Rapid Communications | Editors' Suggestion

Isolated Heisenberg magnet as a quantum time crystal

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We demonstrate analytically and numerically that the paradigmatic model of quantum magnetism, the Heisenberg XXZ spin chain, does not equilibrate. It constitutes an example of persistent nonstationarity in a quantum many-body system that does not rely on external driving or coupling to an environment. We trace this phenomenon to the existence of *extensive dynamical symmetries*. We discuss how the ensuing persistent oscillations that seemingly violate one of the most fundamental laws of physics could be observed experimentally.

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Introduction. Isolated systems consisting of many interacting particles are generally assumed to relax to a stationary equilibrium state whose macroscopic properties are described by the laws of statistical physics. This has been confirmed by a large amount of recent theoretical [1–17] and experimental [18–24] work, particularly focusing on the Heisenberg spin chain in quench setups [25–32]. On the other hand, time crystals describe a phase of mater which never relaxes to stationarity and breaks the continuous time-translation symmetry (TTS) in analogy with the continuous space-translation symmetry breaking in ordinary crystals. Historically the research into quantum time crystals was instigated by an intriguing possibility that a system at zero temperature could exhibit perpetual motion [33]; however, this has subsequently been disputed [34–36], leading to possible generalizations to finite temperature, and systems far from thermal equilibrium. Despite the large amount of work on Floquet (breaking the discrete time-translation symmetry) or dissipation-induced time crystals [37–56] such behavior was believed to be impossible to realize in isolated many-body systems [34–36,57]. These typically relax to stationary states depending only on a few parameters, such as energy and particle number [58,59].

Despite numerous studies on relaxation in many-body quantum systems, there have been no results on spontaneous time-translation symmetry breaking in locally strongly interacting Hamiltonian systems close to equilibrium. Previous examples of TTS breaking include models with a well-defined mean-field limit [60,61], and the case of spin precession. In strongly correlated systems one *a priori* expects relaxation to stationarity [1–17]. The absence of such results might be expected in light of the no-go theorem [36]; however, there is a crucial defining property underlying its derivation, which we relax. It assumes that the system should exhibit long-range spatial correlations, which are not relevant to time-translation symmetry breaking, or nonstationarity.

In this Rapid Communication we show that systems can indeed fail to relax and relate this type of behavior to *extensive dynamical symmetries* that are local in space and have a periodic dependence on time. We find that, as a consequence of the presence of dynamical symmetries, a system can fail to equilibrate after a quantum quench and is instead described by a time-dependent statistical ensemble. Nonstationarity also shows up on the level of dynamical response functions describing the behavior near equilibrium and the stability of the equilibrium state to small perturbations.

We demonstrate the effects of local dynamical symmetries for the one-dimensional Heisenberg spin chain and study its stability under integrability breaking perturbations. This paradigmatic model is used to describe many experimentally relevant situations including organic compounds [62], various materials [63], cold-atom implementations [64], and quantum dots [65].

Quantum time crystals. We introduce two related definitions of spontaneous time-translation symmetry breaking motivated by analogy with previous literature; one will be oscillations in the autocorrelation function at equilibrium, and the other is motivated by relaxation following quenches.

Watanabe and Oshikawa [36] defined quantum time crystals as interacting systems exhibiting persistent oscillations. These can be probed by the autocorrelation function $\tilde{f}(t) = \frac{1}{V^2}\langle O(t)O\rangle$, where V is the volume of the system and O is an extensive observable [36]. They consider this autocorrelation function as a perturbation to equilibrium and show that it is time independent at zero and finite temperature for *local* Hamiltonians (see [66] for nonlocal results). Their definition does not capture all physically measurable persistent oscillations because the time-dependent (connected) part of the function $\tilde{f}(t)$ vanishes in the absence of long-range correlations in the large volume limit at all times t. We instead consider connected autocorrelation functions that are initially normalized $f(t) = \frac{1}{\langle O^2\rangle}\langle O(t)O\rangle$. This probes TTS breaking of measurable local operators if the equilibrium ensemble is perturbed by the extensive operator O, even in the absence of long-range correlations. Following such a perturbation quantum time crystals will never reach stationarity. In contrast *quantum many-body scarred* models [67–69] exhibit oscillations only

for a special set of initial states and are expected to relax to stationarity close to equilibrium.

While there are different ways of identifying the many-body nature of the phenomenon, the definition we use here is that single-body observables relax to stationarity, while some of the many-body observables oscillate persistently. This way oscillations of the higher point correlation functions cannot be attributed to single-body oscillations.

Recently, most of the work on quantum time crystals has focused on the discrete time-translation symmetry breaking in Floquet systems [37,38,41–48]. In this case TTS breaking is studied in an out-of-equilibrium *quench* setup. In this setup the system is prepared in a generic pure state $|\psi\rangle$ and then allowed to evolve under the action of a Hamiltonian. The system is identified as a discrete quantum time crystal if the dynamics breaks the discrete time symmetry of the driving period T with a subharmonic response $\langle\psi|o(t+nT)|\psi\rangle = \langle\psi|o(t)|\psi\rangle$, for some integer $n > 1$, and $\langle\psi|o(t+t_1)|\psi\rangle \neq \langle\psi|o(t)|\psi\rangle$ for $t_1 < nT$ [37]. We make the analogous identification for continuous-time evolution by requiring that $\langle\psi|o(t+T)|\psi\rangle = \langle\psi|o(t)|\psi\rangle$ and $\langle\psi|o(t+t_1)|\psi\rangle \neq \langle\psi|o(t)|\psi\rangle$, for $t_1 < T$ and for some T.

