Asymmetry of the geometrical resonances of composite fermions

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We propose an experiment to test the uniform-Berry-curvature picture of composite fermions. We show that the asymmetry of geometrical resonances observed in a periodically modulated composite fermion system can be explained with the uniform-Berry-curvature picture. Moreover, we show that an alternative way of modulating the system, i.e., modulating the external magnetic field, will induce an asymmetry opposite to that of the usual periodic grating modulation which effectively modulates the Chern-Simons field. The experiment can serve as a critical test of the uniform-Berry-curvature picture and probe the dipole structure of composite fermions proposed by Read.

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I. INTRODUCTION

A two-dimensional electron system (2DES) subjected to a strong perpendicular magnetic field exhibits exotic many-body states, in particular, the fractional quantum Hall states at odd-denominator filling factors [1,2] and the Fermi-liquid-like states at even-denominator fillings [3,4]. Jain's composite fermion (CF) theory provides a unified understanding to these states [5]. A CF can be regarded as an electron attached with $2p$ quantum vortices and feels an effective magnetic field $B^* = B - b_{2p}^{\mathrm{CS}}$, with B being the external magnetic field and $b_{2p}^{\mathrm{CS}} = 2pn_\mathrm{e}\phi_0$ the emergent Chern-Simons (CS) field, where n_e is the density of electrons and $\phi_0 = h/e$ is the quanta of magnetic flux [2,4]. The Halperin-Lee-Read (HLR) theory treats the CF as an electronlike particle and predicts that CFs form a Fermi liquid at even-dominator filling $\nu = 1/2p$, for which the effective magnetic field $B^* = 0$ [4]. The Fermi liquid state is confirmed by various experiments [6]. Though the HLR theory achieves great successes in explaining various observed phenomena, it does not predict a correct CF Hall conductivity $\sigma_{xy}^{\mathrm{CF}} = -e^2/2h$ at half filling as required by the particle-hole symmetry [7]. Motivated by the difficulty, Son proposes that the CF is a Dirac particle [8]. In the Dirac theory, the CF is considered as a vortex dual of a Dirac electron coupling to an emergent gauge field. However, its microscopic basis is not yet clarified [9]. On the other hand, Shi and Ji derive the dynamics of the CF Wigner crystal from the microscopic Rezayi-Read wave function and find that CFs are subjected to a Berry curvature uniformly distributed in momentum space [10]. Based on that, they propose the uniform-Berry-curvature picture of CFs [11]. A calculation of the Berry phase of CFs from a microscopic wave function by Geraedts *et al.* seems to lend support to the Dirac picture [12]. However, a refined calculation suggests otherwise [13]. Actually, the Berry curvature is analytically shown to be uniform for the Rezayi-Read wave function [14]. Although the two pictures look quite different, both predict that a CF accumulates a π Berry phase when it moves around the Fermi circle.

The manifestations of the π Berry phase have been observed in a number of experiments and numerical calculations. In the numerical simulations of the infinite-cylinder density matrix renormalization group, the suppression of $2k_\mathrm{F}$ backscattering off particle-hole symmetric impurities is interpreted as a result of the π Berry phase [15]. In the Shubnikov–de Haas oscillation experiments of CFs at a fixed magnetic field, the π Berry phase is shown to appear in the magnetoresistivity formula [16]. In the geometrical resonance experiments of CFs with periodic grating modulations, the asymmetry of the commensurability condition on the two sides at about half filling observed in Ref. [17] can also be explained as a result of the π Berry phase (see below). Though these studies convincingly show the presence of the π Berry phase, they cannot differentiate the Dirac picture and the uniform-Berry-curvature picture.

