Rapid Communications | Editors' Suggestion

# Quasilocalized dynamics from confinement of quantum excitations

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(Received 29 November 2019; accepted 23 June 2020; published 20 July 2020)

Confinement of excitations induces quasilocalized dynamics in disorder-free isolated quantum many-body systems in one spatial dimension. This occurrence is signaled by severe suppression of quantum correlation spreading and of entanglement growth, long-time persistence of spatial inhomogeneities, and long-lived coherent oscillations of local observables. In this work, we present a unified understanding of these dramatic effects. The slow dynamical behavior is shown to be related to the Schwinger effect in quantum electrodynamics. We demonstrate that it is quantitatively captured for long-time scales by effective Hamiltonians exhibiting Stark localization of excitations and weak growth of the entanglement entropy for arbitrary coupling strength. This analysis explains the phenomenology of real-time string dynamics investigated in a number of lattice gauge theories, as well as the anomalous dynamics observed in quantum Ising chains after quenches. Our findings establish confinement as a robust mechanism for hindering the approach to equilibrium in translationally invariant quantum statistical systems with local interactions.

DOI: 10.1103/PhysRevB.102.041118

*Introduction.* Elementary particles such as quarks experience spatial confinement into composite particles, due to forces acting at arbitrary distances mediated by gauge fields [1]. An analogous effect is also present in condensed-matter systems. In one spatial dimension, confinement typically arises in the ordered phases of systems with a spontaneously broken discrete symmetry: their elementary particle/antiparticle excitations consist of kink/antikink configurations which locally connect different degenerate ground states (vacua). Upon breaking the symmetry via external fields, the various vacua acquire different energy densities. As a result, separating a kink-antikink pair requires a configurational energy cost which grows proportionally to their distance [2–19].

Several recent numerical studies of one-dimensional lattice gauge theories and quantum spin chains have found that confinement may give rise to anomalous real-time dynamics [20–38] and spectral properties [39–41] at finite energy density above the ground state, in contrast with the generically expected thermalization [42–49]. The signatures of these phenomena include extraordinary long-lived coherent oscillations of local observables [20–25,34,50], suppression of the light-cone spreading of quantum correlations [22,34] and of the entanglement growth [22,27,34], and persistent inhomogeneities [26–33,35]. While these observations suggest that confinement is related to a suppression of thermalization, the nature of this connection has not yet been clarified.

In this work we investigate the relationship between the aforementioned dynamical effects of confinement and prototypical aspects of the localization of interacting particles [51–71]. We demonstrate that confinement causes quasilocalized dynamics of states with dilute excitations. In fact, the route towards thermalization involves the decay of these states into entropically favored many-particle states: the energy stored in confining strings has to be converted into mass via the creation of new pairs of excitations from the vacuum. We show that these processes can become dramatically slow, in close analogy with the Schwinger effect, i.e., with the suppressed decay of false vacua in quantum electrodynamics [72]. In this regime, fast spatial propagation of excitations is prevented by their Stark localization [73] in the mutual confining potentials.

Remarkably, these two phenomenona stabilize nonthermal behavior and low entanglement for extremely long times in a thermodynamically relevant portion of the many-body Hilbert space, as illustrated in Fig. 1.

*Confinement and gauge invariance in one dimension.* The occurrence of the phenomena mentioned above relies solely on the presence of confinement, and hence they emerge in both lattice gauge theories (LGTs) and statistical-physics models such as quantum spin chains. An exact correspondence between the two can be formulated in one spatial dimension, via the introduction/elimination of ancillary degrees of freedom together with local dynamical constraints. This leads to a unified framework for this broad class of systems [9,75]. In this Rapid Communication, for concreteness, we focus on the paradigmatic quantum Ising chain. To illustrate the

