Stratonovich-Ito integration scheme in ultrafast spin caloritronics

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The magnonic spin Seebeck effect is a key element of spin caloritronics, a field that exploits thermal effects for spintronic applications. Early studies were focused on investigating the steady-state nonequilibrium magnonic spin Seebeck current, and the underlying physics of the magnonic spin Seebeck effect is now relatively well established. However, the initial steps of the formation of the spin Seebeck current are in the scope of recent interest. To address this dynamical aspect theoretically, we propose here an alternative approach to the time-resolved spin Seebeck effect. Our method exploits the supersymmetric theory of stochastics and the Stratonovich-Ito integration scheme. We found that in the early step the spin Seebeck current has both nonzero transversal and longitudinal components. As the magnetization dynamics approaches the steady state, the transversal components decay through dephasing over the dipole-dipole reservoir. The timescale for this process is typically in subnanoseconds, pointing thus to the potential of an ultrafast control of the dynamical spin Seebeck during its buildup.

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I. INTRODUCTION

Irradiating magnetic samples with electromagnetic fields may result in a variety of phenomena, including subpicosecond magnetic order breakdown, electron-phonon spin-flip scattering [1], electron-magnon scattering in nonequilibrium [2], and superdiffusive spin transport [3]. These observations are related to ultrafast spin dynamics [4,5] and depend on the parameters of the driving fields such as their intensity, duration, and frequencies, as well as on the inherent properties of the magnetic sample. Our interest here is devoted to a particular aspect, namely to the nonequilibrium magnonic current generated by a temperature gradient due to local heating of the sample by a laser pulse [6]. This means that we concentrate on the regime where the phonon temperature profile has already been established and consider the nonequilibrium dynamics of magnons. We note that magnon dynamics is of a particular importance for applications, as magnons are low-energy excitations that can carry information over long distances and can be utilized for logic operations. To deal with nonequilibrium processes under the influence of irregular forces and thermal fluctuations the Fokker-Planck (FP) equation is the method of choice [7–12].

In general, FP equation applies also to nonlinear (chaotic) systems with positive Lyapunov exponents [13]. Treating the thermally activated magnetization dynamics and the steady-state magnonic spin current, the FP equation allows obtaining results beyond the linear response theory [14,15]. However, the corresponding nonstationary case has not yet been treated with the FP equation. In fact, the FP equation is a nonlinear partial differential equation which admits exact analytical time-dependent solution only in few limited cases. Our aim to describe the ultrafast spin dynamics entails access to the time-dependent solution of the FP equation. The available procedures and analytical tools for solving the time-dependent FP equation are limited basically to 1D systems. As an alternative, one can consider a supersymmetric theory of stochastics and the Stratonovich-Ito integration scheme. In this work, we apply the Stratonovich-Ito integration scheme to the system below the Curie temperature.

Here we present an analytical FP-based approach to study thermally activated ultrafast magnonic spin current. Specifically, we focus on the behavior of the nonequilibrium spin current, generated at the interface of a ferromagnetic insulator and a normal metal, [16,17] and calculate how it approaches the nonequilibrium (steady) state. To this end the evaluation of the correlation functions in the nonequilibrium state is needed, and as we show here, this can be achieved by using the FP equation and the Stratonovich-Ito integration scheme for the stochastic noise.

Our choice of the sample is motivated by the recent experiments uncovering the early stage of the spin Seebeck effect [18]. Using terahertz spectroscopy applied to bilayers of ferrimagnetic yttrium iron garnet (YIG) and platinum, the spin Seebeck current is shown to arise on the ∼100 fs timescale.

The work is organized as follows. In Sec. II, we define the magnonic spin current. In Sec. III, we describe the theoretical methods used afterward. In Sec. IV, we present results and conclude the work.