Extensive dynamical symmetries. An important insight into the phenomenon of equilibration is provided by the eigenstate thermalization hypothesis in generic systems [17,58,70,71], or the generalized eigenstate thermalization hypothesis (GETH) in integrable systems [72]. It states that off-diagonal elements of local observables in the eigenbasis of a local Hamiltonian vanish exponentially in the thermodynamic limit, and that their expectation values in a given eigenstate are smooth functions of conserved quantities. Assuming the validity of the GETH the system is expected to locally relax to the maximal entropy, or generalized Gibbs ensemble (GGE) $\rho_{\mathrm{GGE}} = \exp(-\sum_j \mu_j X_j)$, following a quantum quench. The set of chemical potentials μ_j is obtained by matching the expectation values of extensive conservation laws X_j [73] in the ensemble and the initial state [74].

The situation is very different if the system possesses an additional set of *extensive dynamical symmetries* Y and $Y^\dagger$. Such quantities satisfy a simple closure (or eigenoperator) condition

$$[H, Y] = \omega Y, \tag{1}$$

which leads to periodic evolution $Y(t) = \exp(i\omega t)Y(0)$ and $Y(t)^\dagger = \exp(-i\omega t)Y(0)^\dagger$. In any isolated system there is a large number of operators satisfying condition (1). However, in general these are highly nonlocal and as such have no effect on the local physics on large timescales. For simplicity we will discuss the case with a single frequency ω and the single operator Y, but the generalization to multiple frequencies and multiple Y's is straightforward.

Time-dependent generalized Gibbs ensemble. In order to understand how dynamical symmetries affect late time dynamics of local observables after a quantum quench, we consider discrete time dynamics with time steps $2\pi/\omega$ induced by a Hamiltonian H,

$$\mathcal{M}_\omega(Y) = \exp(i2\pi H\omega^{-1})Y\exp(-i2\pi H\omega^{-1}), \tag{2}$$

which renders Y and $Y^\dagger$ conserved. The stationary maximum entropy ensemble for the stroboscopic dynamics can be obtained from the entropy maximization procedure, which has to respect all conserved quantities X_j, as well as dynamical symmetries Y and $Y^\dagger$. More specifically, we study a system initialized in a generic enough (not too highly entangled) pure state and study the time evolution under the influence of dynamics at stroboscopic times. This leads to the GGE description $\rho_{\mathrm{GGE}} = \exp(-\sum_j \mu_j X_j - \mu_Y Y - \bar{\mu}_Y Y^\dagger)$ [74]. The full quantum state is still pure due to the unitarity of time evolution, and both the time-dependent generalized Gibbs ensemble (tGGE) and GGE states should be understood to be the reduced density matrix states of the system *locally*, i.e., for calculating expectation values of local observables. If conserved quantities do not commute this might in principle require redefinition of ensembles [75,76].

Stroboscopic time evolution of the state $|\psi(t)\rangle$ leads to different maximum entropy (GGE) states for each $t \in [0, \frac{2\pi}{\omega})$, which take the form of tGGE for the intermediate times,

$$\rho_{\mathrm{tGGE}} = \exp\left(-\sum_j \mu_j X_j - \mu_Y(t)Y - \bar{\mu}_Y(t)Y^\dagger\right), \tag{3}$$

where the conserved quantities $[H, X_j] = 0$ as usual. The values of the chemical potentials $\mu_j, \mu_Y(t)$ can be fixed in the following way. The maximum entropy nonstationary ensemble which correctly reproduces the initial value of X_k, and the dynamics of dynamical symmetries Y_k is obtained by requiring for an initial state $|\psi\rangle$

$$\begin{aligned}\langle\psi|Y(t)|\psi\rangle &= \mathrm{tr}\,[Y(t)\rho_{\mathrm{tGGE}}],\\ \langle\psi|X_k|\psi\rangle &= \mathrm{tr}\;[X_k\rho_{\mathrm{tGGE}}].\end{aligned} \tag{4}$$

Dynamical response functions in thermal equilibrium. Here we focus on the response of the system in equilibrium. In the large time limit a local observable $O(t)$ can be represented as a linear combination of conserved quantities and dynamical symmetries

$$O(t) \underset{t\to\infty}{=} \alpha_Y^O \exp(i\omega t)Y + \mathrm{H.c.} + \sum_j \alpha_j^O X_j, \tag{5}$$

for calculating the dynamical susceptibilities $\langle O_1(t)O_2\rangle$. This equality is valid only in the *hydrodynamic* level and in the long-time limit. We restrict the discussion to the case with $\langle O_1\rangle = \langle O_2\rangle = 0$, which can be relaxed by considering connected correlation functions. In general the coefficients α_Y^O and α_j^O depend not only on the observable O but also on the thermal ensemble $\langle\cdot\rangle$. If these coefficients vanish, the observable is not expected to oscillate.

