Sign reversal of nonlocal response due to electron collisions

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Electron-electron collisions are known to cause a nonlocal voltage drop in the presence of current flow. The semiphenomenological theory predicts this drop to be opposite to the direction of the current in the ballistic regime. We use a microscopic approach and show that the sign of this drop may be of both signs depending on the temperature and the distance between the source and probe contacts. The change of sign corresponds to the change of the dominant scattering process from head-on collisions to backward scattering of electrons. Our results agree with the experimental data.

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I. INTRODUCTION

Modern technologies allow a fabrication of conducting microstructures smaller than the elastic mean free path of electrons, which makes observable the effects of electron–electron scattering in them. Although this scattering does not contribute to the resistivity of a homogeneous conductor with a parabolic spectrum, it affects the transport properties of finite-size systems in different ways. These effects were investigated in a big number of papers in the hydrodynamic regime where the mean free path of electron-electron scattering l_{ee} is much smaller than the characteristic size of the structure. Among others, they included a temperature-dependent resistance of conducting channels with rough boundaries known as the Gurzhi effect [1–3], but also a negative nonlocal resistance [4–6], which represents a current-induced voltage drop in the direction opposite to the current flow. This negative resistance was even deemed a signature of the hydrodynamic regime [5].

Along with this, the effects of two-particle collisions were also investigated in the ballistic regime where the characteristic size of the structure is much smaller than l_{ee}. In wide ballistic contacts, these collisions were shown to result in a contribution to the current that linearly grows with temperature. This contribution was theoretically predicted in Refs. [7,8] and experimentally observed in Refs. [9,10]. More recently, the negative nonlocal resistance in the ballistic regime was obtained in Ref. [11] using the Boltzmann equation in the semi-phenomenological approximation of a single electron–electron scattering time. However, it was observed [6,12] that at lower temperatures this resistance becomes positive. These authors explained the change of sign by the finite-size effect and attributed it to reflections of the electrons from the opposite boundary of the conductor.

This paper presents a microscopic calculation of the nonlocal response of a ballistic conductor with two-particle scattering. In this approach, the change of sign of the response takes place at low temperatures even in the absence of boundary reflections as a result of competition between different scattering processes in the electron–electron collision integral. The temperature dependence of the effect also appears to be different from that obtained in the semiphenomenological approximation.

II. MODEL AND EQUATIONS

The sketch of the system considered is shown in Fig. 1. It represents a conducting plane separated by thin insulating barriers into three parts, i.e., one grounded half-plane, one grounded quarter-plane, and one more quarter-plane kept at a constant voltage V. The half-plane is connected with the grounded quarter-plane by the probe contact of width $a \gg \lambda_F$ and with the voltage-biased quarter-plane, by the source contact of width $b \gg \lambda_F$. The distance between the contacts is $d \gg \max(a, b)$. A perfect three-dimensional (3D) electrode is attached to the conducting half-plane at a distance $L \gg d$ from the barrier. It is assumed that the electron-electron scattering is so weak that $l_{ee} \gg L$. If $V = 0$, the fluxes of equilibrium electrons from both sides of the probe contact compensate each other and the net electric current through it is zero. At nonzero bias, nonequilibrium electrons are injected into the grounded half-space through the source contact and collide with equilibrium electrons incident on the probe contact so that their flux is changed, which results in a net electric current of either positive or negative sign. If the circuit is disconnected and the current flow through the probe contact is forbidden, this results in the nonlocal voltage that compensates it.