In this paper, we propose an experiment to test the uniform-Berry-curvature picture. First, we show that the uniform-Berry-curvature picture predicts a Fermi wave vector different from the HLR theory but the same as the Dirac theory [8]. The asymmetry of the commensurability conditions in Ref. [17] can be explained with the modified Fermi wave vector. Next, we show that the uniform-Berry-curvature picture is equivalent to the dipole picture initially proposed by Read [18,19]. In the dipole picture, it becomes obvious that the external magnetic field $\boldsymbol{B}$ and the CS field $\boldsymbol{b}^{\mathrm{CS}}$ are coupling to different internal degrees of freedom in a CF, i.e., the electron and the quantum vortices, respectively [see Fig. 1(a)]. We show that a geometrical resonance experiment with a periodically modulated external magnetic field will yield an asymmetry opposite to that of the usual periodic grating modulation. This experiment can serve as a critical test to the uniform-Berry-curvature picture and at the same time, probe the "subatomic" dipole structure of CFs.

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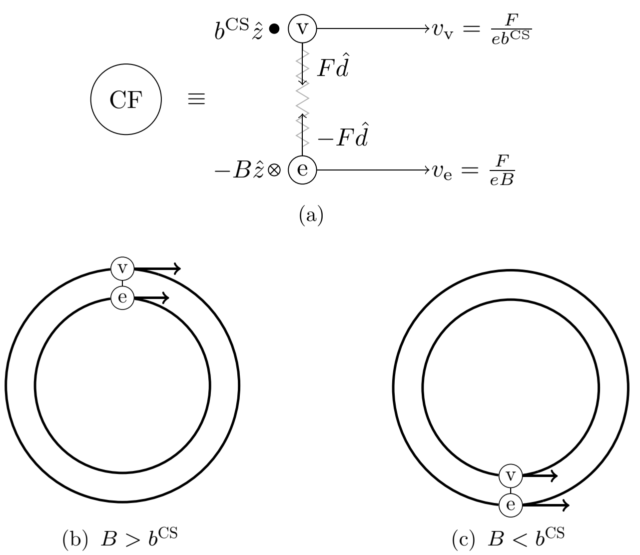

FIG. 1. Cyclotron orbits of a CF under various conditions. (a) The dipole structure of a CF, where a CF consists of an electron (e) and two quantum vortices (v). They are bounded together by a mutual central force $F \propto |\boldsymbol{d}|$. e is coupled to the external magnetic field $-B\hat{z}$, and v is coupled to the Chern-Simons field $b^{\rm CS}\hat{z}$. When $B = b^{\rm CS}$, e and v have the same velocity $v = (-F)/(-eB) = F/eb^{\rm CS}$ and move linearly. (b) When $B > b^{\rm CS}$, e and v have different velocities, resulting in a cyclotron motion. Because v is faster than e, the cyclotron radius of v is larger than that of e, i.e., $R_{\rm c}^{(\rm v)} \equiv R_{\rm c}^{*} > R_{\rm c}^{(\rm e)}$. (c) When $B < b^{\rm CS}$, the opposite is true, i.e., $R_{\rm c}^{(\rm e)} > R_{\rm c}^{(\rm v)}$. The asymmetry between (b) and (c) is responsible for the asymmetry observed in geometrical resonance experiments. By using the usual grating modulation, one measures $R_{\rm c}^{(\rm v)}$. By using the magnetic field modulation, on the other hand, one measures $R_{\rm c}^{(\rm e)}$. It is obvious that the two different approaches will yield opposite asymmetries.

The remainder of the paper is organized as follows. In Sec. II we derive the Fermi wave vector based on the uniform-Berry-curvature picture and show that the uniform-Berry-curvature picture is equivalent to the dipole picture. In Sec. III we study the periodic scalar potential modulation of CFs. In Sec. IV we study the periodic external magnetic field modulation of CFs. In Sec. V we discuss and summarize our results.