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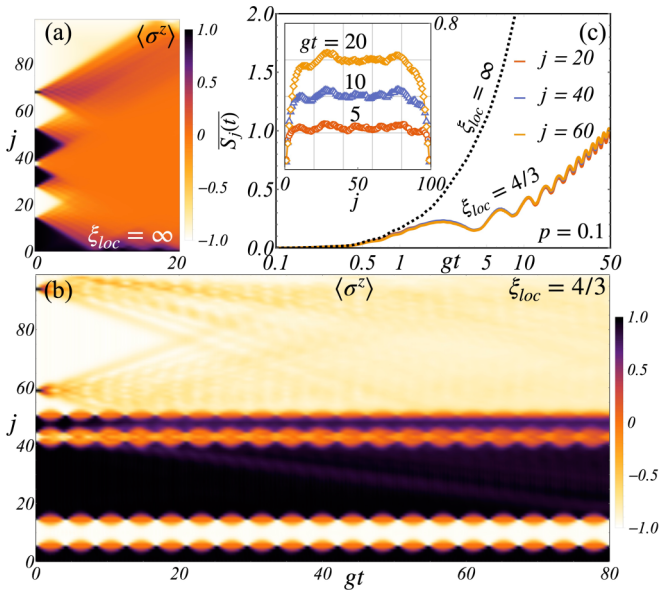


FIG. 1. Effects of confinement: nonequilibrium evolution of the magnetization profile [panels (a) and (b)] and of entanglement (c) in a quantum Ising chain. $L=100$ spins are initialized in a random product state with a density $p=0.1$ of longitudinal domain walls. The quantum evolution is simulated via the time-evolving block-decimation algorithm on matrix-product states with maximum bond dimension $D=300$ [74]. The dynamics are generated by $H$ in Eq. (1) with $J=5g$, and (a) $h=0$: in the absence of confinement, domain walls freely propagate, smoothening out all spatial inhomogeneities; (b) $h=0.75g$: while confined bound states of close-by domain walls diffuse (upper half of the plot), isolated domain walls are Stark-localized by linear confining potentials, and perform coherent Bloch oscillations of spatial amplitude $\xi_{\rm loc}=g/h$ (lower half of the plot). Panel (c): dynamics of the von Neumann entanglement entropy $S_j(t)$ for different position $j$ of the bipartition cut, averaged over 500 initial states. $S_j(t)$ grows linearly in the deconfined limit (a), $\xi_{\rm loc}=\infty$, and logarithmically in the presence of confinement (b), $\xi_{\rm loc}=4/3$, as also emphasized by the inset. These qualitative features are unaltered upon varying the localization length $\xi_{\rm loc}$ while keeping $p\lesssim 1/(2\xi_{\rm loc})$ and $J\gg|g|,\ |h|$.

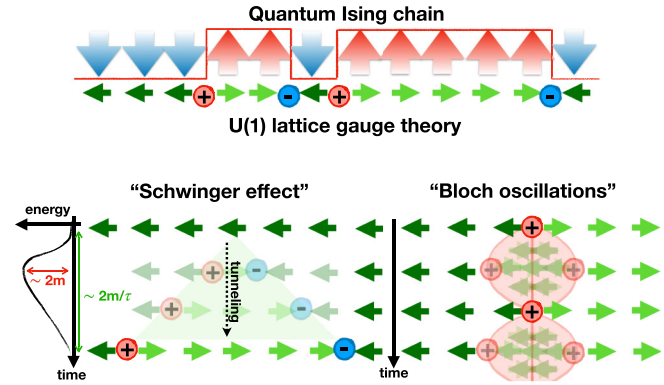


FIG. 2. Mapping between a quantum spin chain and a LGT: Cartoon of the mapping of the quantum Ising chain in Eq. (1) onto the $(1+1)$-dimensional $U(1)$ lattice gauge theory in Eq. (2) (top), and of the two key mechanisms which render the resulting dynamics slow: suppression of false vacuum decay for weak coupling ("Schwinger effect," bottom left), and Stark localization of particles in a linear potential ("Bloch oscillations," bottom right).

general equivalence above, we show how this model can be exactly mapped onto a LGT with local $U(1)$ symmetry. In the Supplemental Material [76], we consider other models with confined excitations, including the lattice Schwinger model of quantum electrodynamics [77–81] and the antiferromagnetic XXZ spin chain in a staggered field, which describes certain anisotropic magnetic insulators [16,18,19,82,83].