To calculate the current through the probe contact, we use an approach similar to that of Ref. [7]. The distribution function of electrons $f(\boldsymbol{p}, \boldsymbol{r})$ in all the three parts of the plane obey the Boltzmann equation [13]

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\,\frac{\partial f}{\partial \boldsymbol{r}} + e\boldsymbol{E}\,\frac{\partial f}{\partial \boldsymbol{p}} = I_{ee}, \qquad (1)$$

where $\mathbf{E} = -\nabla\varphi$ is the electric field. The electron-electron collision integral is given by

$$\hat{I}_{ee}(\boldsymbol{p}) = \frac{\alpha_{ee}}{\hbar v_0^2} \int \frac{d^2 p_1}{(2\pi\hbar)^2} \int \frac{d^2 p_2}{(2\pi\hbar)^2} \int d^2 p_3 \times \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)\,\delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3})$$

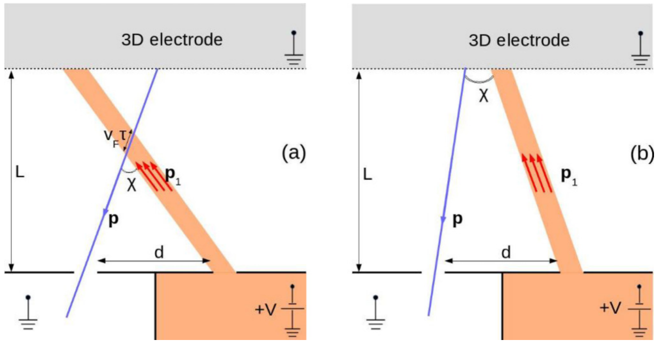

FIG. 1. The layout of the system. (a) Nonequilibrium electrons from the source contact with momentum $\boldsymbol{p}_1$ cross the trajectory of the electron incident on the probe contact with momentum $\boldsymbol{p}$ if the angle χ between $-\boldsymbol{p}$ and $\boldsymbol{p}_1$ is not too small. (b) If χ is too small, the injected beam does not cross the trajectory of incident electron.

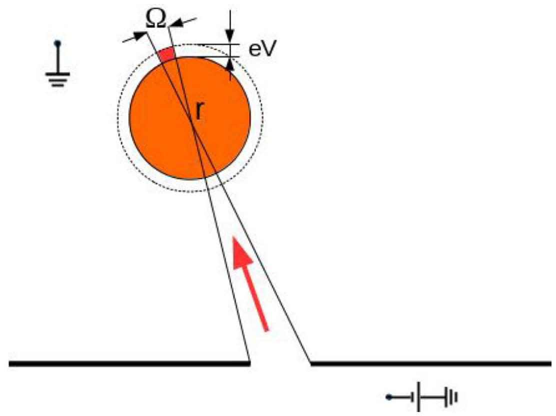

FIG. 2. The distribution function of noninteracting electrons at a given point $\boldsymbol{r}$ in the grounded half-plane. The electrons injected through the source contact form a bump on the Fermi surface.

$$\times \{[1 - f(\boldsymbol{p})]\,[1 - f(\boldsymbol{p}_1)]\,f(\boldsymbol{p}_2)\,f(\boldsymbol{p}_3) - f(\boldsymbol{p})\,f(\boldsymbol{p}_1)\,[1 - f(\boldsymbol{p}_2)]\,[1 - f(\boldsymbol{p}_3)]\}, \quad (2)$$

where α_{ee} is the dimensionless parameter of electron-electron scattering and $\nu_0 = m/\pi\hbar^2$ is the 2D density of states.

In the absence of electron-electron scattering, the nonequilibrium electrons injected through the source contact would travel along their classical trajectories to the depth of the grounded half-plane, and no current would flow through the probe contact. Therefore, the nonlocal current flowing into the grounded half-plane may be written in the form

$$I_n = e \int_a d\rho \int \frac{d^2 p}{(2\pi\hbar)^2}\, v_\perp\, \delta f(\boldsymbol{p}, \boldsymbol{\rho}), \quad (3)$$

where $\boldsymbol{\rho}$ labels points within the probe contact and δf is the correction to the distribution of noninteracting electrons $f^{(0)}(\boldsymbol{p}, \boldsymbol{r})$ from two-particle scattering. It may be obtained by integrating the collision integral in Eq. (1) along the trajectories of electrons that come to point $\boldsymbol{\rho}$ with momentum $\boldsymbol{p}$ from the 3D electrode. Because of the condition $E_F \gg \max(eV, T)$ one may neglect the change of electron velocity caused by the electric field, so [7]

$$\delta f(\boldsymbol{p}, \boldsymbol{\rho}) = \int_0^\infty d\tau\, \hat{I}_{ee}(\boldsymbol{p}, \boldsymbol{\rho} - \tau \boldsymbol{v}), \quad (4)$$

where τ is the time of travel to point $\boldsymbol{\rho}$.