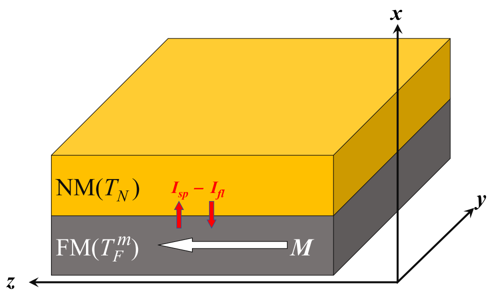

FIG. 1. The schematics of the system. The red arrows show the direction of spin pumping $I_{\rm sp}$ and fluctuating $I_{\rm fl}$ spin currents flowing from the ferromagnetic insulator to the normal metal ($I_{\rm sp}$) and from the normal metal to the ferromagnetic insulator ($I_{\rm fl}$). Equilibrium magnetization in the magnetic insulator is along the z axis. T_F^m is the magnon temperature in the magnetic insulator, and T_N is the temperature of the normal metal.

II. MODELING NONEQUILIBRIUM MAGNONIC SPIN CURRENT

The total spin current $\mathbf{I_{tot}} = \mathbf{I}_{\rm sp} + \mathbf{I}_{\rm fl}$ crossing the normal metal/ferromagnet interface has two contributions: the spin pumping current $\mathbf{I}_{\rm sp}$ flowing from the ferromagnetic insulator to the normal metal and the fluctuating spin current $\mathbf{I}_{\rm fl}$ flowing in the reverse direction (Fig. 1). Equilibration of electronic and phononic degrees of freedom proceeds much faster (subpicoseconds) than magnons (up to nanoseconds). As we are interested in the dynamics of the latter we assume that the temperature T_F in the ferromagnetic layer is set by the equilibrium electrons and the phonon temperature. The same applies to the temperature T_N in the normal metal. The heating is assumed to be induced by a laser pulse. The relation of the pulse parameters and the value of the temperature has been discussed in detail in Ref. [6]. The thermal bias through the mismatch between the magnon temperature T_F^m and the sample temperature T_N drives the magnonic spin current of interest here. We note that magnons are low-energy elementary excitations of the ordered phase. Thus, spin (or electron/lattice) dynamics at (femtosecond) times where the magnetic state is broken down or not yet established is not discussed here.

The spin pumping current flowing from the ferromagnetic insulator into the normal metal reads [19–21]

$$\mathbf{I}_{\rm sp}(t) = \frac{\hbar}{4\pi}[g_r\mathbf{m}(t)\times\dot{\mathbf{m}}(t) + g_i\dot{\mathbf{m}}(t)], \tag{1}$$

where g_r and g_i are the real and imaginary parts of the dimensionless spin mixing conductance of the ferromagnet/normal metal ($F|N$) interface, while $\mathbf{m}(t) = \mathbf{M}(t)/M_s$ is the dimensionless unit vector along the magnetization orientation (here M_s is the saturation magnetization) and $\dot{\mathbf{m}} \equiv d\mathbf{m}/dt$. The spin current is a tensor object characterized by the direction of the current flow and the orientation of the flowing spin (magnetic moment). Due to the geometry of the system (see Ref. [16]), the pumping spin current flows along the x axis while the fluctuating spin current flows in the opposite $(-x)$ direction:

$$\mathbf{I}_{\rm fl}(t) = -\frac{M_sV}{\gamma}\mathbf{m}(t)\times\boldsymbol{\zeta}'(t). \tag{2}$$

Here, V is the total volume of the ferromagnet, γ is the gyromagnetic factor, and $\boldsymbol{\zeta}'(t) = \gamma\mathbf{h}'(t)$, with $\mathbf{h}'(t)$ denoting the random magnetic field. In the classical limit, $k_BT \gg \hbar\omega_0$, the correlation function $\langle\zeta_i'(t)\zeta_j'(t')\rangle$ of $\boldsymbol{\zeta}'(t)$ reads

$$\langle\zeta_i'(t)\zeta_j'(0)\rangle = \frac{2\alpha'\gamma k_BT_N}{M_sV}\delta_{ij}\delta(t) \equiv \sigma'^2\delta_{ij}\delta(t) \tag{3}$$

for $i, j = x, y, z$, where $\langle\cdots\rangle$ denotes the ensemble average, ω_0 is the ferromagnetic resonance frequency, and α' is the contribution to the damping constant due to spin pumping, $\alpha' = \gamma\hbar g_r/4\pi M_sV$. We note that the correlator [Eq. (3)] is proportional to the temperature of the normal metal T_N. The total spin current thus reads

$$\langle\mathbf{I}_{\rm tot}\rangle = \frac{M_sV}{\gamma}[\alpha'\langle\mathbf{m}\times\dot{\mathbf{m}}\rangle - \langle\mathbf{m}\times\boldsymbol{\zeta}'\rangle]. \tag{4}$$