The quantum Ising chain is defined by the Hamiltonian

$$H=-J\sum_{j=1}^{L-1}\sigma_j^z\sigma_{j+1}^z-h\sum_{j=1}^{L}\sigma_j^z-g\sum_{j=1}^{L}\sigma_j^x, \tag{1}$$

where $\sigma_j^{x,y,z}$ are Pauli matrices acting on site $j$. The correspondence is based on the interpretation of the spin polarization operator $s_j^z\equiv\sigma_j^z/2$ as a local "electric flux." Fictitious fermionic matter degrees of freedom are introduced on the sites of the dual chain (i.e., on the bonds of the original chain [80,84,85]): they represent "positrons" and "electrons." Thus, crucially, one enforces local dynamical constraints that associate a kink (antikink) in the spin configuration with the presence of a positron (electron) on the corresponding bond, as described in Fig. 2, top panel. These constraints are interpreted as implementing a discrete Gauss law and result from the $U(1)$ gauge invariance of matter-field interactions.

To make this construction explicit, we define two species of fermions, positively ($p$) and negatively ($e$) charged, respectively, residing on the chain bonds (denoted as half-integer sites), with corresponding creation operators $(c_{j+1/2}^{p,e})^\dagger$ and occupation numbers $n_{j+1/2}^{p,e}=(c_{j+1/2}^{p,e})^\dagger c_{j+1/2}^{p,e}$. We introduce a spin-1/2 $U(1)$-quantum link model [86,87],

$$H_{U(1)}=H_{\rm m}+H_{\rm g}+H_{\rm int}, \tag{2}$$

with

$$H_{\rm m}=m\sum_j\left(n_{j+1/2}^p+n_{j+1/2}^e\right)+U\sum_j n_{j+1/2}^p n_{j+1/2}^e,$$

$$H_{\rm g}=\frac{\tau}{2}\sum_j\sigma_j^z,$$

$$H_{\rm int}=w\sum_j\left\{\left[\left(c_{j-1/2}^p\right)^\dagger+c_{j-1/2}^e\right]\sigma_j^+\left[c_{j+1/2}^p+\left(c_{j+1/2}^e\right)^\dagger\right]+{\rm H.c.}\right\},$$

where $\sigma_j^\pm=(\sigma_j^x\pm i\sigma_j^y)/2$ act as $U(1)$ parallel transporters [80]. $H_{\rm m}$ encodes the fermion mass and on-site Hubbard-like interaction, and $H_{\rm g}$ can be interpreted as the energy shift caused by a background field (or topological $\theta$ angle [33,88]). In $H_{\rm int}$, the various terms describe hopping and pair creation/annihilation of fermions. The $U(1)$ gauge invariance of these interactions is expressed by the local symmetries $[H,G_{j+1/2}]=0$ with $G_{j+1/2}=\sigma_{j+1}^z/2-\sigma_j^z/2-(n_{j+1/2}^p-n_{j+1/2}^e)$. Accordingly, the complete Hilbert space decomposes into dynamically disconnected subspaces, labeled by the set of eigenvalues $\{q_{j+1/2}=0,\pm1,\pm2\}$ of $\{G_{j+1/2}\}$, interpreted as static background charges. Here we focus on the neutral gauge sector, i.e., on the space of the states $|\psi\rangle$ for which the Gauss law $G_{j+1/2}|\psi\rangle\equiv0$ is satisfied at all sites $j$, i.e., $q_{j+1/2}\equiv0$

[80]. This law asserts that the variation of the gauge field strength $\sigma^z/2$ upon crossing a bond $(j, j+1)$ equals the dynamical charge $Q_{j+1/2} = n^p_{j+1/2} - n^e_{j+1/2}$ located on it.

In the presence of a strong Hubbard repulsion $U \to \infty$, each "classical configuration" of the gauge field (eigenstate of all $\sigma^z_j$ operators) fully determines a unique configuration of the matter particles via the Gauss law. This allows one to eliminate the redundant matter degrees of freedom [33,89] and write the model in terms of a *locally* self-interacting gauge field [88,90]. In this case, all matrix elements of the Hamiltonian (2) between two classical gauge-field configurations coincide with the corresponding matrix elements of the quantum Ising chain [Eq. (1)] in the $\sigma^z$ basis, upon identifying $m = 2J$, $\tau = -2h$, $w = -g$, and up to an overall energy shift (see the Supplemental Material [76] for details). Within this LGT picture, the longitudinal field $h$ in the quantum Ising chain plays the role of the electrostatic string tension $\tau$, leading to particle confinement. In passing, we mention that the Ising chain can also be mapped onto a $\mathbb{Z}_2$-LGT (see Ref. [76]).

Below we will analyze in detail the quantum Ising chain, but the conclusions apply to the general class of quantum chains with confinement of excitations [76].