To the first approximation in α_{ee}, the inelastic correction Eq. (4) may be calculated using the distribution function of noninteracting electrons. As the total energy of electron is conserved during its motion along the classical trajectory, $f^{(0)}(\boldsymbol{p}, \boldsymbol{r})$ depends only on the electrode from which the electron trajectory originates. We denote the angular domain that contains all the momenta of electrons that came to point $\boldsymbol{r}$ of the grounded half-plane through the source contact by $\Omega(\boldsymbol{r})$ [14], see Fig. 2. With this notation,

$$f^{(0)}(\boldsymbol{p}, \boldsymbol{r}) = \begin{cases} f_0(\varepsilon_{\boldsymbol{p}} + e\varphi(\boldsymbol{r}) - eV), & \boldsymbol{p} \in \Omega(\boldsymbol{r}) \\ f_0(\varepsilon_{\boldsymbol{p}} + e\varphi(\boldsymbol{r})), & \boldsymbol{p} \notin \Omega(\boldsymbol{r}) \end{cases}, \quad (5)$$

where f_0 is the Fermi distribution. The collision integral Eq. (2) is nonzero only if at least one of the momenta lies in $\Omega(\boldsymbol{r})$. On the other hand, the integral (4) is dominated by large τ and hence small Ω, so the contributions from scattering processes involving more than one momentum in $\Omega(\boldsymbol{r})$ may be neglected. In view of Eqs. (4) and (2), the nonlocal current Eq. (3) may be written in the form

$$\begin{aligned} I_n = ea \frac{\alpha_{ee}}{\hbar\nu_0^2} \iiiint d\varepsilon\, d\varepsilon_1\, d\varepsilon_2\, d\varepsilon_3 \iint \frac{d^2 p}{(2\pi\hbar)^2} \frac{d^2 p_1}{(2\pi\hbar)^2} \\ \times \delta(\varepsilon_{\boldsymbol{p}} - \varepsilon)\, \Theta(v_\perp)\, v_\perp \delta(\varepsilon_{\boldsymbol{p}_1} - \varepsilon_1)\, \tau_m(\boldsymbol{p}, \boldsymbol{p}_1) \\ \times [\delta(\varepsilon + \varepsilon_1 - \varepsilon_2 - \varepsilon_3) A(\varepsilon_2, \varepsilon_3, \boldsymbol{p} + \boldsymbol{p}_1) F(\varepsilon, \varepsilon_1; \varepsilon_2, \varepsilon_3) \\ - \delta(\varepsilon - \varepsilon_1 + \varepsilon_2 - \varepsilon_3) A(\varepsilon_2, \varepsilon_3, \boldsymbol{p} - \boldsymbol{p}_1) F(\varepsilon_3, \varepsilon_1; \varepsilon_2, \varepsilon) \\ - \delta(\varepsilon - \varepsilon_1 + \varepsilon_3 - \varepsilon_2) A(\varepsilon_3, \varepsilon_2, \boldsymbol{p} - \boldsymbol{p}_1) F(\varepsilon_2, \varepsilon_1; \varepsilon_3, \varepsilon)], \end{aligned} \quad (6)$$

where

$$\tau_m(\boldsymbol{p}, \boldsymbol{p}_1) = \int_0^\infty d\tau\, \Theta(\boldsymbol{p}_1 \in \boldsymbol{\rho} - \tau\, \boldsymbol{v}), \quad (7)$$

is the effective dwell time of electrons incident on the probe contact with momentum $\boldsymbol{p}$ in a beam of nonequilibrium electrons with momentum $\boldsymbol{p}_1$ injected through the source contact,

