Effect of Stern-Gerlach force on negative magnetoresistance and Hall resistance in spin-dependent viscous flow

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In two-dimensional electron systems (2DES), a hydrodynamic regime with remarkable viscous effects is recognized when the sample width is small enough and momentum-conserving scattering is dominant. Based on hydrodynamic calculations for a 2DES, we obtain large effective viscosity coefficients, which lead to large negative magnetoresistance (MR) and Hall resistance due to the effect of an in-plane Stern-Gerlach force on the viscosity components. The spin-dependent viscous effect manifests itself via both longitudinal and Hall resistances. The negative MR and Hall resistance are markedly sensitive to the perpendicular magnetic field with amplitude around zero, which would be useful in MR sensors and magnetic recording devices.

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I. INTRODUCTION

Negative magnetoresistance (MR) and Hall resistance in two-dimensional electron systems (2DESs) continue to stimulate intense research interest. They exhibit new features in a hydrodynamic regime when the typical length scale of electron-electron scattering (l_{ee}) is shorter than those of electron-disorder and electron-phonon scatterings (l), i.e., $l_{ee} \ll l$. In this case, the motion of electrons becomes collective, and the electron transport is dominated by a viscous effect. A hydrodynamic approach [1,2] was used to explain the giant negative MR reported in experiments [3–9]. Two-dimensional (2D) viscous electron flows have recently been systematically studied within hydrodynamic theory in Refs. [10–12].

Hall viscosity in the presence of a magnetic field has attracted considerable attention recently due to its quantized nature in the quantum Hall regime [13–15]. For 2DESs in magnetohydrodynamic regimes, transport measurements [15–17] have demonstrated classical Hall viscosity through inhomogeneous flow [1]. The Hall resistance and MR have been studied for viscous electron flow by numerical solutions of the kinetic equation in Ref. [16]. Delacrétaz and Gromov exploited the contribution of Hall viscosity to charge transport, and explained how to determine the Hall viscosity from resistance measurements in a transport experiment [15]. Pellegrino *et al.* [17] proposed an all-electrical scheme to determine the Hall viscosity of a 2D electron liquid based on hydrodynamic equations. The experimental observation of the hydrodynamic effect and Hall viscosity was reported in GaAs mesoscopic samples [9], where a negative Hall resistivity was observed at low magnetic fields.

However, in the hydrodynamic regime, negative MR and Hall resistance have not been studied in spin-related viscous electron flow. It is thus desirable to understand the mechanism of a spin-dependent viscous effect in the 2DES. In addition, the effect of in-plane magnetic fields on negative MR was studied in a GaAs/AlGaAs quantum well [6], where the MR remained essentially unaffected by $B_\parallel < 30$ kG, and it can be suppressed by $B_\parallel \geqslant 30$ kG. Thus an interesting question arises: How does the gradient of an in-plane magnetic field affect the negative MR? In this paper, we propose a two-component hydrodynamic approach to study the effect of an in-plane Stern-Gerlach force on viscosity components on negative MR and Hall resistance. Our hydrodynamic approach is based on the kinetic theory for a two-component system consisting of spin-up and spin-down electrons. The dominant contribution of the Stern-Gerlach force on negative MR and Hall resistance originates from a spin-dependent viscosity stress tensor. The electron viscosity tensor is derived in a shortcut way similar to Ref. [1]. We perform detailed calculations on the effective regular and Hall components of viscosity together with negative MR and Hall resistance for long rectangular GaAs and InSb samples with rough edges in the presence of the Stern-Gerlach force.

II. MODEL AND FORMALISM

The system under consideration is depicted in Fig. 1, which is a 2DES in the (x, y) plane with width w along the y direction. The 2DES is modulated by a uniform electric field E_x along the x direction, a uniform magnetic field B_0 along the z direction, and a constant magnetic field gradient [18] ∇B_y along the y direction. The spin quantization direction is taken to be the y axis. The magnetic field gradient leads to the spin-dependent Stern-Gerlach force $\frac{g^*\mu_B}{2}\nabla B_y s$, where $\mu_B = \frac{e\hbar}{2m_e c}$ is the Bohr magneton, g^* is the effective g-factor, m_e is the free-electron mass, e is the fundamental charge, $\hbar$ is the Planck constant, c is the speed of light, and $s = \pm$

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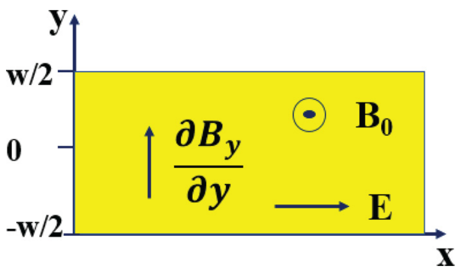

FIG. 1. Schematic illustration of the geometry for the considered 2DES system. The system is modulated by a constant magnetic field gradient ∇B_y along the y direction, an external electric field $\mathbf{E}$ along the x direction, and a uniform magnetic field B_0 along the z direction.

represents up spin and down spin. Unless specified, Gaussian units are adopted for all physical quantities. The electron density of spin-up and spin-down electrons under zero field is uniform, $n_+ = n_- = n/2$.

