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Instability of $j = \frac{3}{2}$ Bogoliubov Fermi surfaces

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Exotic quantum phases including topological states and non-Fermi liquids may be realized by quantum states with total angular momentum $j = \frac{3}{2}$, as manifested in HgTe and pyrochlore iridates. Recently, an exotic superconducting state with a nonzero density of states of zero-energy Bogoliubov (BG) quasiparticles, the Bogoliubov Fermi surface (BG-FS), was also proposed in a centrosymmetric $j = \frac{3}{2}$ system, protected by a Z_2 topological invariant. Here, we consider interaction effects of a centrosymmetric BG-FS and demonstrate its instability by using mean-field and renormalization group analysis. The Bardeen-Cooper-Schrieffer (BCS)-type logarithmical enhancement is shown in fluctuation channels associated with inversion symmetry. Thus, we claim that the inversion-symmetry instability is an intrinsic characteristic of a BG-FS under generic attractive interactions between BG quasiparticles. In drastic contrast to the standard BCS superconductivity, a Fermi surface may generically survive even with the instability. We propose the experimental setup, a second-harmonic-generation experiment with a strain gradient, to detect the instability. Possible applications to iron-based superconductors and heavy-fermion systems, including FeSe, are also discussed.

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Introduction. Electrons on a lattice may form quantum states with a total angular momentum $j = \frac{3}{2}$, especially with strong spin-orbit coupling [1]. Cubic and time-reversal symmetries may protect the degeneracy of the four states as in GaAs and HgTe. A minimal model of the $j = \frac{3}{2}$ band structures was provided by Luttinger, the so-called Luttinger Hamiltonian [2,3], and its low-energy properties are thoroughly understood, being the backbone of semiconductor physics [4].

Recent advances in topology and correlation research unveiled unconventional phases associated with the Luttinger Hamiltonian. Topological insulators may be realized by breaking cubic symmetry, for example, applying uniaxial pressure [5,6], and Weyl semimetals may be formed by breaking time-reversal symmetry, for example, the onset of the all-in-all-out order parameter in pyrochlore iridates [7,8]. In the presence of the long-range Coulomb interaction, either non-Fermi-liquid or topological states with broken symmetries may be realized with renormalized physical quantities [9], and significant advances in experiments have been reported recently [10–13]. Furthermore, quantum phase transitions between the unconventional phases have been investigated, finding new universality classes [14–18].

Exotic superconducting states with $j = \frac{3}{2}$ states were also proposed [19–34]. In the presence of inversion symmetry, it was proven that a noninteracting Bogoliubov Hamiltonian may host a Fermi surface of Bogoliubov quasiparticles, in drastic contrast to standard nodeless, point node, and line node gap structures, named the Bogoliubov Fermi surface (BG-FS). It is characterized by a Z_2 topological invariant of the Hamiltonian [19,35], and several heavy-fermion systems such as URu_2Si_2 and UBe_{13} are suggested as candidate materials, although the presence of BG-FS has not been reported yet [36–39]. Uncovering characteristics of a BG-FS for its discovery is highly desired.

In this work, we propose enhanced fluctuations of an inversion-symmetry-breaking order parameter as a key property of a centrosymmetric BG-FS. It is shown that a centrosymmetric BG-FS becomes unstable at zero temperature under infinitesimally weak interaction between BG quasiparticles, and the Fermi-surface manifold may be changed, as illustrated in Fig. 1. Our results indicate that an inversion-symmetry-breaking order must be included in a phenomenological Ginzburg-Landau functional of a centrosymmetric BG-FS, and we also propose second-harmonic-generation (SHG) experiments with a strain gradient to identify enhanced fluctuations of an inversion-symmetry-breaking order parameter.

One Bogoliubov pair problem. Let us consider a generic BG-FS with inversion symmetry. Two states with α, for example, angular momentum $(|+\vec{k},\alpha\rangle, |-\vec{k},\alpha\rangle)$, are inversion-partner pairs. With an inversion-symmetry unitary operator $U_{\rm Inv}$, the single-particle Hamiltonian with the superscript (1) is characterized by

$$H_B^{(1)}|\vec{k},\alpha\rangle = \epsilon_k|\vec{k},\alpha\rangle, \quad U_{\rm Inv}|+\vec{k},\alpha\rangle = |-\vec{k},\alpha\rangle.$$

The inversion symmetry of the BG-FS, $[H_B^{(1)}, U_{\rm Inv}] = 0$, guarantees $\epsilon_{+\vec{k}}(\alpha) = \epsilon_{-\vec{k}}(\alpha)$.

