z} - A_\mathrm{F} \frac{\partial \mu_{\mathrm{FR},x(z)}}{\partial z}\right)\bigg|_{z=\frac{d}{2}} = 0, \end{aligned} \tag{25}$$

where the normalized dynamic magnetizations $\boldsymbol{\mu}_{\mathrm{H(FL,FR)}} = \boldsymbol{m}_{\mathrm{H(FL,FR)}}/\boldsymbol{M}_{\mathrm{H(FL,FR)}}$ are given by

$$\begin{aligned} \mu_{\mathrm{FL},x} &= i\big(-1)^n\big(1 \cdot e^{ik_{\mathrm{F,p}}(z+\frac{d}{2})} + r_\mathrm{p} e^{-ik_{\mathrm{F,p}}(z+\frac{d}{2})} + r_\mathrm{e} e^{k_{\mathrm{F,e}}(z+\frac{d}{2})}\big), \\ \mu_{\mathrm{FL},z} &= 1 \cdot e^{ik_{\mathrm{F,p}}(z+\frac{d}{2})} + r_\mathrm{p} e^{-ik_{\mathrm{F,p}}(z+\frac{d}{2})} - r_\mathrm{e} e^{k_{\mathrm{F,e}}(z+\frac{d}{2})}, \\ \mu_{\mathrm{H},x} &= i\cos(K_\mathrm{H} z)\big(c_\mathrm{p}^{(+)} e^{ik_{\mathrm{H,p}}z} + c_\mathrm{p}^{(-)} e^{-ik_{\mathrm{H,p}}z} + c_\mathrm{e}^{(+)} e^{-k_{\mathrm{H,e}}z} + c_\mathrm{e}^{(-)} e^{k_{\mathrm{H,e}}z}\big), \\ \mu_{\mathrm{H},z} &= \eta_{\mathrm{H,p}}\big(c_\mathrm{p}^{(+)} e^{ik_{\mathrm{H,p}}z} + c_\mathrm{p}^{(-)} e^{-ik_{\mathrm{H,p}}z}\big) + \eta_{\mathrm{H,e}}\big(c_\mathrm{e}^{(+)} e^{-k_{\mathrm{H,e}}z} + c_\mathrm{e}^{(-)} e^{k_{\mathrm{H,e}}z}\big), \\ \mu_{\mathrm{FR},x} &= i(-1)^n\big(t_\mathrm{p} e^{ik_{\mathrm{F,p}}(z-\frac{d}{2})} + t_\mathrm{e} e^{-k_{\mathrm{F,e}}(z-\frac{d}{2})}\big), \\ \mu_{\mathrm{FR},z} &= t_\mathrm{p} e^{ik_{\mathrm{F,p}}(z-\frac{d}{2})} - t_\mathrm{e} e^{-k_{\mathrm{F,e}}(z-\frac{d}{2})}, \end{aligned} \tag{26}$$

and subscripts "FL" and "FR" stand for the left- and right-hand side ferromagnets, respectively.

If the helimagnet contains a half-integer number of the helicoid's turns, we have $K_\mathrm{H} d = 2\pi(n + 1/2)$, i.e., $d = (n + 1/2)l$. Then, from Cartesian components of Eqs. (22) at the two interfaces, we obtain

$$\begin{aligned} &(\mu_{\mathrm{H},y(z)} - \mu_{\mathrm{FL},y(z)})|_{z=-\frac{d}{2}} = 0, \quad (\mu_{\mathrm{H},y(z)} - \mu_{\mathrm{FR},y(z)})|_{z=\frac{d}{2}} = 0, \\ &\left(A_\mathrm{H} \frac{\partial \mu_{\mathrm{H},y(z)}}{\partial z} - A_\mathrm{F} \frac{\partial \mu_{\mathrm{FL},y(z)}}{\partial z}\right)\bigg|_{z=-\frac{d}{2}} = 0, \quad \left(A_\mathrm{H} \frac{\partial \mu_{\mathrm{H},y(z)}}{\partial z} - A_\mathrm{F} \frac{\partial \mu_{\mathrm{FR},y(z)}}{\partial z}\right)\bigg|_{z=\frac{d}{2}} = 0, \end{aligned} \tag{27}$$

where

$$\begin{aligned} \mu_{\mathrm{FL},y} &= -i(-1)^n\big(1 \cdot e^{ik_{\mathrm{F,p}}(z+\frac{d}{2})} + r_\mathrm{p} e^{-ik_{\mathrm{F,p}}(z+\frac{d}{2})} + r_\mathrm{e} e^{k_{\mathrm{F,e}}(z+\frac{d}{2})}\big), \\ \mu_{\mathrm{FL},z} &= 1 \cdot e^{ik_{\mathrm{F,p}}(z+\frac{d}{2})} + r_\mathrm{p} e^{-ik_{\mathrm{F,p}}(z+\frac{d}{2})} - r_\mathrm{e} e^{k_{\mathrm{F,e}}(z+\frac{d}{2})}, \end{aligned}$$