From a semiclassical viewpoint, the properties of the 2DES can be obtained from the distribution function $f_s = f_s(\mathbf{p}, \mathbf{r})$, which depends on the spin index s, electron quasimomentum $\mathbf{p}$, and position $\mathbf{r}$. The distribution function is governed by the Boltzmann kinetic equation under the relaxation-time approximation [19,20]

$$\mathbf{v}_s \cdot \nabla_{\mathbf{r}} f_s + \left(e\mathbf{E} + \frac{eB_0}{c}\mathbf{v}_s \times \mathbf{e}_z + s\frac{g^*\mu_B}{2}\nabla B_y \right) \cdot \nabla_{\mathbf{p}} f_s = -\frac{f_s}{\tau_s}. \tag{1}$$

Here, $\mathbf{v}_s(\mathbf{p}) = \frac{\partial \epsilon}{\partial \mathbf{p}}$ is the velocity of an electron with spin s and energy ϵ, $\mathbf{e}_z$ is the unit vector along the normal direction, and $\tau_+ = \tau_- = \tau = 1/(1/\tau_{\rm ph} + 1/\tau_0)$ is the normal momentum relaxation time [20] due to electron scattering by acoustic phonons ($\tau_{\rm ph}$) and disorders (τ_0). As in Ref. [1], we take a constant $\tau_0 = 4.5 \times 10^{-10}$ s and an electron-temperature-dependent $1/\tau_{\rm ph} = A_{\rm ph}T$ with $A_{\rm ph} = 10^9\ {\rm s^{-1}\,K^{-1}}$. The macroscopic flow density $\mathbf{j_s}$ and drift velocity $\mathbf{u_s}$ for the spin-s subsystem are calculated from

$$\mathbf{j_s} = \int \frac{d^2\mathbf{p}}{(2\pi)^2}\mathbf{v}_s f_s = n\mathbf{u_s}. \tag{2}$$

Under zero field, the distribution function is given by the Fermi-Dirac function

$$f^0 = \frac{1}{1 + e^{(\epsilon-\mu)/(k_B T)}}, \tag{3}$$

where $\mu = (\hbar\sqrt{2\pi n})^2/(2m)$ is the chemical potential in equilibrium, m is the effective mass of an electron in the 2DES, and k_B is the Boltzmann constant. Under external fields, a small local change of the chemical potential and temperature ($\delta\mu_s$ and δT), together with the current-induced drag, results in a small deviation from the local equilibrium. Such a deviation can be accounted for by introducing a small correction [19–21] δf_s to the distribution function f_s,

$$f_s = f^0 + \delta f_s,$$
$$\delta f_s = -\frac{\partial f^0}{\partial \epsilon}\left(\delta\mu_s + (\epsilon - \mu_s)\frac{\delta T}{T} + \mathbf{p} \cdot \mathbf{u_s} \right). \tag{4}$$

We assume here that thermalization between the electronic system and the lattice is much faster than quasiparticle recombination. This allows us to neglect local temperature fluctuations, i.e., $\delta T = 0$. Further, the electron system is considered to be incompressible [1], $\nabla \cdot \mathbf{u_s} = \mathbf{0}$, which implies $\delta n_s/\delta\mu_s = 0$, i.e., that the system does not respond to changes in μ_s up to exponentially small corrections.