We define the one BG pair problem of the inversion partners as a bound-state quantum mechanics problem between the partners,

$$\left(H_{B,1}^{(1)} + H_{B,2}^{(1)} + V\right)|\Psi^{(2)}\rangle = E|\Psi^{(2)}\rangle,$$

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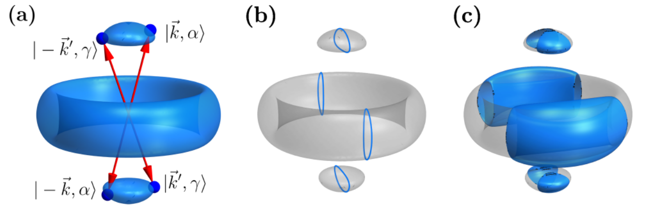

FIG. 1. (a) Example of a BG-FS (blue) in a centrosymmetric system. The inversion partners are illustrated with arrows. (b) Gapped BG-FS (gray) achieved by the fine-tuning condition. Line nodes (blue lines) remain. (c) A noncentrosymmetric BG-FS generically survives in the presence of the inversion instability.

where an interaction between pairs V is introduced. The superscript (2) is to specify a two-body problem. Solving the quantum-mechanical problem is standard. Introducing a gap function, $\Gamma(\vec{k},\alpha,\beta) \equiv [E-\epsilon_{\vec{k}}(\alpha)-\epsilon_{-\vec{k}}(\beta)] \times \langle \vec{k},-\vec{k};\alpha,\beta|\Psi^{(2)}\rangle$, we have the integral equation

$$\Gamma(\vec{k},\alpha,\beta) = \sum_{\vec{k}',\gamma,\delta} \frac{V_{\alpha\beta\gamma\delta}(\vec{k},\vec{k}')}{E-\epsilon_{\vec{k}'}(\gamma)-\epsilon_{-\vec{k}'}(\delta)}\Gamma(\vec{k}',\gamma,\delta), \qquad (1)$$

with $V_{\alpha\beta\gamma\delta}(\vec{k},\vec{k}') \equiv \langle \vec{k},-\vec{k};\alpha,\beta|V|\vec{k}',-\vec{k}';\gamma,\delta\rangle$. Two particle states, $|\vec{k},-\vec{k};\alpha,\beta\rangle$, whose quantum numbers are $(\vec{k},\alpha)$ and $(-\vec{k},\beta)$, are introduced.

We are interested in pairing between quasiparticles with the same energy, and generically, it is safe to consider the case of $\alpha=\beta$ and $\gamma=\delta$. The antisymmetric property of fermions restricts a form of a gap function of the BG pair; only odd-parity functions are allowed. As a proof of concept, we consider a pairing potential $V_{\alpha\alpha\alpha\alpha}(\vec{k},\vec{k}') = \sum_{m=0,\pm1} \lambda^{l=1}_{\alpha\alpha} w^{l=1}_{k,\alpha} w^{l*}_{k',\alpha} Y_{1,m}(\Omega_{\vec{k}})Y^{*}_{1,m}(\Omega_{\vec{k}'})$ assuming SO(3) symmetry. We omit the band index α below, and its generalization is discussed in the Supplemental Material (SM) [40]. Note that the structure of the integral equation is similar to the original Cooper pair problem [41], and the standard self-consistent equation is obtained:

$$\frac{1}{|\lambda^{l=1}|} = -\sum_{\vec{k}} \frac{|w^{l=1}_k|}{E-2\epsilon_{\vec{k}}}, \qquad (2)$$

where the summation on the right-hand side is logarithmically divergent for $E>0$. For $E<0$, a bound-state energy may be determined by $E=-2\Lambda e^{-2/|\lambda^{l=1}|N_F(0)W}$, where the density of states at zero energy $N_F(0)$ of a centrosymmetric BG-FS, the averaged function over a Fermi surface $W \equiv \langle |w^{l=1}_k|^2\rangle_{FS}$, and the UV energy cutoff Λ are introduced.