The temperature-dependent magnetization dynamics is governed by the stochastic Landau-Lifshitz-Gilbert (LLG) equation:

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m}\times(\mathbf{H}_{\rm eff} + \mathbf{h}) + \alpha\mathbf{m}\times\dot{\mathbf{m}}, \tag{5}$$

where α is the Gilbert damping constant, $\mathbf{H}_{\rm eff}$ is the effective field, and the time-dependent random magnetic field in the ferromagnet is described by $\mathbf{h}$. This effective field $\mathbf{H}_{\rm eff}$ contains the anisotropy field H_A and the external magnetic field H_{0z} oriented along the z axis. The total random magnetic field $\mathbf{h}(t)$ has two contributions from independent noise sources: the thermal random field $\mathbf{h}_0(t)$ and the random field $\mathbf{h}'(t)$. The former is related to the finite temperature in the ferromagnetic insulator and the second to the fluctuations in the normal metal. The correlators of the statistically independent noise sources are additive, leading to the effective (enhanced) magnetic damping constant $\alpha = \alpha_0 + \alpha'$ [16] (α_0 is the damping parameter of the ferromagnetic material, meaning without the contributions from the pumping currents),

$$\langle\zeta_i(t)\zeta_j(0)\rangle = \frac{2\alpha\gamma k_BT_F^m}{M_sV}\delta_{ij}\delta(t) = \sigma^2\delta_{ij}\delta(t), \tag{6}$$

where $\boldsymbol{\zeta}(t) = \gamma\mathbf{h}(t)$ and $\alpha T_F^m = \alpha_0T_F + \alpha'T_N$.

III. THEORETICAL METHOD

To find the total spin current [Eq. (4)], we use the FP equation for the distribution function of the magnetization $P(m_z, t)$, which is related to the stochastic equation of the magnetic dynamics [Eq. (5)]

$$\frac{\partial P(m_z,t)}{\partial t} = \frac{\partial}{\partial m_z}\left[\frac{\partial}{\partial m_z} + \beta U'(m_z)\right]P(m_z,t), \tag{7}$$

where $U(m_z) = 2\alpha(\omega_0m_z - \frac{\omega_pm_z^2}{2})$ is the potential, $\beta = 1/\sigma^2$ is the effective inverse temperature, $\omega_p = \gamma H_A$, and H_A is the anisotropy field. As detailed above, in our case $U(m_z)$ is time independent. The stationary solution of Eq. (7) is given by [14]

$$P_0(m_z) = Z^{-1}\exp[-\beta U(m_z)],$$
$$Z = \int\exp[-\beta U(m_z)]d^3\mathbf{m}. \tag{8}$$

For the time-dependent distribution one makes the ansatz

$$P(m_z,t)=\psi(m_z,t)\exp\left[-\frac{\beta}{2}U(m_z)\right]. \tag{9}$$

Using Eqs. (9) and (7) we find that $\psi(m_z,t)$ is a solution of the Schrödinger equation for imaginary time,

$$\frac{\partial\psi(m_z,t)}{\partial t}=-\hat{H}\,\psi(m_z,t), \tag{10}$$

with the Hamiltonian

$$\hat{H}=-\frac{d^2}{dm_z^2}+\left(\frac{U'(m_z)}{2\sigma^2}\right)^2-\frac{U''(m_z)}{2\sigma^2}. \tag{11}$$