A macroscopic equation for the flow densities can be obtained by multiplying the kinetic equation [Eq. (1)] by the quasiparticle velocity and summing over all single-particle states. As a result, we find [19,20]

$$\nabla_i \Pi_{ij} - \frac{en}{2m}E_j - \epsilon_{jk}\omega_c j_{sk} - s\frac{ng^*\mu_B}{4m}\nabla_j B_y = -\frac{j_{sj}}{\tau}. \tag{5}$$

Here, the Einstein summation convention is applied for the repeated index $i, j, k \in \{x, y\}$, $\epsilon_{jk} = 1, -1$ for $(jk) = (xy), (yx)$ and 0 otherwise, Π_{ij} is defined as the viscosity stress tensor, and $\omega_c = eB_0/mc$ is the cyclotron frequency. By means of the changing rate of deformation

$$V_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}, \tag{6}$$

Π_{ij} can be written as

$$\Pi_{ij} = \int \frac{d^2\mathbf{p}}{(2\pi)^2}\frac{v_i v_j}{2} f_s = \frac{n}{2}\langle v_i v_j \rangle$$
$$= -\frac{n}{2}(\eta_{xx}V_{ij} + s\zeta_x u_{sj} + \eta_{yx}\epsilon_{ik}V_{kj} + s\zeta_y\epsilon_{ik}u_{sk}), \tag{7}$$

where η_{xx} and η_{yx} are the same as in Ref. [1], while ζ_x and ζ_y are induced by the effect of the Stern-Gerlach force. The derivations of ζ_x and ζ_y are presented in the Appendix. One has

$$\eta_{xx} = \frac{1}{1+\beta^2}\frac{v_F^2\tau_2}{4}, \quad \eta_{yx} = \frac{\beta}{1+\beta^2}\frac{v_F^2\tau_2}{4},$$
$$\zeta_x = \frac{1}{1+\beta^2}\frac{\tau_2 g^*\mu_B}{2m}\frac{\partial B_y}{\partial y}, \quad \zeta_y = \frac{\beta}{1+\beta^2}\frac{\tau_2 g^*\mu_B}{2m}\frac{\partial B_y}{\partial y}, \tag{8}$$

where τ_2 is the relaxation time for the second moment of the electron distribution function, $v_F = \hbar\sqrt{2\pi n}/m$ is the Fermi velocity, and $\beta = 2\tau_2\omega_c$. As pointed out in Ref. [1], both electron-electron scattering and electron scattering on disorder contribute to τ_2, $1/\tau_2 = 1/\tau_{2,ee} + 1/\tau_{2,0}$ with $1/\tau_{2,ee} = A_{ee}^{\rm FL}T^2/[\ln(\frac{mv_F^2}{2k_BT})]^2$. The values of $\tau_{2,0} = 1.1 \times 10^{-11}$ s and $A_{ee}^{\rm FL} = 1.3 \times 10^{-9}\ {\rm s^{-1}\,K^{-2}}$ are taken as the same in Ref. [1].

In the stationary regime under the external fields, the drift velocity is along the x direction ($u_{sy} = 0$), under which a Hall electric field E_y is established due to the magnetic field. In this case one finds

$$\frac{\eta_{xx}}{2}\frac{\partial^2 u_x}{\partial y^2} + s\frac{\zeta_x}{2}\frac{\partial u_{sx}}{\partial y} + \frac{e}{2m}E_x - \frac{u_{sx}}{\tau} = 0,$$
$$\frac{e}{2m}E_y - \frac{\eta_{yx}}{2}\frac{\partial^2 u_x}{\partial y^2} - s\frac{\zeta_y}{2}\frac{\partial u_{sx}}{\partial y} - \omega_c u_{sx} + s\frac{g^*\mu_B}{4m}\frac{\partial B_y}{\partial y} = 0, \tag{9}$$

where $u_x = u_{+x} + u_{-x}$. The hydrodynamic approach outlined above has been widely used and justified when the electron-electron scattering is the fastest process [1,9,11,20]. Adding and subtracting over the spin index for the first and second identity of Eq. (9), we get

$$\left(\eta_{xx}+\frac{\tau\zeta_y^2}{4}\right)\frac{\partial^2 u_x}{\partial y^2}+\frac{e}{m}E_x-\frac{u_x}{\tau}=0,$$
$$\frac{e}{m}E_y-\left(\eta_{yx}+\frac{\tau\zeta_x\zeta_y}{4}\right)\frac{\partial^2 u_x}{\partial y^2}-\omega_c u_x=0. \quad (10)$$

