Fluctuation-dissipation relations for strongly correlated out-of-equilibrium circuits

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We consider strongly correlated quantum circuits where a dc drive is added on top of an initial out-of-equilibrium (OE) stationary state. Within a perturbative approach, we derive unifying OE fluctuation relations for high-frequency current noise, shown to be completely determined by zero-frequency noise and dc current. We apply them to the fractional quantum Hall effect at arbitrary incompressible filling factors, driven by OE sources, without knowledge of the underlying model. We show that such OE relations provide robust methods for an unambiguous determination of the fractional charge or of key interaction parameters entering in the exploration of anyonic statistics within an anyon collider.

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Out-of-equilibrium (OE) current noise is a valuable tool to explore strongly correlated mesoscopic conductors and circuits, especially in the high-frequency domain, where it unveils underlying dynamics and models [1–9,11–15]. It is a major tool in electron quantum optics [16] where it is essential for characterizing quantum states of electrons [17] or of emitted photons [18,19]. It also unveils fascinating collective phenomena within strongly correlated conductors as fractional charges [14,19–21] and statistics in the fractional quantum Hall effect (FQHE) [22] or charge splitting in the integer quantum Hall effect (IQHE) [6,23,24].

The effect of strong correlations in such systems calls for quantum laws of electronic transport independent of interactions and the microscopic model of the system. At equilibrium, the fluctuation-dissipation theorem (FDT) which uses the differential conductance at zero voltage is such a robust law even in the presence of a nonlinear current at high voltages [25].

In the OE regime, fluctuation-dissipation relations (FDRs) have been long studied but mostly at zero frequency [26,27]. A widely used perturbative OE FDR for high-frequency noise, expressed in terms of the dc current, has been derived by Rogovin and Scalapino [2] for independent particles. Assuming also an initial thermalization, we have extended this FDR to strongly correlated conductors and quantum circuits [7,11,14,15], permitting as well a departure from current inversion symmetry to which Ref. [2] is restricted.

Finally, in a general OE situation, including a multiterminal setup with time-dependent voltages, we have derived universal nonperturbative FDRs [28] which concern only asymmetries between the emission (positive frequency) and absorption (negative frequency) parts of the noise spectrum, expressed in terms of OE nonlinear admittance elements.

However, the recent developments of interferometry experiments involving OE stationary sources in the FQHE, such as the anyon collider shown in Fig. 1 used to probe the nontrivial statistics of anyons emitted by nontrivial sources [29], calls for an in-depth exploration of new FDRs for the full finite-frequency noise, valid in the absence of an initial thermal state. In this Rapid Communication, we derive perturbative OE FDRs for the full finite-frequency noise without assuming initial thermalization [6,30–32]. We show that, as long as the perturbative approach remains valid, high-frequency nonsymmetrized noise is not fully determined by the dc current, as in the initially thermalized case [2,14,15], but also by its zero-frequency counterpart. This relation illustrates the power of the OE perturbative approach, since it can be applied to a variety of situations independently of any underlying microscopic model.

This is especially relevant for the FQHE: The OE FDRs derived for the photoassisted noise [14] and for the high-frequency noise under a dc voltage [14,15] have already provided robust methods implemented in recent experiments to determine the fractional charge [19,21] for filling factors ν which are not simple fractions, though no experimental signature of the validity of the generic effective models was observed [33,34]. We illustrate furthermore the interest of the OE FDRs derived here for the anyon collider. We show that they give access to effective interaction-dependent parameters which are important for the exploration of anyonic statistics, proposed in Ref. [35] and recently implemented in Ref. [29].