A few remarks are as follows. First, the existence of a bound state implies that a centrosymmetric BG-FS becomes unstable under an infinitesimally attractive interaction between BG quasiparticles as in the original Cooper problem. Second, the pair of BG quasiparticles is nontrivial under inversion symmetry, while a Cooper pair of a Fermi liquid is nontrivial under continuous charge-conservation symmetry. Thus, our calculations indicate that a discrete symmetry is enough to induce an instability of a BG-FS. Third, the summation of Eq. (2) gives a logarithmic divergence for $E \geqslant 0$ which may be connected to the standard BCS logarithm as shown below. Fourth, our calculations may be generalized into a system with a symmetry lower than SO(3) and a generic pairing potential form [40]. The former may be achieved by replacing the quantum numbers (l,m) with a generic representation index, and the latter may be argued by relying on the Kohn-Luttinger effect [42]. We stress that the essential part of a pair formation is the presence of a BG-FS, as in the Cooper pair problem on a Fermi liquid [41].

Model Hamiltonian. We consider a model BG-FS Hamiltonian with normal and superconducting parts. For the normal part, a Luttinger Hamiltonian in a cubic system is considered, $H_N = \sum_{\vec{k}} \xi^{\dagger}_{\vec{k}} H_0(\vec{k})\xi_{\vec{k}}$, with

$$H_0(\vec{k}) = (\tilde{c}_0\vec{k}^2-\epsilon_{\rm F})\gamma^0 + \sum_{a=1}^{3}\tilde{c}_1 d_a(\vec{k})\gamma^a + \sum_{a=4}^{5}\tilde{c}_2 d_a(\vec{k})\gamma^a.$$

The 4×4 identity matrix γ^0 and five Dirac gamma matrices γ^a which form a Clifford algebra are introduced. We use four-component spinors $\xi_{\vec{k}} = (f_{\vec{k},\frac{3}{2}}, f_{\vec{k},\frac{1}{2}}, f_{\vec{k},-\frac{1}{2}}, f_{\vec{k},-\frac{3}{2}})^T$ with fermionic annihilation operators $f_{\vec{k},\alpha=\pm3/2,\pm1/2}$. The four parameters of the Luttinger Hamiltonian are the chemical potential $\epsilon_{\rm F}$ and $\tilde{c}_0, \tilde{c}_1, \tilde{c}_2$ for particle-hole and cubic anisotropies. The five functions $d_1(\vec{k})=\sqrt{3}k_xk_y$, $d_2(\vec{k})=\sqrt{3}k_yk_z$, $d_3(\vec{k})=\sqrt{3}k_zk_x$, $d_4(\vec{k})=\frac{\sqrt{3}}{2}(k_x^2-k_y^2)$, and $d_5(\vec{k})=\frac{1}{2}(2k_z^2-k_x^2-k_y^2)$ are used. For the superconducting part, the 8×8 matrix Hamiltonian is introduced with a Nambu spinor $\chi^T_{\vec{k}} = (\xi^T_{\vec{k}}, \xi^{\dagger}_{-\vec{k}})$, whose explicit form is

$$\mathcal{H}^0_{\vec{k}} = \begin{pmatrix} H_0(\vec{k}) & \Delta(\vec{k}) \\ \Delta^{\dagger}(\vec{k}) & -H^T_0(-\vec{k}) \end{pmatrix}. \qquad (3)$$

We choose the chiral pairing channel $\Delta(\vec{k}) = \Delta_0(\gamma^3+i\gamma^2)i\gamma^{12}$ with a SO(3) symmetric band $(c_1=c_2)$ and constant pairing $\Delta_0 \neq 0$ to follow the literature [19]. The contour of zero-energy states is illustrated in Fig. 1(a). Note that the BG-FS Hamiltonian enjoys the particle-hole and inversion symmetries, giving the conditions $H_0(\vec{k}) = H_0(-\vec{k})$ and $\Delta(\vec{k}) = \Delta(-\vec{k}) = -\Delta^T(-\vec{k})$. But the time-reversal symmetry is explicitly broken, as shown in the form of $\Delta(\vec{k})$. The symmetries constrain eight-band spectrums of $\mathcal{H}^0_{\vec{k}}$.

Interaction effects. Our strategy to investigate interaction effects of a BG-FS is as follows. First, we construct an effective two-band model of a BG-FS. Second, we employ standard mean-field and renormalization group analysis. Then, we investigate the implications of our results in terms of a phenomenological theory.