Using the conventional no-slip boundary condition $u_x(y=\pm\frac{W}{2})=0$, we obtain the velocity profile and the Hall voltage from Eq. (10),

$$u_x=\frac{e\tau}{m}E_x\left(1-\frac{\cosh(ky)}{\cosh(kw/2)}\right), \quad (11)$$
$$V_H=\tau\omega_c E_x w\left(1-\frac{\tanh(\xi)}{\xi}\right)-\frac{\eta_{1yx}}{\eta_{1xx}}E_x w\frac{\tanh(\xi)}{\xi}, \quad (12)$$

where $k=(\frac{1}{\tau\eta_{1xx}})^{1/2}$ and $\xi=kw/2$. Finally, the magnetoresistance and Hall resistance are obtained,

$$\rho_{xx}=\frac{m}{e^2n\tau}\frac{1}{1-\tanh(\xi)/\xi},$$
$$\rho_{xy}=\rho_{xy}^{\text{bulk}}\left(1-\frac{1}{\omega_c\tau}\frac{\eta_{1yx}}{\eta_{1xx}}\frac{\tanh(\xi)/\xi}{1-\tanh(\xi)/\xi}\right), \quad (13)$$

where $\rho_{xy}^{\text{bulk}}=-\frac{m\omega_c}{e^2n}$. Particularly, we define the effective viscous coefficients as

$$\eta_{xx1}=\eta_{xx}+\frac{1}{(1+\beta^2)^2}\left(\tau_2\frac{g^*\mu_B}{2mv_F}\frac{\partial B_y}{\partial y}\right)^2\frac{v_F^2\tau}{4},$$
$$\eta_{yx1}=\eta_{yx}+\frac{\beta}{(1+\beta^2)^2}\left(\tau_2\frac{g^*\mu_B}{2mv_F}\frac{\partial B_y}{\partial y}\right)^2\frac{v_F^2\tau}{4}. \quad (14)$$

III. RESULTS AND DISCUSSIONS

Our numerical results in the presence of the Stern-Gerlach force are first compared to those in Refs. [1,6] in a GaAs 2DES with electron density 2.8×10^{11} cm^{-2}, an effective mass $m=0.0665m_e$, and $g^*=0.44$. In the presence of an in-plane magnetic field gradient $\frac{\partial B_y}{\partial y}$, the spin-up and spin-down electron density is taken to be uniform with the value $n_+=n_-=n/2=1.4\times10^{11}$ cm^{-2}. The Stern-Gerlach force is larger for a material with a larger g-factor. We thus then consider a InSb [22] 2DES with $\mu\approx10^5$ cm^2/V s, $g^*=40$, $m=0.018m_e$, and $n=1\times10^{10}$ cm^{-2}. The effective width and temperature are set at $w=10$ μm and $T=1$ K in both cases. Without specification, hereafter $\frac{\partial B_y}{\partial y}$ is normalized by 1 T μm^{-1}.

Figure 2(a) shows ρ_{xx} as a function of B_0 without and with the Stern-Gerlach force for different $\frac{\partial B_y}{\partial y}$ in the GaAs 2DES. The magnetoresistance in GaAs 2DES has been experimentally and theoretically studied in Refs. [1,6] for a long sample with straight boundaries. When the edges are rough and scattering of electrons on them is diffusive, the Poiseuille flow is formed and the magnetoresistance is proportional to the diagonal viscosity such as η_{xx}. The viscosity coefficients under a perpendicular magnetic field differ significantly from those in zero field, leading to a giant negative MR. Without the Stern-Gerlach force, our results are the same as in Ref. [1].

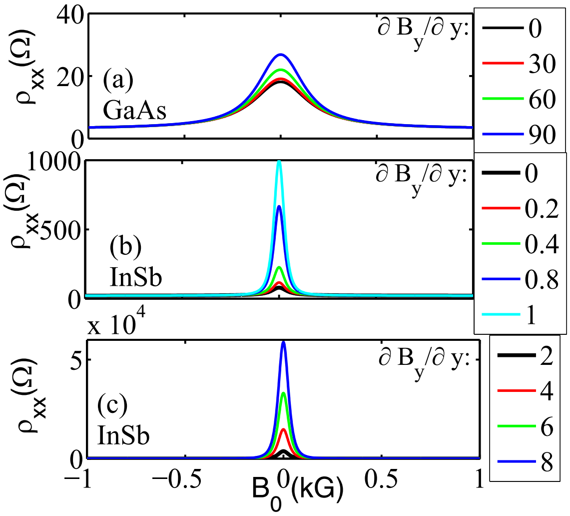

FIG. 2. Longitudinal resistance (ρ_{xx}) vs perpendicular magnetic field B_0 for the (a) GaAs and (b), (c) InSb 2DES.

At a large enough magnetic field gradient 30, ρ_{xx} becomes different from that in Ref. [1]. The peak value of ρ_{xx} increases monotonically with further increasing the magnetic field gradient, leading to stronger negative MR. The negative MR is characterized by $\delta\rho=\rho_c-\rho_0$ or ρ_c/ρ_0, where ρ_c is the value of ρ_{xx} at $B_0=0.5$ kG and ρ_0 is the value of ρ_{xx} at $B_0=0$. Indeed, the negative MR is $\rho_c/\rho_0=0.18$ with $\frac{\partial B_y}{\partial y}=90$, compared to $\rho_c/\rho_0=0.27$ without the Stern-Gerlach force, at $B\approx0.5$ kG.