$$A(\varepsilon_2, \varepsilon_3, \boldsymbol{Q}) = \frac{1}{(2\pi\hbar)^2} \int d^2 p' \int d^2 p''\, \delta(\boldsymbol{p}' \pm \boldsymbol{p}'' - \boldsymbol{Q}) \times \delta(\varepsilon_{\boldsymbol{p}'} - \varepsilon_2)\, \delta(\varepsilon_{\boldsymbol{p}''} - \varepsilon_3) \quad (8)$$

is the phase volume made available for the corresponding scattering process by the momentum conservation, and

$$\begin{aligned} F(\varepsilon, \varepsilon_1; \varepsilon_2, \varepsilon_3) &= [1 - f_0(\varepsilon)][1 - f_0(\varepsilon_1 - eV)]\, f_0(\varepsilon_2)\, f_0(\varepsilon_3) \\ &\quad - f_0(\varepsilon)\, f_0(\varepsilon_1 - eV)\, [1 - f_0(\varepsilon_2)][1 - f_0(\varepsilon_3)] \end{aligned} \quad (9)$$

is the distribution-dependent factor. The first term in square brackets in Eq. (6) describes the scattering of an electron incident on the probe contact by a nonequilibrium electron or reverse process [Fig. 3(a)]. The second and the third terms are equivalent and correspond to the scattering of a nonequilibrium electron into the probe contact by an equilibrium electron or reverse process [Fig. 3(b)]. At long distances from the contacts, these terms correspond to head-on collisions and backward scattering, respectively. They are of opposite signs

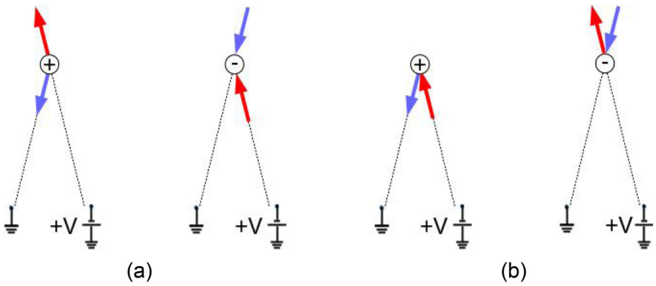

FIG. 3. The scattering processes that contribute to the collision integral. (a) Head-on collisions and (b) backward scattering. Red arrows denote the injected nonequilibrium electrons, and blue arrows denote electrons incident on the probe contact. The "plus" and "minus" signs denote the sign of the corresponding contribution to the nonlocal current.

and the competition between them determines the overall sign of nonlocal current.

It is convenient to describe the dwell time τ_m in terms of the angle ϕ between the normal to the insulating barrier and the trajectory of incident electron and the angle χ between $-\boldsymbol{p}$ and $\boldsymbol{p}_1$. If the electron-electron scattering took place in the whole grounded half-plane, it would increase infinitely at $\chi \to 0$, see Fig. 1(a). But if the trajectories of the incident and the injected electrons intersect further from the barrier than L, the scattering does not take place as shown in Fig. 1(b). Therefore the ϕ-weighted dwell time sharply falls down to zero at $\chi < d/L$ and may be approximately written in the form

$$\bar{\tau}_m(\chi) = \int_{-\pi/2}^{\pi/2} \frac{d\phi}{2\pi} \cos\phi\, \tau_m(\phi,\chi) = \frac{1}{4\pi}\frac{b}{v_F}\Theta(\chi - d/L) \times [(\pi-\chi)\cot\chi + 1]. \quad (10)$$

A sketch of exact $\bar{\tau}_m(\chi)$ is shown in Fig. 4. The phase-space factors A in Eq. (6) also have singularities at $\boldsymbol{p}+\boldsymbol{p}_1 = 0$ of the form

$$A(\varepsilon_2, \varepsilon_3, \boldsymbol{p} \pm \boldsymbol{p}_1) = \frac{(2\pi\hbar v_F)^{-2}\,\Theta(\eta_\pm)}{\cos(\chi/2)\sqrt{\eta_\pm}}, \quad (11)$$

where $\eta_+ = \sin^2(\chi/2) + (\varepsilon_2 - \varepsilon)(\varepsilon_1 - \varepsilon_2)/4E_F^2$ and $\eta_- = \sin^2(\chi/2) + (\varepsilon_1 - \varepsilon_2)/E_F$ account for the thermal smearing at $\chi = 0$. The singularity in the first term of Eq. (6) is well known and results in an additional logarithmic factor in the rate of electron-electron scattering [15–17]. The singularities in the two last terms are smaller at $T \ll E_F$ and normally less important.