Let us construct an effective two-band model. It is crucial to notice that the zero-energy states are doubly degenerate at each momentum because of the particle-hole and inversion symmetries. Therefore, a two-band model is inevitable to capture low-energy excitations, and we introduce an effective low-energy Hamiltonian of a centrosymmetric BG-FS, $\mathcal{H}^{\rm eff}_{\vec{k},0} = E_0(\vec{k})\tau^z$, with a two-component spinor $\Psi_{\vec{k}}$ and Pauli matrices $\tau^{x,y,z}$. The particle-hole and inversion symmetries may have the forms $U_c = \tau^x\mathcal{K}$ and $U_{\rm Inv} = \tau^0 \equiv I_{2\times2}$, respectively, with $\mathcal{K}$ being the complex-conjugation operator. One may obtain the effective two-band Hamiltonian and analyze symmetry properties by projecting the microscopic eight-band model $\mathcal{H}^0_{\vec{k}}$ onto the two-band space [40]. It is useful to study how the inversion-symmetry-breaking order parameter ϕ is coupled in our effective model. The Hermitian properties constrain the coupling significantly,

$$\mathcal{H}^{\text{eff}}_{\vec{k},0} \rightarrow \mathcal{H}^{\text{eff}}_{\vec{k}}(\phi) \equiv \mathcal{H}^{\text{eff}}_{\vec{k},0} - \phi \sum_{\mu=0,x,y,z} \rho_\mu(\vec{k})\tau^\mu. \tag{4}$$

The inversion symmetry imposes the odd-parity conditions, $\rho_\mu(\vec{k}) = -\rho_\mu(-\vec{k})$, and the particle-hole symmetry gives the condition $\rho_z(\vec{k}) = 0$. The energy spectrum of the Hamiltonian is $\sqrt{E_0(\vec{k})^2 + \phi^2[\rho_x(\vec{k})^2 + \rho_y(\vec{k})^2]} - \phi\rho_0(\vec{k})$ at a given $\vec{k}$.

Next, we incorporate a generic short-range interaction of a centrosymmetric BG-FS, whose form may be

$$H_{\text{tot}} = H^{\text{eff}}_0 - \frac{1}{2}\sum_{\mu,\nu;\vec{k},\vec{k}'} g_{\mu\nu}V_{\mu\nu}(\vec{k},\vec{k}')(\Psi^\dagger_{\vec{k}}\tau^\mu\Psi_{\vec{k}})(\Psi^\dagger_{\vec{k}'}\tau^\nu\Psi_{\vec{k}'}),$$

with $H^{\text{eff}}_0 = \sum_{\vec{k}} \Psi^\dagger_{\vec{k}}\mathcal{H}^{\text{eff}}_{\vec{k},0}\Psi_{\vec{k}}$. The form of the interaction is generic, and we consider a separable interaction $V_{\mu\nu}(\vec{k},\vec{k}') = \rho_\mu(\vec{k})\rho_\nu(\vec{k}')$. The particle-hole symmetry imposes the conditions ($g_{z0} = g_{zx} = g_{zy} = 0$), while other mixing terms (g_{x0}, g_{y0}, g_{xy}) are nonzero unless an extra constraint is imposed.

We perform the standard mean-field analysis for the effective two-band model H_{tot} with the ansatz $\langle\Psi^\dagger_{\vec{k}}\tau^\mu\Psi_{\vec{k}}\rangle_{\text{MF}} \equiv \phi\, c^\mu_{\vec{k}}$. The mean-field Hamiltonian is

$$H_{\text{MF}} = H^{\text{eff}}_0 - \phi\sum_{\mu,\vec{k}} d_\mu\rho_\mu(\vec{k})\Psi^\dagger_{\vec{k}}\tau^\mu\Psi_{\vec{k}} + \frac{\phi^2}{2}\sum_{\mu,\nu} d_\mu g^{-1}_{\mu\nu}d_\nu,$$

where $d_\mu \equiv \sum_{\nu,\vec{k}} g_{\mu\nu}\rho_\nu(\vec{k})c^\nu_{\vec{k}}$ and $g_{\mu\nu}$ is a nonsingular matrix. To be specific, we consider a Hamiltonian with two channels (τ_0, τ_x), and a general case is considered in the SM [40]. The Hamiltonian may be diagonalized by