Hereafter we consider the InSb 2DES where a large Stern-Gerlach force is realized under a much smaller $\frac{\partial B_y}{\partial y}$. Figure 2(b) shows ρ_{xx} as a function of B_0 without and with the Stern-Gerlach force for different $\frac{\partial B_y}{\partial y}$. Figure 2(c) shows ρ_{xx} for larger $\frac{\partial B_y}{\partial y}$. It can be seen that ρ_{xx} has a peak at $B_0=0$, exhibiting first a dramatic decrease and then decaying very slowly with the amplitude of B_0, indicating negative MRs. As shown in Figs. 2(b) and 2(c), this dramatic decrease almost terminates at $B_0=0.2$ kG with $\rho_{xx}=\rho_c$. The value of negative MR at $B_0=0.2$ kG is shown in Table I for different $\frac{\partial B_y}{\partial y}$ and compared to that without the Stern-Gerlach force.

With increasing $\frac{\partial B_y}{\partial y}$, the ρ_{xx} peak becomes larger and steeper. The peak value monotonically increases with $\frac{\partial B_y}{\partial y}$ due to Eq. (14). The highest ρ_{xx} peak appears under $\frac{\partial B_y}{\partial y}=8$, where ρ_{xx} decays quickly from $\rho_0=5.9\times10^4$ to $\rho_c=184$ Ω when B_0 changes from 0 to ∼0.2 kG, as shown in Fig. 2(c) (dark blue line). Such a large negative MR could be used for challenging the applications in magnetic recording and understanding spin-dependent viscous phenomena. Note that $\rho_c/\rho_0\approx0.003$ is about two orders of magnitude smaller than that without the Stern-Gerlach force $\rho_c/\rho_0\approx0.3$, as shown in Fig. 2(b) (black line). Consequently, a giant negative MR ($\rho_c/\rho_0\approx0.003$) effect develops at low perpendicular magnetic fields ($\leqslant$0.2 kG), where Landau quantization is not yet important [6]. The high sensitivity of ρ_{xx} to weak magnetic fields could serve as a sensitive probe in MR sensors.

TABLE I. The negative MR ($\delta\rho$ and ρ_c/ρ_0) of an InSb 2DES at $B_0 = 0.2$ kG under different values of $\frac{\partial B_y}{\partial y}$.

$\frac{\partial B_y}{\partial y}$	0	0.2	0.4	0.8	1.0	2.0	4.0	6.0	8.0
$\delta\rho$	−54	−92	−199	−642	−971	−3765	−14932	−32882	−58816
ρ_c/ρ_0	0.3	0.2	0.1	0.04	0.03	0.009	0.0045	0.0035	0.003

The largest negative MR predicted in Ref. [23] is $\rho_c/\rho_0 \approx 0.02$ in a 2DES with $\mu \approx 2.2 \times 10^7$ cm^2/V s at $B \approx 1$ kG. The strongest negative MR reported by Shi *et al.* [6] is about $\rho_c/\rho_0 \approx 0.08$, which was the strongest possible negative MR as a result of classical memory effects. In comparison to those reported in Refs. [6,23], our result exhibits a stronger negative MR with the smallest $\rho_c/\rho_0 \approx 0.003$ in terms of the Stern-Gerlach force, which could be important for memory units.

The tunability of negative MR by the Stern-Gerlach force results from the effective viscosity coefficients such as η_{xx1} and η_{yx1} [see Eq. (14)]. Figures 3(a) and 3(b) present the effective regular component of the kinematic viscosity tensor η_{xx1} (normalized by $\eta = v_F^2\tau_2/4$) as a function of the magnetic field B_0 under several values of $\frac{\partial B_y}{\partial y}$. Without the Stern-Gerlach force, $\eta_{xx1} = \eta_{xx}$ has a narrow positive peak at $B_0 = 0$. With increasing $\frac{\partial B_y}{\partial y}$, the peak value of η_{xx1} increases monotonically, and reaches the highest value of $\approx$1100 for $\frac{\partial B_y}{\partial y} = 8$, which is responsible for the strongest negative MR shown in Fig. 2(c). This can be explained as follows. From the expression of η_{xx1} in Eq. (14), η_{xx1} reaches a maximum value at $B_0 = 0$. The peak value is nearly proportional to $\tau_2 + \alpha^2\tau/(1+\beta^2)$, and the peak width is proportional to $mc/(2\tau_2 e)$ ($\sim$0.05 kG), with $\alpha = \tau_2 \frac{g^*\mu_B}{2mV_F}\frac{\partial B_y}{\partial y}$. Under the Stern-Gerlach force, the obtained η_{xx1} differs obviously from its counterpart η_{xx} in a spin degenerate case. The nontrivial effect of the Stern-Gerlach force on the viscosity of electron fluids leads to unique transport properties of a spin-dependent viscous flow.

