Interaction between moving Abrikosov vortices in type-II superconductors

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(Received 7 April 2020; revised 5 June 2020; accepted 16 June 2020; published 6 July 2020)

The self-energy of a moving vortex is shown to decrease with increasing velocity. The interaction energy of two parallel slowly moving vortices differs from the static case by a small term $\propto v^2$; the "slow" motion is defined as having the velocity $v < v_c = c^2/4\pi\sigma\lambda$, where $\sigma(T)$ is the conductivity of normal excitations and $\lambda(T)$ is London penetration depth. For higher velocities, $v > v_c(T)$, the interaction energy of two vortices situated along the velocity direction is enhanced and in the perpendicular direction is suppressed compared to the static case.

DOI: 10.1103/PhysRevB.102.024506

I. INTRODUCTION

Recent experiments have tracked Abrikosov vortices moving with extremely high velocities well exceeding the speed of sound [1,2]. The time-dependent Ginzburg-Landau equations (GL) were the major tool used to model vortex motion. Although this approach, strictly speaking, is applicable only for gapless systems near the critical temperature [3], it reproduces qualitatively major features of the fast vortex motion.

A much simpler linear London approach had been successfully employed through the years to describe static or nearly static vortex systems. The London equations express the basic Meissner effect and can be used at any temperature for problems where vortex cores are irrelevant. The magnetic structure of moving vortices was commonly considered the same as that of a static vortex displaced as a whole.

It has been shown recently, however, that this is not so for moving vortexlike topological defects in, e.g., neutral superfluids or liquid crystals [4]. Also, this is not the case in superconductors within the time dependent London theory (TDL) which takes into account normal currents, a necessary consequence of moving magnetic structure of a vortex [5]. In this paper we show that the line energy of a moving vortex decreases with increasing velocity. Moreover, the interaction of two vortices moving with the same velocity becomes anisotropic so that it is enhanced when the vector $\boldsymbol{R}$ connecting vortices is parallel to the velocity $\boldsymbol{v}$ and suppressed if $\boldsymbol{R} \perp \boldsymbol{v}$.

A. Outline of time dependent London approach

In time dependent situations, the current consists, in general, of normal and superconducting parts:

$$\boldsymbol{J} = \sigma\boldsymbol{E} - \frac{2e^2|\Psi|^2}{mc}\left(\boldsymbol{A} + \frac{\phi_0}{2\pi}\nabla\chi\right), \qquad (1)$$

where $\boldsymbol{E}$ is the electric field and Ψ is the order parameter.

The conductivity σ approaches the normal state value σ_n when the temperature T approaches T_c in fully gapped s-wave superconductors; it is believed to vanish fast with decreasing temperature along with the density of normal excitations. This is, however, not the case for strong pair breaking when superconductivity becomes gapless while the density of states approaches the normal state value at all temperatures.

Within the London approach $|\Psi|$ is a constant Ψ_0 and Eq. (1) reads:

$$\frac{4\pi}{c}\boldsymbol{J} = \frac{4\pi\sigma}{c}\boldsymbol{E} - \frac{1}{\lambda^2}\left(\boldsymbol{A} + \frac{\phi_0}{2\pi}\nabla\chi\right), \qquad (2)$$

where $\lambda^2 = mc^2/8\pi e^2|\Psi_0|^2$ is the London penetration depth. Acting on this by curl one obtains:

$$-\nabla^2\boldsymbol{H} + \frac{1}{\lambda^2}\boldsymbol{H} + \frac{4\pi\sigma}{c^2}\frac{\partial\boldsymbol{H}}{\partial t} = \frac{\phi_0}{\lambda^2}\boldsymbol{z}\sum_\nu\delta(\boldsymbol{r}-\boldsymbol{r}_\nu), \qquad (3)$$

where $\boldsymbol{r}_\nu(t)$ is the position of the νth vortex, and $\boldsymbol{z}$ is the direction of vortices that coincides with that of $\boldsymbol{H}$ for isotropic infinite type-II superconductors. Equation (3) can be considered as a general form of the time dependent London equation. This form differs from that provided by F. London where contribution of normal quasiparticles to the current was not included [6].

As with the static London approach, the time dependent version (3) has the shortcoming of being valid only outside vortex cores. As such it may produce useful results for materials with large GL parameter κ in fields away from the upper critical field H_{c2}. On the other hand, Eq. (3) is a useful, albeit approximate, tool for low temperatures where GL theory does not work and the microscopic theory is forbiddingly complex.