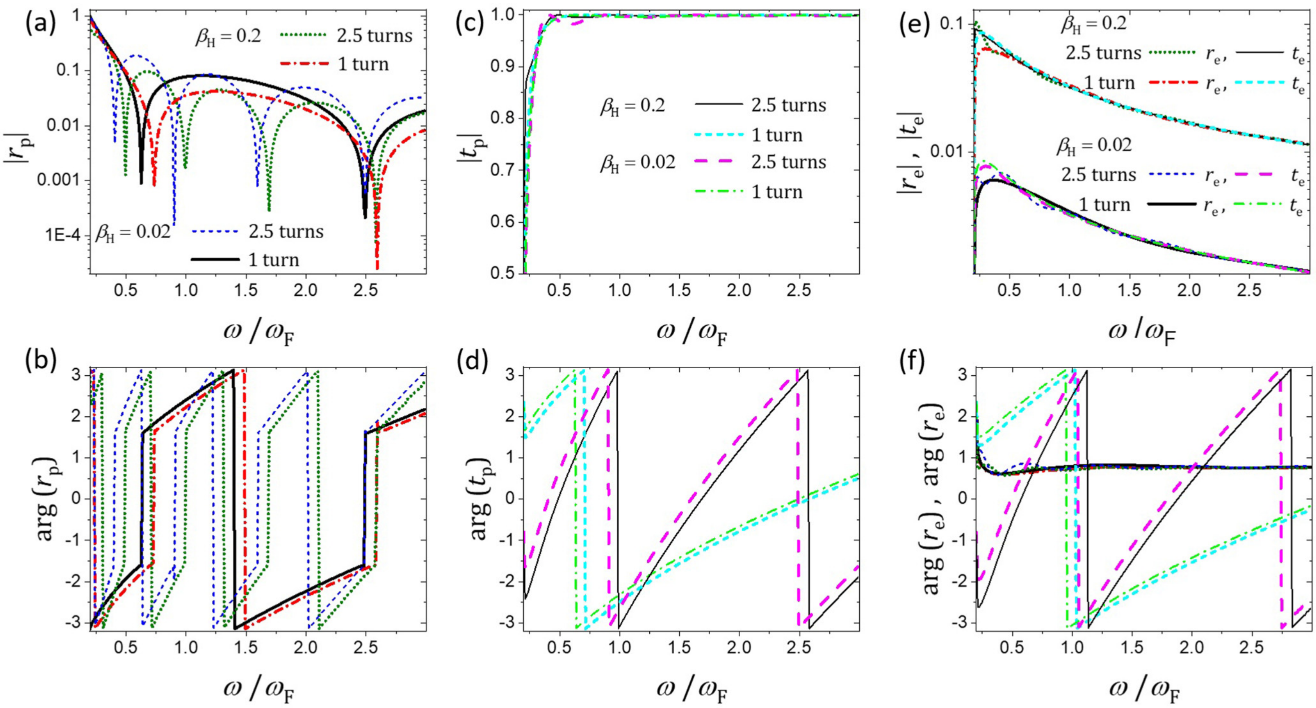

FIG. 3. The frequency dependence of the absolute value (top row) and phase (bottom row) of the spin-wave reflection [panels (a) and (b)] and transmission [panels (c) and (d)] coefficients and of the evanescent wave amplitudes [panels (e) and (f)] in the F media are shown for different values of the uniaxial anisotropy strength and different numbers of the helix turns in the H layer. We assume the values of the exchange length and saturation magnetization to be equal in the helimagnetic and ferromagnetic materials, while $l = 5\lambda_H$ and the damping is zero. The line styles and colors are consistent in the top and bottom panels.