We then present in Figs. 3(c) and 3(d) the normalized effective Hall viscosity coefficient η_{yx1} (normalized by $\eta = v_F^2\tau_2/4$) under the same condition as in Figs. 3(a) and 3(b). Without the Stern-Gerlach force, $\eta_{yx1} = \eta_{yx}$ shows an antisymmetric distribution with respect to $B_0 = 0$ and reaches a positive maximum value of 0.5 at $B_0 = mc/(2\tau_2 e)$. As $\frac{\partial B_y}{\partial y}$ increases from 0 to 8, the peak of η_{yx1} becomes higher, while the peak width of $|\eta_{yx1}|$ is proportional to $mc/(2\tau_2 e)$ in all cases. The positive highest peak of η_{yx1} occurs at $B_0 = mc/(2\tau_2 e)$ ($\sim$0.05 kG) for $\frac{\partial B_y}{\partial y} = 8$, which has an amplitude 350, which is three orders of magnitude larger than that of η_{yx}. The contrast between the variation of η_{yx1} and η_{yx} with the Stern-Gerlach force can be understood from Eq. (14). This equation tells us that η_{yx1} is proportional to B_0.

The enhancement of the effective Hall viscosity will alter greatly the spin-dependent viscous flow from a spin degenerate system. In Fig. 4, the Hall resistance ρ_{xy} is plotted as a function of the perpendicular magnetic field B_0 under several values of $\frac{\partial B_y}{\partial y}$. Due to the time-reversal symmetry, ρ_{xy} is antisymmetric about $B_0 = 0$. In the absence of the Stern-Gerlach force, the profile of ρ_{xy} agrees with that in Ref. [9]. There exists a steep slope of ρ_{xy} at $B_0 = 0$. Such a Hall slope increases with $\frac{\partial B_y}{\partial y}$ and reaches its maximum value at $\frac{\partial B_y}{\partial y} = 8$.

The huge maximum value of ρ_{xy} for $\frac{\partial B_y}{\partial y} = 8$ is comparable to that observed in an InSb 2DES at 1.5 K [22] and is about 30 times as large as that measured in graphene/boron nitride 2DES at 400 K [24]. The drastic change of ρ_{xy} with B_0 is totally unexpected and can be explained from Eq. (13).

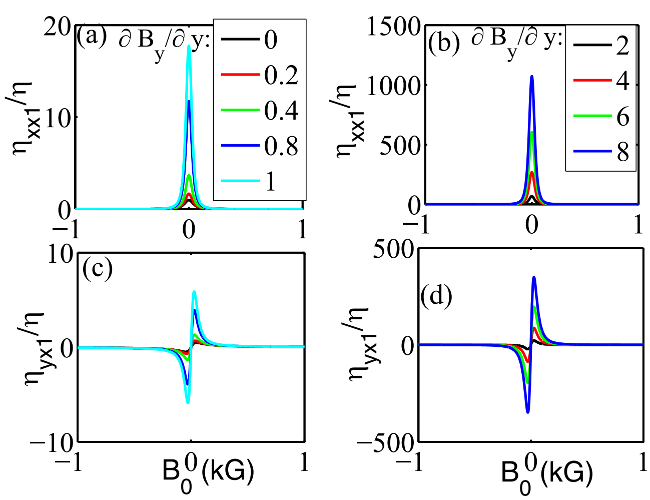

FIG. 3. The effective regular and Hall components of the kinematic viscosity tensor in an InSb 2DES as a function of perpendicular magnetic field B_0 under different values of $\frac{\partial B_y}{\partial y}$. (a), (b) η_{xx1}/η; (c), (d) η_{yx1}/η.