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B. Moving vortex

For a straight vortex along z moving with a constant velocity $\boldsymbol{v}$ in the xy plane Eq. (3) reads:

$$-\lambda^2\nabla^2 H + H + \tau\,\frac{\partial H}{\partial t} = \phi_0\delta(\boldsymbol{r}-\boldsymbol{v}t)\,, \tag{4}$$

where H is the z component of the magnetic field and

$$\tau = 4\pi\sigma\lambda^2/c^2 \tag{5}$$

is the "current relaxation time," the term used in literature on time-dependent GL models. Clearly, the field distribution described by Eq. (4) differs from the solution which would have existed in the absence of relaxation term for $\tau = 0$:

$$H_0(\boldsymbol{r},t) = \frac{\phi_0}{2\pi\lambda^2}\,K_0\left(\frac{|\boldsymbol{r}-\boldsymbol{v}t|}{\lambda}\right), \tag{6}$$

where K_0 is the modified Bessel function.

Equation (4) can be solved by first finding the time dependence of the Fourier transform $H_{\boldsymbol{k}}$, as it is done for the diffusion equation [7]:

$$\tau\,\partial_t H_{\boldsymbol{k}} + (1+\lambda^2k^2)H_{\boldsymbol{k}} = \phi_0\,e^{-i\boldsymbol{k}\boldsymbol{v}t} \tag{7}$$

which yields

$$H_{\boldsymbol{k}} = \frac{\phi_0\,e^{-i\boldsymbol{k}\boldsymbol{v}t}}{1+\lambda^2k^2 - i\boldsymbol{k}\boldsymbol{v}\tau}\,. \tag{8}$$

To find the field distribution in real space for the stationary case of a constant velocity one may consider $t = 0$. This was done in Ref. [5] where it was shown that the moving vortex looses the cylindrical symmetry of vortex at rest, in particular, this distribution is no longer symmetric relative to $x \to -x$ with x being the velocity direction.

Physically, the distortion of the vortex field is due to contribution of the out-of-core normal excitations to vortex currents. At small velocities, the distortion can be disregarded. At low temperatures, the quasiparticles are nearly absent (for the s-wave symmetry) and $\sigma \approx 0$, whereas λ is finite, therefore the vortex field distortion is weak. Hence, the distortion may have an effect at high Ts where the conductivity is close to that of the normal phase. Gapless superconductors are an exception to this rule, since the normal excitations density of states is close to the normal even at low Ts.

II. SELF-ENERGY OF MOVING VORTEX

Given the field distribution of a moving vortex, one readily evaluates the London line energy of a vortex [8,9]:

$$F_1 = \int d^2\boldsymbol{r}[H^2 + \lambda^2(\mathrm{curl}\boldsymbol{H})^2]/8\pi$$
$$= \int \frac{d^2\boldsymbol{k}}{32\pi^3}[|H_{\boldsymbol{k}}|^2 + \lambda^2|\boldsymbol{k}\times\boldsymbol{H}_{\boldsymbol{k}}|^2]\,, \tag{9}$$

where the Fourier transform $H_{\boldsymbol{k}}$ is given in Eq. (8) for $t = 0$. Further, we have $|\boldsymbol{k}\times H_{\boldsymbol{k}}|^2 = k^2|H_{\boldsymbol{k}}|^2$, so that

$$\frac{32\pi^3\lambda^2}{\phi_0^2}F_1 = \int \frac{d^2\boldsymbol{q}\,(1+q^2)}{|1+q^2-i\boldsymbol{q}\boldsymbol{u}|^2}$$
$$= \int \frac{d^2\boldsymbol{q}\,(1+q^2)}{(1+q^2)^2 + q_x^2u^2}\,. \tag{10}$$

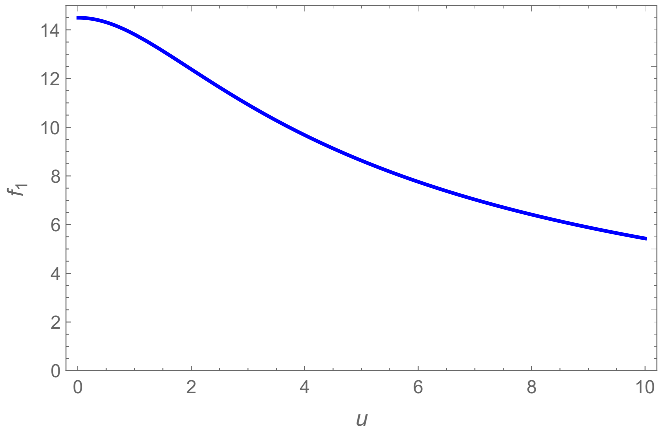

FIG. 1. The line energy f_1 normalized on $\phi_0^2/32\pi^2\lambda^2$ as a function of reduced velocity $u = v/v_c$ for $\kappa = \lambda/\xi = 10$.