II. UNIFORM-BERRY-CURVATURE PICTURE AND DIPOLE PICTURE

In the uniform-Berry-curvature picture, the equations of motion (EOMs) of CFs read

$$\dot{\boldsymbol{x}} = \frac{\boldsymbol{p}}{m_{\rm CF}^{*}} + \frac{1}{eB}\hat{z} \times \dot{\boldsymbol{p}}, \tag{1}$$

$$\dot{\boldsymbol{p}} = -eB^{*}\hat{z} \times \dot{\boldsymbol{x}}, \tag{2}$$

where $\boldsymbol{x}$, $\boldsymbol{p}$, and $m_{\rm CF}^{*}$ are the position, momentum, and effective mass of a CF, respectively [10]. A distinctive feature of the uniform-Berry-curvature picture is the presence of a uniform Berry curvature in the momentum space, which is not presented in the conventional HLR theory [10]. As a result, it predicts a Fermi wave vector different from the HLR theory. In the HLR theory, the CF is treated as an electronlike particle. It predicts a Fermi wave vector $k_{\rm F} = \sqrt{4\pi n_{\rm e}}$. On the other hand, in the uniform-Berry-curvature picture, due to the presence of the Berry curvature $\Omega_z = 1/eB$ in Eq. (1), the phase-space density of states is modified by a factor $D = 1 - B^{*}/B$ [20]. The Fermi wave vector $k_{\rm F}$ of CF can be determined through the condition $\pi k_{\rm F}^2 D/(2\pi)^2 = n_{\rm e}$ and is

$$k_{\rm F} = \sqrt{\frac{eB}{\hbar}}, \tag{3}$$

which is different from the prediction of the HLR theory and independent of $n_{\rm e}$. This result is the same as the Dirac theory. The coincidence is not surprising, because both pictures have a π-Berry phase along the Fermi circle. To differentiate the two pictures, one has to probe deeper.

The uniform-Berry-curvature picture is actually equivalent to the dipole picture initially proposed by Read [18,19]. To see that, we can rewrite the EOMs with the new variables $\boldsymbol{x}^{\rm v} \equiv \boldsymbol{x}$ and $\boldsymbol{x}^{\rm e} = \boldsymbol{x}^{\rm v} - \hat{z} \times \boldsymbol{p}/eB$:

$$-eB\hat{z} \times \dot{\boldsymbol{x}}^{\rm e} = \frac{\partial \varepsilon}{\partial \boldsymbol{x}^{\rm e}}, \tag{4}$$

$$eb^{\rm CS}\hat{z} \times \dot{\boldsymbol{x}}^{\rm v} = \frac{\partial \varepsilon}{\partial \boldsymbol{x}^{\rm v}}, \tag{5}$$

where $\boldsymbol{x}^{\rm e}$ and $\boldsymbol{x}^{\rm v}$ are interpreted as the position of the electron and quantum vortices in a CF, respectively, and $\varepsilon \propto |\boldsymbol{x}^{\rm e} - \boldsymbol{x}^{\rm v}|^2$ is the binding energy between the electron and the quantum vortices [10]. The momentum $\boldsymbol{p}$ of a CF is interpreted as $\boldsymbol{p} = eB\hat{z} \times \boldsymbol{d}$ with the displacement $\boldsymbol{d} \equiv \boldsymbol{x}^{\rm e} - \boldsymbol{x}^{\rm v}$ [see Fig. 1(a)]. From the new form of the EOMs, it is clear that the electron is only coupled to the external electromagnetic field $\boldsymbol{B}$ while the quantum vortices are only coupled to the emergent CS field $\boldsymbol{b}^{\rm CS}$. Moving a CF in the momentum space is equivalent to fixing the quantum vortices and moving the electron in the real space. The Aharonov-Bohm phase accumulated by the electron is nothing but the Berry phase expected from the uniform Berry curvature in Eq. (1) [11]. It also becomes obvious that the external magnetic field and the CS field are not equivalent microscopically, since they are coupling to different internal degrees of freedom. Therefore we anticipate that modulating the external magnetic field $\boldsymbol{B}$ and the CS field $\boldsymbol{b}^{\rm CS}$ have different effects on CFs.

III. SCALAR POTENTIAL MODULATION

Weiss *et al.* show that when a 2DES is weakly modulated by a one-dimensional periodic scalar potential, its magnetoresistance shows an oscillation with respect to $2R_{\rm c}/a$, where $R_{\rm c}$ is the cyclotron radius and a is the period of the modulation [21]. When a 2DES is at an even-dominator filling factor $\nu = 1/2p$, CFs feel a zero effective magnetic field $B^{*} = 0$. It is natural to expect that the Weiss oscillation can also be observed in CF systems when the effective magnetic field deviates from zero. This has been confirmed by a number of geometrical resonance experiments for CFs [22–25]. In experiments, the scalar potential modulation is achieved by imposing a grating pattern [26].