Heisenberg model and extensive dynamical symmetries. We will consider the anisotropic Heisenberg Hamiltonian

$$\begin{aligned}H = J\Bigg[&\sum_j s_j^x s_{j+1}^x + s_j^y s_{j+1}^y + \Delta s_j^z s_{j+1}^z\\ &+ \alpha\left(\sum_j s_j^x s_{j+2}^x + s_j^y s_{j+2}^y + \Delta s_j^z s_{j+2}^z\right)\Bigg] + \sum_j h s_j^z,\end{aligned} \tag{6}$$

as a paradigm example of a system breaking TTS. In (6) we introduced the spin-$\frac{1}{2}$ operators $s^{x,y,z}$, anisotropy Δ, hopping amplitude J, the magnetic field h, and the

integrability breaking parameter α, which is set to 0 except when otherwise specified. One of the crucial aspects of the Heisenberg model, which has a paramount effect on physical properties, are its extensive conservation laws [77–80]. Their known effects range from the absence of thermalization to the ideal (ballistic) energy and spin conductivity at any temperature. Despite the absence of thermalization, the Heisenberg model has in recent years served as a testbed for studying equilibration properties of strongly interacting systems [25–32]. In what follows we will show that in the easy-plane regime $-1 < \Delta < 1$, it does not, in general, reach equilibrium if $h \neq 0$.

This can be seen as a consequence of the existence of semicyclic quantities $Y_\Delta(\phi)$, which were introduced in [78] (see also [81]), with ϕ parametrizing the infinite set of quantities $Y_\Delta(\phi)$ at Δ. We will suppress the dependence on ϕ for the sake of simplicity. While these quantities commute with the Hamiltonian (6) in the absence of the field $h = 0$, the Y_Δ operators do not commute with the total magnetization. Due to them having a surplus of exactly m s^+ operators they rather satisfy $[\sum_j s_j^z, Y_\Delta] = mY_\Delta$ for the anisotropy parameter $\Delta = \cos(\frac{\pi n}{m})$, with $n \in 2\mathbb{N}$, $m \in 2\mathbb{N}+1$. This leads to the Y_Δ operators becoming *dynamical symmetries* of H for nonzero h in the sense of (1). Interestingly, this also means that the frequency of their oscillations $\omega = hm$ is a discontinuous function of Δ (see [81] for details). We emphasize that the set of dynamical symmetries Y_Δ are different for each value of m. This implies that the operators exhibiting persistent oscillations change with Δ. Likewise, the operators $Y_\Delta^\dagger$ have a surplus of m operators s^-.

For the sake of simplicity we will now mainly focus on the oscillations of the transverse correlation function $s_1^x s_2^x s_3^x$ at $\Delta = -\frac{1}{2}$. Physically, such observables correspond to correlations of the three-site measurement statistics; the average measured value of each individual spin relaxes according to standard statistical physics, but the measured values will be such that on average their product oscillates in time. Alternatively, they may be thought of as oscillations of the higher moments of the m-site quantum fluctuations (e.g., $\langle (s_1^x + s_2^x + s_3^x)^3(t)\rangle$). Note that the fact that quantities responsible for oscillations Y do not exist at the noninteracting point $\Delta = 0$ solidifies the argument that the oscillations are a genuine many-body phenomenon.

In order to obtain the dynamics of temporal correlation functions in the Heisenberg model $\langle O_k(t) O_l\rangle$ with $O_k = \sum_i s_i^x \dots s_{i+k-1}^x$, we will use the ansatz (5), specializing to the infinite temperature ensemble $\langle\cdot\rangle = \frac{\mathrm{tr}\,(\cdot)}{\mathrm{tr}\,(\mathbb{1})}$. Conversely, the results provide an asymptotic solution of the quench protocol for any initial state of the form $\rho = \sum_{k\in 2\mathbb{Z}+1} a_k O_k$. A particular choice of O_k was made due to the nonzero overlap with the Y_Δ quantities of the Heisenberg XXZ spin chain [78] in the sense of (5). We emphasize that only a certain (infinite) set of local observables will have overlap with the Y_Δ's at a given value of Δ which can be deduced from their form [78,81].

Results. Using the hydrodynamical projection (5) and the known form of charges, we can calculate the asymptotic value of the autocorrelation function $C(t) = \frac{\langle O(t) O\rangle}{\langle O^2\rangle}$, for the observable $O_3 = \sum_j s_j^x s_{j+1}^x s_{j+2}^x$, and the infinite temperature ensemble $\langle\cdot\rangle = \frac{\mathrm{tr}\,(\cdot)}{\mathrm{tr}\,(\mathbb{1})}$, $C(t)_{t\to\infty} = \frac{1}{64}(\frac{27\sqrt{3}}{\pi} - 8)\cos(3ht)$ [81]. Time-dependent density matrix renormalization group (tDMRG) data presented in Fig. 1 show perfect agreement with the analytical calculations. As predicted by theory, we observe no oscillations for the transverse magnetization $O_1 = \sum_j s_j^x$.