$$
\begin{aligned}
\mu_{H,y} &= i\sin(K_H z)\left(c_p^{(+)}e^{ik_{H,p}z} + c_p^{(-)}e^{-ik_{H,p}z} + c_e^{(+)}e^{-k_{H,e}z} + c_e^{(-)}e^{k_{H,e}z}\right),\\
\mu_{H,z} &= \eta_{H,p}\left(c_p^{(+)}e^{ik_{H,p}z} + c_p^{(-)}e^{-ik_{H,p}z}\right) + \eta_{H,e}\left(c_e^{(+)}e^{-k_{H,e}z} + c_e^{(-)}e^{k_{H,e}z}\right),\\
\mu_{FR,y} &= i(-1)^n\left(t_p e^{ik_{F,p}(z-\frac{d}{2})} + t_e e^{-k_{F,e}(z-\frac{d}{2})}\right),\\
\mu_{FR,z} &= t_p e^{ik_{F,p}(z-\frac{d}{2})} - t_e e^{-k_{F,e}(z-\frac{d}{2})}.
\end{aligned}
\tag{28}
$$

Substituting Eqs. (26) and (28) into Eqs. (25) and (27), respectively, we obtain the same system of eight algebraic equations [Eqs. (A1) in the Appendix] for the cases of integer and half-integer numbers of turns in the H layer. We use MAPLE software [46] to solve the system for the reflection, $r_{p(e)}$, and transmission, $t_{p(e)}$, coefficients, whose frequency dependence is shown in Fig. 3.

V. DISCUSSION

The frequency dependence of the reflection and transmission coefficients for propagating modes is qualitatively similar to that for circularly polarized exchange spin waves [22,32]. Overall, the absolute value of the reflection (transmission) coefficient decreases (increases) as the frequency increases. On top of this general trend, we notice regular dips, the number of which increases with helimagnetic layer thickness. These are due to the Fabry-Perot resonances [47], which occur when waves scattered by the two interfaces interfere constructively (destructively) in the backward (forward) direction. Quantitatively, the ellipticity of the precession in the helimagnetic layer leads to the H layer's interfaces playing the role of a pair of polarizers for the circularly polarized spin waves incident from the ferromagnet. This leads to appearance of the evanescent modes on both sides of each interface and thereby affecting the strength of scattering at each interface. The evanescent modes have negligible amplitudes for $\beta_H = 0.02$ but become more noticeable for $\beta_H = 0.2$. In the latter case, their effect is significant (yet modest) at low frequencies and diminishes quickly at high frequencies, when the spin-wave polarization in the helimagnetic layer becomes more circular [Fig. 2(b)]. It is worthwhile noting however that the ellipticity of precession is generally stronger for spin waves propagating in thin magnetic films, and so, we should expect it to have a more pronounced effect on scattering of such spin waves.

The fact that we have obtained the same system of eight algebraic equations [Eqs. (A1) in the Appendix], for the cases of integer and half-integer numbers of turns in the H layer means that the scattering from the layer is insensitive to its helimagnetic ordering. To explain this peculiarity, we recall that, in general, the wave scattering from a layer with altered properties is determined by two factors: (i) the wave impedance [48] mismatch (i.e., the boundary conditions) at the layer's individual interfaces and (ii) the phase accumulated by the wave after round trips across the layer [14,15]. As we have seen earlier, any information about the helimagnetic order is absent from the boundary conditions (22) due to the geometry of our problem and the linear approximation used here, which lead to Eqs. (23) and (24). The phase accumulated by the propagating spin waves after one round trip across the H layer is also independent of the presence of the helimagnetic order. Indeed, in the "laboratory" frame [3,4], this phase can be calculated as

$$\Delta\varphi = K_{\mathrm{H,p}}^{+}d + K_{\mathrm{H,p}}^{-}d, \tag{29}$$

where $K_{\mathrm{H,p}}^{+}$ and $K_{\mathrm{H,p}}^{-}$ are the wave numbers of spin waves propagating forward and backward, respectively, in the laboratory frame. These are related to the wave numbers $\pm k_{\mathrm{H,p}}$ (defined in the rotating frame and used throughout above) via a frequency-independent shift of K_{H}

$$K_{\mathrm{H,p}}^{\pm} = K_{\mathrm{H}} \pm k_{\mathrm{H,p}}. \tag{30}$$

The dispersion relation of spin waves in the helimagnet is therefore reciprocal in the rotating frame but is nonreciprocal in the laboratory frame. Nonetheless, it follows from Eqs. (29) and (30) that the phase shift accumulated by spin waves after each round trip across the H layer (and therefore, its effect upon the spin-wave scattering) is not influenced by this nonreciprocity (and by extension, by the helimagnetic order in the H layer). This conclusion should also apply to other systems in which the nonreciprocity is described (perhaps approximately) by Eq. (30), which is realized (in some geometries) in the case of the DMI and electric field induced nonreciprocity [11,49].