Here, the dimensionless $\boldsymbol{q} = \lambda\boldsymbol{k}$ is introduced and the normalized velocity $\boldsymbol{u} = \boldsymbol{v}/v_c$, $v_c = c^2/4\pi\sigma\lambda$ (this v_c is by a factor of 2 smaller than v_c used in Ref. [5]). After integration over the angle φ ($q_x = q\cos\varphi$) one obtains the last integral in the form

$$\int_0^\kappa \frac{2\pi\, q dq}{\sqrt{(1+q^2)^2+q^2u^2}}$$
$$= \frac{\pi}{2}\ln\frac{u^4 + 2u^2(1+\kappa^2+\sqrt{(1+\kappa^2)^2+\kappa^2u^2})}{(4+u^2)(2+u^2+2\kappa^2-2\sqrt{(1+\kappa^2)^2+\kappa^2u^2})}, \tag{11}$$

where the logarithmically divergent integral is truncated at $q = \lambda/\xi = \kappa$. The reduced line energy $f_1 = F_1/(\phi_0^2/32\pi^2\lambda^2)$ for $\kappa = 10$ is shown in Fig. 1.

It is worth noting that large values of the reduced velocity $u = v/v_c$ not necessarily imply a large actual velocity because v_c depends on temperature, in particular, $v_c \to 0$ when $T \to T_c$. For a "fast" motion $u^2 \gg 1$, Eq. (11) gives

$$F_1 \approx \frac{\phi_0^2}{16\pi^2\lambda^2}\,\frac{\kappa}{u}\,. \tag{12}$$

Near $u = 0$ we have

$$f_1 \approx 2\pi\ln\kappa - \pi u^2/4\,, \tag{13}$$

where the first term gives the standard line energy of a vortex at rest.

III. INTERVORTEX INTERACTION

For two parallel vortices moving with the same velocity, one at the origin at $t = 0$ and the other at $\boldsymbol{R} = (x, y)$, the field is given by the Fourier transform:

$$H_{\boldsymbol{q}} = \frac{\phi_0(1+e^{-i\boldsymbol{q}\boldsymbol{R}})}{1+q^2-iq_xu}\,. \tag{14}$$

Using Eqs. (9), one obtains the total energy F of two vortices and the interaction energy $F_{\mathrm{int}} = F - 2F_1$ where F_1 is the line energy of a single vortex given in Eq. (10):

$$\frac{16\pi^3\lambda^2}{\phi_0^2}F_{\mathrm{int}}(\boldsymbol{R}) = \int \frac{d^2\boldsymbol{q}\,(1+q^2)\cos\boldsymbol{q}\boldsymbol{R}}{(1+q^2)^2+q_x^2u^2}\,. \tag{15}$$

For $u = 0$ this yields the static interaction energy [8]:

$$F_{\rm int} = \frac{\phi_0^2}{16\pi^3\lambda^2}\int \frac{d^2\boldsymbol{q}\,\cos \boldsymbol{qR}}{1+q^2} = \frac{\phi_0^2}{8\pi^2\lambda^2}K_0\left(\frac{R}{\lambda}\right). \tag{16}$$

This energy is commonly written as $F_{\rm int} = \phi_0 H_{12}(R)/4\pi$, where $H_{12}(R)$ is the field generated by the first vortex at the location of the second.

The double integral (15) can be evaluated numerically. The integral over q, however, diverges logarithmically. One can isolate effects of motion by subtracting and adding the result (16) for the interaction of vortices at rest:

$$f_{\rm int} = 2\pi K_0(R) - \int \frac{u^2\,q_x^2\cos(\boldsymbol{qR})\,d^2\boldsymbol{q}}{(1+q^2)\big[(1+q^2)^2+q_x^2u^2\big]}, \tag{17}$$

where the reduced interaction $f_{\rm int} = (16\pi^3\lambda^2/\phi_0^2)F_{\rm int}$. Another benefit of this step is that the logarithmic divergence is now incorporated in the exact first term, while the integral here is convergent.