In Fig. 1(b) we plot the time dependence of a local three-point correlation function $o_3 = s_j^+ s_{j+1}^+ s_{j+2}^+ + s_j^- s_{j+1}^- s_{j+2}^-$ starting from the ferromagnetic initial state maximally polarized in the x direction,

$$|\psi\rangle = 2^{-N/2}(|\uparrow\rangle + |\downarrow\rangle)^{\otimes n}, \tag{7}$$

as well as the one-point function $o_1 = s_j^+ + s_j^-$ (single - body observable). It shows the relaxation of the one-point function and persistent oscillations in the three-point function, illustrating the many-body nature of the time-crystalline behavior.

As shown in Fig. 1(c) we find numerically that the oscillations of relevant observables persist for a long time with an altered amplitude, following small to intermediate perturbations $\delta\Delta$ of the anisotropy. At $\Delta \neq -0.5$ the three-point operator $s_j^x s_{j+1}^x s_{j+2}^x$ no longer has an overlap with the Y_Δ [$\mathrm{tr}\,(Y_{\Delta\neq -1/2} s_j^x s_{j+1}^x s_{j+2}^x) = 0$] and thus does not pertain to any dynamical symmetry [see (3) and (5)]. Despite this, only at a relatively large value of perturbation $\delta\Delta = -0.2$ do we see significant damping of the amplitude on the accessible timescales. This is a hint of the potential stability of the oscillations for this observable. The oscillations are also stable with respect to the integrability-breaking term α on accessible timescales as shown in Fig. 1(d). We use the general results of [82,83] to find that the Y operators are conserved under stroboscopic time evolution in (2) up to at least second order in perturbation strength (see also [84]). This implies that the oscillations decay no faster than $\exp[-tJO(\gamma^2)]$, with respect to *any* integrability-breaking perturbation γ. We have tested this prediction by fitting the long-time parts of our numerical solutions to exponentially decaying oscillations. These fits indicate that the oscillations may be more, potentially even exponentially [83,85,86], stable.

Generic Δ. As remarked before the extensive dynamical symmetries $Y_\Delta(\phi)$ exist for a dense set of anisotropies $-1 < \Delta < 1$ which can be parametrized as $\Delta = \cos(\frac{\pi n}{m})$, with $n \in 2\mathbb{N}$, $m \in 2\mathbb{N}+1$. The infinite set of these operators (parametrized by ϕ) consist of an infinite sum of an infinite number of different local operators [81]. The observables with a finite overlap with these operators will oscillate at a frequency $\omega = hm$ (see Fig. 2 for some examples). The observables become "less" local with increasing m, the smallest one being supported on m consecutive sites.

Experimental realization. Due to the demonstrated stability, we expect that in current quantum cold-atom simulations of the XXZ spin chain, such as the ones in Bloch's group [87], the lattice depth can be sufficiently tuned to make the dynamics fast enough compared to integrablity-breaking effects to observe oscillations.

Measurement of local on-site equal-time many-body correlation functions is available through quantum gas microscopes for cold-atom systems [64]. For experiments an important