$$H_{\text{MF}} = \sum_{\vec{k},\alpha=\pm} E(\vec{k};\phi)\,\gamma^\dagger_\alpha(\vec{k})\gamma_\alpha(\vec{k}) + \mathcal{E}_0(\phi), \tag{5}$$

with

$$E(\vec{k};\phi) \equiv \sqrt{E_0(\vec{k})^2 + \phi^2 d_x^2\rho_x(\vec{k})^2} - \phi d_0\rho_0(\vec{k})$$

and

$$\mathcal{E}_0(\phi) \equiv -\sum_{\vec{k}}\sqrt{E_0(\vec{k})^2 + \phi^2 d_x^2\rho_x(\vec{k})^2} + \frac{\phi^2}{2}\sum_{\mu,\nu} d_\mu g^{-1}_{\mu\nu}d_\nu.$$

The unitary transformation of $\Psi_{\vec{k}}$ determines the annihilation operator $\gamma_\alpha(\vec{k})$, with $\alpha = \pm$.

The ground-state energy is $E^{\text{MF}}_G[\phi] = \mathcal{E}_0(\phi) + 2\sum_{\vec{k}\in M^-} E(\vec{k};\phi)$, where the negative-energy manifold M^- is specified by the condition $E(\vec{k};\phi) < 0$. A zero energy is

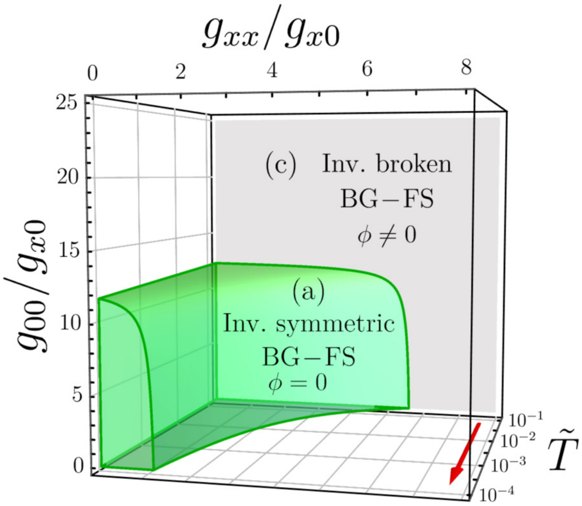

FIG. 2. Schematic mean-field phase diagrams at different temperatures. Parameters of the separable interactions are $\rho_0(\vec{k}) = k_y/|\vec{k}|$, $\rho_x(\vec{k}) = k_x/|\vec{k}|$. The relative coupling constants (g_{xx}/g_{x0}) and (g_{00}/g_{x0}) and dimensionless temperature $\tilde{T} \equiv T/\Lambda$ with a UV cutoff scale are introduced for each axis. A centrosymmetric BG-FS is stable for weak-coupling regions (green) at each temperature and becomes unstable for strong-coupling regimes, where its inversion symmetry is broken ($\phi \neq 0$). The regions of a BG-FS shrink with decreasing temperature (red arrow) and eventually vanish at $T = 0$.

obtained by the condition

$$E_0(\vec{k})^2 + \phi^2 d_x^2\rho_x(\vec{k})^2 = \phi^2 d_0^2\rho_0(\vec{k})^2, \tag{6}$$

which gives a Fermi surface. In Fig. 2, a mean-field phase diagram is obtained by minimizing the mean-field free energy, $F_{\text{MF}} = -T\ln[\text{Tr}(e^{-H_{\text{MF}}/T})]$.

The main results of our mean-field calculations are summarized as follows. First, the centrosymmetric BG-FS is absent at zero temperature. The ground-state energy difference, $\Delta E^{\text{MF}}_G[\phi] \equiv E^{\text{MF}}_G[\phi] - E^{\text{MF}}_G[0]$, is

$$\Delta E^{\text{MF}}_G[\phi] = \sum_{\vec{k}}\left[|E_0(\vec{k})| - \sqrt{E_0(\vec{k})^2 + \phi^2 d_x^2\rho_x(\vec{k})^2}\right] + \cdots.$$