Our results correspond to a rather stringent requirement that the thickness of the H layer be equal an integer or half-integer number of the helicoid periods given by Eq. (5). The period of the helicoid varies continuously with temperature and might not necessarily be commensurate with the lattice constant of the H layer [3,4,50]. This could lead to magnetic relaxation at the interfaces and associated deviation of the magnetic configuration from that given by Eq. (4) in the H layer and uniform magnetization in the F media. However, the angle of such deviation would be limited, in any case, by that between the spins in the nearest neighbor unit cells [3,4,51]. For example, the lattice constant of $CrNb_3S_6$ is 1.21 nm and the period of the helicoid is about 48 nm [52–54]. The corresponding angle between the nearest-neighbor spins of about 9° is rather small, while any deviation from the expected static configuration would be even smaller. Hence, we expect our results to hold for such "long-periodic" [3,4] magnetic structures.

VI. CONCLUSIONS

In summary, we have developed a theory of scattering of linear exchange spin waves normally incident upon a helimagnetic layer sandwiched between two semi-infinite ferromagnetic media. The scattering is shown to be reciprocal and insensitive to the presence of the DMI induced helimagnetic order in the layer. The magnetic precession in the helimagnetic layer is elliptical, and its interfaces act as a system of two semicrossed polarizers for the spin waves that have circular polarization in the ferromagnetic media. The account of the evanescent solutions proves to be essential to correctly describe the spin-wave scattering in this case.

Experimentally, the spin-wave response of helimagnets has recently started to be studied using ferromagnetic resonance [55–57] (FMR) and optical pump-probe techniques [58]. The FMR-based measurements aided by suitable microwave-to-spin-wave transducers [59] have recently enabled studies of propagating exchange spin waves in uniformly magnetized ferromagnetic films [60,61] and could also be expected to be applied to helimagnet-based systems such as ours. Once this is achieved, our findings will prove essential for design and development of magnonic devices containing helimagnetic constituents.

ACKNOWLEDGMENTS

The authors gratefully acknowledge fruitful discussions with V. N. Krivoruchko and F. B. Mushenok at early stages of this work. The research leading to these results has received funding from the European Union's Horizon 2020 research and innovation program under Marie Skłodowska-Curie Grant Agreement No. 644348 (MagIC).

APPENDIX: SYSTEM OF EQUATIONS FOR DETERMINATION OF THE SCATTERING COEFFICIENTS

The scattering coefficients of spin waves from the helimagnetic layer are calculated from the following system of algebraic equations:

$$\begin{aligned}
&c_{\mathrm{p}}^{(+)}e^{-ik_{\mathrm{H,p}}\frac{d}{2}} + c_{\mathrm{p}}^{(-)}e^{ik_{\mathrm{H,p}}\frac{d}{2}} + c_{\mathrm{e}}^{(+)}e^{k_{\mathrm{H,e}}\frac{d}{2}} + c_{\mathrm{e}}^{(-)}e^{-k_{\mathrm{H,e}}\frac{d}{2}} - r_{\mathrm{p}} - r_{\mathrm{e}} = 1,\\
&\eta_{\mathrm{H,p}}\big(c_{\mathrm{p}}^{(+)}e^{-ik_{\mathrm{H,p}}\frac{d}{2}} + c_{\mathrm{p}}^{(-)}e^{ik_{\mathrm{H,p}}\frac{d}{2}}\big) + \eta_{\mathrm{H,e}}\big(c_{\mathrm{e}}^{(+)}e^{k_{\mathrm{H,e}}\frac{d}{2}} + c_{\mathrm{e}}^{(-)}e^{-k_{\mathrm{H,e}}\frac{d}{2}}\big) - r_{\mathrm{p}} + r_{\mathrm{e}} = 1,\\
&A_{\mathrm{H}}\big(ik_{\mathrm{H,p}}\big(c_{\mathrm{p}}^{(+)}e^{-ik_{\mathrm{H,p}}\frac{d}{2}} - c_{\mathrm{p}}^{(-)}e^{ik_{\mathrm{H,p}}\frac{d}{2}}\big) - k_{\mathrm{H,e}}\big(c_{\mathrm{e}}^{(+)}e^{k_{\mathrm{H,e}}\frac{d}{2}} - c_{\mathrm{e}}^{(-)}e^{-k_{\mathrm{H,e}}\frac{d}{2}}\big)\big) - A_{\mathrm{F}}(-ik_{\mathrm{F,p}}r_{\mathrm{p}} + k_{\mathrm{F,e}}r_{\mathrm{e}}) = iA_{\mathrm{F}}k_{\mathrm{F,p}},\\
&A_{\mathrm{H}}\big(ik_{\mathrm{H,p}}\eta_{\mathrm{H,p}}\big(c_{\mathrm{p}}^{(+)}e^{-ik_{\mathrm{H,p}}\frac{d}{2}} - c_{\mathrm{p}}^{(-)}e^{ik_{\mathrm{H,p}}\frac{d}{2}}\big) - k_{\mathrm{H,e}}\eta_{\mathrm{H,e}}\big(c_{\mathrm{e}}^{(+)}e^{k_{\mathrm{H,e}}\frac{d}{2}} - c_{\mathrm{e}}^{(-)}e^{-k_{\mathrm{H,e}}\frac{d}{2}}\big)\big) - A_{\mathrm{F}}(-ik_{\mathrm{F,p}}r_{\mathrm{p}} - k_{\mathrm{F,e}}r_{\mathrm{e}}) = iA_{\mathrm{F}}k_{\mathrm{F,p}},\\
&c_{\mathrm{p}}^{(+)}e^{ik_{\mathrm{H,p}}\frac{d}{2}} + c_{\mathrm{p}}^{(-)}e^{-ik_{\mathrm{H,p}}\frac{d}{2}} + c_{\mathrm{e}}^{(+)}e^{-k_{\mathrm{H,e}}\frac{d}{2}} + c_{\mathrm{e}}^{(-)}e^{k_{\mathrm{H,e}}\frac{d}{2}} - t_{\mathrm{p}} - t_{\mathrm{e}} = 0,\\
&\eta_{\mathrm{H,p}}\big(c_{\mathrm{p}}^{(+)}e^{ik_{\mathrm{H,p}}\frac{d}{2}} + c_{\mathrm{p}}^{(-)}e^{-ik_{\mathrm{H,p}}\frac{d}{2}}\big) + \eta_{\mathrm{H,e}}\big(c_{\mathrm{e}}^{(+)}e^{-k_{\mathrm{H,e}}\frac{d}{2}} + c_{\mathrm{e}}^{(-)}e^{k_{\mathrm{H,e}}\frac{d}{2}}\big) - t_{\mathrm{p}} + t_{\mathrm{e}} = 0,\\
&A_{\mathrm{H}}\big(ik_{\mathrm{H,p}}\big(c_{\mathrm{p}}^{(+)}e^{ik_{\mathrm{H,p}}\frac{d}{2}} - c_{\mathrm{p}}^{(-)}e^{-ik_{\mathrm{H,p}}\frac{d}{2}}\big) - k_{\mathrm{H,e}}\big(c_{\mathrm{e}}^{(+)}e^{-k_{\mathrm{H,e}}\frac{d}{2}} - c_{\mathrm{e}}^{(-)}e^{k_{\mathrm{H,e}}\frac{d}{2}}\big)\big) - A_{\mathrm{F}}(ik_{\mathrm{F,p}}t_{\mathrm{p}} - k_{\mathrm{F,e}}t_{\mathrm{e}}) = 0,\\
&A_{\mathrm{H}}\big(ik_{\mathrm{H,p}}\eta_{\mathrm{H,p}}\big(c_{\mathrm{p}}^{(+)}e^{ik_{\mathrm{H,p}}\frac{d}{2}} - c_{\mathrm{p}}^{(-)}e^{-ik_{\mathrm{H,p}}\frac{d}{2}}\big) - k_{\mathrm{H,e}}\eta_{\mathrm{H,e}}\big(c_{\mathrm{e}}^{(+)}e^{-k_{\mathrm{H,e}}\frac{d}{2}} - c_{\mathrm{e}}^{(-)}e^{k_{\mathrm{H,e}}\frac{d}{2}}\big)\big) - A_{\mathrm{F}}(ik_{\mathrm{F,p}}t_{\mathrm{p}} + k_{\mathrm{F,e}}t_{\mathrm{e}}) = 0.
\end{aligned} \tag{A1}$$

[1] I. Dzyaloshinsky, A thermodynamic theory of weak ferromagnetism of antiferromagnetics, J. Phys. Chem. Solids **4**, 241 (1958).

[2] T. Moriya, Anisotropic superexchange interaction and weak ferromagnetism, Phys. Rev. **120**, 91 (1960).

[3] Y. A. Izyumov, Modulated, or long-periodic, magnetic structures of crystals, Sov. Phys. Usp. **27**, 845 (1984).

[4] Y. A. Izyumov, *Diffraction of Neutrons on Long-Periodic Structures* (Energoatom-Izdat, Moscow, 1987), in Russian.

[5] J. Kishine and A. S. Ovchinnikov, Theory of monoaxial chiral helimagnet, Solid State Phys. **66**, 1 (2015).