To exclude large $|k| > 1/\xi$ (where the London theory breaks down, ξ is the vortex core size), we introduce a factor $e^{-k^2\xi^2} = e^{-q^2/\kappa^2}$ in the integrand of Eq. (17) and integrate over region $-q_m < q_x < q_m$ and $-q_m < q_y < q_m$ with q_m exceeding κ substantially, so that the square shape of the integration domain does not matter. The result is shown in Fig. 2. The upper panel shows that at low velocities, the interaction energy $F_{\rm int}(x,y) = F_{\rm int}(R,\varphi)$ is nearly azimuth independent. In particular, this means that the interaction force is nearly radial, as is the case of vortices at rest. With increasing velocity the situation changes drastically, and the force $-\boldsymbol{\nabla} F_{\rm int}(x,y)$, which is perpendicular to contours $F_{\rm int}(x,y) =$ const has a complicated distribution.

Clearly, the interaction energy (15) remains the same if $\boldsymbol{R} \to -\boldsymbol{R}$; also, it is symmetric with respect to reflection $y \to -y$. Since the interaction force is $-\nabla F_{\rm int}$, the lower panel of Fig. 2 shows that the force direction deviates from the direction of $\boldsymbol{R}$, unless $\boldsymbol{R}$ is parallel or perpendicular to the velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$. It is worth noting that the field distribution of the first vortex is asymmetric with respect to $x \to -x$, so that the interaction energy is not proportional to the field of the first vortex at the location of the other.

To have a better view of the energy $F_{\rm int}(x,y)$ we provide a three-dimensional plot in Fig. 3. It is seen that the interaction energy of a pair of vortices at the x axis is larger than for a pair situated at the same distance at the y axis, i.e., along the direction perpendicular to $\boldsymbol{v}$. This difference is pronounced for high velocities as shown in Fig. 3 for $u = 10$. This graph suggests a possibility of a shallow minimum at $x = 0$ and some finite y^*. To see this minimum we plot in Fig. 4 $f_{\rm int}(0,y)$ for a few high velocities. Hence, the minimum indeed exists; it deepens and moves to smaller intervortex distance y^* for increasing velocities. Numerical tests show that for $u \lesssim 2$ the minimum disappears within exponentially small tails of $f_{\rm int}(0,y)$, see the Appendix. We thus conclude that a pair of vortices situated at the y axis and moving fast with $u > 2$ repel at distances $y < y^*$ and attract when separated farther than y^*. In Fig. 5 the region of negative interaction energies is plotted for $u = 20$. It is seen that the minimum of the interaction energy is near the point $(0, 0.8)$ and the two-dimensional region of negative $f_{\rm int}(x,y)$ extends in all directions on distances of the order λ.