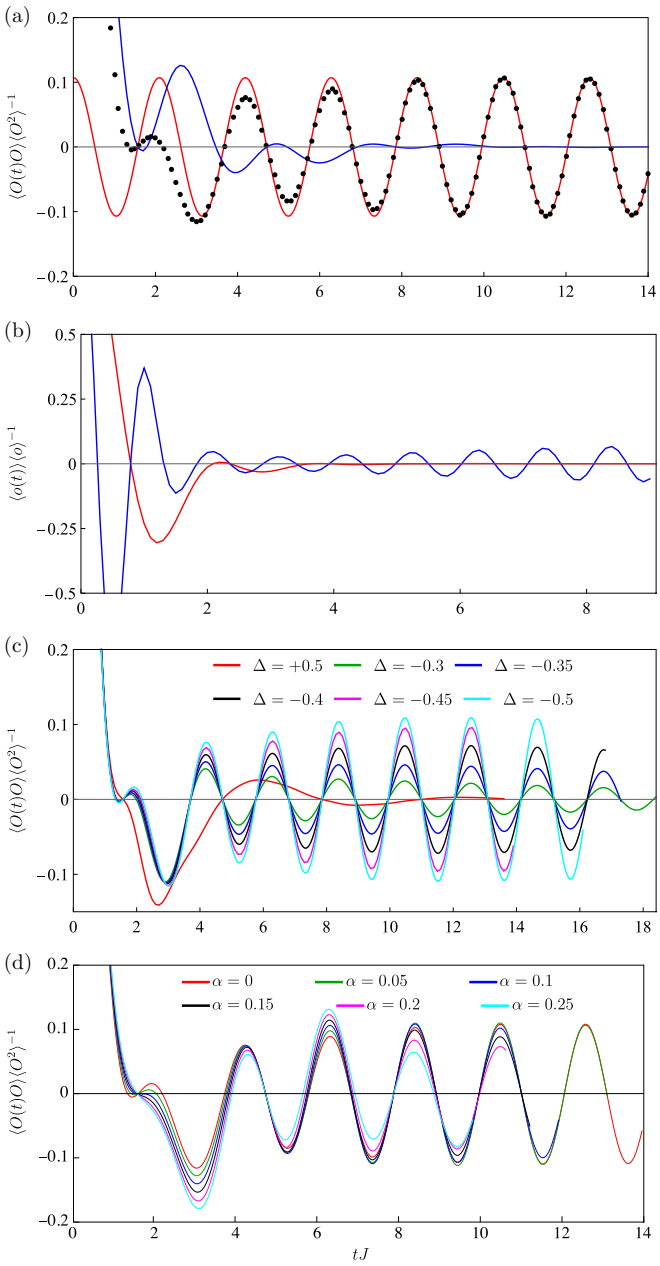

FIG. 1. (a) Nonstationary behavior of the numerically computed dynamical susceptibility of $O = O_3 = \sum_j s^x_j s^x_{j+1} s^x_{j+2}$ for $\Delta = -1/2$, $h = J$ (dots) and compare it to the exact asymptotic result (red curve). As expected, the single-body observable $O = O_1 = \sum_j s^x_j$ relaxes to stationarity (blue curve). In (b) we plot the data for the quench from the ferromagnetic state, showing nonstationarity of the three-point function $o = o_3 = s^+_j s^+_{j+1} s^+_{j+2} + s^-_j s^-_{j+1} s^-_{j+2}$ (blue) and relaxation of a single-point function $o = o_1 = s^+_j + s^-_j$ (red), for $\Delta = -0.5$, $h = 2J$. In both (a) and (b) we choose four different observables, though qualitatively the same two kinds of behavior (persistent oscillations or relaxation) would be seen for any choice of observables. In (c) we present the effects of perturbing Δ at $h = J$ on the dynamical susceptibility for $O = O_3$. In (d) we present the effects that the integrability breaking next-to-nearest neighbor interaction α has on the dynamical susceptibility for $O = O_3$ at $\Delta = -1/2$, $h = J$. All simulations were performed using DMRG with the system size $N = 100$.

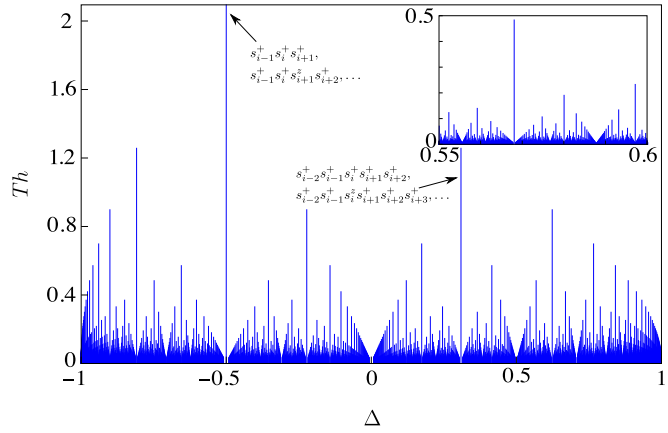

FIG. 2. The period $T = 2\pi/\omega$ of the persistent oscillations of dynamical symmetries depending on Δ. Inset shows a close-up illustrating the nowhere continuous (fractal) nature of the curve. Crucially, as the families of the Y_Δ operators are different at each Δ the observables that have overlap with them and thus persistently oscillate also change via (4) and (5). The arrows show examples of operators that have overlap with the Y_Δ at a given Δ and thus persistently oscillate.

discovery is that oscillations can be observed for a quench from the ferromagnetic initial state (7), which can be engineered [64]. Preparation and measurement of autocorrelation functions is more involved, but can be achieved through the use of ancilla qubits in Rydberg atoms [81]. Our results could potentially also have far-reaching applications in quantum metrology [88], due to the sensitivity of the dynamical symmetries Y_Δ to the anisotropy. In cold-atom simulations this can be directly related to the strength of the external magnetic field used to achieve Feshbach resonance of the spin-spin interaction [64]. In this regard, an important observation is that the amplitude seems to be less affected by integrability breaking, than by a change of the anisotropy.

Conclusion. Numerous questions remain open. While we have addressed the question of stability to perturbations from a practical perspective, stability to all orders remains an open problem, related to the long-standing question of the existence of the Kolmogorov-Arnold-Moser (KAM) theorem in systems with infinitely many degrees of freedom [89]. That being said, the crucial ingredient for oscillations is not integrability itself, but rather local or extensive quantities satisfying the relation (1). Importantly, we were able to identify similar quantities in topological models [90], conformal field theories [91], two-dimensional cold-atom systems [92], and approximately in mean-field models [60,61], and certain localized systems [93]. The glimmers of similar dynamical symmetries have also been identified in locally constrained models exhibiting quantum many-body scars, preventing the systems from relaxing for certain special initial conditions [67–69]. Indeed, a possible relation to integrability has also been drawn [94].