Model. The underlying Hamiltonian of the OE perturbative approach in the stationary regime [14,36],

$$\mathcal{H}(t) = \mathcal{H}_0 + e^{-i\omega_J t}\hat{A} + e^{i\omega_J t}\hat{A}^\dagger, \tag{1}$$

involves an unspecified Hamiltonian $\mathcal{H}_0$ and perturbing operator $\hat{A}$. The Josephson-like frequency ω_J must enter only through $e^{i\omega_J t}$ in $\mathcal{H}(t)$ and is added on top of other dc drives already present in the system. For concreteness, we will focus here on charge transport, though the theory extends beyond that. We thus assume that there is a charge operator $\hat{Q}$, conserved by $\mathcal{H}_0$, translated by a model-dependent charge e^* when acting upon by $\hat{A}$. Then, Eq. (1) implies that

$$\partial_t \hat{Q} = \hat{I}(t) = \frac{ie^*}{\hbar}(e^{-i\omega_J t}\hat{A} - e^{i\omega_J t}\hat{A}^\dagger). \tag{2}$$

This is true when $\hat{A}$ contains the unitary operator $e^{i\hat{\varphi}}$ where the phase operator $\hat{\varphi}$ obeys $[\hat{\varphi}, \hat{Q}] = e^*$. In many situations, $\partial_t\hat{\varphi}$ obeys a Josephson-type relation with e^* instead of $2e$

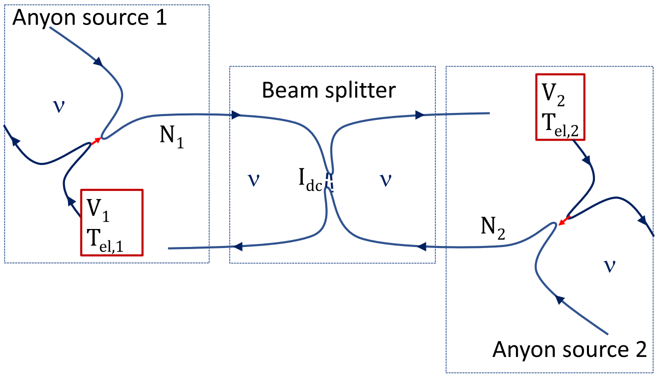

FIG. 1. An anyon collider setup in the FQHE. Two QPCs at possibly different temperatures $T_{\rm el,1}$ and $T_{\rm el,2}$, are subject to dc biases V_1 and V_2. They inject N_1 and N_2 anyons into the upper/down edges which collide at the central QPC. The finite-frequency noise of the backscattering current I_{dc} obeys the OE FDRs, independently of the (incompressible) fractional filling factor ν and the microscopic model.

[19,21,37] therefore,

$$\omega_J = \frac{e^*}{\hbar} V_{\rm dc}, \tag{3}$$

where $V_{\rm dc}$ is the voltage bias. Quantum averages, denoted by $\langle \cdots \rangle$, are taken over a stationary OE initial density operator ρ_0 ($[\rho_0, \mathcal{H}_0] = 0$), thereby corresponding to nonthermal occupation probabilities of many-body $\mathcal{H}_0$ eigenstates. These can, for example, arise from temperature and dc-voltage biases.

Let us give some examples. In tunneling junctions between two similar or different (hybrid) conductors, such as NIN or SIN junctions [27], $\hat{A}$ and $\hat{I}(t)$ respectively correspond to the tunneling and electrical current operators. In Josephson junctions, $\hat{I}(t)$ is either the quasiparticle ($e^* = e$) or the pair current ($e^* = 2e$). But the form in Eq. (1) goes beyond the transfer Hamiltonian approach, as $\mathcal{H}_0$ is not split into right and left terms [38], so that it can incorporate all relevant screened Coulomb interactions. One can also include in $\mathcal{H}_0$ and $\hat{A}$ strong coupling to a linear or a nonlinear electromagnetic environment.

In the IQHE or the FQHE at arbitrary incompressible filling factors ν, $\hat{A}$ corresponds to a weak spatially extended backscattering of electrons or quasiparticles with a fractional charge e^* through a QPC, acting as a beam splitter, and $\hat{I}(t)$ is the backscattering current. The unperturbed Hamiltonian $\mathcal{H}_0$ may include edge reconstruction or inhomogeneous Coulomb interactions [23], or even extended tunneling processes between counterpropagating edges. As those emanate from different contacts, such processes may not be sufficient to ensure their equilibration [33], a situation one could address as well.