*Exponential suppression of pair creation.* When a particle and an antiparticle in the vacuum are adiabatically separated at a distance $d$, the energy $E(d) \sim \tau d$ associated with the gauge-field string linking them grows proportionally to $d$ and eventually it overcomes the threshold $E_{\min} \sim 2m$ for the creation of a new pair. We argue that the *dynamical* breaking of strings after a quench of the interactions takes anomalously long times for large values of the mass. The mechanism for this suppression may be essentially understood as a tunneling process across a high-energy barrier. In fact, the decay process which converts the large amount of potential energy stored in long gauge-field strings into the energy of additional particle-antiparticle pairs is energetically allowed and entropically favorable, because a string state is very atypical compared to many-particle states with the same total energy. Accordingly, thermalization requires string breaking. However, due to the energy conservation, the created particle and antiparticle of a pair must be separated at such a distance $d$ that the energy $\tau d$ they subtract from the broken string portion equals their mass, i.e., $\tau d \sim 2m$. If the string tension $\tau$ is small compared to the particle mass $m$, local pair creation is not possible, and virtual particles have to tunnel across a distance $d \sim 2m/\tau \gg 1$ in order for the string to decay [see the bottom left panel of Fig. 2 for an illustration (here the lattice spacing is the unit length)]. This occurs through increasingly high-order processes in the interactions, and hence the decay is extremely slow.

The above qualitative picture is made quantitative by constructing the effective Hamiltonian in perturbation theory in $1/m$. We formally split the Hamiltonian into the mass term $H_0$, possessing highly degenerate blocks, and the rest $V$, which involves gauge field and interactions. $H_0$ defines sectors of the Hilbert space labeled by the number of particles and well-separated in energy. $V$ may contain block-diagonal matrix elements $H_1$, describing particle/antiparticle energy and motion, and block-off-diagonal ones $R_1 = V - H_1$, corresponding to particle-antiparticle pair creation or annihilation. The latter processes are eliminated through a unitary transformation $e^{S_1}$. For the quantum Ising chain, the resulting effective Hamiltonian is $H^{(1)}_{\rm eff} = -J\sum_j \sigma^z_j\sigma^z_{j+1} - h\sum_j \sigma^z_j - g\sum_j (P^\uparrow_{j-1}\sigma^x_j P^\downarrow_{j+1} + P^\downarrow_{j-1}\sigma^x_j P^\uparrow_{j+1})$, where $P^\uparrow_j$ ($P^\downarrow_j$) projects onto the "up" ("down") state of the $j$th spin along $z$.

This standard procedure [91–93] (often termed Schrieffer-Wolff transformation) can be carried out to any arbitrary order $n$ in perturbation theory: One introduces higher-order terms $S_2, S_3, \ldots$ in the generator of the unitary transformation $e^{S_{\leqslant n}}$, with $S_{\leqslant n} = -S^\dagger_{\leqslant n} = S_1 + S_2 + \cdots + S_n$. These terms are determined order by order in such a way that the transformed Hamiltonian commutes with $H_0$ up to the $(n+1)$th power of the perturbation strength, i.e., $H' = e^{S_{\leqslant n}} H e^{-S_{\leqslant n}} = H^{(n)}_{\rm eff} + V_{>n}$, with $H^{(n)}_{\rm eff} \equiv H_0 + H_1 + \cdots + H_n$, and $[H_j, H_0] = 0$ (see the Supplemental Material [76] for details). The effective Hamiltonian $H^{(n)}_{\rm eff}$ preserves the block-diagonal structure of $H_0$ and accounts for all transitions within each sector of $H_0$ occurring through up to $n$ intermediate transitions involving states in different blocks (*virtual* particle pairs).