Some questions remain also from the standpoint of integrable systems. Here we only focused on the lowest frequency of oscillations at a given Δ, while in general the state ρ_{tGGE} should support a complete harmonic spectrum $\omega = khm$, for

$k \in \mathbb{N}$. Furthermore, due to the noncommutativity of the conserved quantities and dynamical symmetries, subtleties might arise in obtaining the correct form of ρ_{tGGE} [75]. The answers to these questions should be attainable by extending the thermodynamic Bethe ansatz description [95], to include additional quantities Y. The existence of these quantities implies that the standard GGE description is in fact incomplete even in the absence of persistent oscillations ($h = 0$). We found that microscopic nearest-neighbor correlation functions do not relax to stationarity, whereas the single-particle functions do. Thus, another exciting question is whether the dynamical symmetries, and the absence of many-body equilibration has a counterpart in the realm of classical physics. Otherwise, the phenomenon would constitute one of the first many-body quantum effects that can be observed in macroscopic systems on large space and time scales solely due to the extensive many-body nature of the Y operators.

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[1] J. Eisert, M. Friesdorf, and C. Gogolin, Nat. Phys. **11**, 124 (2015).
[2] S. Sotiriadis and G. Martelloni, J. Phys. A: Math. Theor. **49**, 095002 (2016).
[3] M. Collura, S. Sotiriadis, and P. Calabrese, Phys. Rev. Lett. **110**, 245301 (2013).
[4] S. Sotiriadis and P. Calabrese, J. Stat. Mech.: Theory Exp. (2014) P07024.
[5] P. Calabrese, F. H. L. Essler, and G. Mussardo, J. Stat. Mech.: Theory Exp. (2016) 064001.
[6] P. Calabrese, F. H. L. Essler, and M. Fagotti, Phys. Rev. Lett. **106**, 227203 (2011).
[7] S. R. Manmana, S. Wessel, R. M. Noack, and A. Muramatsu, Phys. Rev. Lett. **98**, 210405 (2007).
[8] D. Rossini, A. Silva, G. Mussardo, and G. E. Santoro, Phys. Rev. Lett. **102**, 127204 (2009).
[9] J.-S. Caux and R. M. Konik, Phys. Rev. Lett. **109**, 175301 (2012).
[10] M. Rigol, Phys. Rev. Lett. **112**, 170601 (2014).
[11] M. Mestyán, B. Bertini, L. Piroli, and P. Calabrese, J. Stat. Mech.: Theory Exp. (2017) 083103.
[12] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, Rev. Mod. Phys. **83**, 863 (2011).
[13] C. Gogolin and J. Eisert, Rep. Prog. Phys. **79**, 056001 (2016).
[14] M. Eckstein, M. Kollar, and P. Werner, Phys. Rev. Lett. **103**, 056403 (2009).
[15] V. Alba and P. Calabrese, Proc. Natl. Acad. Sci. USA **114**, 7947 (2017).
[16] P. Calabrese, F. H. L. Essler, and M. Fagotti, J. Stat. Mech.: Theory Exp. (2012) P07022.
[17] M. Rigol, V. Dunjko, and M. Olshanii, Nature (London) **452**, 854 (2008).
[18] M. Cheneau, P. Barmettler, D. Poletti, M. Endres, P. Schauß, T. Fukuhara, C. Gross, I. Bloch, C. Kollath, and S. Kuhr, Nature (London) **481**, 484 (2012).
[19] T. Langen, R. Geiger, M. Kuhnert, B. Rauer, and J. Schmiedmayer, Nat. Phys. **9**, 640 (2013).
[20] M. Gring, M. Kuhnert, T. Langen, T. Kitagawa, B. Rauer, M. Schreitl, I. Mazets, D. A. Smith, E. Demler, and J. Schmiedmayer, Science **337**, 1318 (2012).
[21] D. Chen, M. White, C. Borries, and B. DeMarco, Phys. Rev. Lett. **106**, 235304 (2011).
[22] F. Meinert, M. J. Mark, E. Kirilov, K. Lauber, P. Weinmann, A. J. Daley, and H.-C. Nägerl, Phys. Rev. Lett. **111**, 053003 (2013).
[23] T. Langen, R. Geiger, and J. Schmiedmayer, Annu. Rev. Condens. Matter Phys. **6**, 201 (2015).
[24] G. Clos, D. Porras, U. Warring, and T. Schaetz, Phys. Rev. Lett. **117**, 170401 (2016).
[25] B. Wouters, J. De Nardis, M. Brockmann, D. Fioretto, M. Rigol, and J.-S. Caux, Phys. Rev. Lett. **113**, 117202 (2014).
[26] B. Pozsgay, M. Mestyán, M. A. Werner, M. Kormos, G. Zaránd, and G. Takács, Phys. Rev. Lett. **113**, 117203 (2014).
[27] B. Bertini, M. Collura, J. De Nardis, and M. Fagotti, Phys. Rev. Lett. **117**, 207201 (2016).
[28] M. Mestyán, B. Pozsgay, G. Takács, and M. A. Werner, J. Stat. Mech.: Theory Exp. (2015) P04001.
[29] L. Piroli, E. Vernier, and P. Calabrese, Phys. Rev. B **94**, 054313 (2016).
[30] E. Ilievski, J. De Nardis, B. Wouters, J.-S. Caux, F. H. L. Essler, and T. Prosen, Phys. Rev. Lett. **115**, 157201 (2015).
[31] M. Collura, P. Calabrese, and F. H. L. Essler, Phys. Rev. B **92**, 125131 (2015).
[32] E. Ilievski, E. Quinn, and J.-S. Caux, Phys. Rev. B **95**, 115128 (2017).
[33] F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012).
[34] P. Bruno, Phys. Rev. Lett. **110**, 118901 (2013).
[35] P. Bruno, Phys. Rev. Lett. **111**, 029301 (2013).
[36] H. Watanabe and M. Oshikawa, Phys. Rev. Lett. **114**, 251603 (2015).
[37] D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).
[38] A. Lazarides, A. Das, and R. Moessner, Phys. Rev. Lett. **112**, 150401 (2014).