One may also consider OE quasiparticle sources, such as quantum dots acting as energy filters or biased QPCs. As will be illustrated later in the anyon collider depicted in Fig. 1, the Josephson-type relation in Eq. (3) may break down, motivating us to keep ω_J as a free parameter.

Main OE relations. Letting $\delta\hat{I}_{\mathcal{H}}(t) = \hat{I}_{\mathcal{H}}(t) - I_{\rm dc}(\omega_{\rm J})$, where the subscript $\mathcal{H}$ refers to the Heisenberg representation with respect to $\mathcal{H}(t)$ in Eq. (1), we focus on the current noise,

$$S(\omega_J; t) = \langle \delta\hat{I}_{\mathcal{H}}(0)\delta\hat{I}_{\mathcal{H}}(t)\rangle. \tag{4}$$

To express S at second order in $\hat{A}$, we replace $\delta\hat{I}_{\mathcal{H}}(t)$ by $\hat{I}_{\mathcal{H}_0}(t)$, or, in Eq. (2), $\hat{A}_{\mathcal{H}}(t)$ by $\hat{A}_{\mathcal{H}_0}(t) = e^{i\mathcal{H}_0 t}\hat{A}\, e^{-i\mathcal{H}_0 t}$. We obtain these two building blocks,

$$\hbar^2 X_{\rightarrow}(t) = \big\langle \hat{A}^{\dagger}_{\mathcal{H}_0}(t)\hat{A}_{\mathcal{H}_0}(0)\big\rangle, \tag{5a}$$

$$\hbar^2 X_{\leftarrow}(t) = \big\langle \hat{A}_{\mathcal{H}_0}(0)\hat{A}^{\dagger}_{\mathcal{H}_0}(t)\big\rangle. \tag{5b}$$

Being evaluated in the OE regime characterized by $\mathcal{H}_0$ and $\hat{\rho}_0$, these are OE correlators which do not satisfy any kind of detailed balance equations. They determine the current noise in Eq. (4) and its Fourier transform at ω,

$$S(\omega_J; t)/e^{*2} \simeq e^{-i\omega_J t} X_{\rightarrow}(-t) + e^{i\omega_J t} X_{\rightarrow}(t), \tag{6a}$$

$$S(\omega_J; \omega)/e^{*2} \simeq X_{\rightarrow}(\omega_J - \omega) + X_{\leftarrow}(\omega_J + \omega)\,. \tag{6b}$$

In particular, the zero-frequency noise reads

$$S(\omega_J; \omega = 0)/e^{*2} \simeq X_{\rightarrow}(\omega_J) + X_{\leftarrow}(\omega_J), \tag{7}$$

and the dc average current

$$I_{\rm dc}(\omega_{\rm J}) = \langle \hat{I}_{\mathcal{H}}(t)\rangle \simeq e^*[X_{\rightarrow}(\omega_{\rm J}) - X_{\leftarrow}(\omega_{\rm J})] \tag{8}$$

can be interpreted as the difference of two transfer rates $X_{\rightarrow}, X_{\leftarrow}$ in opposite directions [14].

Then, at a finite frequency ω, the rescaled noise in Eq. (6b) is a sum of these transfer rates evaluated at two effective potential drops in two opposite directions $\pm\omega_J - \omega$. A transfer of a charge e^* in each direction is associated with the emission (absorption) of a photon if $\omega > 0$ ($\omega < 0$) by the correlated many-body eigenstates, thus the effective potential $\pm\omega_J - \omega$ decreases (increases) with respect to $\pm\omega_J$.