[6] M. Garst, J. Waizner, and D. Grundler, Collective spin excitations of helices and magnetic skyrmions: Review and perspectives of magnonics in non-centrosymmetric magnets, J. Phys. D **50**, 293002 (2017).

[7] V. V. Kruglyak, S. O. Demokritov, and D. Grundler, Magnonics, J. Phys. D **43**, 264001 (2010).

[8] S. A. Nikitov, D. V. Kalyabin, I. V. Lisenkov, A. Slavin, Y. N. Barabanenkov, S. A. Osokin, A. V. Sadovnikov, E. N. Beginin, M. A. Morozova, Y. P. Sharaevsky *et al.*, Magnonics: A new research area in spintronics and spin wave electronics, Phys. Usp. **58**, 1002 (2015).

[9] A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, *Spin Waves* (North-Holland, Amsterdam, 1968).

[10] L. Udvardi and L. Szunyogh, Chiral Asymmetry of the Spin-Wave spectra in Ultrathin Magnetic Films, Phys. Rev. Lett. **102**, 207204 (2009).

[11] J.-V. Kim, R. L. Stamps, and R. E. Camley, Spin Wave Power Flow and Caustics in Ultrathin Ferromagnets With the Dzyaloshinskii-moriya Interaction, Phys. Rev. Lett. **117**, 197204 (2016).

[12] S. A. Diaz, J. Klinovaja, and D. Loss, Topological Magnons and Edge States in Antiferromagnetic Skyrmion Crystals, Phys. Rev. Lett. **122**, 187203 (2019).

[13] V. G. Bar'yakhtar, A. G. Danilevich, and V. N. Krivoruchko, Magnetochiral nonreciprocity of spin wave damping in long-period structures, Phys. Rev. B **99**, 104407 (2019).

[14] M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1964).

[15] L. M. Brekhovskikh and O. A. Godin, *Acoustics of Layered Media* (Springer, Berlin, Heidelberg, 1997).

[16] S. O. Demokritov, A. A. Serga, A. Andre, V. E. Demidov, M. P. Kostylev, B. Hillebrands, and A. N. Slavin, Tunneling of Dipolar Spin Waves Through a Region of Inhomogeneous Magnetic Field, Phys. Rev. Lett. **93**, 047201 (2004).

[17] R. Hertel, W. Wulfhekel, and J. Kirschner, Domain-wall Induced Phase Shifts in Spin Waves, Phys. Rev. Lett. **93**, 257202 (2004).

[18] M. P. Kostylev, A. A. Serga, T. Schneider, T. Neumann, B. Leven, B. Hillebrands, and R. L. Stamps, Resonant and nonresonant scattering of dipole-dominated spin waves from a region of inhomogeneous magnetic field in a ferromagnetic film, Phys. Rev. B **76**, 184419 (2007).

[19] S. V. Vasiliev, V. V. Kruglyak, M. L. Sokolovskii, and A. N. Kuchko, Spin wave interferometer employing a local nonuniformity of the effective magnetic field, J. Appl. Phys. **101**, 113919 (2007).

[20] D. R. Birt, B. O'Gorman, M. Tsoi, X. Q. Li, V. E. Demidov, and S. O. Demokritov, Diffraction of spin waves from a submicrometer-size defect in a microwaveguide, Appl. Phys. Lett. **95**, 122510 (2009).

[21] T. Schneider, A. A. Serga, A. V. Chumak, B. Hillebrands, R. L. Stamps, and M. P. Kostylev, Spin-wave tunnelling through a mechanical gap, EPL **90**, 27003 (2010).

[22] M. Dvornik, A. N. Kuchko, and V.V. Kruglyak, Micromagnetic method of s-parameter characterization of magnonic devices, J. Appl. Phys. **109**, 07D350 (2011).

[23] Y. Au, M. Dvornik, O. Dmytriiev, and V. V. Kruglyak, Nanoscale spin wave valve and phase shifter, Appl. Phys. Lett. **100**, 172408 (2012).

[24] R. Gieniusz, V. D. Bessonov, U. Guzowska, A. I. Stognii, and A. Maziewski, An antidot array as an edge for total non-reflection of spin waves in yttrium iron garnet films, Appl. Phys. Lett. **104**, 082412 (2014).

[25] J. Iwasaki, A. J. Beekman, and N. Nagaosa, Theory of magnon-skyrmion scattering in chiral magnets, Phys. Rev. B **89**, 064412 (2014).

[26] C. Schütte and M. Garst, Magnon-skyrmion scattering in chiral magnets, Phys. Rev. B **90**, 094423 (2014).