As U is time independent, the general solution of Eq. (11) is $\psi(m_z,t)=\sum_n C_n\exp(-\lambda_n t)\psi_n(m_z)$, where $\psi_n(m_z)$ and λ_n are the eigenfunctions and eigenvalues of the stationary equation, $\hat{H}\psi_n(m_z)=\lambda_n\psi_n(m_z)$. Hence, the problem of solution of the time-dependent FP equation reduces to the determination of λ_n and the corresponding eigenfunctions of the Hamiltonian $\hat{H}$. The calculations can be substantially simplified due to the hidden supersymmetry of this problem [22]. Indeed, one can introduce the supersymmetric Hamiltonian (for the supersymmetry, see Refs. [23–28]) $\hat{H}_{\rm susy}=Q^\dagger Q+QQ^\dagger=\mathrm{diag}\,(\hat{H}_+,\hat{H}_-)$, where

$$Q=\begin{bmatrix}0&0\\A&0\end{bmatrix},\quad Q^\dagger=\begin{bmatrix}0&A^\dagger\\0&0\end{bmatrix}, \tag{12}$$

and $A=\frac{1}{2}\beta U'(m_z)-\partial/\partial m_z$. Note that Q and $Q^\dagger$ are nilpotent operators, $Q^2=(Q^\dagger)^2=0$, and the commutator $[Q,\hat{H}_{\rm susy}]=0$. As a result, $\hat{H}_+$ and $\hat{H}_-$ have common eigenfunctions: if Ψ_n is an eigenfunction of $\hat{H}_+$, then $Q\Psi_n$ is the eigenfunction of $\hat{H}_-$ (except for the ground state corresponding to $\lambda_0=0$). The operator $\hat{H}_{\rm susy}$ acts in the space of Bose and Fermi fields. Namely, $\hat{H}_+=d^2/dm_z^2+V_+(m_z)$ is the operator for bosons and $\hat{H}_-=d^2/dm_z^2+V_-(m_z)$ is the operator for fermions, where $V_\pm(m_z)=(U'/2\sigma^2)^2\pm U''/2\sigma^2$ are the corresponding potentials. The operator Q transforms bosons to fermions and vice versa.

$\hat{H}_+$ coincides with Hamiltonian (11). It is, however, more convenient to solve the Schrödinger equation with the fermionic Hamiltonian $\hat{H}_-$ because the corresponding potential $V_-(m_z)$ is close to the parabolic form. Owing to the supersymmetry, the eigenfunctions and eigenvalues are the same. Using this approach we find $\lambda_1=\frac{\sigma^2}{2\pi}\exp(-\alpha\omega_p/\sigma^2)$, and in the limit of strong anisotropy, we find

$$\lambda_n\approx 4\alpha\omega_p(n-1)/\sigma^2. \tag{13}$$

The first nonvanishing $1/\lambda_2$ defines the characteristic relaxation timescale. For more details on the supersymmetry theory of stochastics, we refer the reader to Refs. [29–35].

To explore the time dependence of the nonequilibrium spin current $\langle\mathbf{I}(t)_{\rm tot}\rangle$, we utilize the Stratonovich-Ito integration scheme [36–38] and construct a reductive perturbation theory valid in the low-temperature limit (specified below). We briefly recall the main concepts of the stochastic Ito-Stratonovich integration. The time integral from the stochastic noise is equal to the function $W(t)$, which has no time derivative [$W(t)$ is not a smooth function] $\int_0^t\xi(\tau)d\tau=W(t)$. Therefore, the stochastic integration is performed using the mean-square (ms) convergence of the sequence of the random variable $X_n(\omega)$, meaning that

$$\mathrm{ms}\{\lim_{n\to\infty}X_n\}=X \tag{14}$$

is equivalent to

$$\lim_{n\to\infty}\int_{-\infty}^{\infty}p(\omega)[X_n(\omega)-X(\omega)]^2$$
$$=\mathrm{ms}\{\lim_{n\to\infty}\langle(X_n-X)^2\rangle\}=0. \tag{15}$$