For CF systems, a periodic scalar potential modulation is equivalent to a modulation of the CS field for CFs. In such a modulation, CFs are subjected to a weak electrostatic potential modulation $\delta V^{\text{ext}}(x) = V^{\text{ext}} \cos(2\pi x/a)$. The electrostatic potential will induce a modulation of the electron density δn_{e}, which in turn induces a modulation of the CS field $\delta b^{\text{CS}} = 2\phi_0 \delta n_{\text{e}}$. The energy corrections associated with the electrostatic potential and the CS field are $-e\delta V^{\text{ext}}$ and $-e\dot{\boldsymbol{x}} \cdot \delta \boldsymbol{a}^{\text{CS}}$, respectively. By assuming a noninteracting CF model, the ratio of these two contributions is $\pi/ak_{\text{F}} \ll 1$ (e.g., for $B = 14$ T and $a = 200$ nm, $\pi/ak_{\text{F}} \approx 0.1$) [27]. As a result, the effect of the CS field modulation dominates in this case.

The commensurability condition can be derived semiclassically as shown in Refs. [27,28], in which the modulation is treated as a perturbation. In the absence of the modulation, for a CF on the Fermi circle, the solutions of Eqs. (1) and (2) are

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + R_{\text{c}}^*[-\cos(\omega_{\text{c}}^* t + \varphi), \sin(\omega_{\text{c}}^* t + \varphi)], \quad (6)$$

$$\boldsymbol{p}(t) = \hbar k_{\text{F}}[\sin(\omega_{\text{c}}^* t + \varphi), \cos(\omega_{\text{c}}^* t + \varphi)], \quad (7)$$

where $\boldsymbol{x}_0$, $R_{\text{c}}^* = \hbar k_{\text{F}}/eB^*$, and $\omega_{\text{c}}^* = eB^*/Dm_{\text{CF}}^*$ are the center coordinate, radius, and frequency of the cyclotron orbit, respectively, and φ is a phase factor. Without the periodic modulation, all orbits have a degenerate energy. In the presence of the weak periodic modulation, the degeneracy is split. The correction to the energy, to the first order, is the average energy change due to the CS field modulation during a period of the cyclotron motion $T = 2\pi/\omega_{\text{c}}^*$:

$$\begin{aligned}\delta U &\approx \frac{1}{T}\int_0^T dt(-e\dot{\boldsymbol{x}} \cdot \delta \boldsymbol{a}^{\text{CS}}) \\ &= (2ek_{\text{F}}V^{\text{ext}}/q)J_1(qR_{\text{c}}^*)\cos qx_0,\end{aligned} \quad (8)$$

where $q = 2\pi/a$, and $J_1(x)$ is the first Bessel function [27]. In the weak effective magnetic field limit $qR_{\text{c}}^* \gg 1$, $\delta U \approx -\sqrt{2/\pi qR_{\text{c}}^*}(2ek_{\text{F}}V^{\text{ext}}/q)\cos qx_0 \cos(qR_{\text{c}}^* + \pi/4)$. The energy correction depends on the center position x_0, resulting in the broadening of the Landau level. The broadening caused by the modulation is proportional to $\cos(qR_{\text{c}}^* + \pi/4)$, which vanishes when the commensurability condition $2R_{\text{c}}^*/a = i + \gamma$ with $\gamma = 1/4$ is fulfilled. One may assume that the conductivity along the direction transverse to the modulation is proportional to the broadening [27,29]. As a result, the commensurability condition is manifested in experiments as a series of the minimum of the longitudinal magnetoresistance. For the Fermi wave vector shown in Eq. (3), the commensurability condition can be written as

$$\frac{B_0}{|B_i^*|} \approx \frac{a}{2}\sqrt{\frac{eB_0}{\hbar}}(i+\gamma) + \begin{cases} -\frac{1}{2} & B^* > 0 \\ \frac{1}{2} & B^* < 0 \end{cases} \quad (9)$$

for $|B_i^*| \ll B_0$, where $B_0 \equiv 2n_{\text{e}}\phi_0$ is the magnetic field at the half filling, and B_i^* is the effective magnetic field of the ith magnetoresistance minima. We see that the commensurability condition shows an asymmetry between the particle ($B^* > 0$) and hole ($B^* < 0$).