The perturbative series generated by this transformation are generally divergent at finite energy density, pointing to an asymptotic hybridization of the various blocks and hence thermalization. However, by adapting the rigorous theory in Ref. [94] (see also the recent Ref. [95]), one finds that by truncating the series at an "optimal order" $n^*$ that scales linearly with the particle mass $m$, the rest $V_{>n^*}$ can be made *exponentially* small in $m$. Consequently, the effect of the latter can be neglected for exponentially long times. Denoting $H^{(n^*)}_{\rm eff} \equiv H_{\rm eff}$ and $S_{\leqslant n^*} \equiv S$, the nonequilibrium evolution of the system is accurately described by

$$|\Psi(t)\rangle \simeq e^{-S} e^{-itH_{\rm eff}} e^{S} |\Psi(t=0)\rangle. \tag{3}$$

Within this transformed picture, the number of particles is exactly conserved by $H_{\rm eff}$, and hence it is approximately conserved by $H$ in the original picture at least for exponentially long times. This implies the emergence of nonthermal behavior in highly excited states, signaled by the time evolution of local observables such as the particle mass density, the gauge field, and the energy density. In fact, the analysis above shows that the bulk of a long gauge string is stable against pair creation, since the "string-breaking" (or "vacuum-decay") timescale is exponentially long in $m$. This bulk stability persists in the continuum limit [10], and, within the mapping in Eq. (2), it is reminiscent of the Schwinger effect in quantum electrodynamics [72], in that the decay rate $\Gamma(\mathcal{E})$ per unit volume of a false vacuum in the presence of a background electric field $\mathcal{E}$ into particle pairs, is exponentially small in the ratio between the electron mass $m$ and the electrostatic energy $|e\mathcal{E}| \times 1/m$ contained within a Compton length, i.e., $\Gamma(\mathcal{E}) \propto (e\mathcal{E})^2 \exp(-\frac{\pi m^2}{|e\mathcal{E}|})$, where $e$ is the electron charge and $\hbar = c = 1$ [72,96].

In the Supplemental Material [76], we provide the details of the construction of $H_{\rm eff}$ and discuss the quantum Ising chain in Eq. (1) and the lattice Schwinger model as specific cases. In the former, for $J \gg |g|, |h|$ the estimates adapted from Refs. [94,97] lead to the quasiconservation of the spatial

density of domain walls at times $t \ll T_{\rm sb}$, where

$$T_{\rm sb} \geqslant g^{-1} \exp(\text{const} \times J/\sqrt{h^2+g^2}), \tag{4}$$

and the constant is independent of the parameters [76].

*Stark localization.* The nonequilibrium dynamics starting from a generic initial state may be expected to undergo *prethermalization* to the Gibbs ensemble $e^{-\beta H_{\rm eff}}/Z$ defined by the effective (nonintegrable) Hamiltonian $H_{\rm eff}$ discussed above, at the inverse temperature $\beta$ uniquely determined by the energy density of the initial state [94,98]. Contrarily to this expectation, we demonstrate that the combination of confinement and lattice effects leads to a dramatic slowdown of prethermalization in a thermodynamically significant portion of the many-body Hilbert space. This phenomenon is due to the Stark localization of particles [73] in their mutual linear confining potential, which suppresses spatial propagation and energy transport for *arbitrary* interaction strength.

We consider below many-particle states, with a diluteness parameter $p$, i.e., with an average separation of $1/p$ lattice sites between consecutive particles. To disentangle the effect of having a finite particle mass—leading to exponentially slow pair creation—from the intrinsic slow dynamics of $H_{\rm eff}$, we analyze the nonequilibrium dynamics generated by the latter truncated at the lowest order. The effective picture consists of a system of hopping hard-core particles in a constant electric field, subject to interactions. Higher-order terms in $H_{\rm eff}$ do not alter the physics qualitatively, as they just renormalize the hopping amplitudes with small longer-range terms [76].

In the extremely dilute limit $p \ll 1$ the system consists of isolated particles moving in a linear potential, a so-called Wannier-Stark ladder. This problem can be solved exactly [99]: eigenstates are product states of localized orbitals with equispaced energy levels $E_n \propto n$. For the quantum Ising chain in Eq. (1), $E_n = 2hn$ and the localized wave function centered around the site $n$ reads $\Psi_j^{(n)} = \mathcal{J}_{n-j}(g/h)$, where $\mathcal{J}_\nu$ is the Bessel function of order $\nu$ [76]. The tails of these localized orbitals decay faster than exponentially for $|n-j| \gg g/h \equiv \xi_{\rm loc}$. If the distance between consecutive particles is much larger than $\xi_{\rm loc}$, transport and thermalization are suppressed, and particles perform coherent (Bloch) oscillations around their initial position, with spatial amplitude $\xi_{\rm loc}$ and temporal period $\pi/h$ [26].