[39] V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

[40] V. Khemani, R. Moessner, and S. L. Sondhi, arXiv:1910.10745.

[41] S. Choi, J. Choi, R. Landig, G. Kucsko, H. Zhou, J. Isoya, F. Jelezko, S. Onoda, H. Sumiya, V. Khemani, C. von Keyserlingk, N. Y. Yao, E. Demler, and M. D. Lukin, Nature (London) **543**, 221 (2017).

[42] P. Bordia, H. Lüschen, U. Schneider, M. Knap, and I. Bloch, Nat. Phys. **13**, 460 (2017).

[43] K. Sacha and J. Zakrzewski, Rep. Prog. Phys. **81**, 016401 (2017).

[44] W. C. Yu, J. Tangpanitanon, A. W. Glaetzle, D. Jaksch, and D. G. Angelakis, Phys. Rev. A **99**, 033618 (2019).

[45] B. Zhu, J. Marino, N. Y. Yao, M. D. Lukin, and E. A. Demler, New J. Phys. **21**, 073028 (2019).

[46] F. M. Gambetta, F. Carollo, A. Lazarides, I. Lesanovsky, and J. P. Garrahan, Phys. Rev. E **100**, 060105(R) (2019).

[47] K. Giergiel, A. Dauphin, M. Lewenstein, J. Zakrzewski, and K. Sacha, New J. Phys. **21**, 052003 (2019).

[48] F. M. Gambetta, F. Carollo, M. Marcuzzi, J. P. Garrahan, and I. Lesanovsky, Phys. Rev. Lett. **122**, 015701 (2019).

[49] J. G. Cosme, J. Skulte, and L. Mathey, Phys. Rev. A **100**, 053615 (2019).

[50] B. Buča, J. Tindall, and D. Jaksch, Nat. Commun. **10**, 1730 (2019).

[51] F. Iemini, A. Russomanno, J. Keeling, M. Schirò, M. Dalmonte, and R. Fazio, Phys. Rev. Lett. **121**, 035301 (2018).

[52] N. Dogra, M. Landini, K. Kroeger, L. Hruby, T. Donner, and T. Esslinger, Science **366**, 1496 (2019).

[53] D. Barberena, R. J. Lewis-Swan, J. K. Thompson, and A. M. Rey, Phys. Rev. A **99**, 053411 (2019).

[54] C. Lledó, T. K. Mavrogordatos, and M. H. Szymańska, Phys. Rev. B **100**, 054303 (2019).

[55] B. Buča and D. Jaksch, Phys. Rev. Lett. **123**, 260401 (2019).

[56] J. Tindall, C. S. Muñoz, B. Buča, and D. Jaksch, New J. Phys. **22**, 013026 (2020).

[57] G. E. Volovik, JETP Lett. **98**, 491 (2013).

[58] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Adv. Phys. **65**, 239 (2016).

[59] L. Vidmar and M. Rigol, J. Stat. Mech.: Theory Exp. (2016) 064007.

[60] A. Syrwid, J. Zakrzewski, and K. Sacha, Phys. Rev. Lett. **119**, 250602 (2017).

[61] B. Sciolla and G. Biroli, Phys. Rev. Lett. **105**, 220401 (2010).

[62] S. J. Blundell and F. L. Pratt, J. Phys.: Condens. Matter **16**, R771 (2004).

[63] O. Breunig, M. Garst, E. Sela, B. Buldmann, P. Becker, L. Bohatý, R. Müller, and T. Lorenz, Phys. Rev. Lett. **111**, 187202 (2013).

[64] C. Gross and I. Bloch, Science **357**, 995 (2017).

[65] F. A. Zwanenburg, A. S. Dzurak, A. Morello, M. Y. Simmons, L. C. L. Hollenberg, G. Klimeck, S. Rogge, S. N. Coppersmith, and M. A. Eriksson, Rev. Mod. Phys. **85**, 961 (2013).

[66] V. K. Kozin and O. Kyriienko, Phys. Rev. Lett. **123**, 210602 (2019).

[67] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Nat. Phys. **14**, 745 (2018).