The asymmetry had actually been observed in experiments. We adapt and fit the experimental results of Ref. [17] and show them in Fig. 2. One can see that for all index i's, the value of $B_0/|B_i^*|$ with $B_i^* < 0$ (hole) sits above that with $B_i^* > 0$ (particle). The vertical shift of the two lines is $\Delta(B_0/|B_i^*|) = -1.33 \pm 0.39$, close to the prediction $\Delta(B_0/|B_i^*|) = -1$.

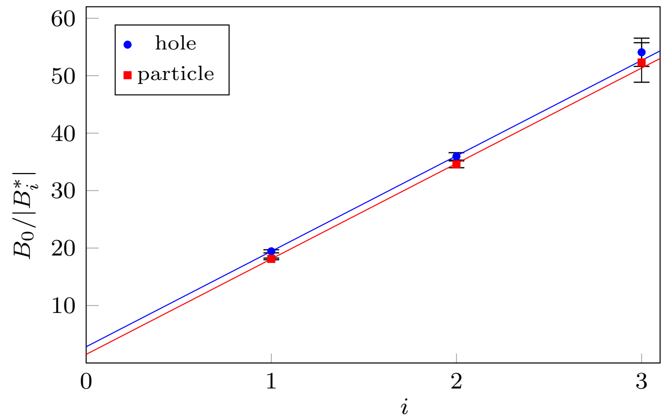

FIG. 2. $B_0/|B_i^*|$ vs i. The circles (squares) are data for holes (particles), and the error bar is also shown. The slope of both lines is 16.61 ± 0.11. The difference of the intercepts of two lines is -1.33 ± 0.39. The fitting value of phase factor is $\gamma = 0.13 \pm 0.01$. Points are data adapted from Ref. [17] with $B_0 = 14.383$ T.

IV. EXTERNAL MAGNETIC FIELD MODULATION

In this section we show that a weak periodic modulation of the external magnetic field will induce an asymmetry opposite to that of the periodic scalar potential modulation. First, we derive the commensurability condition with respect to the external magnetic field modulation. We then consider the effect of induced density modulation and determine when the unwanted density modulation effect can be suppressed.

A. Direct modulation effect

When a weak periodic modulation of the external magnetic field $\delta \boldsymbol{B}(\boldsymbol{x}) = \delta B \cos \boldsymbol{q} \cdot \boldsymbol{x}\hat{z}$ is applied to a 2DES, it couples to the electron in the CF and the energy correction is $\delta U = -e\dot{\boldsymbol{x}}^{\text{e}} \cdot \delta \boldsymbol{A}(\boldsymbol{x}^{\text{e}})$, with $\delta \boldsymbol{A}(\boldsymbol{x})$ being the vector potential with respect to $\delta \boldsymbol{B}(\boldsymbol{x})$. Note that δU in the current case is related to the electron coordinate $\boldsymbol{x}^{\text{e}}$ instead of $\boldsymbol{x}$, as in the previous case. We can determine the commensurability condition just as we do in the last section. In the absence of the modulation, from Eqs. (6) and (7) we determine

$$\begin{aligned}\boldsymbol{x}^{\text{e}}(t) &= \boldsymbol{x}(t) - \hat{z} \times \boldsymbol{p}(t)/eB \\ &= \boldsymbol{x}_0 + R_{\text{c}}^{(\text{e})}[-\cos(\omega_{\text{c}}^* t + \varphi), \sin(\omega_{\text{c}}^* t +)],\end{aligned} \quad (10)$$