[27] V. Ashokan, A. Khater, M. Abou Ghantous, and D. Ghader, Spin wave ballistic transport properties of nanojunctions between Co leads, J. Magn. Magn. Mater. **384**, 18 (2015).

[28] L. C. Parsons, Influence of spin wave attenuation on a ferromagnetic nanowire-based magnonic Bragg mirror, J. Magn. Magn. Mater. **426**, 794 (2017).

[29] A. Khater, L. Saim, R. Tigrine, and D. Ghader, Fabry-Perot magnonic ballistic coherent transport across ultrathin ferromagnetic lamellar bcc Ni nanostructures between Fe leads, Surf. Sci. **672**, 47 (2018).

[30] O. V. Dobrovolskiy, R. Sachser, S. A. Bunyaev, D. Navas, V. M. Bevz, M. Zelent, W. Smigaj, J. Rychly, M. Krawczyk, R. V. Vovk, M. Huth, and G. N. Kakazei, Spin-wave phase inverter upon a single nanodefect, ACS Appl. Mater. Interfaces **11**, 17654 (2019).

[31] N. J. Whitehead, S. A. R. Horsley, T. G. Philbin, and V. V. Kruglyak, Graded index lenses for spin wave steering, Phys. Rev. B **100**, 094404 (2019).

[32] P. Borys, N. Qureshi, C. Ordoez-Romero, and O. Kolokoltsev, Scattering of exchange spin waves from regions of modulated magnetization, EPL **128**, 17003 (2019).

[33] J. Lan, W. C. Yu, R. Q. Wu, and J. Xiao, Spin-Wave Diode, Phys. Rev. X **5**, 041049 (2015).

[34] M. F. Limonov, M. V. Rybin, A. N. Poddubny, and Y. S. Kivshar, Fano resonances in photonics, Nat. Photon. **11**, 543 (2017).

[35] F. J. Buijnsters, Y. Ferreiros, A. Fasolino, and M. I. Katsnelson, Chirality-dependent Transmission of Spin Waves Through Domain Walls, Phys. Rev. Lett. **116**, 147204 (2016).

[36] P. Bak and M. H. Jensen, Theory of helical magnetic structures and phase transitions in MnSi and FeGe, J. Phys. C: Solid State Phys. **13**, L881 (1980).

[37] J. Müller, A. Rosch, and M. Garst, Edge instabilities and skyrmion creation in magnetic layers, New J. Phys. **18**, 065006 (2016).

[38] V. V. Kruglyak, O. Y. Gorobets, Y. I. Gorobets, and A. N. Kuchko, Magnetization boundary conditions at a ferromagnetic interface of finite thickness, J. Phys.: Condens. Matter **26**, 406001 (2014).

[39] A. Raeliarijaona, R. Nepal, and A. A. Kovalev, Boundary twists, instabilities, and creation of skyrmions and antiskyrmions, Phys. Rev. Mater. **2**, 124401 (2018).

[40] N. M. Salansky and M. S. Erukhimov, *Physical Properties and Applications of Magnetic Films* (Nauka, Novosibirsk, 1975), in Russian.

[41] P. Grünberg, M. G. Cottam, W. Vach, C. Mayr, and R. E. Camley, Brillouin scattering of light by spin waves in thin ferromagnetic films (invited), J. Appl. Phys. **53**, 2078 (1982).

[42] G. Dieterle, J. Förster, H. Stoll, A. S. Semisalova, S. Finizio, A. Gangwar, M. Weigand, M. Noske, M. Fähnle, I. Bykova, J. Gräfe, D. A. Bozhko, H. Y. Musiienko-Shmarova, V. Tiberkevich, A. N. Slavin, C. H. Back, J. Raabe, G. Schütz, and S. Wintz, Coherent Excitation of Heterosymmetric Spin Waves with Ultrashort Wavelengths, Phys. Rev. Lett. **122**, 117202 (2019).

[43] N. J. Whitehead, S. A. R. Horsley, T. G. Philbin, A. N. Kuchko, and V. V. Kruglyak, Theory of linear spin wave emission from a bloch domain wall, Phys. Rev. B **96**, 064415 (2017).

[44] V. D. Poimanov, V. V. Kruglyak, and V. G. Shavrov, Scattering of spin waves by the interface of biaxial ferromagnets, Zh. Radioelek. (J. Radio Electron.) **11** (2018), in Russian, http://jre.cplire.ru/jre/nov18/17/text.pdf.

[45] R. Verba, V. Tiberkevich, and A. Slavin, Spin-wave transmission through an internal boundary: Beyond the scalar approximation, Phys. Rev. B **101**, 144430 (2020).