The logarithmic divergence manifests the BCS-type instability,

$$\frac{\partial\Delta E^{\text{MF}}_G[\phi]}{\partial\phi^2}\Big|_{\phi=0} = -\frac{d_x^2}{2}\sum_{\vec{k}}\frac{\rho_x(\vec{k})^2}{|E_0(\vec{k})|} \propto -\ln\left(\frac{\Lambda}{\mu}\right)$$

where the UV and infrared energy cutoffs Λ and μ are introduced. Thus, the inversion symmetry must be broken at $T = 0$. Second, a centrosymmetric BG-FS whose regime diminishes at lower temperatures survives at nonzero temperature, as shown in Fig. 2. Third, the original Fermi surface is transformed by the inversion-symmetry breaking, and a Fermi surface of BG quasiparticles survives unless it is fine-tuned. More details of the fine-tuning condition are discussed in the SM [40]. In Fig. 1(b), one example of a fine-tuned case $[\rho_x(\vec{k}) = \rho_0(\vec{k}) = k_x/|\vec{k}|]$ is illustrated. Generically, the two functions $[\rho_0(\vec{k}), \rho_x(\vec{k})]$ are independent even for a separable interaction. We stress that the survival of a Fermi surface after the instability of inversion symmetry is drastically different from the standard BCS superconductivity due to the presence of the τ^0 channel. In Fig. 1(c), one example of a noncentrosymmetric BG-FS is illustrated [$\rho_x(\vec{k}) = \rho_0(\vec{k}) = k_x/|\vec{k}|$].

To go beyond the mean-field analysis, we perform the renormalization group analysis. For simplicity, we illustrate the case with two channels (τ_0, τ_x), and the generic cases are discussed in the SM [40]. Introducing dimensionless coupling constants $\tilde{g}_{\mu\nu}$ which are averaged quantities over a Fermi surface weighted by $\rho_\mu(\vec{k})$, we find

$$\frac{d\tilde{g}_{xx}}{dl} = \tilde{g}_{xx}^2, \quad \frac{d\tilde{g}_{x0}}{dl} = \tilde{g}_{x0}\tilde{g}_{xx}, \quad \frac{d\tilde{g}_{00}}{dl} = \tilde{g}_{x0}^2, \tag{7}$$

with l being the scale variable of the renormalization group analysis. In the long-wavelength limit ($l \to \infty$), the BCS-type logarithmic dependence manifests in $\tilde{g}_{xx}$. It is obvious that the first two equations have positive eigenvalues, and the right-hand side of the third one is always positive. Thus, the original BG-FS is unstable at $T = 0$ for attractive bare interactions, which is consistent with the mean-field results.

The above instability calculations indicate that the inversion-symmetry-breaking order parameter should be included in a phenomenological Ginzburg-Landau theory of BG-FSs from the beginning. The Ginzburg-Landau functional is obtained by integrating out fermions,

$$\mathcal{F}[\Delta, \phi] = r_\Delta \mathrm{Tr}[\Delta^\dagger \Delta] + r_\phi \phi^2 + \cdots . \tag{8}$$

A BG-FS may be considered by the condition $r_\Delta < 0$, and our instability calculation indicates $r_\phi = r_\phi^0 - \langle \mathcal{O} \rangle_{\mathrm{FS}} \ln(\frac{\Lambda}{T})$ with a positive-definite quantity averaged over a BG-FS, $\langle \mathcal{O} \rangle_{\mathrm{FS}} \propto d_x^2 \langle \rho_x^2 \rangle_{\mathrm{FS}} + d_y^2 \langle \rho_y^2 \rangle_{\mathrm{FS}}$, if a negative interaction channel exists [40]. Note that the sign of higher-order Ginzburg-Landau coefficients may determine either the natures of transitions or whether the two orders compete or cooperate.

Let us consider a schematic phase diagram of the Ginzburg-Landau functional. Adjusting the parameters (r_Δ, r_ϕ), one may obtain four possible phases: (A) $r_\Delta > 0, r_\phi > 0$, a centrosymmetric metal; (B) $r_\Delta < 0, r_\phi > 0$, a centrosymmetric BG-FS; (C) $r_\Delta > 0, r_\phi < 0$, a polar metal; or (D) $r_\Delta < 0, r_\phi < 0$, a noncentrosymmetric superconductor. Note that an intermediate phase between (A) and (B) may be present. For example, a time-reversal-symmetric superconductor may appear if (A) is a time-reversal-symmetric metal. Our instability calculations indicate that phase (D) always appears at low temperature. As discussed above, a Fermi surface generically survives in a noncentrosymmetric BG-FS similar to the ones in the literature [20,43–45]. Furthermore, Ginzburg-Landau theory indicates that a phase transition from (A) to (D) generically happens with two-step transitions unless it is fine-tuned to go through O.