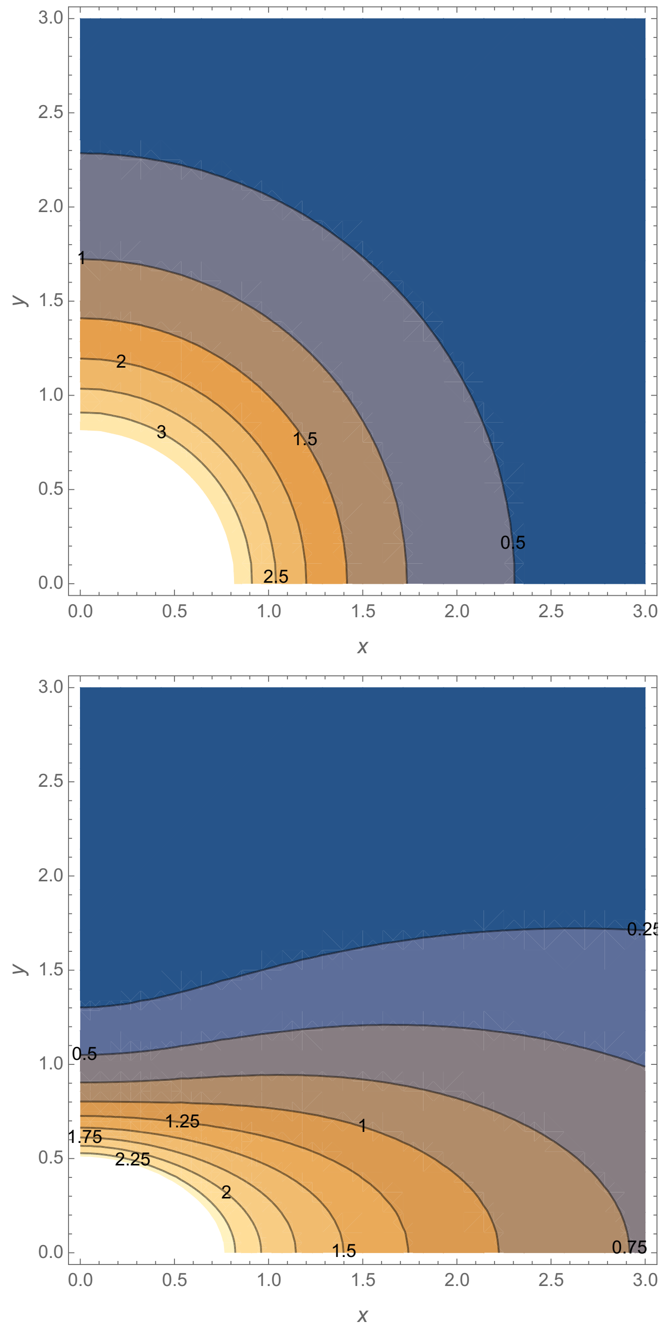

FIG. 2. Contours of constant interaction energy for a vortex at the origin (0, 0) and another one at (x, y) for a small velocity $u = v/v_c = 0.2$ (the upper panel) and for fast vortices with $u = v/v_c = 4$ (the lower panel). x, y are measured in units of λ. In this calculation $\kappa = 8$ and $q_m = 12$.

IV. ELECTRIC FIELD AND DISSIPATION

Having the magnetic field (8) of a moving vortex, one gets for two vortices of our interest:

$$H_{\boldsymbol{q}} = \frac{\phi_0(1+e^{-i\boldsymbol{qR}})e^{-iq_x ut/\tau}}{1+q^2-iq_x u}. \tag{18}$$

The moving nonuniform distribution of the vortex magnetic field causes an electric field $\boldsymbol{E}$ out of the vortex core, which in turn causes the normal currents $\sigma\boldsymbol{E}$ and the dissipation $\sigma\boldsymbol{E}^2$.

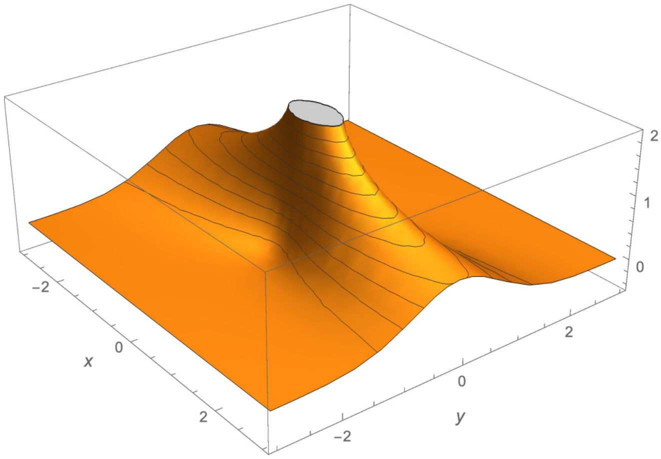

FIG. 3. The interaction energy $f_{\rm int}(x, y)$ for a vortex at the origin (0, 0) and another one at (x, y) both moving along x with velocity $u = v/v_c = 10$. In this calculation $\kappa = 10$ and $q_m = 20$.

Usually this dissipation is small relative to Bardeen-Stephen core dissipation [10], but for fast vortex motion it can become substantial [5].

The field $\boldsymbol{E}$ is expressed in terms of known $\boldsymbol{H}$ with the help of the Maxwell equations $i(\boldsymbol{k}\times\boldsymbol{E}_{\boldsymbol{k}})_z = -\partial_t H_{z\boldsymbol{k}}/c$ and $\boldsymbol{k}\cdot\boldsymbol{E}_{\boldsymbol{k}} = 0$:

$$E_{x\boldsymbol{k}} = -\frac{\phi_0 v}{c}\,\frac{q_x q_y(1+e^{-i\boldsymbol{qR}})}{q^2(1+q^2-iq_x u)}\,, \tag{19}$$

$$E_{y\boldsymbol{k}} = \frac{\phi_0 v}{c}\,\frac{q_x^2(1+e^{-i\boldsymbol{qR}})}{q^2(1+q^2-iq_x u)}\,. \tag{20}$$