The relative magnitude of the terms in Eq. (6) depends on the ratio between d/L and T/E_F. If the temperature is high or the contacts are so close that $d/L \ll T/E_F$, both the cutoff in the dwell time and the thermal smearing of the phase volume are essential, and therefore upon the integration over χ, the most singular contribution at low temperatures from the first term

$$I_n = -e\frac{\alpha_{ee}v_0}{32\pi^2\hbar}ab\ln\left(\frac{d}{L}\right)\iiiint d\varepsilon\, d\varepsilon_1\, d\varepsilon_2\, d\varepsilon_3 \times \delta(\varepsilon+\varepsilon_1-\varepsilon_2-\varepsilon_3)\frac{\Theta[(\varepsilon_2-\varepsilon)(\varepsilon_1-\varepsilon_2)]}{\sqrt{(\varepsilon_2-\varepsilon)(\varepsilon_1-\varepsilon_2)}} \times F(\varepsilon,\varepsilon_1;\varepsilon_2,\varepsilon_3) \quad (12)$$

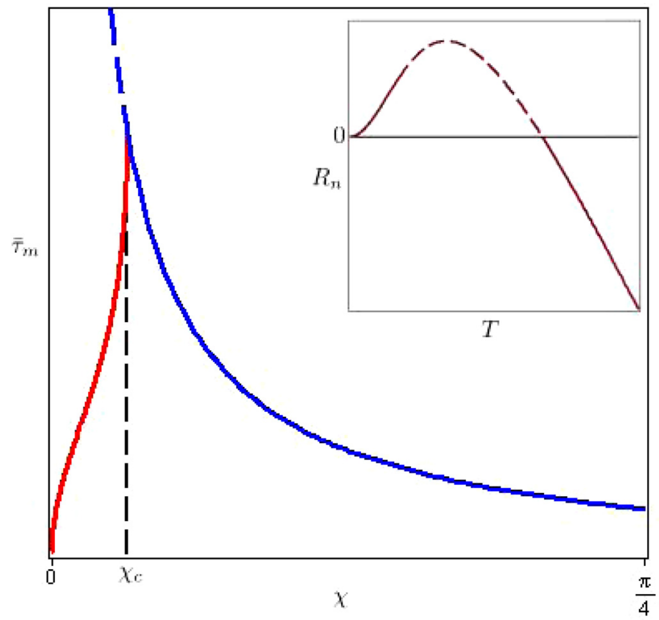

FIG. 4. The $\bar{\tau}_m(\chi)$ curve for $L/d = 10$. The blue dashed curve shows this dependence in the limit $L \to \infty$. The suppression takes place at $\chi < \chi_c \sim d/L$. The inset shows the approximate theoretical $R_n(T)$ dependence.

is $(E_F/T)^{1/2}$ times larger than that from the second and third terms. In the limit $eV \ll T$, Eq. (12) reduces to

$$I_n = \frac{C_0\alpha_{ee}}{32\pi^2}\frac{e^2}{\hbar}Vab\,v_0T\,\ln\left(\frac{L}{d}\right), \quad C_0 \approx 3.46. \quad (13)$$