Here $p(\omega)$ is the probability distribution function. The stochastic integral is defined as follows:

$$\int_{t_0}^{t}G(\tau)\,dW(\tau)$$
$$=\mathrm{ms}\left\{\lim_{n\to\infty}\sum_{i=1}^{n}G(t_{i-1})[W(t_i)-W(t_{i-1})]\right\}, \tag{16}$$

where $G(t)$ is an arbitrary function of time. We assume that the magnon temperature in the system is low, which means that the thermal energy is smaller than the anisotropy barrier. Therefore, the appropriate ansatz for the solution of the stochastic LLG equation is

$$\mathbf{m}(t)=\mathbf{m}_0(t)+\varepsilon\mathbf{m}_1(t), \tag{17}$$

where $\mathbf{m}_0(t)$ is the deterministic solution and $\mathbf{m}_1(t)$ is the correction due to the stochastic field. The equation for the stochastic part reads

$$d\mathbf{m}_1(t)=-A[\mathbf{m}_0(t)]\,\mathbf{m}_1(t)\,dt$$
$$+B[\mathbf{m}_0(t)]\,d\mathbf{W}(t), \tag{18}$$

where $d\mathbf{W}(t)=\boldsymbol{\xi}(t)dt$, and for brevity we introduced the notations

$$A[\mathbf{m}_0(t)]=\begin{bmatrix}0&\omega_{\rm eff}(t)&0\\-\omega_{\rm eff}(t)&0&0\\A&0&0\end{bmatrix},$$
$$B[\mathbf{m}_0(t)]=\begin{bmatrix}0&m_{0z}(t)&-m_{0y}(t)\\-m_{0z}(t)&0&m_{0x}(t)\\m_{0y}(t)&-m_{0x}(t)&0\end{bmatrix}, \tag{19}$$

with $\omega_{\rm eff}(\omega_0+\omega_p m_z)$. Taking into account Eqs. (14)–(19), after relatively involved analytical calculations for the correlation functions and the nonequilibrium spin current, we deduce

$$\langle\mathbf{I}_s(t)\rangle=2\alpha' k_B\varepsilon^2\mathbf{m}_0(t)\big(T_F^m-T_N\big),$$
$$\langle m_{1i}(t)\xi_j(t)\rangle=\sigma^2\varepsilon_{ijk}m_{0k}(t),$$
$$\langle m_{1i}(t)\xi'_j(t)\rangle=\sigma'^2\varepsilon_{ijk}m_{0k}(t). \tag{20}$$

In the case of a weak anisotropy, Eq. (20) simplifies, and for the nonequilibrium magnonic spin current components we obtain the following:

$$\langle I_s^x(t)\rangle=2\alpha' k_B\varepsilon^2\frac{\cos(\varphi_0+\omega_0 t)}{\cosh\alpha\omega_0 t}\big(T_F^m-T_N\big),$$
$$\langle I_s^y(t)\rangle=2\alpha' k_B\varepsilon^2\frac{\sin(\varphi_0+\omega_0 t)}{\cosh\alpha\omega_0 t}\big(T_F^m-T_N\big),$$
$$\langle I_s^z(t)\rangle=2\alpha' k_B\varepsilon^2\tanh(\alpha\omega_0 t)\big(T_F^m-T_N\big). \tag{21}$$

From Eq. (21) it follows that in the asymptotic, long-time limit the only component of the magnonic spin current that survives is $\langle I_s^z(t)\rangle$, and we recover the classical result of Xiao *et al.* [16]. For short times, however, the other components are sizable and even dominant and thus can be exploited for ultrafast picosecond magnonics.

We applied the Stratonovich-Ito integration scheme to the system below the Curie temperature. Nevertheless, our method can be extended to the Landau-Lifshitz-Bloch equation as well. Note that Eq. (18), in the coefficients $A[\mathbf{m}_0(t)]$ and $B[\mathbf{m}_0(t)]$, contains the solution of the deterministic Landau-Lifshitz-Gilbert equation. One can replace the solution of the deterministic LLG equation $\mathbf{m}_0(t)$ by the solution of the deterministic Landau-Lifshitz-Bloch equation with extra longitudinal damping parameter [39]. After this replacement, we can again perform the Stratonovich-Ito integration.

The result, Eq. (21), is obtained in the single macrospin approximation but can be generalized to an extended system using the ensemble averaging over the dipole-diploe reservoir. We note that the transversal spin current components in Eq. (21) contain the rotating terms. In the case of extended systems, each spin rotates with a slightly different frequency due to the broadening of the resonance frequency ω_0. Precession with different frequencies leads to the dephasing of the signal in time. We assume that the dephasing of the transversal magnetization and current components have the same nature.