For the stationary motion, one can consider the dissipation at $t = 0$. The dissipation power per unit length is:

$$W = \sigma\int d\boldsymbol{r}E^2 = \sigma\int\frac{d^2\boldsymbol{k}}{4\pi^2}(|E_{x\boldsymbol{k}}|^2+|E_{y\boldsymbol{k}}|^2) = \frac{\phi_0^2\sigma v^2}{\pi^2c^2}\int d^2\boldsymbol{q}\,\frac{q_x^2\cos^2(\boldsymbol{qR}/2)}{q^2\big[(1+q^2)^2+q_x^2u^2\big]}\,. \tag{21}$$

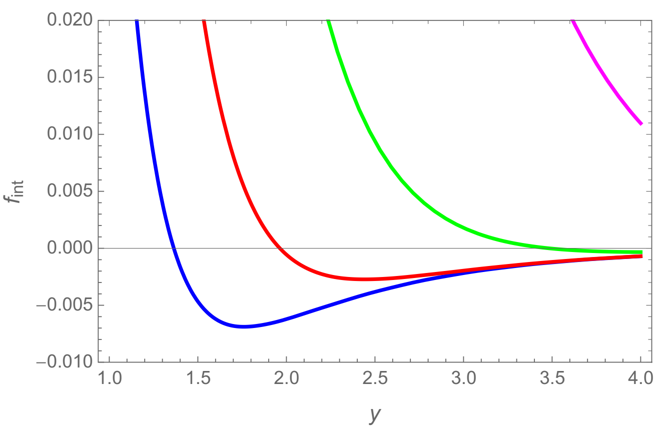

FIG. 4. The interaction energy $f_{\rm int}(0, y)$ for a vortex at the origin (0, 0) and another one at $(0, y)$ for $u = v/v_c = 8, 6, 4, 2$ in left-to-right order. In this calculation $\kappa = 10$ and $q_m = 20$.

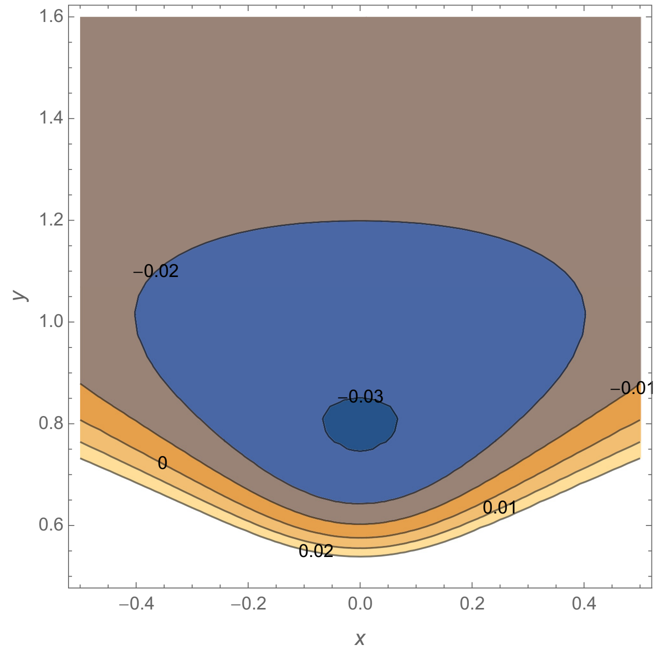

FIG. 5. The interaction energy $f_{\rm int}(x, y)$ for a vortex at the origin (0, 0) and another one at (x, y) for $u = v/v_c = 20$. In this calculation $\kappa = 10$ and $q_m = 20$.

Treating this integral numerically in the same way as was done for the energy integral in Eq. (17), we calculate the reduced quantity $w(x, y) = W(\pi c^2\lambda^2/\phi_0^2\sigma v_c^2)$ shown in Fig. 6.

An interesting feature of this result is that the dissipation $w(x, y)$ develops a shallow ditch along the x axis. An example of this ditch is better seen if we plot a cross section $w(2, y)$ as shown in Fig. 7. It is seen that for vortices separated by $x \approx 2\lambda$, the ditch width is $\Delta y \approx 2\lambda$, although the dissipation in the minimum is only about 3% less than at the maxima.