[68] S. Choi, C. J. Turner, H. Pichler, W. W. Ho, A. A. Michailidis, Z. Papić, M. Serbyn, M. D. Lukin, and D. A. Abanin, Phys. Rev. Lett. **122**, 220603 (2019).

[69] H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, M. Endres, M. Greiner, V. Vuletić, and M. D. Lukin, Nature (London) **551**, 579 (2017).

[70] J. M. Deutsch, Phys. Rev. A **43**, 2046 (1991).

[71] M. Srednicki, Phys. Rev. E **50**, 888 (1994).

[72] A. C. Cassidy, C. W. Clark, and M. Rigol, Phys. Rev. Lett. **106**, 140405 (2011).

[73] E. Ilievski, M. Medenjak, T. Prosen, and L. Zadnik, J. Stat. Mech.: Theory Exp. (2016) 064008.

[74] F. H. L. Essler and M. Fagotti, J. Stat. Mech.: Theory Exp. (2016) 064002.

[75] B. Doyon, Commun. Math. Phys. **351**, 155 (2017).

[76] M. Fagotti, J. Stat. Mech.: Theory Exp. (2014) P03016.

[77] T. Prosen, Phys. Rev. Lett. **106**, 217206 (2011).

[78] L. Zadnik, M. Medenjak, and T. Prosen, Nucl. Phys. B **902**, 339 (2016).

[79] E. Ilievski, M. Medenjak, and T. Prosen, Phys. Rev. Lett. **115**, 120601 (2015).

[80] T. Prosen and E. Ilievski, Phys. Rev. Lett. **111**, 057203 (2013).

[81] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.102.041117 for the discussion of the semi-cyclic periodic quantities, the time dependent dynamical susceptibilities, and on the proposal for the experimental implementation, which includes Refs. [64,73,78,80,97–107].

[82] M. Kollar, F. A. Wolf, and M. Eckstein, Phys. Rev. B **84**, 054304 (2011).

[83] T. Mori, T. N. Ikeda, E. Kaminishi, and M. Ueda, J. Phys. B: At., Mol. Opt. Phys. **51**, 112001 (2018).

[84] K. Mallayya, M. Rigol, and W. De Roeck, Phys. Rev. X **9**, 021027 (2019).

[85] D. Abanin, W. De Roeck, W. W. Ho, and F. Huveneers, Commun. Math. Phys. **354**, 809 (2017).

[86] G. P. Brandino, J.-S. Caux, and R. M. Konik, Phys. Rev. X **5**, 041043 (2015).

[87] T. Fukuhara, P. Schauß, M. Endres, S. Hild, M. Cheneau, I. Bloch, and C. Gross, Nature (London) **502**, 76 (2013).

[88] V. Giovannetti, S. Lloyd, and L. Maccone, Nat. Photonics **5**, 222 (2011).

[89] T. Kappeler and J. Pöschel, *KdV & KAM* (Springer Science & Business Media, Berlin, 2013), Vol. 45.

[90] X.-G. Wen, Phys. Rev. Lett. **90**, 016803 (2003).

[91] P. Calabrese and J. Cardy, J. Stat. Mech.: Theory Exp. (2016) 064003.

[92] L. P. Pitaevskii and A. Rosch, Phys. Rev. A **55**, R853 (1997).

[93] A. A. Michailidis, M. Žnidarič, M. Medvedyeva, D. A. Abanin, T. c. v. Prosen, and Z. Papić, Phys. Rev. B **97**, 104307 (2018).

[94] V. Khemani, C. R. Laumann, and A. Chandran, Phys. Rev. B **99**, 161101(R) (2019).

[95] E. Ilievski, E. Quinn, J. D. Nardis, and M. Brockmann, J. Stat. Mech.: Theory Exp. (2016) 063101.

[96] ITensor Library (version 2.0.11), http://itensor.org.
[97] C. Kassel, *Quantum Groups* (Springer-Verlag, New York, 1995).
[98] C. Korff, J. Phys. A: Math. Gen. **36**, 5229 (2003).
[99] T. Prosen, Nucl. Phys. B **886**, 1177 (2014).
[100] E. Ilievski and J. De Nardis, Phys. Rev. Lett. **119**, 020602 (2017).
[101] A. De Luca, M. Collura, and J. De Nardis, Phys. Rev. B **96**, 020403(R) (2017).
[102] M. Ljubotina, L. Zadnik, and T. Prosen, Phys. Rev. Lett. **122**, 150605 (2019).
[103] R. Zwanzig, *Nonequilibrium Statistical Mechanics* (Oxford University Press, New York, 2001).
[104] M. F. Parsons, A. Mazurenko, C. S. Chiu, G. Ji, D. Greif, and M. Greiner, Science **353**, 1253 (2016).
[105] A. W. Glaetzle, R. M. van Bijnen, P. Zoller, and W. Lechner, Nat. Commun. **8**, 15813 (2017).
[106] R. Dorner, S. R. Clark, L. Heaney, R. Fazio, J. Goold, and V. Vedral, Phys. Rev. Lett. **110**, 230601 (2013).
[107] J. Kreula, S. R. Clark, and D. Jaksch, Sci. Rep. **6**, 32940 (2016).