Comparing Eq. (6b) to Eqs. (7) and (8), we derive the central result of this Rapid Communication, an OE FDR expressing the OE current noise at finite frequency in terms of OE current average and noise at zero frequency [39],

$$\begin{aligned} 2S(\omega_J; \omega) &= S(\omega_J + \omega; 0) + S(\omega_J - \omega; 0) \\ &\quad - e^* I_{\rm dc}(\omega_J + \omega) + e^* I_{\rm dc}(\omega_J - \omega). \end{aligned} \tag{9}$$

Note that the first and second lines on the right-hand side yield the symmetric and antisymmetric parts of the noise $2S^{\pm}(\omega_J; \omega) = S(\omega_J, \omega) \pm S(\omega_J, -\omega)$. The high-frequency behavior of S^+ is indeed totally determined by its dependence on the dc bias at zero frequency,

$$2S^+(\omega_J; \omega) = S^+(\omega_J + \omega; 0) + S^+(\omega_J - \omega; 0). \tag{10}$$

Moreover, using the exact relation [7,28] $S^-(\omega_J; \omega) = -2\hbar\omega\, {\rm Re}[Y(\omega_J, \omega)]$ connecting the antisymmetric part of the noise to the OE admittance $Y(\omega_J, \omega)$ [40], Eq. (9) enables us to extend the validity of the relation

$$2\hbar\omega\, {\rm Re}[Y(\omega_J, \omega)] = e^*[I_{\rm dc}(\omega_J - \omega) - I_{\rm dc}(\omega_J + \omega)] \tag{11}$$

beyond the hypothesis of initial thermalization adopted in Refs. [14,15,36]. Since the Kramers-Kronig relation also yields ${\rm Im}[Y(\omega_J; \omega)]$ in terms of $I_{\rm dc}$, the admittance $Y(\omega_J, \omega)$ is totally determined by the dc-current/voltage characteristic.

The heart of our perturbative approach, underlying the previous relations, is the fact that OE current and noise can be expressed only through the two OE correlators $X_{\rightarrow}$ and $X_{\leftarrow}$ in Eqs. (5a) and (5b). These are generally independent, as we do not impose any of two hypotheses generically adopted: an odd dc current and thermalization. We can formulate separately these two restrictions, not adopted here, through two links between $X_{\rightarrow}$ and $X_{\leftarrow}$. The first one extends the particle-hole symmetry to strongly correlated systems [14],

$$X_{\rightarrow}(\omega_J) = X_{\leftarrow}(-\omega_J). \quad (12)$$

Thus the transfer rate in one direction is obtained by reversing the sign of the dc drive, so that the dc current in Eq. (8) becomes odd, $I_{dc}(\omega_J) = -I_{dc}(-\omega_J)$, and the noise in Eq. (6b) is even/ω_J, $S(\omega_J;\omega) = S(-\omega_J;\omega)$.

The second link expresses thermalization at an electronic temperature $T_{el} = 1/k_B\beta$, $X_{\rightarrow}(\omega) = e^{\beta\omega}X_{\leftarrow}(\omega)$. In that case, the OE FDR (9) reduces to the previously obtained [14,15] FDR,

$$S(\omega_J;\omega)/e^{*2} = [1 + N(\omega_J + \omega)]I_{dc}(\omega_J + \omega) + N(\omega_J - \omega)I_{dc}(\omega_J - \omega), \quad (13)$$

in which $N(\omega) = (e^{\beta\omega} - 1)^{-1}$, thereby repositioning a long stream of model-dependent derivations of this relation [3,6–8,11,18] into a unified framework. Note that Rogovin and Scalapino's FDR [2] is recovered from (13) by considering the symmetric noise, $S^+(\omega_J;\omega) = e^* \sum_{\pm} \coth[\beta(\omega_J \pm \omega)/2]I_{dc}(\omega_J \pm \omega)$, which we have extended beyond its original context and without assuming an odd current Eq. (12) [41]. Indeed, for an initial thermal state, the dc current in Eq. (8), though not odd, has the sign of the dc bias [14],

$$\omega_J I_{dc}(\omega_J) \geqslant 0, \quad (14)$$

but in the general OE case, the current may have the opposite sign of ω_J [27].