Following Ref. [40], we write down the equation for the transversal magnetization component,

$$-i\hbar\frac{dm_x(t)}{dt} = [\hat{H}_d(t), m_x(t)], \tag{22}$$

or in the matrix form

$$-i\hbar\frac{d[m_x(t)]_{nn'}}{dt} = \hbar\Delta\omega(t)_{nn'}[m_x(t)]_{nn'}. \tag{23}$$

The Hamiltonian $\hat{H}_d(t)$ in Eqs. (22) and (23) describes the dipole-dipole reservoir, and the time dependence of the Heisenberg operators is governed through the Zeeman Hamiltonian $\hat{H}_Z$ (see Ref. [40] for more details). Let us quantify the fluctuations of the local field through the function

$$\langle\Delta\omega(t)_{nn'}\Delta\omega(t+\tau)_{nn'}\rangle = M_2\Psi(\tau), \tag{24}$$

where

$$M_2 = -\frac{\mathrm{Tr}\{[\hat{H}_d, m_x]^2\}}{\hbar^2\mathrm{Tr}\{m_x^2\}} - \omega_0^2 \tag{25}$$

is the second moment of the transversal component. We assume that the dephasing mechanism of the transversal spin current components is the same. Taking into account Eqs. (22)–(25) for the ensemble-averaged dephasing transversal spin currents, we infer

$$\langle\langle I_s^x(t)\rangle\rangle = 2\alpha' k_B\varepsilon^2\frac{\cos(\varphi_0+\omega_0 t)}{\cosh\alpha\omega_0 t}\exp\left[-M_2\int_0^t(t-\tau)\Psi(\tau)d\tau\right]\left(T_F^m - T_N\right),$$
$$\langle\langle I_s^y(t)\rangle\rangle = 2\alpha' k_B\varepsilon^2\frac{\sin(\varphi_0+\omega_0 t)}{\cosh\alpha\omega_0 t}\exp\left[-M_2\int_0^t(t-\tau)\Psi(\tau)d\tau\right]\left(T_F^m - T_N\right). \tag{26}$$

In the limit of the white noise the dephasing exponent takes the simpler form: $\langle\langle I_s^{x,y}(t)\rangle\rangle \approx \langle\langle I_s^{x,y}(0)\rangle\rangle\exp[-t/T_2]$, where the transversal relaxation time is given by $T_2 = -M_2 t\int_0^\infty\Psi(\tau)d\tau$. Thus, the decay of the transversal magnonic spin current components in the ultrafast spin Seebeck effect is solely determined by the dipole-dipole interactions.

IV. RESULTS AND DISCUSSIONS

In the numerical simulation, the motion of $\mathbf{m}$ is governed by the LLG equation (5). The adopted numerical parameters are $M_s = 1.4\times10^5$ A/m, the damping constant $\alpha = 0.001$, the external magnetic field $H_{0z} = 2\times10^5$ A/m, and the spin-mixing conductance $g_r = 3\times10^{15}$ 1/m^2. In the equilibrium state, the local magnetization points along the $+z$ direction. We set the temperature $T_F^m = 5K$ and $T_N = 0$. The time-dependent magnonic spin pumping currents $I_s^x(t)$, $I_s^y(t)$, and $I_s^z(t)$ are plotted in Fig. 2. The spin current is calculated using Eq. (1). For the transversal components $I_s^x(t)$ and $I_s^y(t)$ we consider the averaging procedure through the exponential factors $\langle\langle I_s^x(t)\rangle\rangle = e^{-t/T_2}\langle I_{s,0}^x\rangle$ and $\langle\langle I_s^y(t)\rangle\rangle = e^{-t/T_2}\langle I_{s,0}^y\rangle$, with the transversal relaxation time $T = N/\omega_0$, $N = 50$, and the values of $\langle I_{s,0}^x\rangle$ and $\langle I_{s,0}^y\rangle$ are calculated from Eq. (1). The numerical solution plotted in Fig. 2 is in good agreement with the analytical results expressed by Eqs. (20) and (26). Calculations done for the anisotropy field $H_z = \frac{2K_z}{\mu_0 M_s}$ (along the z axis) with constant $K_z = 1.8\times10^4$ J/m^3 (not shown) lead to similar conclusions. To explore the dephasing problem for an extended sample, we performed micromagnetic simu-