with

$$R_{\text{c}}^{(\text{e})} = DR_{\text{c}}^*. \quad (11)$$

Therefore the electron has a cyclotron radius different from that of the quantum vortices. In this case, the average energy change of a CF due to the external magnetic field modulation during a period of the cyclotron motion is

$$\begin{aligned}\delta U &\approx \frac{1}{T}\int_0^T dt[-e\dot{\boldsymbol{x}}^{\text{e}} \cdot \delta \boldsymbol{A}(\boldsymbol{x}^{\text{e}})] \\ &= \left(e\omega_{\text{c}}^* R_{\text{c}}^{(\text{e})}\delta B/q\right)J_1\left(qR_{\text{c}}^{(\text{e})}\right)\cos(qx_0).\end{aligned} \quad (12)$$

In the weak effective magnetic field limit $qR_c^{(e)} \gg 1$, $\delta U \approx -\sqrt{2/\pi q R_c^{(e)}}(e\omega_c^* R_c^{(e)} \delta B/q)\cos(qx_0)\cos(qR_c^{(e)}+\pi/4)$.

As a result, the commensurability condition becomes $2R_c^{(e)}/a = i+\gamma$ and can be written as

$$\frac{B_0}{|B_i^*|} \approx \frac{a}{2}\sqrt{\frac{eB_0}{\hbar}}(i+\gamma) + \begin{cases} \frac{1}{2} & B^* > 0 \\ -\frac{1}{2} & B^* < 0 \end{cases} \tag{13}$$

for $|B_i^*| \ll B_0$. We see that the value of $B_0/|B_i^*|$ with $B_i^* > 0$ (electron) now sits above that with $B_i^* < 0$ (hole). The asymmetry is opposite to the asymmetry induced by the CS field modulation.

Based on the result, we propose a new geometrical resonance experiment with a modulating external magnetic field. The inverse of the asymmetry would be the signature confirming the underlying "subatomic" structure of the CF.

B. Induced CS field modulation

However, there is still a complexity for the proposed experiment. This is because the energy of the lowest Landau level (LLL) is proportional to B, and the modulation of B will introduce a modulation of the effective potential felt by CFs [30]. While the direct effect of the effective potential is negligible, the CS field induced by modulation may not be small. To estimate the modulation amplitude of the induced CS field, we apply the density functional approach [31–33]. By ignoring the effect of density gradient, the grand canonical energy functional $\mathcal{E}$ of the system can be approximated as

$$\mathcal{E}[n] = \int d\boldsymbol{r}\Bigg[-\mu n(\boldsymbol{r}) + \left(\frac{\hbar e}{2m_b}+\frac{g\mu_B}{2}\right)B(\boldsymbol{r})n(\boldsymbol{r}) + v_{xc}[n(\boldsymbol{r})]n(\boldsymbol{r}) + \frac{e^2}{8\pi\epsilon}\int d\boldsymbol{r}'\frac{\Delta n(\boldsymbol{r})\Delta n(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\Bigg], \tag{14}$$

where μ is the chemical potential, the second term is the kinetic and Zeeman energy of electrons in the LLL, with m_b, g, and μ_B being the band mass, the effective Landé factor, and the Bohr magneton, respectively, $v_{xc}[n(\boldsymbol{r})]$ is the exchange-correlation energy per particle, and the last term is the Coulomb energy due to the density modulation $\Delta n(\boldsymbol{r})$. We adopt the interpolation formula of the exchange-correlation energy presented in Ref. [34], which is a function of the filling factor $\nu = n/(eB/h)$. Under the local density approximation, the exchange-correlation energy can be written as

$$v_{xc}[n(\boldsymbol{r})] = (e^2/4\pi\epsilon l_B(\boldsymbol{r}))u(\nu(\boldsymbol{r})), \tag{15}$$

with

$$u(\nu) = -(\pi/8)^{1/2}\nu - 0.782\nu^{1/2}(1-\nu)^{3/2} + 0.683\nu(1-\nu)^2 - 0.806\nu^{3/2}(1-\nu)^{5/2}, \tag{16}$$

where $l_B \equiv \sqrt{\hbar/eB(\boldsymbol{r})}$ is the magnetic length and ϵ is the static permittivity [34].