However, delocalization gradually occurs as $\ell = 1/p$ is made comparable with twice the localization length $2\xi_{\rm loc}$. To understand this phenomenon and the associated timescales, we consider an isolated string with a particle (kink) at site $n_1$ and an antiparticle (antikink) at site $n_2 > n_1$. In the center-of-mass frame described by the relative coordinate $n_- = n_2 - n_1 > 0$, the problem reduces to a single-particle Wannier-Stark ladder with hopping $2g\cos K$, where $K$ is the center-of-mass momentum, and subject to a hard wall at the origin, i.e., to the boundary condition $\psi_{n_-=0} \equiv 0$ [76,100]. The solution consists of a discrete sequence of particle-antiparticle bound states ("mesons") labeled by $\ell = 1, 2, \ldots$ with dispersion relations $E_\ell(K)$. The wave functions $\Psi_{n_-}^{(\ell,K)} = \mathcal{J}_{\ell-n_-}(2g\cos K/h)$ with $\ell \gg 2\xi_{\rm loc}$ are localized far away from the boundary $n_- = 0$: they are hardly affected by it, and hence their energy $E_\ell = 2h\ell$ is independent of $K$. This implies that spatially extended bound states have asymptotically flat bands: the two particles perform uncorrelated Bloch oscillations at the edges of the string connecting them, while the quantum diffusion of their center of mass is suppressed. However, the presence of the boundary bends the dispersion relation $E_\ell(K)$ of bound states with an extension comparable to that of the Bloch oscillations. This leads to correlated ("rigid") motion of the string edges, and hence spatial delocalization and entanglement growth.

The correction $\delta E_\ell(K)$ to the energy level $E_\ell$ in the dilute regime $\ell \gtrsim 2\xi_{\rm loc}$ is found to be approximately $-2g\cos K\,\mathcal{J}_\ell(2\xi_{\rm loc}\cos K)\mathcal{J}_{\ell-1}(2\xi_{\rm loc}\cos K)$ [76]. The analysis of the resulting spreading velocities $v_\ell^{\rm max} = \text{Max}_K\,|\partial_K E_\ell(K)|$ of bound states for varying quantum number $\ell$, leads to a sequence of delocalization timescales $T_{\rm dloc}(\ell, \xi_{\rm loc})$ rapidly increasing as the ratio $\ell/\xi_{\rm loc}$ increases; for large $\ell \gtrsim \xi_{\rm loc}^2$, one has [76]

$$T_{\rm dloc}(\ell, \xi_{\rm loc}) \sim g^{-1}(\ell!)^2 \ell^{-3/2} \xi_{\rm loc}^{-2\ell+1}. \tag{5}$$

As a result, the typical delocalization timescale is state dependent via the diluteness parameter $p$, unlike the string-breaking timescale $T_{\rm sb}$ in Eq. (4). We stress that the above equations are nonperturbative in $g/h = \xi_{\rm loc}$ and hence valid for arbitrarily large values of this ratio.

*Slow entanglement growth.* The scenario outlined above sheds light on the effects of confinement on the nonequilibrium evolution of entanglement. While the entanglement entropy $S(t)$ is expected to increase linearly in time in generic quantum many-body systems which dynamically relax to equilibrium [101–107], the quasilocalization discussed above is expected to cause a severe suppression of the growth of