Also, two important generic features, obtained at zero and finite frequencies, follow from Eq. (13) at a very low temperature: the Poissonian statistics and the existence of a threshold for the emitted noise at $\omega > \omega_J$ [28]. We now exploit Eq. (9) to show their common origin and their breakdown for initial OE states. For this, we use the properties of $X_{\rightarrow,\leftarrow}(\omega_J)$ in Eqs. (5a) and (5b) derived from their spectral decomposition [14]. Indeed, $X_{\rightarrow,\leftarrow}(\omega_J) \geqslant 0$, so that the zero-frequency noise in Eq. (7), compared to Eq. (8), obeys

$$S(\omega_J;0) \geqslant e^*|I_{dc}(\omega_J)|. \quad (15)$$

This leads to a lower bound on the high-frequency noise in Eq. (9) (Θ is the Heaviside function),

$$2S(\omega_J;\omega) \geqslant \sum_{\pm} e^*\Theta(\mp I_{dc}(\omega_J \pm \omega))|I_{dc}(\omega_J \pm \omega)|. \quad (16)$$

Let us consider first the case when the system is initially in the ground many-body eigenstate of $\mathcal{H}_0$. Then, by spectral decomposition, we can show that only one transfer rate survives [$X_{\rightarrow}(\omega_J < 0) = X_{\leftarrow}(\omega_J > 0) = 0$], so that Eq. (14) holds, and Eqs. (6b) and (8) imply that the inequality (15) reduces to an equality: Zero-frequency noise is Poissonian. As a consequence, (16) is also saturated, from which one infers the threshold for the emission noise at $\omega_J > 0$, $S(\omega_J;\omega > \omega_J) = 0$. Therefore, single charge-transfer processes are Poissonian and impose energy conservation underlying the threshold.

These two features are violated when considering OE initial states: The inequality in Eq. (15) is strict, leading to a super-Poissonian zero-frequency noise. So is the inequality in Eq. (16), smoothing out the threshold at ω_J, due to the nonvanishing emission noise above ω_J, $S(\omega_J;\omega > \omega_J) > 0$. These purely OE effects persist even at vanishing temperatures. In order to distinguish them from thermal fluctuations, which also lead to strict inequalities in Eqs. (15) and (16) [see Eqs. (14) and (13) with a finite T_{el}], let us deduce the OE noise at $\omega_J = 0$ from Eq. (9). For simplicity, we assume that current inversion symmetry, thus Eq. (12), holds, so that we get

$$S(\omega_J = 0;\omega) = S(\omega_J = \omega;\omega = 0) - e^*I_{dc}(\omega_J = \omega). \quad (17)$$

This shows that a finite emission noise $S(\omega_J = 0;\omega > 0)$ quantifies deviations both from the Poissonian regime and from initial thermalization, for which it would vanish.

Applications. The FDRs are alternative laws in the OE regime to the equilibrium FDT, thus providing similarly a robust test of analytical, numerical, or experimental results for OE noise. One can, inversely, test the validity of the underlying hypotheses of our perturbative approach by checking Eqs. (9) and (10) [5], whereas the signature of a departure from initial thermalization [42] would be a violation of Eq. (13). In strongly correlated conductors with OE initial many-body states, a key issue is to determine ω_J in terms of the experimentally controlled parameters, such as dc voltages and temperatures when $e^* \neq e$ or when the Josephson-type relation Eq. (3) breaks down.

This can be achieved either by measuring the admittance, using Eq. (11), or by measuring the noise both at finite and zero frequency, using Eqs. (9) and (10). One can infer ω_J from the coincidence of the functions of ω on both sides of these OE FDRs.