To determine the density modulation due to the modulation of the external magnetic field, we minimize the energy functional with respect to the density and obtain

$$\mu = \left(\frac{\hbar e}{2m_b}+\frac{g\mu_B}{2}\right)B(\boldsymbol{r}) + \frac{e^2}{4\pi\epsilon l_B(\boldsymbol{r})}\frac{\partial[u(\nu)\nu]}{\partial\nu}\bigg|_{\nu=\frac{n(\boldsymbol{r})}{eB(\boldsymbol{r})/h}} + \frac{e^2}{4\pi\epsilon}\int d\boldsymbol{r}'\frac{\Delta n(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}. \tag{17}$$

For a weak periodic modulation of the magnetic field $B(\boldsymbol{r}) = B + \delta B(\boldsymbol{r})$ and $\delta B(\boldsymbol{r}) = \delta B\cos(2\pi x/a)$, we have $\Delta n(\boldsymbol{r}) \approx \delta n\cos(2\pi x/a)$. By assuming that both δn and δB are small quantities, it is easy to obtain

$$\frac{\delta n}{n} \approx -\frac{a_c/l_B - \beta_1}{a/l_B - \beta_2}\frac{\delta B}{B}, \tag{18}$$

where $a_c \equiv 2\pi(1+gm_b/2m_e)a_B^*$, with a_B^* being the effective Bohr radius and m_e being the bare electron mass, $\beta_1 = 2\pi[(\nu u(\nu))'' - (\nu u(\nu))'/2\nu]$, $\beta_2 = -2\pi[(\nu u(\nu))'']$. At $\nu \approx 1/2$, we have $\beta_1 \approx 2.3$ and $\beta_2 \approx 1.6$.

To observe the asymmetry inverse predicted in Eq. (13), we require $|\alpha| \ll 1$. It is not difficult to fulfill the requirement in a GaAs-based 2DES, for which $a_c \approx 62$ nm. For the experimental parameters of Ref. [17], i.e., $a = 200$ nm and $B = 14$ T, the value of α is 0.24, fulfilling the requirement. In the strong-field limit $B \to \infty$, $l_B \to 0$, we have $\alpha = a_c/a$. Therefore one can always fulfill the requirement by choosing a modulation period a much larger than 62 nm. We further note that the modulation of the external magnetic field had already been achieved for electrons by placing a ferromagnet or superconductor microstructure on top of a 2DES [35–37]. We expect that similar techniques can be implemented for CF systems.

V. DISCUSSION AND SUMMARY

A natural question is what the Dirac CF theory would predict for the asymmetry. Dirac CF theory proposed by Son also captures the effect of the π-Berry phase, with a Berry curvature singularly distributed in the center of the momentum space [8]. In Refs. [38,39], Mulligan *et al.* conclude that for Dirac CFs, the difference between the scalar potential modulation and the magnetic field modulation is in the factor γ, i.e., $\gamma = 1/4$ for the scalar potential modulation and $\gamma = -1/4$ for the magnetic field modulation. It would predict the interchange of the positions of the magnetoresistance minimum and maximum. This is different from our prediction of the inverse of the asymmetry. Our prediction is based on the dipole picture of the "subatomic" structure of the CF, which is a result of the microscopic Rezayi-Read wave function. However, the prediction for Dirac CFs is based upon an effective field theory. Unfortunately, for the Dirac CF theory there is still no consensus on the microscopic wave function and the "subatomic" structure.

In summary, we theoretically study the manifestations of the uniform-Berry-curvature picture in the geometrical resonance experiments for CFs. We show that the modulation of an externally applied magnetic field will induce an asymmetry opposite to that induced by a periodic scalar potential modulation. This experiment can serve as a critical test to the uniform-Berry-curvature picture. Since the effect originates from the dipole structure of CFs, its successful observation will also provide an experimental confirmation to the dipole picture of CFs initially proposed by Read.

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