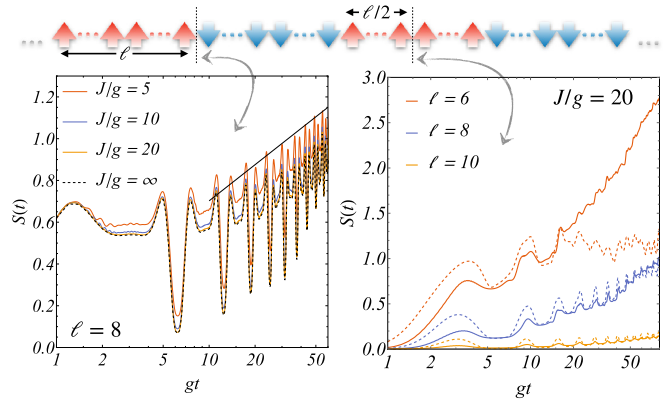


FIG. 3. Signatures of slow dynamics: Growth of the von Neumann entanglement entropy $S(t)$ in the nonequilibrium dynamics of the quantum Ising chain in Eq. (1), numerically simulated via the TEBD algorithm, starting from a state with equally spaced domain walls at a distance $\ell$; the cartoon above the plots indicates the position of the bipartition cuts along the chain. Left: $S(t)$ exhibits pronounced coherent oscillations with frequency $2h$ superimposed to a slow growth (the straight line is a guide for the eye). Right: The growth of $S(t)$ slows down upon increasing the diluteness. Dotted lines represent the growth of $S(t)$ in the evolution of a single isolated string formed by the two domain walls adjacent to the cut. The latter can be obtained analytically [76], is upper-bounded by $\log \ell + \text{const}$, and reaches its maximum around the time $T_{\rm dloc}$ [cf., Eq. (5)]. Parameters: $\xi_{\rm loc} = 2$, $L = 120$.

$S(t)$ despite the finite energy density, in analogy with disordered and glassy quantum systems [54–56,58,63,66–68,108–110]. This expectation is confirmed by numerical simulations using the time-evolving-block-decimation (TEBD) algorithm on matrix-product states [74], with maximum bond dimension $D = 300$. In particular, we initialize a quantum Ising chain of $L = 100$ spins in nonentangled product states with a spatial density $p$ of domain walls: in Fig. 1 these states are drawn from a thermal ensemble $\rho_0 = e^{-\mu H_0}/Z$ of the "unperturbed" classical Ising chain with $p = [1 - \tanh(\mu J)]/2$. (Similar dilute states with tunable $p$ can be experimentally realized via the quantum Kibble-Zurek mechanism [111,112].) The numerical results reported in Fig. 1 are compatible with a logarithmic growth of the bipartite entanglement entropy $S_j(t)$ superimposed to coherent oscillations of period $\pi/h$, ascribed to Bloch oscillations. In Fig. 3, instead, regularly arranged initial states are considered with equispaced domain walls at a distance $\ell = 1/p$ and $L = 120$. The fast convergence of $S(t)$ to that generated by the effective Hamiltonian $H_{\text{eff}}$ upon increasing $J$ (Fig. 3, left panel) leads us to rule out the hypothesis that the slow vacuum decay is responsible for the entanglement growth. Furthermore, the bottom right panel of Fig. 3 shows that the initial growth of $S(t)$ is captured by the delocalization of individual strings described in Eq. (5) above [76]; however, at longer times, many-particle effects lead to a slow unbounded growth.

*Outlook.* In the framework of localization phenomena in disorder-free quantum systems [60–71,113,114], this work establishes the role of confinement as a robust mechanism capable of dramatically slowing down the approach to equilibrium [22,26,33–37,109,110,115]. It is interesting to highlight the connection with the recently proposed "Stark many-body localization" [70,71,116,117], in that the effective dynamics of the systems considered in the present work may be viewed as that of interacting particles in a constant field. Our preliminary numerical results suggest that rare high-density regions embedded in dilute systems do *not* thermalize the rest of the system within the relevant timescales in this work; however, a complete analysis of this problem and of the various stages of the dynamics [118] calls for further investigations, which we leave to future studies.

Our discussion applies to generic one-dimensional lattice models with confined excitations, including Abelian and non-Abelian LGTs [27,78,119]. The extension of our work to confining theories in higher dimensions stands as a challenging direction for future work, inasmuch as their real-time dynamics has hardly been explored in the framework of nonequilibrium statistical mechanics.

*Acknowledgments*. We thank F. Alet, R. Verdel, Y. Baum, J. Berges, W. De Roeck, E. Ercolessi, F. H. L. Essler, A. V. Gorshkov, M. Heyl, R. M. Konik, C. J. Lin, O. I. Motrunich, R. Nandkishore, E. van Nieuwenburg, S. Parameswaran, N. J. Robinson, A. Scardicchio, M. Schulz, J. Sisti, and X. Turkeshi for interesting discussions and particularly D. A. Abanin, M. Dalmonte, and M. Müller for sharing with us their insights on the subject matter of this work. This work is partly supported by the ERC under Grant No. 758329 (AGEnTh). A.L., F.M.S., and P.P.M. thank the Galileo Galilei Institute for Theoretical Physics for their hospitality during the workshop "Breakdown of Ergodicity in Isolated Quantum Systems: From Glassiness to Localization" and the INFN for partial support during the completion of this work.

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