First, these methods could be especially relevant for thermoelectricity [31]. The determination of ω_J provides the voltage drop across a strongly correlated junction in the presence of a temperature gradient ΔT. In particular, by imposing $I_{dc}(\omega_J) = 0$, it offers a method based on current noise measurement to infer the Seebeck coefficient from $\omega_J/\Delta T$. Note that at zero-bias voltage, the temperature gradient ΔT generates a thermoelectric current $I_{dc}(\omega_J = 0) \neq 0$ [14].

Second, the determination of ω_J is an especially acute question in the FQHE context, which goes beyond that of the fractional charge e^* using Eq. (3) when valid, as in recent experiments [19,21]. The important point is that, at a given incompressible filling factor ν, for example, 2/3, the theoretical description by effective models cannot favor one among multiple competing candidates, which may even predict different values of e^* [34]. As of now, because of Coulomb-induced nonuniversal effects such as edge reconstruction, there is no clear agreement between experiments [19,21] and effective models, predicting power laws [33]. In this context, the OE FDR can help us sort out, among the various models, the most suitable one for the experimental data.

Let us illustrate this point in an anyon collider, to show how the determination of ω_J can help us to pinpoint the best candidate model. As depicted on Fig. 1, two dc-biased QPCs inject anyons with a fractional charge e^*, characterized by number operators $\hat{N}_{1,2}$ and averages $N_{1,2}$, which collide on the central QPC. Since equilibrium reservoirs are replaced by OE sources, the backscattering noise obeys the OE FDRs given by Eqs. (9) and (10), but not that given by Eq. (13)

[41]. Let us adopt for the edge states, as in Ref. [35], an effective model characterized by two free parameters λ, δ which need to be known to fix the model [34]. While λ refers to an effective dimensionless charge, δ monitors the statistical phase of quasiparticles. In the case $\nu = 1/(2n+1)$, one has $\lambda = \delta = \nu$, but λ, δ may be renormalized by Coulomb interactions and edge reconstruction, whose role can be evaluated by determining experimentally λ, δ. Importantly, λ, δ intervene directly in the cross correlations of the anyon collider [29,35], thus affecting the interpretation of the latter in terms of anyonic statistics.

In Ref. [35], λ renormalizes $\hat{N}_1, \hat{N}_2$ in the OE part of $\hat{A}$, $\hat{A} \to e^{2i\pi\lambda(\hat{N}_1-\hat{N}_2)}\hat{A}$. This derives from the equation of motion method for bosonic fields with boundary conditions fixed by $\hat{N}_1, \hat{N}_2$ [23]. Indeed, we can show that λ describes plasmonic propagation between the injection point and the central QPC, thus we can relate it to the dc conductance by using the scattering approach for plasmons [23]. The OE bosonization extends the latter by taking into account higher cumulants of $\hat{N}_1, \hat{N}_2$ [32]. If we assume weak transmission coefficients of the two OE source QPCs, $\hat{N}_1, \hat{N}_2$ are Poissonian, so that their cumulants are proportional to their small average values N_1, N_2. We notice that a perturbative analysis with respect to N_1, N_2 fails when both temperatures of the sources are low and their voltages become close, $V_1 \simeq V_2$; the origin of this failure, to which Ref. [43] was faced, has been explained in Ref. [10]. It is circumvented by incorporating higher cumulants of the Poissonian $\hat{N}_1, \hat{N}_2$ [32], which are proportional to the injected average currents $I_{1,2} = dN_{1,2}/dt$, leading to an effective dc drive given by [35]

$$\omega_J = \frac{2\pi}{e^*}\sin(2\pi\lambda)I_-, \tag{18}$$

with $I_- = I_1 - I_2$. Due to the strongly correlated Hall liquid in the sources, I_1, I_2 have a nonlinear behavior on V_1, V_2, and so does ω_J, which violates the Josephson-type relation in Eq. (3) (with $V_{\rm dc} = V_1 - V_2$). By using the OE FDR to determine ω_J, and assuming e^* is already inferred from intrinsic noise of the QPCs, one can determine $\sin(2\pi\lambda)$, thus λ, from Eq. (18), as I_1, I_2, thus I_-, can be measured directly in the outgoing edges [29]. One can infer the second parameter δ from the model-dependent expressions of the dc current and zero-frequency noise in Ref. [35],