[46] https://www.maplesoft.com/.

[47] C. Fabry and A. Pérot, Théorie et applications d'une nouvelle méthode de spectroscopie interférentielle, Ann. de ChimPhys. **16**, 115 (1899).

[48] The use of the wave impedance in its conventional meaning for elliptically polarized spin waves encounters difficulties, as discussed, e.g., in Ref. [44].

[49] V. N. Krivoruchko, A. S. Savchenko, and V. V. Kruglyak, Electric-field control of spin-wave power flow and caustics in thin magnetic films, Phys. Rev. B **98**, 024427 (2018).

[50] Y. Togawa, J. Kishine, P. A. Nosov, T. Koyama, G. W. Paterson, S. McVitie, Y. Kousaka, J. Akimitsu, M. Ogata, and A. S. Ovchinnikov, Anomalous Temperature Behavior of the Chiral Spin Helix in $CrNb_3S_6$ thin Lamellae, Phys. Rev. Lett. **122**, 017204 (2019).

[51] J. Kishine, K. Inoue, and Y. Yoshida, Synthesis, structure, and magnetic properties of chiral molecule-based magnets, Prog. Theor. Phys. Suppl. **159**, 82 (2005).

[52] T. Moriya and T. Miyadai, Evidence for the helical spin structure due to antisymmetric exchange interaction in $Cr_{1/3}NbS_2$, Sol. State. Commun. **42**, 209 (1982).

[53] T. Miyadai, K. Kikuchi, H. Kondo, S. Sakka, M. Arai, and Y. Ishikawa, Magnetic properties of $Cr_{1/3}NbS_2$, J. Phys. Soc. Jpn. **52**, 1394 (1983).

[54] Y. Togawa, T. Koyama, K. Takayanagi, S. Mori, Y. Kousaka, J. Akimitsu, S. Nishihara, K. Inoue, A. S. Ovchinnikov, and J. Kishine, Chiral Magnetic Soliton Lattice on a Chiral Helimagnet, Phys. Rev. Lett. **108**, 107202 (2012).

[55] T. Schwarze, J. Waizner, M. Garst, A. Bauer, I. Stasinopoulos, H. Berger, C. Pfleiderer, and D. Grundler, Universal helimagnon and skyrmion excitations in metallic, semiconducting and insulating chiral magnets, Nat. Mater. **14**, 478 (2015).

[56] F. J. T. Goncalves, T. Sogo, Y. Shimamoto, Y. Kousaka, J. Akimitsu, S. Nishihara, K. Inoue, D. Yoshizawa, M. Hagiwara, M. Mito, R. L. Stamps, I. G. Bostrem, V. E. Sinitsyn, A. S. Ovchinnikov, J. Kishine, and Y. Togawa, Collective resonant dynamics of the chiral spin soliton lattice in a monoaxial chiral magnetic crystal, Phys. Rev. B **95**, 104415 (2017).

[57] J. Kishine, V. E. Sinitsyn, I. G. Bostrem, I. Proskurin, F. J. T. Goncalves, Y. Togawa, and A. S. Ovchinnikov, Theory of standing spin waves in a finite-size chiral spin soliton lattice, Phys. Rev. B **100**, 024411 (2019).

[58] J. D. Koralek, D. Meier, J. P. Hinton, A. Bauer, S. A. Parameswaran, A. Vishwanath, R. Ramesh, R. W. Schoenlein, C. Pfleiderer, and J. Orenstein, Observation of Coherent Helimagnons and Gilbert Damping in an Itinerant Magnet, Phys. Rev. Lett. **109**, 247204 (2012).

[59] Y. Au, E. Ahmad, O. Dmytriiev, M. Dvornik, T. Davison, and V. V. Kruglyak, Resonant microwave-to-spin-wave transducer, Appl. Phys. Lett. **100**, 182404 (2012).

[60] J. L. Chen, T. Yu, C. P. Liu, T. Liu, M. Madami, K. Shen, J. Y. Zhang, S. Tu, M. S. Alam, K. Xia, M. Z. Wu, G. Gubbiotti, Y. M. Blanter, G. E. W. Bauer, and H. M. Yu, Excitation of unidirectional exchange spin waves by a nanoscale magnetic grating, Phys. Rev. B **100**, 104427 (2019).

[61] P. Che, K. Baumgaertl, A. Kukol'ova, C. Dubs, and D. Grundler, Efficient wavelength conversion of exchange magnons below 100nm by magnetic coplanar waveguides, Nat. Commun. **11**, 1445 (2020).