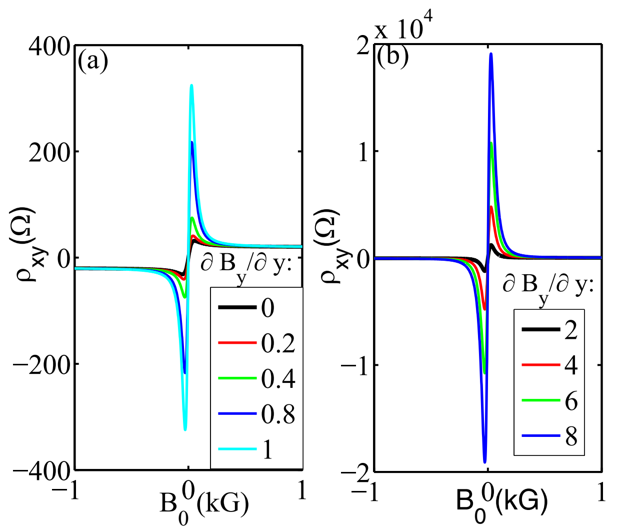

FIG. 4. Hall resistance (ρ_{xy}) of an InSb 2DES as a function of perpendicular magnetic field B_0 under different values of $\frac{\partial B_y}{\partial y}$.

The second term in this equation is proportional to η_{yx1}/η_{xx1} which determines the profile of ρ_{xy}, where $|\eta_{yx1}|/\eta$ (as well as ρ_{xy}) takes its maximum value at $B_0 = mc/(2\tau_2 e)$ ($\sim$0.05 kG) while η_{xx1} shows a relatively low value (see Fig. 3 above). In other words, the $|\eta_{yx1}|/\eta$ takes its minimum value at $B_0 = 0$ and rapidly increases to its maximum value at $B_0 = mc/(2\tau_2 e)$, while η_{xx1}/η has a maximum value at $B_0 = 0$ and quickly decreases as B_0 increases. The fact is that they increase/decrease at the same rate. This is consistent with Eq. (9), where the Hall viscosity coefficient (η_{yx1}) determines the dispersion of the 2D electron collective waves, while the regular viscosity coefficient (η_{xx1}) determines their dissipation, resulting in negative MR. These characteristics have been reported in Ref. [1]. However, they are enhanced in the presence of the Stern-Gerlach force. Our results and analysis indicate that this spin-dependent viscous mechanism is responsible for the outstanding negative MR, the Hall viscosity, and Hall resistance.

Giant negative MR has been observed in viscous electron flow in Refs. [3–9]. The Stern-Gerlach force can be obtained by applying an in-plane magnetic field gradient to the 2DES sample, which has been realized experimentally [18]. Therefore, it is demonstrated that the large negative MR and Hall resistance obtained in this theoretical work is within the scope of current experimental technology.

IV. CONCLUSIONS

In conclusion, we have used a hydrodynamic model to investigate the negative magnetoresistance and Hall resistance for long rectangular GaAs and InSb 2DESs under the modulations of an in-plane magnetic-field gradient and a weak perpendicular magnetic field. Our model is correlated with the predictions of spin-dependent viscous flow in a hydrodynamic regime. In this regime, the effective viscous coefficients can be altered greatly by the Stern-Gerlach force, resulting in large negative magnetoresistance and Hall resistance. At a proper Stern-Gerlach force, both longitudinal and Hall resistances are markedly sensitive to the perpendicular magnetic field around the zero value. Such a sensitivity could be useful for MR sensors and data storage devices.

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APPENDIX: SPIN-DEPENDENT VISCOSITY COMPONENTS

In this Appendix, we derive the spin-dependent viscosity components in the presence of the Stern-Gerlach force and the perpendicular weak magnetic field, in a shortcut way similar to the Drude conductivity.

The viscosity terms in the hydrodynamic theory can be expressed through the viscous stress tensor of per one particle, $\Pi_{i,j} = m\langle v_i v_j\rangle$, where $\mathbf{v} = (v_x, v_y)$ is the velocity of an electron and the angular brackets represent averaging over the electron velocities at a position $\mathbf{r} = (x, y)$. The motion equation without a magnetic field and Stern-Gerlach force reads

$$m\frac{\partial u_i}{\partial t} = -\frac{\partial \Pi_{ij}}{\partial x_j} - \frac{m u_i}{\tau} + eE_i, \tag{A1}$$

where $\mathbf{u} = \langle \mathbf{v}\rangle$ is the hydrodynamic velocity. Following Ref. [1], the expression for Π_{ij} is given [21] for a timescale much larger than the relaxation time for the second moment τ_2,

$$\Pi_{ij} = \Pi_{ij}^0 = -m\eta u_{ij}, \quad u_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j}. \tag{A2}$$