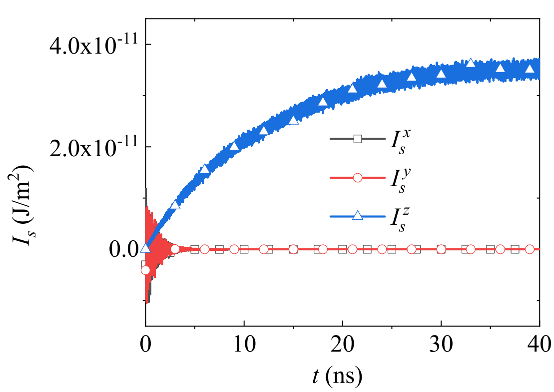

FIG. 2. Time-dependent nonequilibrium transversal and longitudinal magnonic spin current components $I_s^x(t)$, $I_s^y(t)$, and $I_s^z(t)$. The magnon temperature is equal to $T_F^m = 5$ K and temperature of the normal metal $T_N = 0$. The external magnetic field $H_{0z} = 2\times10^5$ A/m is applied in the $+z$ direction.

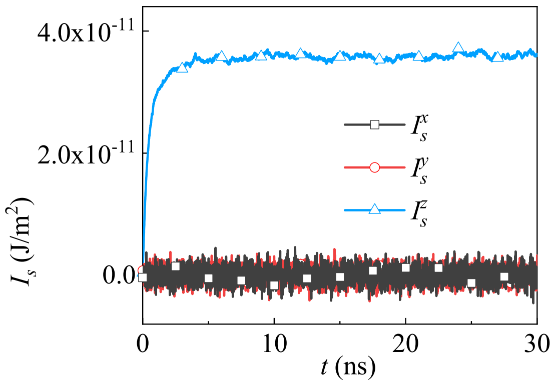

FIG. 3. Time-dependent, nonequilibrium transversal and longitudinal magnonic spin current components $I_s^x(t)$, $I_s^y(t)$, and $I_s^z(t)$ for a finite ferromagnetic sample. The short timescale longitudinal magnonic spin currents are important, whereas in the long-time limit $I_s^z(t)$ is dominant.

lations. We performed numerical simulations for an extended ferromagnetic sample. Our simulations include the effects of the dipole-dipole and exchange interactions as well as the magnetic anisotropy. The geometry of the ferromagnetic sample is as follows: the length is 350 nm (along the z axis), the width 50 nm (along the y axis), and the thickness is 5 nm (along the x axis). In this case, the equilibrium magnetization points to the in-plane along the $+z$ direction. The magnon temperature $T_F^m = 5$ K and the temperature of the normal metal is set to zero $T_N = 0$. The time-dependent magnonic spin pumping currents $I_s^x(t)$, $I_s^y(t)$, and $I_s^z(t)$ are plotted in Fig. 3. For extended samples, the dephasing of the transversal spin current components is faster and can hardly be captured through the micromagnetic simulations. As is evident, the transversal components of the current oscillate randomly close to the zero value, leading to $\langle I_s^x(t)\rangle = 0$ and $\langle I_s^y(t)\rangle = 0$. The longitudinal component $I_s^z(t)$ increases in time and saturates in the stationary regime.

Summarizing, we proposed a theoretical approach to the time evolution of the spin Seebeck current. The approach is based on the time-dependent Fokker-Planck equation and supersymmetry arguments. We managed to derive the analytical formula for the initial step of the buildup of the spin current in the spin Seebeck effect. The results are confirmed by full numerical calculations. The current experimental interest shows that ultrafast spin dynamics will play an increasingly significant role in spin caloritronics in the foreseeable future. Analytical tools for the time-dependent FP equation are quite limited. Therefore, the alternative method proposed in our work should be useful for spin caloritronic studies.

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