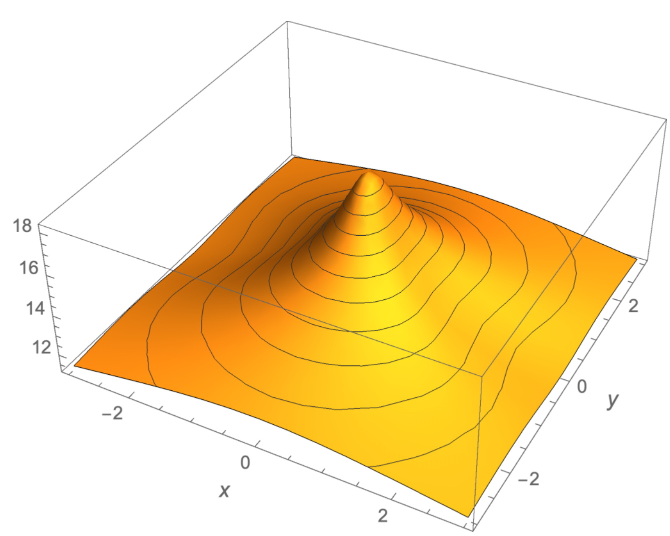

FIG. 6. The reduced dissipation $w(x, y)$ for a vortex at the origin (0, 0) and another one at (x, y) both moving along x with velocity $u = v/v_c = 10$. In this calculation $\kappa = 10$ and $q_m = 15$.

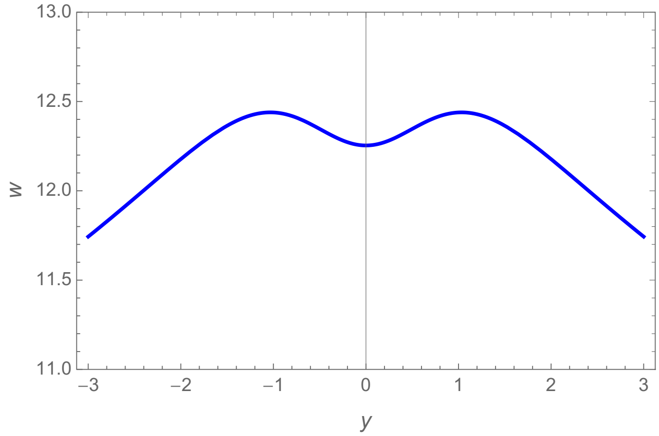

FIG. 7. The reduced dissipation $w(2, y)$ for a vortex at the origin $(0, 0)$ and another one at $(2, y)$ for $u = v/v_c = 10$. In this calculation $\kappa = 10$ and $q_m = 15$.

V. SUMMARY AND DISCUSSION

The time-dependent London equations, formulated to include normal currents around a moving vortex, show that the vortex field distribution differs from the static distribution displaced as a whole [5]. Hence, dynamics of vortices in superconductors with a short-range field distribution is similar to dynamics of topological singularities in overdamped systems with long-range interaction potentials [4].

We argue that the self-energy of a moving vortex is reduced as compared to the static case and decreases with increasing velocity. Moreover, the interaction energy of two parallel vortices, moving with the same velocity, one at the origin at $t = 0$ and another at $\boldsymbol{R} = (x, y)$, is symmetric relative to $x \to -x$ (x is along the velocity) notwithstanding the asymmetric field distribution of the first. In other words, the common rule stating that the interaction energy of two vortices is proportional to the field of the first vortex at the location of the other holds only for the vortices at rest.

As in any London based approach, our results are applicable only out of the vortex cores. The only relevant parameter of the theory, in addition to the penetration depth λ, is the reduced vortex velocity $u = v/v_c$ with the crossover velocity $v_c = c^2/4\pi\sigma\lambda$. We have shown that only for $u > 1$ distortions of the field in a moving vortex become considerable and even large. Hence, the estimate of v_c is crucial for relevance of our calculations.

To estimate maximum possible vortex velocity $v_{\max}$, one can set the transport current causing the vortex motion equal to the maximum possible, i.e., to the depairing value $j_{dp} = c\phi_0/16\pi^2\lambda^2\xi$. Then, the equation of motion $\eta v_{\max} = \phi_0 j_{dp}/c$ and Bardeen-Stephen's $\eta = \phi_0^2\sigma_n/2\pi c^2\xi^2$ provide $v_{\max}$ and

$$u_{\max} = \frac{v_{\max}}{v_c} = \frac{1}{2\kappa} < 1\,, \tag{22}$$

because conductivities from the expressions for the drug coefficient and of the crossover velocity cancel out. If this estimate is correct, effects of the vortex motion should be weak.