Based on the Drude-like equation, Π_{ij} defined in Eq. (A2) is obtained during τ_2,

$$\frac{\partial \Pi_{ij}}{\partial t} = -\frac{\Pi_{ij} - \Pi_{ij}^0}{\tau_2}. \tag{A3}$$

The terms of the magnetic field and Stern-Gerlach force can be added in the equations for $\partial u_i/\partial t$ and $\partial \Pi_{ij}/\partial t$,

$$\begin{aligned}
\left(\frac{\partial\langle v_i\rangle}{\partial t}\right)_{\text{mag}} &= \omega_c \varepsilon_{ik}\langle v_k\rangle,\\
\left(\frac{\partial\langle v_i v_j\rangle}{\partial t}\right)_{\text{mag}} &= \omega_c(\varepsilon_{ik}\langle v_k v_j\rangle + \varepsilon_{jk}\langle v_j v_k\rangle),\\
\left(\frac{\partial\langle v_i\rangle}{\partial t}\right)_{\text{spin}} &= sF_i,\\
\left(\frac{\partial\langle v_i v_j\rangle}{\partial t}\right)_{\text{spin}} &= s(F_i\langle v_j\rangle + F_j\langle v_i\rangle).
\end{aligned} \tag{A4}$$

Here, $F = \frac{g^*\mu_B}{2m}\frac{\partial B_y}{\partial y}$, $F_i = F$ for $i = y$ and 0 otherwise. The terms in Eq. (A4) are added to the right-hand side of Eq. (A3), which yields

$$\begin{aligned}
\frac{\partial \Pi_{ij}}{\partial t} = &-\frac{\Pi_{ij} - \Pi_{ij}^0}{\tau_2} + m\omega_c(\varepsilon_{zik}\langle v_k v_j\rangle\\
&+ \varepsilon_{zjk}\langle v_i v_k\rangle) + sm(F_i\langle v_j\rangle + F_j\langle v_i\rangle).
\end{aligned} \tag{A5}$$

In the stationary regime under the magnetic field and Stern-Gerlach force, we derive from Eqs. (A4) and (A5) the following equation,

$$\Pi_{ij} - \tau_2\omega_c(\varepsilon_{zik}\Pi_{kj} + \varepsilon_{zjk}\Pi_{ik}) - s\tau_2 m(F_i\langle v_j\rangle + F_j\langle v_i\rangle) = \Pi_{ij}^0. \tag{A6}$$

The components of the tensor Π_{ij} satisfy

$$\begin{aligned}
&\Pi_{xx} - \tau_2\omega_c(\Pi_{yx} + \Pi_{xy}) = \Pi_{xx}^0,\\
&\Pi_{yy} - \tau_2\omega_c(-\Pi_{yx} - \Pi_{xy}) = \Pi_{yy}^0,\\
&\Pi_{xy} - \tau_2\omega_c(\Pi_{yy} - \Pi_{xx}) = \Pi_{xy}^0 + s\tau_2 mF\langle v_x\rangle,\\
&\Pi_{yx} - \tau_2\omega_c(\Pi_{yy} - \Pi_{xx}) = \Pi_{yx}^0 + s\tau_2 mF\langle v_x\rangle,
\end{aligned} \tag{A7}$$

which leads to $\Pi_{yy} = -\Pi_{xx}$, $\Pi_{xy} = \Pi_{yx}$, and

$$\Pi_{xx} = \frac{1}{1+\beta^2}\Pi^0_{xx} + \frac{\beta}{1+\beta^2}\Pi^0_{yx} + sm\zeta_y\langle v_x\rangle,$$
$$\Pi_{yx} = \frac{1}{1+\beta^2}\Pi^0_{yx} - \frac{\beta}{1+\beta^2}\Pi^0_{xx} + sm\zeta_x\langle v_x\rangle, \tag{A8}$$

with

$$\eta_{xx} = \frac{1}{1+\beta^2}\frac{v_F^2\tau_2}{4}, \quad \eta_{yx} = \frac{\beta}{1+\beta^2}\frac{v_F^2\tau_2}{4},$$
$$\zeta_x = \frac{1}{1+\beta^2}\frac{\tau_2 g^*\mu_B}{2m}\frac{\partial B_y}{\partial y}, \quad \zeta_y = \frac{\beta}{1+\beta^2}\frac{\tau_2 g^*\mu_B}{2m}\frac{\partial B_y}{\partial y}. \tag{A9}$$

Here, τ_2 is the relaxation time for the second moment of the electron distribution function, $v_F = \hbar\sqrt{4\pi n}/m$ is the Fermi velocity, and $\beta = 2\tau_2\omega_c$.

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