This expression is very similar to the one obtained in Ref. [7] for the inelastic correction to the current through a ballistic contact except that the square of the contact width is replaced by the product of widths of source and probe contacts. At the same time, the nonlocal current Eq. (13) exhibits a linear temperature dependence, while in Ref. [11] it is quadratic with a similar logarithmic factor. One power of T is eliminated due to the superposition of the two singularities in the dwell time and in the phase volume available for the scattering of electrons with almost opposite momenta. In the opposite limit $eV \gg T$, T is effectively replaced by eV, so

$$I_n = \operatorname{sgn}V\,\frac{(4-\pi)\,\alpha_{ee}}{64\pi^2}\frac{e^2}{\hbar}V^2ab\,v_0\,\ln\left(\frac{L}{d}\right). \quad (14)$$

If $d/L \gg \max(T, eV)/E_F$, finite dwell time cuts off the integration at small χ before the thermal smearing of the Fermi surface comes into play, so the phase-volume factors may be written simply as $A = (2\pi^2\hbar^2v_F^2\sin\chi)^{-1}$. Therefore, all the three terms in Eq. (6) give the contributions to I_n with the same absolute values, but the signs of the two last terms are opposite to the first one, so the total nonlocal current

$$I_n = -\frac{\alpha_{ee}}{192\pi^2}\frac{e^2}{\hbar}V\,abv_0\frac{e^2V^2+4\pi^2T^2}{E_F}\frac{L}{d} \quad (15)$$

changes the sign. Therefore, at $eV \ll T$, the nonlocal current exhibits a nonmonotonic temperature dependence. In the absence of boundary reflections, it starts with a quadratic growth at zero temperature, then reaches a maximum and decreases to become negative, while its absolute value linearly grows with temperature until l_{ee} becomes smaller than L.

If the quarter-plane on the opposite side of the probe contact is electrically isolated instead of being grounded, a compensating nonlocal voltage of opposite sign to the calculated nonlocal current arises. The quantity measured in experiments of Braem *et al.* [6] was $R_n = V_n/I$, where $V_n = -R_p I_n$ is the voltage drop across the probe contact with resistance R_p and $I = V/R_s$ is the current through the source contact with resistance R_s. The geometry investigated there is different from ours and the nonlocal response R_n contains a significant contribution from electron scattering at the opposite boundary of the channel. However, this contribution should be temperature-independent as long as the width of the channel is smaller than l_{ee}. If one subtracts from $R_n(T)$ its value at $T = 0$, the resulting curves are in good agreement with our predictions shown in Fig 4, inset. In particular, these curves exhibit clear maxima at low temperatures. Note also that the negative-slope linear portion of $R_n(T)$ corresponds to the temperature range about $T = 2$ K where a positive correction to the conductance of a contact from electron-electron scattering was observed [9,10]. At these temperatures, $l_{ee} \sim 10\,\mu$m while the width of the channel in Ref. [6] is 5 μm, so the assumption of ballistic regime is justified.

In the lowest approximation in α_{ee}, one may calculate the resistances R_p and R_s using the Sharvin expressions $\pi^2\hbar^2/(e^2 p_F a)$ and $\pi^2\hbar^2/(e^2 p_F b)$ [18]. Assuming that the electron concentration is $n = 1.2 \times 10^{11}$ cm^{-2} and mobility is $\mu = 6 \times 10^6$ cm^2/(V s) as in Ref. [6], and that $\alpha_{ee} \sim 1$ [19], one obtains from Eq. (13) the estimate of temperature slope of R_n normalized to the sheet resistance ρ of the conducting plane about $-2\,\mathrm{K}^{-1}$. This is of the same order of magnitude as the temperature slope of R_n/ρ in Ref. [6]. It would be of interest to set up an experiment in which the reflections from the boundaries would play no role, so that there would be no need to subtract their contribution from the nonlocal response, and to investigate the low-temperature portions of $R_n(T)$ in more detail. These measurements could provide an insight into microscopic processes of electron-electron scattering in 2D conductors

III. SUMMARY

In summary, the spatial confinement changes the relative importance of different scattering channels in the electron-electron collision integral. While for large systems the singularities in the available phase-space volume are smeared by the finite temperature and head-on collisions dominate over backward scattering, for small enough systems they are cut off by a finite dwell time of an electron in the system and the backward scattering becomes the dominating process. This results in the change of sign of the nonlocal response, which may be experimentally observable.

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