We note, however, that Bardeen-Stephen's η describes the dissipation within the normal vortex core and thus involves the normal state conductivity σ_n, whereas the conductivity σ entering v_c is of quasiparticles in the superconducting phase. These two are not necessarily the same, i.e., the above estimate should be

$$u_{\max} = \frac{1}{2\kappa}\,\frac{\sigma}{\sigma_n}\,. \tag{23}$$

In a recent work [11], Smith, Andreev, and Spivak argue that the conductivity of normal quasiparticles could be strongly enhanced by inelastic processes in time dependent situations. If this is the case, the crossover velocity $v_c \propto 1/\sigma$ could be suppressed and high values of the reduced velocity $u = v/v_c$ become available.

In experiments [1,2], at velocities exceeding 10^6 cm/s, vortices are reported to form chains along the velocity. The moving vortex core has a tail of suppressed order parameter in the $-\boldsymbol{v}$ direction which at large enough velocities may cause the following vortex to trail the first one. A moving vortex generates heat due to normal currents and the changing in time order parameter. This complicated process is discussed in Refs. [1,2] within the time dependent GL theory.

In this paper we consider a less ambitious and simple model of Abrikosov vortices moving with a constant velocity within linear time-dependent London theory. Whereas distances $\sim\xi$ are inaccessible within this approach, the interaction of vortices at distances of the order of $\lambda \gg \xi$ are well described by the London-type theory. Hence, although the TDL approach cannot describe all features of fast moving vortices, it may provide an extra useful insight. E.g., our results on the interaction energy of two fast-moving vortices suggest that vortices in a single chain along the velocity should have a tendency to slip aside, in other words, such a chain is unstable. Thus, we have shown that usual models treating a moving vortex as a static structure displaced as a whole, miss nontrivial changes in the vortex field structure and in the intervortex interaction that become relevant for fast motion.

ACKNOWLEDGMENTS

This work was supported by the US Department of Energy (DOE), Office of Science, Basic Energy Sciences, Materials Science and Engineering Division. Ames Laboratory is operated for the US DOE by Iowa State University under Contract No. DE-AC02-07CH11358.

APPENDIX: POSITION OF INTERACTION MINIMUM

As mentioned above, numerical tests show that the minimum of the interaction energy at $(0, y^*)$ practically disappears for $u \lesssim 2$. Here, we examine this question in more detail. The minimum position $(0, y^*)$ is determined by $\partial f_{\text{int}}(0, y)/\partial y = 0$. Differentiating Eq. (17) over y and setting $x = 0$ we obtain an equation for y^*:

$$-2\pi K_1(y^*) + u^2 \int \frac{q_x^2 q_y \sin(q_y y^*) d^2\boldsymbol{q}}{(1+q^2)\left[(1+q^2)^2 + q_x^2 u^2\right]} = 0. \tag{A1}$$

Let us consider the case of slow motion $u \ll 1$. Since we have u^2 in front of the integral here, in the lowest approximation one can set $u = 0$ in the integrand. We then have:

$$\int \frac{q_x^2 q_y \sin(q_y y^*) d^2\boldsymbol{q}}{(1+q^2)^3}$$
$$= \int_0^\infty \frac{q^4 dq}{(1+q^2)^3} \int_0^{2\pi} d\varphi \cos^2\varphi \sin\varphi \sin(qy^* \sin\varphi)$$
$$= \frac{2\pi}{y^*} \int_0^\infty \frac{q^3 J_2(qy^*) dq}{(1+q^2)^3} = \frac{\pi y^*}{4} K_0(y^*). \quad \text{(A2)}$$

Hence for $u \ll 1$, y^* satisfies

$$K_1(y^*) = \frac{y^* u^2}{8} K_0(y^*). \quad \text{(A3)}$$

Clearly, for vortices at rest, $y^* = \infty$ as is should be. Since the leading terms in asymptotic expansions of K_0 and K_1 are the same, we obtain for small velocities (and large y^*):

$$y^* = 8/u^2. \quad \text{(A4)}$$

This relation can also be confirmed by a direct numerical evaluation.

It is worth noting that this result implies that the minimum of the interaction energy $f_{\text{int}}(x, y)$ at the y axis in fact exists for any nonzero velocity. However for small u it is so far from the origin where f_{int} is exponentially small anyway and, therefore, details of the interaction coordinate dependence become irrelevant.

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