Discussion and conclusion. Based on our instability results, we propose enhanced fluctuations of an inversion-symmetry-breaking order parameter are a key property of a centrosymmetric BG-FS. This is analogous to the fact that a Fermi liquid is always susceptible to a superconducting instability, as shown by the seminal work by Kohn and Luttinger [42]. In Fig. 3(b), we illustrate a schematic phase diagram with a tuning parameter of quantum fluctuations of an inversion-symmetry-breaking order parameter. Our results indicate that

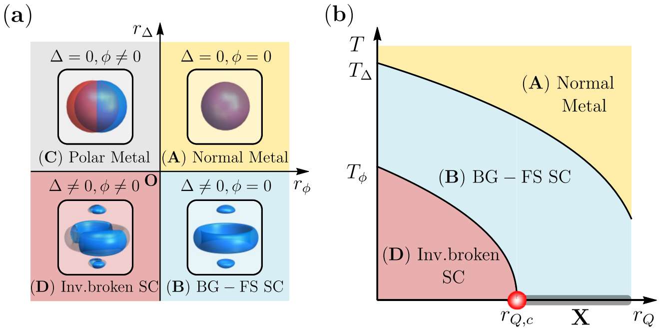

FIG. 3. (a) Generic phase diagram of the four phases. O = (0, 0) is the multicritical point. The phase (B) becomes unstable at low temperatures. (b) Schematic phase diagram with the two parameters, a quantum fluctuation parameter r_Q and temperature T. The phase **X** is BG quasiparticle excitations at zero temperature. Our results indicate that the phase **X** is *absent* if BG quasiparticles are well defined on a BG-FS.

a weakly interacting centrosymmetric BG-FS is unstable, and the phase **X** is *absent*. On the other hand, it is an interesting question whether strongly interacting BG quasiparticles stabilize a centrosymmetric BG-FS. The recent work of a pairing instability in a non-Fermi liquid [46] suggests that a stable BG-FS may be possible down to zero temperature if its excitations lose their quasiparticle nature.

Enhanced fluctuations of an inversion-symmetry-breaking order parameter ϕ may be captured by inversion susceptibility. Motivated by recent advances in flexoelectricity, we note that a strain gradient on a sample breaks inversion symmetry and plays the role of an external field of ϕ. Moreover, it is well known that the SHG experiment is a probe to identify an inversion-symmetry-breaking order parameter, providing information about its onset, $\phi \sim (T_c - T)^\beta$, with T_c being the inversion-symmetry-breaking critical temperature [47]. Combining the two methods, we propose a SHG experiment with a strain gradient to measure inversion susceptibility and expect to obtain information about the susceptibility, $\chi_\phi \sim |T_c - T|^{-\gamma}$. Note that the susceptibility has a nontrivial signature even at higher temperatures, $T > T_c$, in sharp contrast to the absence of an order parameter at higher temperatures. We believe the SHG with a strain gradient may be applied in both superconducting and normal states with enhanced inversion fluctuations since inversion symmetry acts in the same way. It is desired to test the experiment in the candidate heavy-fermion materials, including URu_2Si_2 and UBe_{13}. Recently, FeSe was also proposed to be a candidate system of a BG-FS [48], and we believe that inversion-symmetry-breaking order parameter fluctuations may be enhanced in FeSe.

In conclusion, we investigated the interaction effects of a centrosymmetric BG-FS and found its instability in the inversion-symmetry channel if a negative interaction channel exists. Condensation of BG pairs induces the instability, similar to the BCS instability of Fermi liquids where Cooper pairs condense and break U(1) symmetry. On the other hand, in contrast to the standard BCS superconductivity [49], a Fermi surface generically survives unless it is fine-tuned. The instability enforces a phenomenological Ginzburg-Landau functional to include an inversion-symmetry-breaking order parameter from the beginning. Future works including disorder and strong quantum fluctuations are highly desired. Microscopic calculations of SHG with a strain gradient and analysis to find intriguing phases near the BG-FS based on quantum Monte Carlo simulation would also be useful.

Note added in proof. Recently, we became aware of a preprint by Tamura *et al.* which considered multiple order instabilities of BG-FS [34].

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