$$I_{\rm dc}(\omega_J) = C'\sin(\pi\delta){\rm Im}(\omega_+ + i\omega_J)^{2\delta-1}, \tag{19a}$$

$$S(\omega_J;\omega=0) = e^*C'\cos(\pi\delta){\rm Re}(\omega_+ + i\omega_J)^{2\delta-1}. \tag{19b}$$

Here, C' is a prefactor, ω_J given by Eq. (18), and $\omega_+ = 4\pi\sin^2(\pi\lambda)I_+/e^*$, with $I_+ = I_1 + I_2$. Though δ controls the power law, this is not an easy way to extract it, so we propose an alternative way. We notice first that, compared to equilibrium reservoirs, the validity domain of perturbation is extended: For high enough ω_+, one can lower ω_J down to 0 by injecting equal currents $I_1 = I_2$ through tuning $V_1 \simeq V_2$. This is precisely the regime where anyonic statistics is best revealed [35]. Then using Eqs. (19a) and (19b), one has

$$S(\omega_J=0;\omega=0) = e^*\frac{\cot(\pi\delta)}{1-2\delta}\cot(\pi\lambda)I_+\left(\frac{\partial I_{\rm dc}}{\partial I_-}\right)_{I_-=0},$$

proportional to the total injected current I_+ and the derivative of I_{dc} at $\omega_J = 0$ (depending on I_+). The atypical "Fano factor" $\cot(\pi\delta)\cot(\pi\lambda)/(1-2\delta)$ then provides δ once λ is determined. Now we can express explicitly the high-frequency backscattering noise in Eq. (10), by injecting the dc expressions in Eqs. (19a) and (19b). In particular, at $I_- = 0$, as current inversion symmetry now holds, we can use Eq. (17) with a fixed ω_+, $S(\omega_J = 0;\omega) = -C'\,{\rm Im}(-\omega + i\omega_+)^{2\delta-1}$.

Conclusion. In this Rapid Communication, we have derived perturbative FDRs showing that high-frequency noise is completely determined by zero-frequency transport. Due to OE initial states, zero-frequency noise is super-Poissonian, and washes out the threshold for the emitted spectrum above the dc drive. The OE FDRs offer experimental tests of their underlying hypothesis [5], in particular the breakdown of initial thermalization. They provide a noise measurement method of the Seebeck coefficient in a strongly correlated junction. In the FQHE, the OE FDRs permit one to probe the fractional charges without relying on the microscopic model [14,19,21] or on initial thermal equilibrium. The latter breaks down in the anyon collider used to prove anyonic statistics [29,35]. The high-frequency backscattering noise does not obey the previously derived FDRs [14,15] but the OE FDR obtained herein, which offers a protocol to extract a nonuniversal parameter that depends on the structure of the edge channels and enters anyonic statistics. This may prove useful in forthcoming investigations of anyonic statistics through finite-frequency correlations. Future perspectives include using the OE FDRs for shot-noise thermometry [31,44], as well as for thermoelectricity in the anyon collider. Beyond current noise, they can be applied to the voltage noise across a phase-slip Josephson junction [45] as well as to the spin current noise in spin Hall insulators [46,47].

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[40] $Y(\omega_J,\omega)$ is the response of average current to a small ac modulation $\delta V(t)=v_{\rm ac}e^{i\omega t}$ superimposed on $V_{\rm dc}$. In particular, $Y(\omega_J,\omega=0)=G_{\rm dc}(\omega_J)$.

[41] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.102.041113 for the minimal conditions of the perturbative approach, for the case where inversion symmetry or initial thermalization holds, and for the microscopic description of the anyon collider.

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